The Multiscale Generalized Method of Cells and its Utility in Predicting the Deformation and Failure of Woven CMCs

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Outline

- Integrated multiscale Micromechanics Analysis Code (ImMAC)
- Multiscale Generalized Method of Cells (MSGMC)
- Modeling of Woven Fabrics (Plain & 5HS)
- Results
  - Tensile (Deterministic, Stochastic)
    - Load and Unloading
  - Creep
- Concluding Remarks

Presentation Objective:

Apply a synergistic multiscale modelling technique to woven composites to determine underlying reasons for nonlinear response

- Understand influence (i.e., primary, secondary, etc.) of architectural parameters (e.g., fiber/void volume fraction, weave geometry, tow geometry, void geometry) at multiple length scales on the mechanical response of CMCs.
- Analyze the significance of effects and compare to material scatter
Goal is to Balance Efficiency vs Fidelity

Hierarchical (One-Way) Multiscale

Model Fidelity vs Model Efficiency

Analytical
Semi-Analytical
Numerical

FEA, MD

R&T

Goal

Engineering

Science

Concurrent Multiscale
NASA’s Integrated multiscale Micromechanics Analysis Code (ImMAC) Suite

**Micro Scale**
- Fiber/Inclusion
- Interphase
- Matrix

**Meso Scale**
- Tow
- Ply
- Woven/Braided
  - RUC
- Laminate
- Stiffened Panel

**Global Scale**
- Homogenized Material Element
- Structure

**Stand-alone MAC/GMC**
- Multiscale CLT
- Multiscale GMC

**HyperMAC** (Implemented within HyperSizer)

**FEAMAC** (Implemented within Abaqus)
MAC/GMC is Evolving Anisotropic Thermoelastic Inelastic and Damage Constitutive Model

Micro-level Field Equations (subcell)

\[
\varepsilon^{(\alpha \beta \gamma)} = \mathbf{A}^{(\alpha \beta \gamma)} \varepsilon + \mathbf{D}^{(\alpha \beta \gamma)} (\varepsilon^{I} + \varepsilon^{T})
\]

\[
\sigma^{(\alpha \beta \gamma)} = \mathbf{C}^{(\alpha \beta \gamma)} \left[ \mathbf{A}^{(\alpha \beta \gamma)} \varepsilon + \mathbf{D}^{(\alpha \beta \gamma)} (\varepsilon^{I} + \varepsilon^{T}) - \left( \varepsilon^{I(\alpha \beta \gamma)} + \varepsilon^{T(\alpha \beta \gamma)} \right) \right]
\]

Macro-level Constitutive Equations

\[
\bar{\sigma} = B^* (\bar{\varepsilon} - \bar{\varepsilon}^{I} - \bar{\varepsilon}^{T})
\]

\[
B^* = \frac{1}{dhl} \sum_{\alpha=1}^{N_\alpha} \sum_{\beta=1}^{N_\beta} \sum_{\gamma=1}^{N_\gamma} d_{\alpha \beta \gamma} h_{\alpha \beta \gamma} \mathbf{C}^{(\alpha \beta \gamma)} (\mathbf{A}^{(\alpha \beta \gamma)})
\]

\[
\bar{\varepsilon}^{I} = \frac{-[B^*]^{-1}}{dhl} \sum_{\alpha=1}^{N_\alpha} \sum_{\beta=1}^{N_\beta} \sum_{\gamma=1}^{N_\gamma} d_{\alpha \beta \gamma} h_{\alpha \beta \gamma} \mathbf{C}^{(\alpha \beta \gamma)} (\mathbf{D}^{(\alpha \beta \gamma)} \varepsilon^{I} - \varepsilon^{T(\alpha \beta \gamma)})
\]

\[
\bar{\varepsilon}^{T} = \frac{-[B^*]^{-1}}{dhl} \sum_{\alpha=1}^{N_\alpha} \sum_{\beta=1}^{N_\beta} \sum_{\gamma=1}^{N_\gamma} d_{\alpha \beta \gamma} h_{\alpha \beta \gamma} \mathbf{C}^{(\alpha \beta \gamma)} (\mathbf{D}^{(\alpha \beta \gamma)} \varepsilon^{T} - \varepsilon^{I(\alpha \beta \gamma)})
\]
Fidelity vs. Efficiency in Composite Micromechanics
Comparison of Local Stress Invariants
Transverse Loading; 50% Glass/Epoxy

