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A review of spectral methods for variable amplitude fatigue prediction and new results

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Abstract

A comprehensive review of the available methods for estimating fatigue damage from variable amplitude loading is presented. The dependence of fatigue damage accumulation on power spectral density (psd) is investigated for random processes relevant to real structures such as in offshore or aerospace applications. Beginning with the Rayleigh (or narrow band) approximation, attempts at improved approximations or corrections to the Rayleigh approximation are examined by comparison to rainflow analysis of time histories simulated from psd functions representative of simple theoretical and real world applications. Spectral methods investigated include corrections by Wirsching and Light, Ortiz and Chen, the Dirlik formula, and the Single-Moment method, among other more recent proposed methods. Good agreement is obtained between the spectral methods and the time-domain rainflow identification for most cases, with some limitations. Guidelines are given for using the several spectral methods to increase confidence in the damage estimate.
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*Keywords:*damage accumulation; rainflow analysis; random processes; stationary Gaussian; spectral methods; stochastic fatigue

1. Introduction

Rainflow analysis is a method for identifying and counting fatigue stress reversal cycles from a time history [1,2,3]. The rainflow method allows the application of the Palmgren-Miner rule [4] in order to assess the fatigue life of a structure subject to complex loading and is recognized in the technical community as the "gold standard" for estimating fatigue damage from variable amplitude loading or for comparison to other estimate techniques. Fatigue life assessment can also be performed in the frequency domain using semi-empirical methods. The Rayleigh or narrowband approximation [5] is the fundamental of these methods, while various attempts at improved approximations have been made over the years using different moments of the power spectral density (psd) for stationary Gaussian processes and fitting of empirical parameters to the results of rainflow analysis of a corresponding random stress time history. In the present work rainflow analysis is used to compare to a number of spectral methods, those proposed some time ago in the literature and those more recently proposed. Mršnik et al. [6] provided a recent comprehensive review and comparison of available spectral

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methods, but two older methods in particular were not discussed: the method proposed by Ortiz and Chen [7] and the Single-Moment method [8,9]. These methods are worth some examination as follows in comparison to the best methods as observed by the work of Mršnik et al. [6]. For further background, the reader is referred to the following excellent texts on random vibration and stochastic fatigue [10,11,12,13,14]

Nomenclature

A	fatigue strength coefficient from the material S-N curve, $A=NS^m$
$D(\tau)$	the accumulated damage function due to stresses or strains occurring up to the time τ
D_{NB}	the narrowband or Rayleigh damage
D_{SM}	the Single-Moment method estimate of damage
E[X]	expectation of the random variable X
f	frequency (Hz)
m	fatigue strength exponent from the material S-N curve
n_p	rate of peak occurrences from rainflow analysis
M_j	the jth moment of the one-side spectral density
psd	power spectral density
S	stress cycle range = (peak-valley)
W_s	one-sided stress power spectral density ($stress^2$/Hz)
α	irregularity factor
β_k	generalized spectral bandwidth
ε	spectral width
Γ	gamma function
ν_o^+	equivalent frequency based on rate of positive slope zero crossings
ν_p	rate of peak occurrences
σ_S	response stress overall RMS level
τ	exposure time (seconds)

1.1. Defining equations

The fatigue strength coefficient A for the purpose of this paper is taken as the stress level on the S-N curve where N=1/2 cycles, assuming no reduction in slope due to strain hardening in the low-cycle range. An alternate method is to calculate the A value using fracture mechanics for the case of a pre-existing crack [10]. Assuming a zero-mean, stationary stress process, the following defining equations are useful:

The moments of a one-sided (positive frequency) power spectral density are defined as:

$$M_j = \int_0^\infty f^j W_S(f)\,df \tag{1}$$

In which the index j may be non-integer.

