On the Minimum Induced Drag of Wings

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Imagination vs Knowledge

- Requirements and Assumptions
- Concepts and Solutions

Lift

Where does lift come from?
Personal Air Vehicles

Birds
The Four Ways Birds Differ from Aircraft

- Birds turn and maneuver without a vertical tail
- Birds have slender tips that carry little load
- Birds gracefully fly formation with overlapped wingtips
- Birds have narrow wingtips without tip stall

Wilbur & Orville Wright

- Flying experiments 1899 to 1905
Prandtl Lifting Line Theory

- Prandtl’s “vortex ribbons”

- Elliptical spanload for a given span (1920)

- “the downwash produced by the longitudinal vortices must be uniform at all points on the aerofoils in order that there may be a minimum of drag for a given total lift.” $y = c$

Minimum Induced Drag & Bending Moment

- Prandtl (1932)
  Constrain minimum induced drag
  Constrain integrated wing bending moment
  22% increase in span with 11% decrease in induced drag
Horten Applies the Bbell Spanload

- Horten Spanload (1934-1954)
  use twist to achieve spanload
  induced thrust at tips
  no structural implications

Horten Sailplanes (Germany & Argentina)
Prandtl & Horten

**Jones Spanload**

- Minimize induced drag (1950)
  - Constrain wing root bending moment
  - 30% increase in span with 17% decrease in induced drag

- "Hence, for a minimum induced drag with a given total lift and a given bending moment the downwash must show a linear variation along the span."  \( y = bx + c \)
Klein and Viswanathan

- Minimize induced drag (1975)
  - Constrain bending moment
  - Constrain shear stress
  - 16% increase in span with 7% decrease in induced drag

- "Hence the required downwash-distribution is parabolic." \( y = ax^2 + bx + c \)

Winglets

- Richard Whitcomb’s Winglets
  - Induced thrust on wingtips
  - Induced drag decrease is about half of the span “extension”
  - Reduced wing root bending stress
Whitcomb’s Winglets

Prandtl (1920) vs Prandtl (1932)

Prandtl 1920
Elliptical Spanload

Prandtl 1932
Bell Spanload
Spanload, Downwash, Induced Drag

- All wings dictate 3 solutions
- Spanload
- Downwash
- Induced Drag

Horten H Xc Example

- Horten H Xc footlaunched ultralight sailplane 1950
- 24 degree leading edge sweep angle
- Chord:
  root - 63 inches
  tip - 15.75 inches
- Span: 49.2 feet
Prandtl Wing

- 24 degree leading edge sweep angle
- Chord:
  - root – 15.75 inches
  - tip – 3.875 inches
- Span: 147.6 inches

Calculation Method

- Taper
- Twist
- Control Surface Deflections
- Central Difference Angle

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Dr Edward Uden’s Results

- Spanload and Induced Drag
- Elevon Configurations
- Induced Yawing Moments

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<tr>
<th>Elevon Config</th>
<th>Cn\delta</th>
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Elliptical Half-Lemniscate

- Minimum induced drag for given control power (roll)
- Dr Richard Eppler: FS-24 Phoenix
"Mitteleffekt"

- Artifact of spanload approximations
- Effect on spanloads
  - Increased load at tips
  - Decreased load near centerline
- Upwash due to sweep unaccounted for

Symmetrical Spanloads

- Elevon Trim
- CG Location
Asymmetrical Spanloads

- $C_{l\alpha}$ (roll due to aileron)
- $C_{n\alpha}$ (yaw due to aileron)
- Induced component
- Profile component change with lift
- $C_{n\alpha}/C_{l\alpha}$
- $C_L$(Lift Coefficient)
  - Increased lift:
    - Increased $C_{\beta}$
    - Increased $C_{n\beta}$
  - Decreased lift:
    - Decreased $C_{\beta}$
    - Decreased $C_{n\beta}$

<table>
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<th>$C_L$</th>
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Performance Comparison

- Max $L/D$: 31.9
- Min sink: 89.1 fpm
- Does not include pilot drag
- Predicted $L/D$: 30
- Predicted sink: 90 fpm
Prandtl’s Bell Spanload

Results of Prandtl’s Spanload

\[
\lim_{x: \to 0}^{b/2} L(x) = 0 \tag{1}
\]

\[
\lim_{x: \to 0}^{b/2} \frac{dL(x)}{dx} = 0 \tag{2}
\]

\[
\lim_{x: 0 \to b/2} \frac{dW(x)}{dx} = \lim_{x: \to \infty}^{b/2} \frac{dW(x)}{dx} \tag{3}
\]
Spanload

Spedding’s Gliding Falcon

- Spedding photographs a gliding falcon’s wake with He bubbles
- Vortex cores are 0.76 b apart
- Elliptical spanload is assumed, so the vortex cores are assumed to come from the wingtips
Portugal, et al 2014 (Nature)

Upwash and Wing Beats

Portugal 2014

Hainsworth 1988

Cutts & Speakman 1994

Speakman & Banks 1995
Upwash and Wing Beats

Wing Stall

Local Cl

Portugal

Unknown

Wing Stall

Local Cl
Nachtigall 1966 (J of Exp Bio)

Effect of Sideslip

- Wing twist
- Sideslip is imposed
- Distorts the bell spanload and the induced drag/thrust profile
PRANDTL-D Proverse Yaw?

