Green’s Functions for Prediction of Noise From Non-Axisymmetric Jets

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April 11-12, 2012
Cleveland, Ohio

Work Supported by Fundamental Aeronautics Program
Supersonics Project
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Jet Noise Prediction Needs

Next generation aircraft will involve complex exhaust system geometries

- Non-circular exits
- Multiple Streams
- Nearby Solid Surfaces

In order to make noise predictions for Next GEN exhaust systems, need to extend physics-based methods (JeNo) to be able to handle:
  - Non-axisymmetric mean flows
  - Interactions with solid surfaces
Acoustic Analogy Approach

• Many versions:
  – Lighthill (1952), Lilley (1972), ...
• Current work based on Goldstein (2003) formulation.
• Rearrange the Navier-Stokes equations to obtain:
  – A linear wave operator which governs sound propagation (mean flow interaction effects) through a specified base flow.
  – Nonlinear source terms.
• Model source terms.
• Compute propagation effects.
  – Solve linear inhomogeneous equations using a Green’s function.
Acoustic Analogy Approach
Formula for the Acoustic Spectrum

\[ I_\omega (x) = 2\pi \int_V \int_V \Gamma_\lambda_j (x | y; \omega) \Gamma_{kl}^* (x | y + \eta; \omega) e^{-i\omega \tau} R_{\lambda j \kappa l} (y, \eta, \tau) d\eta d\tau dy \]

\( \Gamma_\lambda_j \) Propagator Function (Green's Function) -- Computed

\( R_{\lambda j \kappa l} = \epsilon_{\lambda j, \sigma m} R_{\sigma m \gamma \gamma} \epsilon_{\kappa l, \gamma \gamma} \); \( \epsilon_{\lambda j, \sigma m} \equiv \delta_{\lambda \sigma} \delta_{j m} - \frac{\gamma - 1}{2} \delta_{\lambda j} \delta_{\sigma m} \)

\( R_{\sigma m \gamma \gamma} \) Reynolds Stress Auto-Covariance Tensor -- Modeled
Acoustic Analogy Approach

Adjoint Vector Green’s Function

\[
\frac{\tilde{D}g_{i4}^a}{D\tau} - g_{j4}^a \frac{\partial \tilde{v}_j}{\partial y_i} + c^2 \frac{\partial g_{44}^a}{\partial y_i} + \frac{\gamma - 1}{\bar{\rho}} \frac{\partial \tilde{\theta}_{ij}}{\partial y_j} g_{44}^a + \frac{\partial g_{54}^a}{\partial y_i} = 0
\]

\[
\frac{\tilde{D}g_{44}^a}{D\tau} + \frac{\partial g_{i4}^a}{\partial y_i} - (\gamma - 1) g_{44}^a \frac{\partial \tilde{v}_j}{\partial y_j} = -\delta(x - y)\delta(t - \tau)
\]

\[
\frac{\tilde{D}g_{54}^a}{D\tau} + \frac{1}{\bar{\rho}} \frac{\partial \tilde{\theta}_{ij}}{\partial y_j} g_{i4}^a = 0
\]

Propagator \( \Rightarrow \Gamma_{\lambda j} \equiv \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{i\omega(t-\tau)} \left[ \frac{\partial g_{\lambda 4}^a(y, \tau|x,t)}{\partial y_j} - (\gamma - 1) \frac{\partial \tilde{v}_{\lambda}}{\partial y_j} g_{44}^a(y, \tau|x,t) \right] d(t - \tau)

- For design and concept-evaluation purposes, desire at most ‘overnight’ run times.
- Green’s function computation most time-consuming part of prediction. Need for reduced-order models.
Acoustic Analogy Approach
Weakly Non-Parallel Mean Flow

• Major simplification if mean flow is assumed weakly non-parallel.
  – Locally parallel approximation is leading-order solution.
    • Away from singularity in propagator at supersonic speeds and polar angles near the jet axis.

