On the Minimum Induced Drag of Wings

Albion H. Bowers
NASA Armstrong Chief Scientist

NASA Neil A. Armstrong Flight Research Center
Imagination vs Knowledge

- Requirements and Assumptions

- Concepts and Solutions

Lift

Where does lift come from?
Personal Air Vehicles

Birds
The Four Ways Birds Differ from Aircraft

• Birds turn and maneuver without a vertical tail
• Birds have slender tips that carry little load
• Birds gracefully fly formation with overlapped wingtips
• Birds have narrow wingtips without tip stall

Wilbur & Orville Wright

• Flying experiments 1899 to 1905
**Prandtl Lifting Line Theory**

- Prandtl's "vortex ribbons"

- Elliptical spanload for a given span (1920)

- "the downwash produced by the longitudinal vortices must be uniform at all points on the aerofoils in order that there may be a minimum of drag for a given total lift." \( y = c \)

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**Minimum Induced Drag & Bending Moment**

- Prandtl (1932)
  - Constrain minimum induced drag
  - Constrain integrated wing bending moment
  - 22% increase in span with 11% decrease in induced drag

This is the accepted theory and the standard for the minimum drag of wings. But what is a wing? Is it only aerodynamic? What about the structure?
Horten Applies the Bbell Spanload

- Horten Spanload (1934-1954)
  use twist to achieve spanload
  induced thrust at tips
  no structural implications

Horten Sailplanes (Germany & Argentina)
Prandtl & Horten

Jones Spanload

- Minimize induced drag (1950)
  Constrain wing root bending moment
  30% increase in span with 17% decrease in induced drag

- "Hence, for a minimum induced drag with a given total lift and a given bending moment the downwash must show a linear variation along the span." $y = bx + c$
Klein and Viswanathan

- Minimize induced drag (1975)
  - Constrain bending moment
  - Constrain shear stress
  - 16% increase in span with 7% decrease in induced drag

- "Hence the required downwash-distribution is parabolic." \( y = ax^2 + bx + c \)

Winglets

- Richard Whitcomb's Winglets
  - Induced thrust on wingtips
  - Induced drag decrease is about half of the span "extension"
  - Reduced wing root bending stress
Whitcomb's Winglets

Prandtl (1920) vs Prandtl (1932)

Prandtl (1920)

- Elliptical Spanload

Prandtl (1932)

- Bell Spanload
Spanload, Downwash, Induced Drag

- All wings dictate 3 solutions
- Spanload
- Downwash
- Induced Drag

Horten H Xc Example

- Horten H Xc footlaunched ultralight sailplane 1950
- 24 degree leading edge sweep angle
- Chord:
  - root - 63 inches
  - tip - 15.75 inches
- Span: 49.2 feet
Prandtl Wing

- 24 degree leading edge sweep angle
- Chord:
  - root - 15.75 inches
  - tip - 3.875 inches
- Span: 147.6 inches

Calculation Method

- Taper
- Twist
- Control Surface Deflections
- Central Difference Angle

Twist

Span

| R0  | 8.3274 |
| R1  | 8.3524 |
| R2  | 8.7259 |
| R3  | 8.8441 |
| R4  | 8.9030 |
| R5  | 8.9984 |
| R6  | 8.8257 |
| R7  | 8.6801 |
| R8  | 8.4956 |
| R9  | 8.1492 |
| R10 | 7.7522 |
| R11 | 7.2592 |
| R12 | 6.6534 |
| R13 | 5.9570 |
| R14 | 5.1362 |
| R15 | 4.1927 |
| R16 | 3.1253 |
| R17 | 1.9394 |
| R18 | 0.6680 |
| R19 | -0.6417 |
| R20 | -1.6726 |
Dr Edward Uden’s Results

- Spanload and Induced Drag
- Elevon Configurations
- Induced Yawing Moments

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Elliptical Half-Lemniscate

- Minimum induced drag for given control power (roll)
- Dr Richard Eppler: FS-24 Phoenix
"Mitteleffekt"

- Artifact of spanload approximations
- Effect on spanloads
  - Increased load at tips
  - Decreased load near centerline
- Upwash due to sweep unaccounted for