The rate of zero up-crossings can be estimated as:

$$\nu_0^+ = \sqrt{M_2/M_0} \tag{2}$$

The rate of peak occurrences is:

$$\nu_p = \sqrt{M_4/M_2} \tag{3}$$

The irregularity factor is a measure of bandwidth defined as:

$$\alpha_i = \frac{M_i}{\sqrt{M_0 M_{2i}}} \tag{4}$$

In which a commonly used special case is for i=2:

$$\alpha_2 = \frac{M_2}{\sqrt{M_0 M_4}} = \frac{\nu_0^+}{\nu_p} \quad (5)$$

2. Spectral Damage Estimate Methods

2.1. Rayleigh or narrowband damage approximation

The Rayleigh approximation, also referred to as the narrowband approximation, assumes that the stress ranges are distributed as the Rayleigh distributed peaks of the limiting narrowband process [5,10,11]:

$$\mathrm{D_{NB}} = \frac{\nu_0^+ \tau}{\mathrm{A}} \left(2\sqrt{2}\sigma_{\mathrm{S}}\right)^{\mathrm{m}} \Gamma\left(\frac{1}{2}\mathrm{m} + 1\right) \quad (6)$$

Note that failure is assumed to occur when $D_{NB} \geq 1.0$, but in practice many use thresholds which are conservatively lower.

The damage due to a wideband stress process may be estimated from the narrowband damage as:

$$D = \lambda D_{NB} \quad (7)$$

In which λ is a generic scale or correction factor. Specific forms for λ that have been suggested in the literature follow.

2.2. Wirsching and Light correction

Wirsching and Light developed the following correction factor by simulating processes having a variety of spectral shapes [5].

$$\lambda_W(\varepsilon, m) = a(m) + \left[1 - a(m)\right]\left(1 - \varepsilon\right)^{b(m)} \quad (8)$$

In which:

a(m) = 0.0926 - 0.033 m

b(m) = 1.587 m – 2.323

Note that Wirsching and Light used S-N slope values of m= 3, 4, 5 and 6 for the rainflow analysis of the simulations and the basis of the above correction.

2.3. Ortiz and Chen correction

Ortiz and Chen developed the following correction factor by applying the generalized spectral bandwidth to the Rayleigh distribution [7].

$$\lambda_k = \frac{\beta_k^m}{\alpha_2} \quad (9)$$

In which the generalized spectral bandwidth is:

$$\beta_k = \sqrt{\frac{M_2 M_k}{M_0 M_{k+2}}} \tag{10}$$

for which k = 2.0/m (note: may be non-integer)

2.4. Benasciutti and Tovo

The first Benasciutti & Tovo correction factor from [6] is:

$$\lambda_{BT} = \left[b + (1-b)\,\alpha_2^{m-1} \right] \alpha_2 \tag{11}$$

In which:

$$b = \frac{(\alpha_1 - \alpha_2)\left[1.112\left(1 + \alpha_1\alpha_2 - (\alpha_1 + \alpha_2)\right)\exp\left(2.11\alpha_2\right) + \left(\alpha_1 - \alpha_2\right)\right]}{\alpha_2} \tag{12}$$

2.5. $\alpha_{0.75}$ Method

The $\alpha_{0.75}$ correction factor from [6] proposed by Benasciutti & Tovo is:

$$\lambda_\alpha = \alpha_{0.75}^2 \tag{13}$$

2.6. Dirlik

The Dirlik method [6] approximates the cycle-amplitude distribution by using a combination of one exponential and two Rayleigh probability densities.

2.7. Zhao-Baker

Zhao and Baker combined theoretical assumptions and simulation results to give an expression for the cycle distribution as a linear combination of the Weibull and Rayleigh probability density functions [6].

2.8. Single-Moment method

Larsen and Lutes proposed an empirical relation for D(τ), referred to in the literature as the Single-Moment method [8,9,10,11]. Benasciutti et al. [15] more recently provided a mathematical interpretation of the method.

$$D_{SM} = \frac{\tau}{A}\left(2\sqrt{2}\right)^m \left(M_{2/m}\right)^{m/2} \Gamma\left(\frac{1}{2}m + 1\right) \tag{14}$$

3. Time history simulation

A time history with a normal distribution can be synthesized to match an applied force or base excitation power spectral density. The excitation can then be applied to a structural model. The structure's response can then be calculated via a modal transient analysis. The resulting stress can then be calculated as a post-processing step from the strain response. The rainflow cycles can then be calculated using the method in [1]. The cumulative damage is then calculated via the Palmgren-Miner formula, per [4].

4. Spectra studied

For purposes of comparison, the authors studied the above methods for four of the same spectra types as those used by Mršnik et al. [6], referred to as the multi-modal (MM), the background noise (BN), spectral width (SW) and the close-modes (CM) spectra. Table 1 summarizes the bandwidth range of the spectra studied in terms of the α_2 parameter, the irregularity factor.

Table 1. Irregularity factor α_2 for spectra studied.