Time = 1
Time ≈ 1×10^{-4}
Time ≈ 1×10^{-1}

von Mises stress \((J_2)\)
Pressure
\((= -\sigma_{\text{mean}})\)
(MPa)

~11,000 GPS Elements
676 Subcells
1024 Subcells

Simpler Methods
Mean Field

FEA
GMC
HFGMC
Individual Stress Components

Axial stress (MPa)

Transverse stress in loading direction (MPa)

Transverse stress (MPa) normal to loading direction

Transverse shear stress (MPa)
Failure Criterion for Strength and Durability

**Subcell Elimination Criterion**
- Max. Stress Theory
- Max. Strain Theory
- Tsai-Hill Theory
- Tsai-Wu
- SIFT
- Elastic Allowables

**Progressive Damage Criterion**
- Scalar Damage (Triaxial)
- MMCDM
- Smeared Crack Band

Experiment
- Max Stress - Initial
- Max Strain - Initial
- Tsai-Hill - Initial
- Tsai-Wu (Hahn) - Initial
- Max Stress - Final
- Max Strain - Final
- Tsai-Hill - Final
- Tsai-Wu (Hahn) - Final

(0°/±45°/90°) laminate
AS4/3501-6
**Integrated Multiscale Analysis of Arbitrary Composite Structures with FEAMAC**

**Synergistic Multiscale Modeling**
- Embed micromechanics within FEA at element integration points
- New tool for micro/macro analysis of composite structures: **FEAMAC**
- Localize/homogenize on the fly

Structure-Scale FEA

Element/Integration Point

MAC/GMC micromechanics analysis
Utilize Novel Multiscale Generalized Method of Cells (MSGMC) For Concurrent Analysis of Woven/Braided Composite Systems
Problem Definition

<table>
<thead>
<tr>
<th>Tow Properties</th>
</tr>
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<tbody>
<tr>
<td><strong>Type</strong></td>
</tr>
<tr>
<td><strong>Overall Fiber Volume Fraction</strong></td>
</tr>
<tr>
<td><strong>Tow Volume Fraction</strong></td>
</tr>
<tr>
<td><strong>Tow Width</strong></td>
</tr>
<tr>
<td><strong>Tow Spacing</strong></td>
</tr>
<tr>
<td><strong>Thickness</strong></td>
</tr>
<tr>
<td><strong>Matrix</strong></td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Tow Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Fiber Vol Fraction within Tow</strong></td>
</tr>
<tr>
<td><strong>Tow Packing Structure</strong></td>
</tr>
<tr>
<td><strong>Fiber</strong></td>
</tr>
<tr>
<td><strong>Matrix</strong></td>
</tr>
<tr>
<td><strong>Interface</strong></td>
</tr>
</tbody>
</table>

Current Multiscale Analysis Involves 4 Scales And 3 Homogenizations/Localizations
Multiscale Generalized Method of Cells (MSGMC) Overview

- Newly developed recursive GMC methodology
  - Each length scale in each subcell can call a separate GMC analysis
- Works for any arbitrary multiphase material
  - Elastic / Inelastic / Damage

Multiscale analysis can determine local stresses at different length scales

\[
\sigma^{\{\alpha \beta \gamma\}\{\beta \gamma\}} = C^{\{\alpha \beta \gamma\}\{\beta \gamma\}} A^{\{\alpha \beta \gamma\}\{\beta \gamma\}} A_{tt}^{\{\alpha \beta \gamma\}} A_{ip}^{\{\beta \gamma\}} \Delta \varepsilon
\]
Macroscale (Weave) Two Step Homogenization