• Single equation for reduced Green’s function:

\[
\hat{G}_o(y_T | x_T; k, \omega) = i(\omega - U k) \hat{g}^a_4
\]

\[
\frac{\partial}{\partial y_j} \frac{\tilde{c}^2}{(\omega - k U)^2} \frac{\partial \hat{G}_o}{\partial y_j} + \left[ 1 - \frac{k^2 \tilde{c}^2}{(\omega - k U)^2} \right] \hat{G}_o = \frac{\delta(x_T - y_T)}{(2\pi)^2} \quad j = 2, 3
\]

• For observer in far field: \( k = \frac{\omega}{c_\infty} \cos \vartheta \)
**Green’s Functions**

**Reduced-Order Models**

**Conformal Mapping**

- Assume (approximate) levels surfaces of mean velocity and temperature to coincide, $U = U(u)$; $T = T(u)$, and be concentric.

- For certain shapes, can choose $u$, and the corresponding orthogonal curves, $v = \text{constant}$, such that

$$W(y_2 + iy_3) = u(y_2, y_3) + iv(y_2, y_3)$$

maps the plane $y_2 + iy_3$ into a strip in the $(u + iv)$ plane where the Green’s function:
- Is periodic in $v$
- Satisfies the appropriate far-field boundary conditions as $u \to \infty$

- Express Green’s function in terms of a Fourier series.
- Reduce partial differential equation to coupled system of ordinary differential equations for Fourier modes.
Application: Rectangular Jets

- Mean flow in cross-flow planes approximated by concentric ellipses.
- Conformal mapping to cylindrical elliptical coordinates.

\[ y_2 + iy_3 = C \cosh(u + iv), \quad C \text{ is a real constant} \]
Application: Rectangular Jets Noise Prediction Scheme

• Conformal mapping to elliptical coordinates for Green’s function.
• Hybrid (space-time/frequency) source model of Leib and Goldstein (2011).
  – Scalings from low-speed flow surveys (from K. Zaman)
• WIND US RANS solutions as input (from F. Frate)
• Compare with acoustic data (from J. Bridges)
Application: Rectangular Jets
EXTENSIBLE RECTAGULAR NOZZLES

• Aspect ratios two, four and eight baseline rectangular nozzles.

• Equivalent diameter = 2.13 inches.
• NPRs = 1.197, 1.439, 1.856
  – Acoustic Mach numbers at exit = 0.5, 0.7, 0.9
  – Unheated (Cold)
Application: Rectangular Jets
Mean Flow Model – 4:1 Aspect Ratio
Application: Rectangular Jets Noise Predictions

Aspect Ratio Two
Set Point 5:
NPR=1.439
MJ=0.7

θ = 90°
θ = 120°
θ = 150°
Application: Rectangular Jets
Noise Predictions

Aspect Ratio Four
Set Point 5: NPR=1.439, MJ=0.7

\( \phi = 0^\circ \)
\( \phi = 90^\circ \)

\( \theta = 90^\circ \)
\( \theta = 120^\circ \)
\( \theta = 150^\circ \)
Application: Rectangular Jets Noise Predictions

Aspect Ratio Eight
Set Point 5:
NPR=1.439
MJ=0.7

\[ \phi = 0^\circ \]
\[ \phi = 90^\circ \]
Application: Twin Round Jets

- Conformal mapping to Cassinian ovals

\[ u + iv = \ln \left[ \left( y_2 + iy_3 \right)^2 - C^2 \right], \quad C \text{ is a real constant} \]
Application: Twin Round Jets
Test Case for Green’s Function Solver

Analytic Function for Mean Flow

\[ M(u) = \frac{M_0}{2} \left[ 1 - \tanh u \right] \]

\[ \tilde{c}^2(u) \equiv c_\infty^2 \]

\[ u = \ln \left| (y_2 + iy_3)^2 - C^2 \right| \]

\[ C = 1 \quad M_0 = 0.5 \]
Application: Twin Round Jets

Sample Results

$\phi = 90^\circ$

$\phi = 180^\circ$

$\phi = 0^\circ$

$\theta = 120^\circ$

$\phi$

$\omega^{3/2}/|g|$
Application: Twin Round Jets
Green’s Function Directivity

\[ \varphi = 180^\circ \quad \varphi = 0^\circ \]

\[ S_t = 0.625 \quad S_t = 0.75 \]
Green’s Function
Reduced-Order Models
Orthogonal Function Expansion

• Represent mean flow by sum of orthogonal functions.

\[ M(y_2, y_3) = \sum_{m=0}^{l} a_m(\rho) \Psi_m(\varphi) \quad \rho = \rho(y_2, y_3) ; \quad \varphi = \varphi(y_2, y_3) \]

  – For computational efficiency, a relatively small number of functions is desired.

