On the Minimum Induced Drag of Wings

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Imagination vs Knowledge

- Requirements and Assumptions
- Concepts and Solutions

Lift

Where does lift come from?
Personal Air Vehicles

Birds
The Four Ways Birds Differ from Aircraft

- Birds turn and maneuver without a vertical tail
- Birds have slender tips that carry little load
- Birds gracefully fly formation with overlapped wingtips
- Birds have narrow wingtips without tip stall

Wilbur & Orville Wright

- Flying experiments 1899 to 1905
Prandtl Lifting Line Theory

- Prandtl's "vortex ribbons"

- Elliptical spanload for a given span (1920)

- "the downwash produced by the longitudinal vortices must be uniform at all points on the aerofoils in order that there may be a minimum of drag for a given total lift." \( y = c \)

Minimum Induced Drag & Bending Moment

- Prandtl (1932)
  Constrain minimum induced drag
  Constrain integrated wing bending moment
  22% increase in span with 11% decrease in induced drag
Horten Applies the Bbell Spanload

- Horten Spanload (1934-1954)
  use twist to achieve spanload
  induced thrust at tips
  no structural implications

Horten Sailplanes (Germany & Argentina)
Prandtl & Horten

Minimize induced drag (1950)
Constrain wing root bending moment
30% increase in span with 17% decrease in induced drag

"Hence, for a minimum induced drag with a given total lift
and a given bending moment the downwash must show a
linear variation along the span." y = bx + c
Klein and Viswanathan

- Minimize induced drag (1975)
  - Constrain bending moment
  - Constrain shear stress
  - 16% increase in span with 7% decrease in induced drag

- "Hence the required downwash-distribution is parabolic." \( y = ax^2 + bx + c \)

Winglets

- Richard Whitcomb's Winglets
  - Induced thrust on wingtips
  - Induced drag decrease is about half of the span "extension"
  - Reduced wing root bending stress
Prandtl's Winglets

Whitcomb's Winglets

Prandtl (1920) vs Prandtl (1932)

Prandtl (1920)
Elliptical Spanload

Prandtl (1932)
Bell Spanload
Spanload, Downwash, Induced Drag

- All wings dictate 3 solutions
- Spanload
- Downwash
- Induced Drag

Horten H Xc Example

- Horten H Xc 
  footlaunched 
  ultralight sailplane 
  1950
- 24 degree leading edge 
  sweep angle
- Chord: 
  root - 63 inches 
  tip - 15.75 inches
- Span: 49.2 feet
• 24 degree leading edge sweep angle
• Chord:
  root – 15.75 inches
  tip – 3.875 inches
• Span: 147.6 inches

Calculation Method

• Taper
• Twist
• Control Surface Deflections
• Central Difference Angle

<table>
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<th>Span</th>
<th>Twist</th>
<th>R0</th>
<th>R1</th>
<th>R2</th>
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<th>R16</th>
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<th>R19</th>
<th>R20</th>
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</table>
Dr Edward Uden's Results

- Spanload and Induced Drag
- Elevon Configurations
- Induced Yawing Moments

<table>
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<tr>
<th>Elevon Config</th>
<th>Cnδa</th>
<th>Spanload</th>
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<tbody>
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<td>I</td>
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<tr>
<td>XI</td>
<td>0.005455</td>
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</tbody>
</table>

Elliptical Half-Lemniscate

- Minimum induced drag for given control power (roll)
- Dr Richard Eppler: FS-24 Phoenix
"Mitteleffekt"

- Artifact of spanload approximations
- Effect on spanloads
  increased load at tips
  decreased load near centerline
- Upwash due to sweep unaccounted for

Symmetrical Spanloads

- Elevon Trim
- CG Location
Asymmetrical Spanloads

- $C_{l_{\text{a}}}$ (roll due to aileron)
- $C_{n_{\text{a}}}$ (yaw due to aileron)
  - induced component
  - profile component
  - change with lift
- $C_{n_{\text{a}}}/C_{l_{\text{a}}}$
- $C_{L_{(\text{Lift Coefficient)}}}$
  - Increased lift:
    - increased $C_{l_{\beta}}$
    - increased $C_{n_{\beta}}$
  - Decreased lift:
    - decreased $C_{l_{\beta}}$
    - decreased $C_{n_{\beta}}$

### Performance Comparison

- Max $L/D$: 31.9
- Min sink: 89.1 fpm
- Does not include pilot drag
- Predicted $L/D$: 30
- Predicted sink: 90 fpm
Prandtl's Bell Spanload

Results of Prandtl's Spanload

\[ \lim_{x \to 0} L(x) = 0 \]  \hspace{1cm} (1)

\[ \lim_{x \to 0} \frac{dL(x)}{dx} = 0 \]  \hspace{1cm} (2)

\[ \lim_{x \to 0} \frac{dW(x)}{dx} = \lim_{x \to \infty} \frac{dW(x)}{dx} \]  \hspace{1cm} (3)
Mike Allen
Spedding's Gliding Falcon