	α_2			
MM1-4	0.5463	0.7817	0.8061	0.8056
BN1-4	0.8252	0.8714	0.8939	0.9316
SW1-4	0.8097	0.8889	0.8818	0.7861
CM1-4	0.7384	0.7577	0.8388	0.8679

In this study the materials considered were also those used by [16] and Mršnik et al. [6] to insure that results could be compared on a one-for-one basis. In particular, the values of the S-N curve exponent parameter m were 3.324 (steel), 7.3 (aluminum 2219-T851), and 11.76 (spring steel). This results in some difficulty in the use of the Wirsching and Light correction in that it was not developed with data beyond m=6 [5].

5. Results

In this work computations were made of the damage rate by each of the spectral methods, given by $A\dot{D}$ (units of stress^m/second) and compared to the damage rate computed by rainflow analysis and Miner's rule, as given by $n_pE[R^m]$ (following the convention of [8]).

Comparison of the spectral methods to the rainflow analysis is shown in Figures 1 – 4 in terms of the relative error of the spectral method with respect to the rainflow analysis. Note that it is unconservative to underestimate the damage rate as the time to failure will thus be overestimated.

In these figures the relative errors for each psd have been connected by a smooth curve to aid visualization of the trends occurring. No implication is intended that the errors follow any smooth function with respect to the psds or any bandwidth parameter, although the values of the irregularity factor for each psd have also been co-plotted to attempt to ascertain if any of the variation in relative error observed in the spectral methods could be related to bandwidth as indicated by α_2.

6. Discussion

As shown in Figure 1 for m = 3.324, all the spectral methods studied exhibit less than 20% absolute error, except for one case of the narrowband or Rayleigh approximation. Some particularly strong correlation to bandwidth (α_2) is noted for many of the methods for the most extreme wideband psds (MM-1, CM-1, CM-2). However, the Ortiz Chen method in particular exhibits the least error for the CM-1, CM-2 cases, which may be explained by the method's dependence on higher order moments, but it has a rather large error for the MM-1 case, which is the most wideband of all the cases studied as measured by α_2. This indicates that the moments and bandwidth parameters normally in use are likely insufficient to describe the psd shape affects that influence the distribution of ranges. The Dirlik, $\alpha_{0.75}$, Single Moment, and Benasciutti Tovo methods generally exhibit less error with changing bandwidth, and along with the Ortiz Chen method, are shown in Figure 2 as the best methods for more detailed study. There is still some correlation with α_2 apparent in this figure, especially for the wideband MM-1 case, but the exaggerated errors of the deleted methods are not apparent. The Dirlik and Benasciutti Tovo methods underestimate or approach from below to the rainflow damage rate, with the Dirlik method being somewhat more accurate. The Single Moment method under- and overestimates within a band of less than 4% absolute error, while the $\alpha_{0.75}$ method generally overestimates the damage rate, but with less than 4% error. The Ortiz Chen method