To compensate for lack of normal-shear coupling within GMC a two-step homogenization scheme is employed for woven composites. (Bednarcyk & Arnold, *IJSS*, 41, 2003)
Constituent Constitutive Model and Strain Localization

Microscale

Assume Fiber and Interface Linear Elastic Hashin Fiber Failure Criteria (1980)

\[ f = \frac{\sigma_{11}^2}{\sigma_{\text{axial}}} + \frac{1}{2} \left( \frac{\tau_{13}^2 + \tau_{12}^2}{\tau_{\text{axial}}} \right) \]

Matrix damage driven by magnitude of triaxiality

If \( \sigma_H > \sigma_{\text{critical}} \)

\[ f = 3\varepsilon_H n K - \sigma_H = 0 \]

\[ 1 - \phi^{i+1} = \lambda^{i+1} = \frac{n \Delta\varepsilon_H^{i+1} + \lambda^i \varepsilon_H^{i+1}}{\left( \Delta\varepsilon_H^{i+1} + \varepsilon_H^{i+1} \right)} \]

\[ \left( \varepsilon^{i+1} = \varepsilon^i + \Delta\varepsilon^{i+1} \right) \]

\( K^0 = \) initial bulk modulus
Full Multiscale Modeling of 5HS Weave with Porosities

5HS and most other orthogonal weaves can be discretized into 8 unique subcell groups. Furthermore model tow and matrix with voids using lower scale RUCs.
Three Void Modeling Schemes Considered

Voided Matrix Response Achieved via Separate GMC Analysis

No Voids
Evenly Distributed Voids
Localized Voids

Gold = 12.7% voids ; Red = 90% voids; Blue = 5% voids

Load direction

- Localization of porosity significantly influences failure response
  a) Knee – 33% delta
  b) Strain to Failure – 25-55% delta
- Assuming uniform distribution of voids similar to no voids

Fiber

<table>
<thead>
<tr>
<th>Name</th>
<th>iBN-Sylramic</th>
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<tbody>
<tr>
<td>Modulus</td>
<td>400 GPa</td>
</tr>
<tr>
<td>Poisson's Ratio</td>
<td>0.2</td>
</tr>
<tr>
<td>Axial Strength</td>
<td>2.2 GPa</td>
</tr>
<tr>
<td>Shear Strength</td>
<td>900 MPa</td>
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</table>

Interface

<table>
<thead>
<tr>
<th>Name</th>
<th>Boron Nitride</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulus</td>
<td>22 GPa</td>
</tr>
<tr>
<td>Poisson's Ratio</td>
<td>0.2</td>
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Matrix

<table>
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<tr>
<th>Name</th>
<th>CVI-SiC</th>
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</thead>
<tbody>
<tr>
<td>Modulus</td>
<td>420 GPa</td>
</tr>
<tr>
<td>Poisson's Ratio</td>
<td>0.2</td>
</tr>
<tr>
<td>$\sigma_{\text{dam}}$</td>
<td>180 MPa</td>
</tr>
<tr>
<td>$n$</td>
<td>0.04</td>
</tr>
</tbody>
</table>
Simulation Identifies Local Damage Events / Mechanisms
Explaining Nonlinearities in Macro Stress Strain Curve
Assuming 5HS RUC with Localized Porosities

Fiber: Elastic, Hashin Fiber Failure Criteria (includes shear stress)
Interface: Elastic (very compliant 1/20th)
Matrix: Elastic, Hydrostatic-Driven Damage

G.N. Morscher, Comp. Sci. Tech., 2004, 64, 1311-1319
Study Effects Of Micro, Meso, And Macro Parameters on Macroscale Response