What would Proverse Yaw look like?
Flight Data

- Measurement of proverse yaw would be the final hurdle to achieve
- Icing on the cake: measure Cnda (yawing moment due to aileron deflection)

- NOT ONE SECOND OF FLIGHT DATA EXISTS TO PROVE ANY OF THIS IS TRUE

Proverse Yaw

- ...until June 26th, 2013
- Roll and Yaw are the same sign
- From Uden: Cnda is +ve
- Uncertainty

Inertias; configuration changes, turbulence, and slope of Cnda
You Have Three Choices:

1/ drag a vertical tail around with you all the time to create a yawing moment

2/ manipulate drag at the wing tips to control yaw

-OR-

3/ manipulate THRUST at the wing tips to control yaw

Biological vs Mechanical Flight
Biological Flight

- Mechanical Flight (110 yrs)
- Vertabrate Flight (128 My)

Prandtl, Horten, Jones, and Birds
Efficiency

- Efficiency: 12.5% increase in wing efficiency
- 20-30% increase in efficiency by eliminating the tail
- 15.4% increase in propulsive efficiency
- TOTAL EFFICIENCY INCREASE: 69%

- CY2011: world jet fuel consumption $134B
- $55B in jet fuel saved

- CY2011 World GDP: $69.7T
- World power production: $12.0T
- $1.85T savings in world power production

Concluding Remarks

- Birds as as the first model for flight
- Applied approach gave immediate solutions, departure from bird flight
- Eventual meeting of theory and applications (applied theory)
- Spanload evolution (Prandtl/Horten/Jones/Klein/Viswanathan/Whitcomb/Bowers)
- Solve performance, structure and control with ONE spanload solution!
- 12.5% increase in L/D, -2% increase in prop efficiency, 20-30% decrease in drag eliminating the tail, -43-62% reduction in total aero efficiency

- Assumptions and Solutions

- The Wrights disintegrated the flight of birds, and Prandtl/Horten/Jones reintegrated the flight of birds...

- Thanks: Red Jensen, Brian Eslinger, Dr Christian Gelzer, Dr Oscar Murillo, Hayley Foster & Steve Craft, Dr Bob Liebeck, Nalin Ratnayake, Mike Allen, Walter Horten, Georgy Dez-Falvy, Rudi Optiz, Bruce Cermichael, R. T. Jones, Russ Lee, Bob Hoey, Phil Barnes, Dan & Jan Armstrong, Dr Phil Burgers, Ed Lockhart, Andy Kesckes, Dr Paul MacCready, Reinhold Stadler, Dr Edward Uden, & Dr Karl Nickel
NASA Aero Academies & Others

- 2014 NASA Aero Academy
  - Brian Plank, Joe Lorenzetti, Kathleen Glasheen, Bryce Doerr, Cynthia Farr, Nancy Pinon, Heather Laffoon, Jack Toth, Leo Banuelos
- 2013 NASA Aero Academy
  - Eric Gutierrez, Louis Edelman, Kristyn Kadala, Nancy Pinon, Cody Karcher, Andy Puch, Hovig Yaralian, Jacob Hall
- 2012 NASA Aero Academy
  - Steffi Volkov, Juliana Plumb (Ulrich), Luis Andrade, Stephanie Reynolds, Joey Wagster, Kimmy Callan, Javier Rocha, Sanel Horozovic, Ronalyyn Ramos, Nancy Pinon

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- Uden, Edward; unpublished personal notes.
- Klein, Armin and Mavranathan, Sathy; "Approximate Solution for Minimum Induced Drag of Wings with a Given Structural Weight"; Journal of Aircraft, Feb 1978, Vol 15 No 2, AIAA.
- Kordel, Carl; "California Condor"; Audobon Special Report No. 4, 1950, Dover, NY.

29
PRANDTL-D

- Videos
  - TEDxNASA 2011
    http://www.youtube.com/watch?v=2230maQ9uLY
  - NASA Aero Academy 2013
    http://www.youtube.com/watch?v=Hr0I6wBFGpY

Red Jensen: pilot, engineer
If you want to build a ship, don't drum up people to collect wood and don't assign them tasks and work, but rather teach them to long for the endless immensity of the sea...

- Antoine de Saint-Exupery