An Empirical State Error Covariance Matrix Orbit Determination Example

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Short Abstract: The concept of the empirical state error covariance matrix for batch estimation is investigated. This matrix represents a direct and timely sampled data estimate of state errors, under unknown or mismodeled error sources. Of special interest is the problem of orbit determination under modeling errors. The problem investigated here is that of the empirical covariance matrix behavior and performance under simple but nontrivial errors for gravity, drag and tracking uncertainty. Comparisons are made to the usual, theoretical error covariance associated with the batch estimate. Results, as compared to ideal modeling, are presented for epoch and a 24 hour propagation interval.

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Long Abstract: State estimation techniques serve effectively to provide mean state estimates. However, the state error covariance matrices provided as part of these techniques suffer from some degree of lack of confidence in their ability to adequately describe the uncertainty in the estimated states. A specific problem with the traditional form of state error covariance matrices is that they represent only a mapping of the assumed observation error characteristics into the state space. Any errors that arise from other sources (environment modeling, precision, etc.) are not directly represented in a traditional, theoretical state error covariance matrix. First, consider that an actual observation contains only measurement error and that an estimated observation contains all other errors, known and unknown. Then it follows that a measurement residual (the difference between expected and observed measurements) contains all errors for that measurement. Therefore, a direct and appropriate inclusion of the actual measurement residuals in the state error covariance matrix of the estimate will result in an empirical

state error covariance matrix. This empirical state error covariance matrix will fully include all of the errors in the state estimate.

The empirical error covariance matrix is determined from a literal reinterpretation of the equations involved in the weighted least squares estimation algorithm. It is a formally correct, empirical state error covariance matrix obtained through use of the average form of the weighted measurement residual variance performance index rather than the usual total weighted residual form. Based on its formulation, this matrix will contain the total uncertainty in the state estimate, regardless as to the source of the uncertainty and whether the source is anticipated or not. It is expected that the empirical error covariance matrix will give a better, statistical representation of the state error in poorly modeled systems or when sensor performance is suspect. In its most straight forward form, the technique only requires supplemental calculations to be added to existing batch estimation algorithms.

In the current problem being studied a truth model making use of gravity with spherical, J2 and J4 terms plus a standard exponential type atmosphere with simple diurnal and random walk components is used. The ability of the empirical state error covariance matrix to account for errors is investigated under four scenarios during orbit estimation. These scenarios are: exact modeling under known measurement errors, exact modeling under corrupted measurement errors, inexact modeling under known measurement errors, and inexact modeling under corrupted measurement errors. For this problem a simple analog of a distributed space surveillance network is used. The sensors in this network make only range measurements and with simple normally distributed measurement errors. The sensors are assumed to have full horizon to horizon viewing at any azimuth. For definiteness, an orbit at the approximate altitude and inclination of the International Space Station is used for the study.

The comparison analyses of the data involve only total vectors. No investigation of specific orbital elements is undertaken. The total vector analyses will look at the chi-square values of the error in the difference between the estimated state and the true modeled state using both the empirical and theoretical error covariance matrices for each of scenario.