<table>
<thead>
<tr>
<th>Architectural Parameter</th>
<th>Relevant Length Scale</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tow Fiber Volume Fraction</td>
<td>Meso</td>
<td>0.46, 0.48, 0.50</td>
</tr>
<tr>
<td>Tow Void Volume Fraction</td>
<td>Meso</td>
<td>0.01, 0.05, 0.07</td>
</tr>
<tr>
<td>Tow Aspect Ratio</td>
<td>Macro</td>
<td>8, 10, 12</td>
</tr>
</tbody>
</table>
Influence of Varying Matrix Material Parameters on the Macroscale Response

$\sigma_{\text{crit}}$: UTS $\uparrow$ 8%; $e_f$ $\downarrow$ 24%; 1$^{\text{st}}$ matrix cracking $\uparrow$ 94%

$E$: Initial Modulus $\uparrow$ 10%; UTS $\uparrow$ 2%; $e_f$ $\downarrow$ 10%; 1$^{\text{st}}$ matrix cracking $\uparrow$ 10%

$n$: UTS $\uparrow$ 10%; $e_f$ $\downarrow$ 12%; post $E$ $\uparrow$ 120%

Baseline

- Variation of $n$ $\uparrow$ 200%
- Variation of Critical Stress $\uparrow$ 100%
- Variation of Modulus $\uparrow$ 50%
Depicts Entire Range Of Macro Response Curves Given the 27 Variations In Architectural Parameters

Utilized Localized Void Model

Architectural Variations clearly contribute to variation in measured material response.

- Initial Modulus \( \approx 24\% \)
- UTS \( \approx 2\% \)
- 1\text{st} matrix cracking \( \approx 16\% \)
- Post matrix cracking Modulus \( \approx 24\% \)
- \( \varepsilon_f \) impacted \( \approx 16\% \)
Assumed Normal Distributions for Architectural Parameters

Normal Distribution Probability Plot*

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Mean</th>
<th>Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tow Fiber Volume Fraction</td>
<td>0.48</td>
<td>0.033</td>
</tr>
<tr>
<td>Tow Aspect Ratio</td>
<td>8</td>
<td>.533</td>
</tr>
<tr>
<td>Tow Void Volume Fraction</td>
<td>0.05</td>
<td>0.01</td>
</tr>
<tr>
<td>Localized Weave Void Volume Fraction</td>
<td>0.75</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Note: Material Properties held fixed at Baseline Values; Void shape – sheet like

Procedure for Incorporating Stochastics Requires Significant Computation Resources

<table>
<thead>
<tr>
<th>Weave Type</th>
<th>Time / Increment (sec)</th>
<th>Typical Increments</th>
<th>Total Time (sec)</th>
<th>No. of Subcells</th>
</tr>
</thead>
<tbody>
<tr>
<td>5HS (1x1)</td>
<td>12</td>
<td>200</td>
<td>4000 (1.1hrs)</td>
<td>93,800</td>
</tr>
<tr>
<td>PW (1x1)</td>
<td>1.5</td>
<td>150</td>
<td>225</td>
<td>18,840</td>
</tr>
<tr>
<td>PW (6x6)</td>
<td>53</td>
<td>200</td>
<td>10600 (2.9 hrs)</td>
<td>678,276</td>
</tr>
</tbody>
</table>
Macro Stress-Strain Response Curves Given Stochastic Assumption of Architectural Parameters

Utilized Localized Void Model

Statistical Reconstruction 95% Confidence*

Lower Strength $\rightarrow$ Higher Weave Void Volume Fraction

* determined from normality assumption using bilinear approximation

27 cases evaluated
Secant Through Thickness Moduli (E_{zz}) Degrades With Loading As Does In-plane (E_{11})

Note: In composites many material “properties” evolve with loading history!

Normal Probability Plot of E_{zz} → Has some skewness towards right…maybe log normal?

