A Microwave Radiometric Method to Obtain the Average Path Profile of Atmospheric Temperature and Humidity Structure Parameters and its Application to Optical Propagation System Assessment

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Turbulence Effects on Optical Beams

- Scintillation
- Beam broadening
- Spatial coherence
- Angle of arrival
- Temporal pulse stretching

Temporal and spatial intensity fluctuations at the receiving aperture results in power surges and fades
Temperature (or humidity) structure function

\[ D_T(\vec{r}, \vec{r} + \Delta \vec{r}) = \left\langle \left[ T(\vec{r}) - T(\vec{r} + \Delta \vec{r}) \right]^2 \right\rangle \]

Contains the spatial statistics of the temperature field

Within a range of certain \( \Delta \vec{r} \), the well-known Kolmogorov “2/3” law holds

\[ D_T(\vec{r}, \vec{r} + \Delta \vec{r}) = C_T^2 |\Delta \vec{r}|^{2/3} \]

Requires simultaneous measurements at \( \vec{r} \) and \( \vec{r} + \Delta \vec{r} \)
Single Radiometer Turbulence Characterization

Temperature (or humidity) measurements are taken at a fixed location.

Wind velocity field shifts the turbulent air mass by a distance \( \Delta \vec{r} = \vec{U} \Delta t \) between measurements.

For a particular altitude, the structure function is now

\[
D_T(t, t + U \Delta t) = \left\langle \left[ T(t) - T(t + U \Delta t) \right]^2 \right\rangle
\]

Due to the radiometer integration time \( \Delta t \), Kolmogorov-Obukhov turbulence theory and the Taylor frozen flow hypothesis must be modified.
Energy Transfer Spectra

- From the Boussinesq approximation (eddy viscosity model) of the Navier-Stokes equations, it is possible to obtain equations involving the Fourier spectra of the turbulent energy, wind velocity and temperature fluctuations*

\[
\begin{align*}
F(k) - \phi_{13}(k) \frac{d\bar{U}}{dz} + \beta \phi_{3T}(k) - 2\nu k^2 \phi(k) &= 0 \\
F_{TT}(k) - \phi_{3T}(k) \frac{d\bar{T}}{dz} - 2\nu_T k^2 \phi_{TT}(k) &= 0
\end{align*}
\]

Modified Turbulence Spectrum

- **Case 1: Near the boundary surface**
  - Significant stratification and shear

  \[ \phi_{TT}(k) = Ak^{-1} \]
  \[ A = 2^{1/2} \left| \frac{d\tilde{U}}{dz} \right| \left| \frac{d\tilde{T}}{dz} \right|^{-2} \gamma^{-1} b^{-2} N^2 \epsilon^{-1} \]

- **Case 2: Free atmosphere**
  - No stratification or shear

  \[ \phi_{TT}(k) = Bk^{-5/3} \]
  \[ B = \left( \frac{2}{3} \right) 4^{1/3} \gamma^{-2/3} b^{-1} N \epsilon^{-1/3} \]

- **General:**
  - Asymptotically reduces to either Case 1 or Case 2

  \[ V_{TT}(k) = \frac{AB}{B|k| + A|k|^{5/3}} \]
Relating to the Structure Functions

The connection of the temporal statistics of the temperature $T(t)$ to the spatial spectrum $V_{TT}(k)$ is through the Fourier-Stieltjes transform

$$D_T(\Delta t) = 2 \int_{-\infty}^{\infty} \left(1 - \left\langle e^{-ik(\bar{U} + v)\Delta t}\right\rangle\right) V_{TT}(k) dk$$

Evaluation of the integral is analytical in terms of Meijer G functions, however two useful series expansions can be obtained for the asymptotic cases

$$D_T(\Delta t) \approx C_T^2(\bar{U} \Delta t)^{2/3} \left[1 - 0.11 \frac{\langle v^2 \rangle}{\bar{U}^2}\right]$$

Kolmogorov “2/3” law

Crossover frequency $k_c = (B/A)^{3/2}$

$$D_T(\Delta t) \approx \frac{C_T^2}{4} k_c^{-2/3} \left[0.57722 + \log(k_c) + \log(\bar{U} \Delta t) - \frac{1}{2} \frac{\langle v^2 \rangle}{\bar{U}^2}\right]$$