Symmetrical Spanloads

- Elevon Trim
- CG Location
Asymmetrical Spanloads

- $\text{Cl}_{\alpha}$ (roll due to aileron)
- $\text{Cn}_{\alpha}$ (yaw due to aileron)
  - induced component
  - profile component
  - change with lift
- $\text{Cn}_{\alpha}/\text{Cl}_{\alpha}$
- $\text{CL (Lift Coefficient)}$
  - increased lift:
    - increased $\text{Cl}_{\beta}$
    - increased $\text{Cn}_{\beta}^*$
  - decreased lift:
    - decreased $\text{Cl}_{\beta}$
    - decreased $\text{Cn}_{\beta}^*$

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Performance Comparison

- Max $\text{L/D}$: 31.9
- Min sink: 89.1 fpm
- Does not include pilot drag
- Predicted $\text{L/D}$: 30
- Predicted sink: 90 fpm
Prandtl’s Bell Spanload

\[ L(x) = 0 \quad (1) \]

\[ \lim_{x: 0 \to b/2} \frac{dL}{dx} = 0 \quad (2) \]

\[ \lim_{x: 0 \to b/2} \frac{dW}{dx} = \lim_{x: \to \infty} \frac{dW}{dx} \quad (3) \]
Spanload

Spedding's Gliding Falcon

- Spedding photographs a gliding falcon's wake with He bubbles
- Vortex cores are 0.76 b apart
- Elliptical spanload is assumed, so the vortex cores are assumed to come from the wingtips

![Graph showing spanload](image-url)
Portugal, et al 2014 (Nature)

Upwash and Wing Beats

Hainsworth 1988
Cutts & Speakman 1994
Speakman & Banks 1995
Upwash and Wing Beats

Wing Stall

Local Cl

Portugalia

Unknown

Wing / Thrust

Surface & Enk

Prandtl 1932

Extension of 1932 theorem

Portugal

Heaviside

Cutts and Speakman

Speakman and Enk

Photo: Jeff Jennings
Effect of Sideslip

- Wing twist
- Sideslip is imposed
- Distorts the bell spanload and the induced drag/thrust profile
What would Proverse Yaw look like?
Flight Data

- Measurement of proverse yaw would be the final hurdle to achieve
- Icing on the cake: measure Cnda (yawing moment due to aileron deflection)

- NOT ONE SECOND OF FLIGHT DATA EXISTS TO PROVE ANY OF THIS IS TRUE

Proverse Yaw

- ...until June 26th, 2013
- Roll and Yaw are the same sign
- From Uden: Cnda is +ve
- uncertainty

Inertias; configuration changes, turbulence, and slope of Cnda
Control of Yaw

- You Have Three Choices:
  - 1/ drag a vertical tail around with you all the time to create a yawing moment
  - 2/ manipulate drag at the wing tips to control yaw
    - OR -
  - 3/ manipulate THRUST at the wing tips to control yaw

- Biological vs Mechanical Flight
Biological Flight

- Mechanical Flight (110 yrs)
- Vertebrate Flight (128 My)

Prandtl, Horten, Jones, and Birds
Efficiency

- Efficiency: 12.5% increase in wing efficiency
- 20-30% increase in efficiency by eliminating the tail
- 15.4% increase in propulsive efficiency
- TOTAL EFFICIENCY INCREASE: 69%

- CY2011: world jet fuel consumption $134B
- $55B in jet fuel saved
- CY2011 World GDP: $69.7T
- World power production: $12.0T
- $1.85T savings in world power production

Concluding Remarks

- Birds as the first model for flight
- Applied approach gave immediate solutions, departure from bird flight
- Eventual meeting of theory and applications (applied theory)
- Spanload evolution (Prandtl/Horten/Jones/Klein/Viswanathan/Whitcomb/Bowers)
- Solve performance, structure and control with ONE spanload solution!
- 12.5% increase in L/D, -2% increase in prop efficiency, 20-30% decrease in drag eliminating the tail, -43-62% reduction in total aero efficiency

- Assumptions and Solutions
- The Wrights disintegrated the flight of birds, and Prandtl/Horten/Jones reintegrated the flight of birds...

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- Jones, Robert T.; "Minimum Induced Drag"; Soaring, October 1979, Soaring Society of America.
- Kordi, Caril; "California Condor"; Audubon Society Report No. 4, 1950, Dover, NY.
PRANDTL-D

- Videos
  - TEDxNASA 2011
    http://www.youtube.com/watch?v=223OmaQ9uLY
  - NASA Aero Academy 2013
    http://www.youtube.com/watch?v=Hr0I6wBFGpY

Red Jensen: pilot, engineer
If you want to build a ship, don't drum up people to collect wood and don't assign them tasks and work, but rather teach them to long for the endless immensity of the sea...

- Antoine de Saint-Exupery