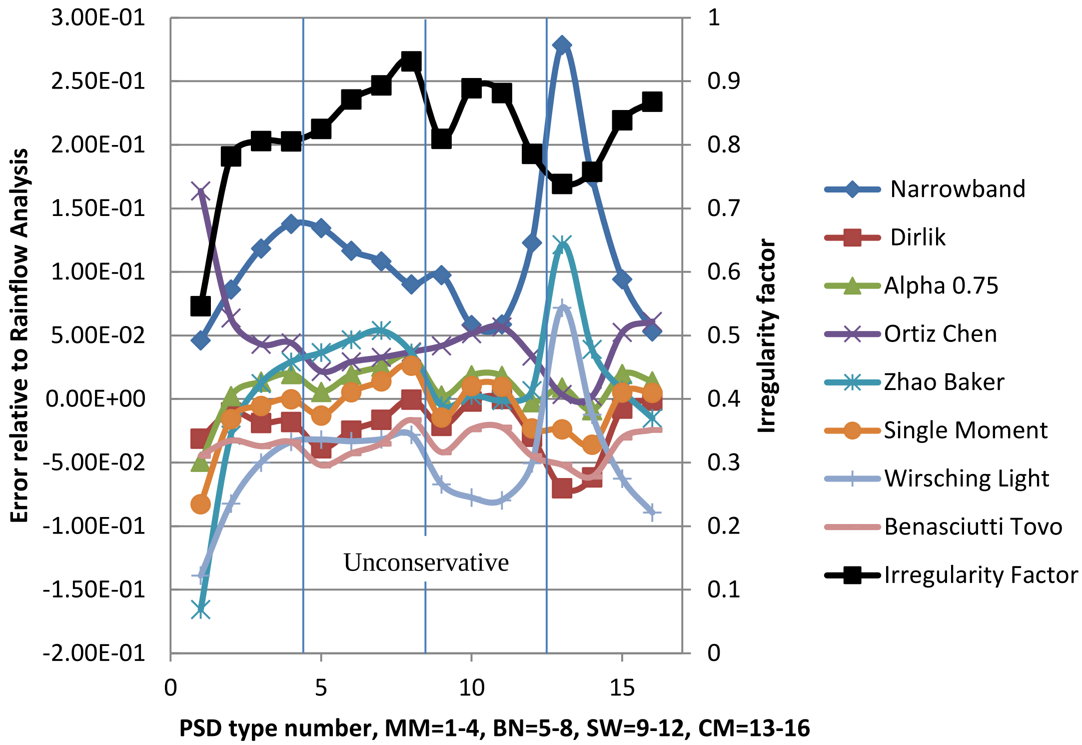

Fig. 1. Relative error of spectral methods compared to rainflow analysis for m=3.324.

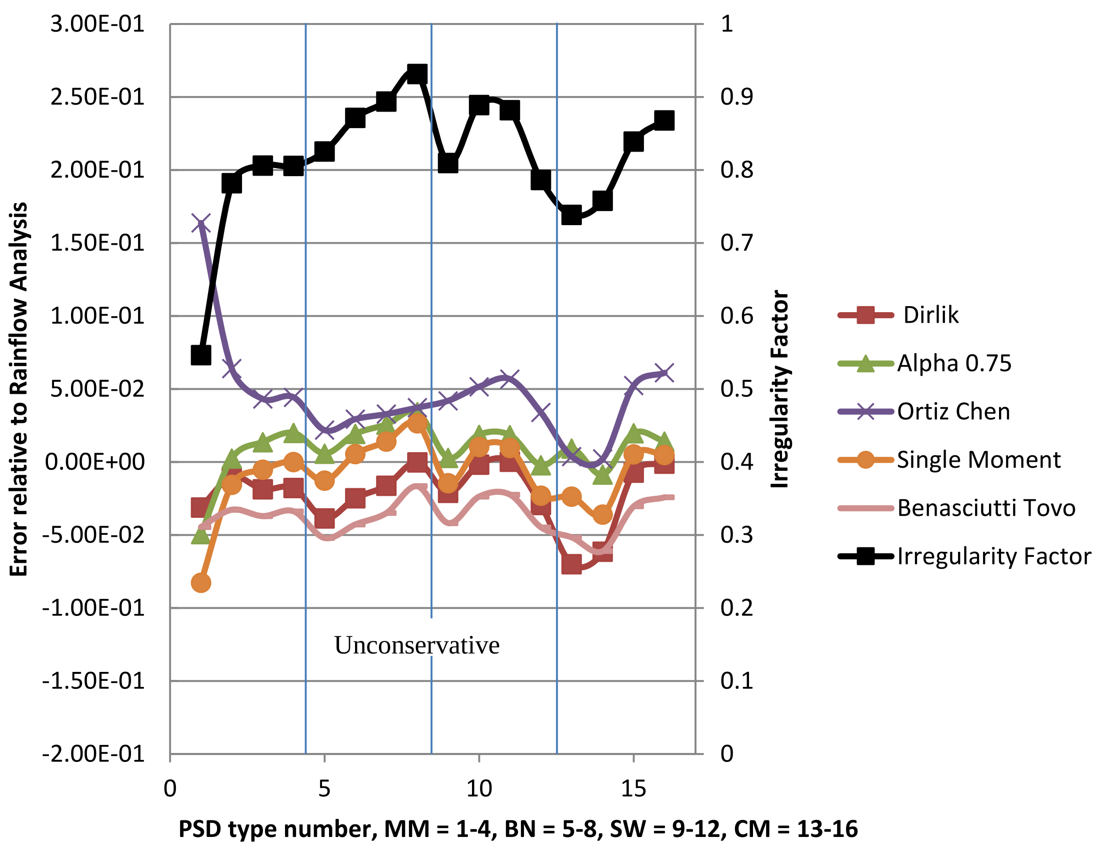

Fig. 2. Relative error of best spectral methods compared to rainflow analysis for m=3.324.