Average wind velocity

Wind velocity fluctuations

$
\bar{U} \Delta t \ll 1$

$\bar{U} \Delta t \gg 1$
**Instrumentation**

- NASA TDRSS ground terminal site located at White Sands, NM
- Radiometrics MP-3000A
- 35 calibrated channels
  - 300 MHz bandwidth/channel
  - 21 K-band (22 to 30 GHz)
  - 14 V-band (51 to 59 GHz)
- 1.1 second integration time per channel
- Total $\Delta t \approx 40$ second sample period
- Temperature resolution $\approx 0.1$ K
Temperature Data Analysis

- Measurements taken in January, 2013
- Dataset comprised of about 2100 profiles taken over a 24-hour period
- Each temperature time series divided into 10-minute moving average windows

**Example fluctuations over a 2-hour time period**

Fluctuation standard deviation, \( \sigma \approx 0.5 \text{ K} - 3 \text{ K} \)

**Average profiles for each 10 min window**

Average lapse rate of \( \approx -5.474 \text{ K/km} \)
• Vertical profiles of horizontal wind speed
• SPARC Data Center High-resolution radiosonde measurements at Santa Theresa, NM
• Statistics derived from 2376 wind profiles
• Principal component analysis (PCA) used for data reduction and retention of key features of the wind behavior

Ground wind speeds typically between 5 and 8 m/s
Max tropopause wind speeds typically between 30 and 40 m/s
Results for $C_n^2$

- At optical wavelengths the refractive index structure parameter is a function of $C_T^2$ only.

\[
C_n^2 = \left(10^{-6} \times \frac{77.689 \langle P \rangle}{\langle T \rangle^2} \right)^2 C_T^2
\]

Free atmospheric estimates of $C_n^2$ are about 10x – 100x larger than expected.

Ground estimates of $C_n^2 \approx 10^{-13} \text{ m}^{2/3}$

Specific atmospheric conditions during data compilation were not available, thus a nominal value of $k_c = 15 \text{ m}^{-1}$ was assumed.
Results – Coherence Diameter

- Coherence diameter, also known as the Fried parameter

\[ r_0 = \left[ 0.423k^2 \int_0^H C_n^2(h)dh \right]^{-3/5} \]

- Determines resolution limitations of telescopes

- Also determines the spacing of actuators in adaptive optical systems

\( D > r_0 \) atmosphere limited
\( D < r_0 \) diffraction limited

Night time variation between 5.2 cm – 7.5 cm
Day time variation between 4.3 cm – 6 cm

\( \lambda = 1550 \text{ nm} \)
Results - Greenwood Frequency

- The Greenwood frequency specifies the response characteristic required of an atmospheric adaptive optics system to mitigate the refractive index perturbations.

\[ f_G = 0.255 \left[ k^2 \int_0^H C_n^2(h) (\bar{U}(h))^{5/3} \, dh \right]^{3/5} \]

These values are about a factor of 3 larger than expected for the experimental site in January.

Night time variation between 150 – 290 Hz

Day time variation between 200 – 350 Hz

\( \lambda = 1550 \text{ nm} \)
Results – Five Days in October 2012

- Diurnal variations are easily resolved
- Verification that the resolution requirements of the radiometer are sufficient for this method

Coherence diameter varies between about 1 cm (day) and 10 – 18 cm (night)

Greenwood frequency varies between ≈ 2.5 kHz (day) and 200 Hz (night)
• Atmospheric remote sensing method using a single microwave profiling radiometer to obtain temperature and humidity turbulence structure parameters
• Augmented Kolmogorov turbulence theory to account for boundary effects in a general stratified atmosphere
• Test case shows promising results; however Greenwood frequencies and coherence diameters are over/under estimated
• A more rigorous turbulence spectrum derivation is required
• Ground-based measurements of the gradient Richardson numbers are required for better estimation of the crossover frequency $k_c$
• Concurrent radiosonde measurements of structure parameters along with the radiometer is needed for appropriate comparison