- Spedding photograph's a gliding falcon's wake with He bubbles
- Vortex cores are 0.76 b apart
- Elliptical spanload is assumed, so the vortex cores are assumed to come from the wingtips
Portugal, et al 2014 (Nature)

Upwash and Wing Beats

Portugal 2014

Hainsworth 1988

Cutts & Speakman 1994

Speakman & Banks 1995
Upwash and Wing Beats

Wing Stall

Local Cl

Portugal

Unknown

Wing Stall

Local Cl

Portugal

Unknown

Wing Stall

Local Cl
Effect of Sideslip

- Wing twist
- Sideslip is imposed
- Distorts the bell spanload and the induced drag/thrust profile
PRANDTL-D Proverse Yaw?

What would Proverse Yaw look like?
Flight Data

- Measurement of proverse yaw would be the final hurdle to achieve
- Icing on the cake: measure Cnda (yawing moment due to aileron deflection)

NOT ONE SECOND OF FLIGHT DATA EXISTS TO PROVE ANY OF THIS IS TRUE

Proverse Yaw

- ...until June 26th, 2013
- Roll and Yaw are the same sign
- From Uden: Cnda is +ve
- uncertainty

Inertias; configuration changes, turbulence, and slope of Cnda
You Have Three Choices:

1/ drag a vertical tail around with you all the time to create a yawing moment

2/ manipulate drag at the wing tips to control yaw

-OR-

3/ manipulate THRUST at the wing tips to control yaw

Biological vs Mechanical Flight
Biological Flight

- Mechanical Flight (110 yrs)
- Vertebrate Flight (128 My)

Prandtl, Horten, Jones, and Birds
Efficiency

- Efficiency: 12.5% increase in wing efficiency
- 20-30% increase in efficiency by eliminating the tail
- 15.4% increase in propulsive efficiency
- TOTAL EFFICIENCY INCREASE: 69%

- CY2011: world jet fuel consumption $134B
- $55B in jet fuel saved

- CY2011 World GDP: $69.7T
- World power production: $12.0T
- $1.85T savings in world power production

Concluding Remarks

- Birds as the first model for flight
- Applied approach gave immediate solutions, departure from bird flight
- Eventual meeting of theory and applications (applied theory)
- Spanload evolution (Prandtl/Horten/Jones/Klein/Viswanathan/Whitcomb/Bowers)
- Solve performance, structure and control with ONE spanload solution!
- 12.5% increase in L/D, -2% increase in prop efficiency, 20-30% decrease in drag eliminating the tail, -43-62% reduction in total aero efficiency
- Assumptions and Solutions

- The Wrights disintegrated the flight of birds, and Prandtl/Horten/Jones reintegrated the flight of birds...
- Thanks: Red Jensen, Brian Eslinger, Dr Christian Gelzer, Dr Oscar Murillo, Hayley Foster & Steve Craft, Dr Bob Liebeck, Nalin Ratnayake, Mike Allen, Walter Horten, George Dez-Falvy, Rudi Opliz, Bruce Carmichael, R.T. Jones, Russ Lee, Bob Hoey, Phil Barnes, Dan & Jan Armstrong, Dr Phil Burgers, Ed Lockhart, Andy Kesckes, Dr Paul MacCready, Reinhold Stadler, Dr Edward Uden, & Dr Karl Nickel
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- Horten, Reimar; unpublished personal notes.
- Uden, Edward; unpublished personal notes.
- Klein, Armin and Mavranathan, Sathy; "Approximate Solution for Minimum Induced Drag of Wings with a Given Structural Weight"; Journal of Aircraft, Feb 1979, Vol. 16 No 2, AIAA.
- Jones, Robert T.; "Minimum Inducing Drag"; Soaring, October 1979; Soaring Society of America.
- Koldor, Carl; "California Condor"; Audubon Special Report No 4, 1950, Dover, NY.
- Lee, Russell; "Only the Wing: Reimar Horten’s Solo Quest to Stabilize and Control the All-Wing Aircraft," Smithsonian Institution Scholarly Press (Rowman & Littlefield), Washington D.C., 2011.
PRANDTL-D

- Videos

  - TEDxNASA 2011
    http://www.youtube.com/watch?v=2230maQ9uLY

  - NASA Aero Academy 2013
    http://www.youtube.com/watch?v=Hr0I6wBFGpY

Red Jensen: pilot, engineer
If you want to build a ship, don't drum up people to collect wood and don't assign them tasks and work, but rather teach them to long for the endless immensity of the sea...

- Antoine de Saint-Exupery