• Expand Green’s function in series of these orthogonal functions

\[ \tilde{g}(y_2, y_3) = \sum_{m=0}^{\infty} g_m(\rho) \Psi_m(\varphi) \]

• Solve system of coupled ordinary differential equations for Green’s function modes:
  – Direct solution of banded system.
  – Iterative solution (Mani).
Green’s Function
Reduced-Order Models
Orthogonal Function Expansion

Example: Polar coordinates and Fourier series expansion

\[-i(\omega - Uk)\hat{g}_1^{a} + ike^2 \hat{g}_4^{a} = 0\]

\[-i(\omega - Uk)\hat{g}_r^{a} + \frac{\partial U}{\partial r} \hat{g}_1^{a} - e^2 \frac{\partial \hat{g}_4^{a}}{\partial r} = 0,\]

\[-i(\omega - Uk)\hat{g}_\phi^{a} + \frac{1}{r} \frac{\partial U}{\partial \phi} \hat{g}_1^{a} - e^2 \frac{1}{r} \frac{\partial \hat{g}_4^{a}}{\partial \phi} = 0,\]

\[-i(\omega - Uk)\hat{g}_4^{a} - \frac{1}{r} \frac{\partial \hat{g}_r^{a}}{\partial r} + \frac{1}{r} \frac{\partial \hat{g}_\phi^{a}}{\partial \phi} - \frac{1}{r} \hat{g}_r^{a} \right] + ike^{a} = \frac{1}{(2\pi)^2} \delta(r - r_0, \phi - \phi_0)\]

Solution of the form:

\[U(r, \phi) = \sum_{n=-l}^{l} U_n(r) e^{in\phi}\]

\[\hat{g}_4^{a}(r,\phi) = \sum_{n=-\infty}^{\infty} G_{\sigma 4}^{(n)}(r) e^{in\phi}\]
Application: Offset Stream

\[ M(r, \phi) = M_C e^{-(1+\alpha - 2\alpha \cos \phi)r^2} \]

Fourier Expansion of Mean Flow

\[ M(r, \phi) = \sum_{n=-\infty}^{\infty} M_n(r) e^{in\phi} \]

\[ M_n = M_C e^{-(1+\alpha)r^2} I_n(2\alpha r^2) \]
Application: Fluid Shield

\[ M(r, \varphi) = M_C \left[ e^{-ar^2} + br^2 e^{-c(r-1)^4} g(\varphi) \right] \]

\[ g(\varphi) = \begin{cases} 
0 & , \ 0 \leq \varphi < \pi \\
-\sin \varphi & , \ \pi \leq \varphi < 2\pi 
\end{cases} \]

Fourier Expansion of Mean Flow

\[ M(r, \varphi) = \sum_{n=-\infty}^{\infty} M_n(r) e^{in\varphi} \]

\[ M_n = M_C \left[ e^{-ar^2} \delta_{n,0} + br^2 e^{-c(r-1)^4} \left( \frac{1}{2\pi(1-n^2)} \right) \left[ (-1)^n + 1 \right] \right] \ , \ n \neq \pm 1 \]

\[ M_{\pm 1} = \pm M_C \frac{i}{4} br^2 e^{-c(r-1)^4} \]
Green’s Function
Numerical Methods

• Develop code for numerical solution of the acoustic analogy equations Green’s function.
  – Collaboration with John Goodrich, GRC

• Use for:
  – Validation of reduced-order models.
  – High-resolution calculations for cases of special interest.
  – Study effects of non-parallel mean flow.
Green’s Function
Numerical Methods

• Time-domain, finite-difference method.
• Computational domain surrounded by damping layers.
• High-order boundary conditions (related to Giles and Thompson) on outer boundary of damping layers.

• Status:
  – Current code is written for the Linearized Euler Equations.
  – Have validated a two-dimensional version of code with analytical solution for a uniform flow.

• Plans:
  – Extend to three dimensions.
  – Extend to non-uniform mean flow.
  – Adapt for solution of acoustic analogy equations.
Summary

• To support development of noise-reduction concepts:
  – Develop reduced-order models for Green’s function of acoustic analogy equations.
    • Rectangular Jets
    • Twin Round Jets
    • Offset Jets
    • FLADE
  – Integrate with source model for noise prediction.
    • Rectangular Jets
  – Develop code for numerical solution of Green’s function of acoustic analogy equations.