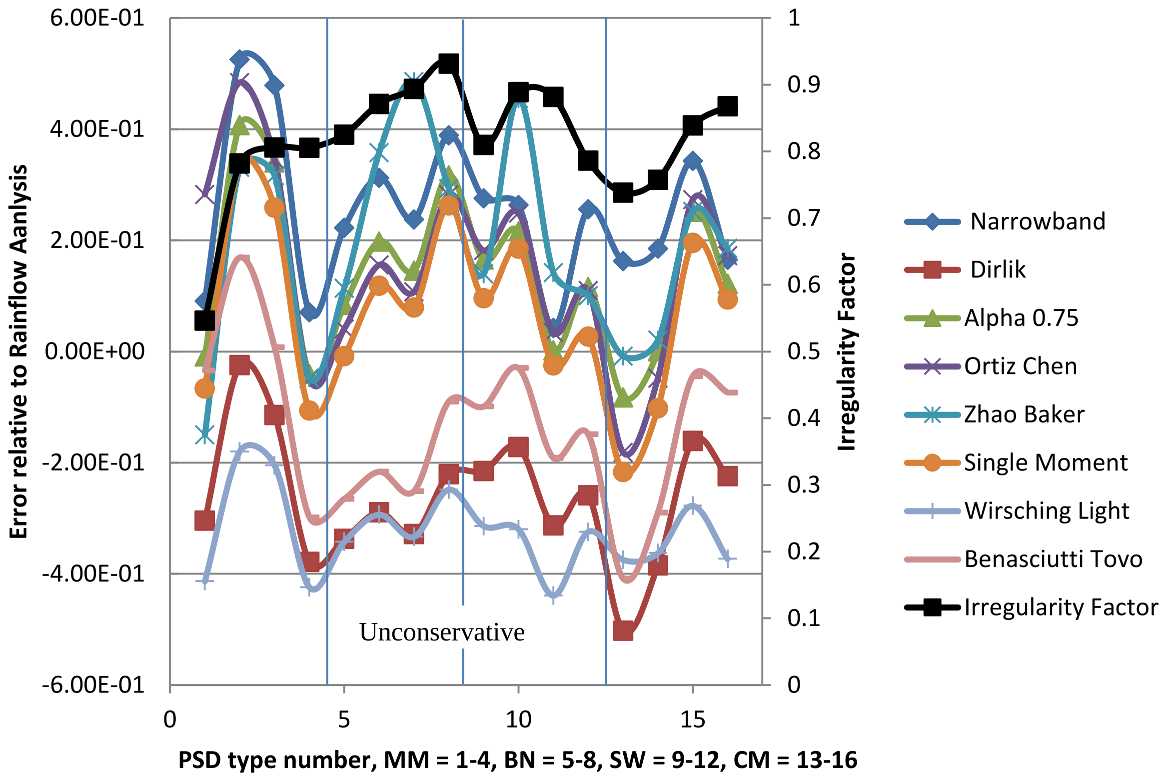

Fig. 3. Relative error of spectral methods compared to rainflow analysis for m=11.76.