E_{zz} = 59.02 \pm 12.5 ; 68.2\% confidence
E_{zz} = 59.02 \pm 25 ; 95.4\% confidence
Loading Histories with Unloading Are Critical For Deducing Mechanisms Driving Nonlinear Response

Morscher, G.: 2008
Experimental Unloading Response Returns to Zero – indicating nonlinearity due to damage

MSGMC Simulation, \( V_f = 36\% \)

Morscher, G.: 2008
Experimental Unloading Response Returns to Zero – indicating nonlinearity due to damage

\( V_f = 40\% \)
Examine Plain Weave Discretization to Study Architectural Parameters on Structural Scale

Subcell group properties determined from lower length scales

Tow Aspect Ratio = width/thickness
Macroscale – Plain Weave Discretization
Assumes Normal Distribution for all Architectural Parameters

58 Cases Evaluated

PW slightly less stiff and more nonlinear than 5HS

* determined from normality assumption using bilinear approximation

Macroscale (1x1)

Statistical Reconstruction 95% Confidence*

5HS

PW

* determined from normality assumption using bilinear approximation
Sensitivity To Architectural Features Changes With Increasing Structural Scale: Plain Weave

Structural Scale (2x2) RUC
2x2 RUC
10 cases

Structural Scale (3x3) RUC
3x3 RUC
10 cases

Structural Scale (4x4) RUC
4x4 RUC
8 cases

Structural Scale (5x5) RUC
5x5 RUC
6 cases
Sensitivity To Architectural Features Changes With Increasing Structural Scale: Plain Weave

**Graphs:**
- **Structural Scale (6x6):**
  - 6x6 RUC
  - 30 cases

- **Statistical Reconstruction 95% Confidence:**

**Table:**

<table>
<thead>
<tr>
<th>Property</th>
<th>E11 (GPa)</th>
<th>PLS</th>
<th>H (GPa)</th>
<th>σ_{UTS}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>211</td>
<td>111</td>
<td>73.7</td>
<td>460</td>
</tr>
<tr>
<td>± σ</td>
<td>9.5</td>
<td>10</td>
<td>6.5</td>
<td>42.5</td>
</tr>
<tr>
<td>± 2σ</td>
<td>19</td>
<td>20</td>
<td>13</td>
<td>85</td>
</tr>
</tbody>
</table>
Comparison of Reconstructed 95% (2σ) Confidence Plain Weave Stress-Strain Response
Blue = 1x1, Yellow = 6x6

- Composite Stiffness, PLS (first matrix cracking), Secondary Modulus, statistically unaffected by increasing size of RUC
- Failure stress/strain is the only value that we can say with 95% confidence is influenced by architectural details

<table>
<thead>
<tr>
<th>Property</th>
<th>E11 (GPa) 1x1</th>
<th>E11 (GPa) 6x6</th>
<th>PLS 1x1</th>
<th>PLS 6x6</th>
<th>H (GPa) 1x1</th>
<th>H (GPa) 6x6</th>
<th>σ_{UTS} 1x1</th>
<th>σ_{UTS} 6x6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>209.5</td>
<td>211</td>
<td>116</td>
<td>111</td>
<td>74.5</td>
<td>73.7</td>
<td>512.5</td>
<td>460</td>
</tr>
<tr>
<td>± σ</td>
<td>13</td>
<td>9.5</td>
<td>10</td>
<td>10</td>
<td>6.5</td>
<td>6.5</td>
<td>15</td>
<td>42.5</td>
</tr>
<tr>
<td>± 2σ</td>
<td>26</td>
<td>19</td>
<td>20</td>
<td>20</td>
<td>13</td>
<td>13</td>
<td>30</td>
<td>85</td>
</tr>
</tbody>
</table>
Conclusion

1. Demonstrated that a synergistic analysis using the multiscale generalized method of cells (MSGMC) can accurately represent woven CMC tensile behavior (loading/unloading)
   - 4 level of scales analyzed
   - Nonlinear behavior due to damage – demonstrated by unloading
   - