On the Minimum Induced Drag of Wings

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Imagination vs Knowledge

• Requirements and Assumptions

• Concepts and Solutions

Lift

Where does lift come from?
Personal Air Vehicles

Birds
The Four Ways Birds Differ from Aircraft

- Birds turn and maneuver without a vertical tail
- Birds have slender tips that carry little load
- Birds gracefully fly formation with overlapped wingtips
- Birds have narrow wingtips without tip stall

Wilbur & Orville Wright

- Flying experiments 1899 to 1905
Prandtl Lifting Line Theory

- Prandtl's "vortex ribbons"

- Elliptical spanload for a given span (1920)

- "the downwash produced by the longitudinal vortices must be uniform at all points on the aerofoils in order that there may be a minimum of drag for a given total lift." \( y = c \)

Minimum Induced Drag & Bending Moment

- Prandtl (1932)
  Constrain minimum induced drag
  Constrain integrated wing bending moment
  22% increase in span with 11% decrease in induced drag
Horten Applies the Bbell Spanload

- Horten Spanload (1934-1954)
  use twist to achieve spanload
  induced thrust at tips
  no structural implications

Horten Sailplanes (Germany & Argentina)
Minimize induced drag (1950)
Constrain wing root bending moment
30% increase in span with 17% decrease in induced drag

“Hence, for a minimum induced drag with a given total lift and a given bending moment the downwash must show a linear variation along the span.” $y = bx + c$
Klein and Viswanathan

- Minimize induced drag (1975)
  - Constrain bending moment
  - Constrain shear stress
  - 16% increase in span with 7% decrease in induced drag

- "Hence the required downwash-distribution is parabolic." $y = ax^2 + bx + c$

Winglets

- Richard Whitcomb's Winglets
  - Induced thrust on wingtips
  - Induced drag decrease is about half of the span "extension"
  - Reduced wing root bending stress
Whitcomb's Winglets

Prandtl (1920) vs Prandtl (1932)

Elliptical Spanload

Bell Spanload
Spanload, Downwash, Induced Drag

- All wings dictate 3 solutions
- Spanload
- Downwash
- Induced Drag

Horten H Xc Example

- Horten H Xc
tootlaunched
ultralight sailplane
1950

- 24 degree leading edge
sweep angle

- Chord:
  root - 63 inches
tip - 15.75 inches

- Span: 49.2 feet
Prandtl Wing

- 24 degree leading edge sweep angle
- Chord:
  - root – 15.75 inches
  - tip – 3.875 inches
- Span: 147.6 inches

Calculation Method

- Taper
- Twist
- Control Surface Deflections
- Central Difference Angle

Twist

Span

| R0 | 8.3274 |
| R1 | 8.5624 |
| R2 | 8.7259 |
| R3 | 8.8441 |
| R4 | 8.9030 |
| R5 | 8.8984 |
| R6 | 8.8257 |
| R7 | 8.6801 |
| R8 | 8.4065 |
| R9 | 8.1492 |
| R10| 7.7522 |
| R11| 7.2592 |
| R12| 6.6854 |
| R13| 5.9579 |
| R14| 5.1362 |
| R15| 4.1227 |
| R16| 3.1253 |
| R17| 1.9394 |
| R18| 0.6986 |
| R19| -0.6417 |
| R20| -1.6726 |
Dr Edward Uden’s Results

- Spanload and Induced Drag
- Elevon Configurations
- Induced Yawing Moments

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<th>Elevon Config</th>
<th>$C_{n_{\delta}}$</th>
<th>Spanload</th>
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Elliptical Half-Lemniscate

- Minimum induced drag for given control power (roll)
- Dr Richard Eppler: FS-24 Phoenix
"Mitteleffekt"

- Artifact of spanload approximations
- Effect on spanloads
  - Increased load at tips
  - Decreased load near centerline
- Upwash due to sweep unaccounted for

Symmetrical Spanloads

- Elevon Trim
- CG Location
Asymmetrical Spanloads

- $C_l\delta_a$ (roll due to aileron)
- $C_n\delta_a$ (yaw due to aileron)
  induced component
  profile component
  change with lift
- $C_{n\delta_a}/C_l\delta_a$
- $C_L$ (Lift Coefficient)
  Increased lift:
  increased $C_l\beta$
  increased $C_{n\beta}^*$
  Decreased lift:
  decreased $C_l\beta$
  decreased $C_{n\beta}^*$

<table>
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Performance Comparison

- Max $L/D$: 31.9
- Min sink: 89.1 fpm
- Does not include pilot drag
- Predicted $L/D$: 30
- Predicted sink: 90 fpm
Prandtl’s Bell Spanload

Results of Prandtl’s Spanload

\[ \lim_{x: 0 \to b/2} L(x) = 0 \]  \hspace{1cm} (1)

\[ \lim_{x: 0 \to b/2} \frac{dL(x)}{dx} = 0 \]  \hspace{1cm} (2)

\[ \lim_{x:0 \to b/2} \frac{dD(x)}{dx} = \lim_{x: \infty \to b/2} \frac{dD(x)}{dx} \]  \hspace{1cm} (3)
Spanload

Spedding's Gliding Falcon

- Spedding photograph's a gliding falcon's wake with He bubbles
- Vortex cores are 0.76 b apart
- Elliptical spanload is assumed, so the vortex cores are assumed to come from the wingtips
Portugal, et al 2014 (Nature)

Upwash and Wing Beats

Portugal 2014

Hainsworth 1986

Cutts & Speakman 1994

Speakman & Banks 1995
Upwash and Wing Beats

Wing Stall

Local Cl

Portugal

Unknown

Wing & Thrust

Sculpture & Brakes

Prandtl 1932
— Extension of 1932 theorem
— Portugal
— Heatworth
— Cutts and Speakman
— Speakman and Binks
Nachtigall 1966 (J of Exp Bio)

Effect of Sideslip

- Wing twist
- Sideslip is imposed
- Distorts the bell spanload and the induced drag/thrust profile
What would Proverse Yaw look like?

PRANDTL-D Proverse Yaw?
Flight Data

- Measurement of proverse yaw would be the final hurdle to achieve
- Icing on the cake: measure Cnda (yawing moment due to aileron deflection)

- NOT ONE SECOND OF FLIGHT DATA EXISTS TO PROVE ANY OF THIS IS TRUE

Proverse Yaw

- ...until June 26th, 2013
- Roll and Yaw are the same sign
- From Uden: Cnda is +ve
- Uncertainty

Inset: inertias; configuration changes, turbulence, and slope of Cnda
Control of Yaw

- You Have Three Choices:
  - 1/ drag a vertical tail around with you all the time to create a yawing moment
  - 2/ manipulate drag at the wing tips to control yaw
  - 3/ manipulate THRUST at the wing tips to control yaw

Biological vs Mechanical Flight
Biological Flight

- Mechanical Flight (110 yrs)
- Vertebrate Flight (128 My)

Prandtl, Horten, Jones, and Birds
Efficiency

- Efficiency: 12.5% increase in wing efficiency
- 20-30% increase in efficiency by eliminating the tail
- 15.4% increase in propulsive efficiency
- TOTAL EFFICIENCY INCREASE: 69%

- CY2011: world jet fuel consumption $134B
  - $55B in jet fuel saved

- CY2011 World GDP: $69.7T
- World power production: $12.0T
- $1.85T savings in world power production

Concluding Remarks

- Birds as the first model for flight
- Applied approach gave immediate solutions, departure from bird flight
- Eventual meeting of theory and applications (applied theory)
- Spanload evolution (Prandtl/Horten/Jones/Klein/Viswanathan/Whitcomb/Bowers)
- Solve performance, structure and control with ONE spanload solution!
- 12.5% increase in L/D; ~2% increase in prop efficiency, 20-30% decrease in drag eliminating the tail, ~43-62% reduction in total aero efficiency
- Assumptions and Solutions
- The Wrights disintegrated the flight of birds, and Prandtl/Horten/Jones reintegrated the flight of birds...

- Thanks: Red Jensen, Brian Estinger, Dr Christian Gelzer, Dr Oscar Murillo, Hayley Foster & Steve Craft, Dr Bob Liebeck, Nalin Ratnayake, Mike Allen, Walter Horten, George Dez-Falvy, Rudi Optiz, Bruce Carmichael, R T. Jones, Russ Lee, Bob Hoey, Phil Barnes, Dan & Jan Armstrong, Dr Phil Burgers, Ed Lockhart, Andy Kesckes, Dr Paul MacCready, Reinhold Stadler, Dr Edward Uden, & Dr Karl Nickel
**NASA Aero Academies & Others**

- 2014 NASA Aero Academy
  - Brian Plank, Joe Lorenzetti, Kathleen Glasheen, Bryce Doerr, Cynthia Farr, Nancy Pinon, Heather Laflloon, Jack Toth, Leo Banuelos
- 2013 NASA Aero Academy
  - Eric Gutierrez, Louis Edelman, Kristyn Kadala, Nancy Pinon, Cody Karcher, Andy Putch, Hovig Yaralian, Jacob Hall
- 2012 NASA Aero Academy
  - Steffi Valkov, Juliana Plumb (Ulrich), Luis Andrade, Stephanie Reynolds, Joey Wagster, Kimmy Callan, Javier Rocha, Sanel Horozovic, Ronalynn Ramos, Nancy Pinon

**References**

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- Uden, Edward; unpublished personal notes.
- Klien, Armin and Mavranan, Sathy; "Approximate Solution for Minimum Induced Drag of Wings with a Given Structural Weight"; Journal of Aircraft, Feb 1979, Vol 16 No 2, AIAA.
- Korol, Carl; "California Condor"; Audubon Special Report No. 4, 1950, Dover, NY.
PRANDTL-D

- Videos
  - TEDxNASA 2011
    http://www.youtube.com/watch?v=223OmaQ9uLY
  - NASA Aero Academy 2013
    http://www.youtube.com/watch?v=Hr0l6wBFgBY

Red Jensen: pilot, engineer
If you want to build a ship, don't drum up people to collect wood and don't assign them tasks and work, but rather teach them to long for the endless immensity of the sea...

- Antoine de Saint-Exupery

Questions?