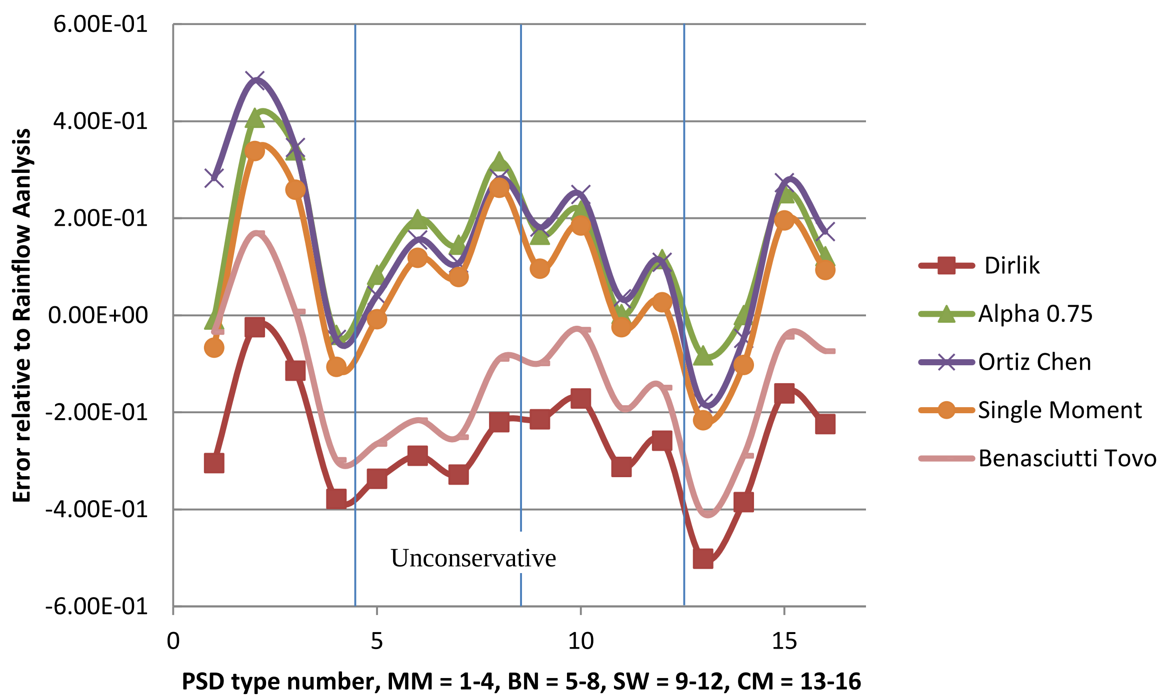

Fig. 4. Relative error of best spectral methods compared to rainflow analysis for m=11.76

generally overestimates the damage rate within less than 6.5% error. In practice for this low value of the fatigue exponent m, any of these five best methods will give acceptable damage or life estimates within generally accepted engineering accuracy for fatigue design problems.

Plots of the relative errors for m = 7.3 are omitted for brevity, but the trends are as follows. The errors in the methods are more exaggerated due to the higher value of the fatigue exponent, with several cases for some methods exceeding 20% absolute error. In general the trends for the better methods are the same as those observed for m = 3.324. Both the Dirlik and Benasciutti Tovo methods significantly underestimate the damage rate for one or two wideband cases (CM-1, CM-2), with the Dirlik method again generally exhibiting less error. The Ortiz Chen method overestimates by more than 20% for the three cases MM-2, MM-2 and MM-3, but again is among the more accurate for the CM-1, CM-2 cases. If an acceptable absolute error in engineering practice for the time to failure is about 20%, then due to the reciprocal relationship an acceptable damage rate error may be within +25% to -17%. If a method is expected to remain in these bounds for all cases studied, then only the Ortiz Chen, $\alpha_{0.75}$ and Single Moment methods might be deemed acceptable for practical use: the Ortiz Chen having an error band of –5% to +24%, the $\alpha_{0.75}$ method exhibiting an error band of –4% to +20%, and the Single Moment method within an error of -10% to +14%.

For m = 11.76, the damage rate errors with respect to rainflow analysis are greatly magnified, as shown in Figure 3, with the absolute worst cases of about 50%. A closer review of the five "best" methods in Figure 4 shows that the Dirlik method is now the worst performing, in almost all cases, exhibiting more than -17% underestimation of the damage rate. The Benasciutti Tovo method is only slightly more accurate but still has most cases also outside this bound. The Ortiz Chen method generally overestimates the damage rate, but only does slightly better than Benasciutti Tovo in absolute accuracy, with 6 of 16 psd cases outside of +25% to -17% error bounds. The $\alpha_{0.75}$ method is better, generally overestimating the damage rate, with only 4 cases having greater than +25% error, no cases with worse than -8% error, and 3 cases essentially exact with the rainflow damage rate, for an overall error range of +41% to -8%. The Single Moment method generally is less conservative than the $\alpha_{0.75}$ method, with the two worst errors of +34% and -22% and the remaining errors in a range of +26% to -10%.

7. Conclusion

For small values of the S-N exponent (m~3) any of the five best methods studied give damage estimates within acceptable engineering accuracy as compared to rainflow analysis. As the value of m becomes larger (m~7) the acceptable spectral methods are reduced to the Ortiz Chen, $\alpha_{0.75}$ and Single Moment methods. For an extreme value of m (~12) all the spectral methods exhibit large errors for all spectra types compared to rainflow, but the $\alpha_{0.75}$ and Single Moment methods may still be acceptable.

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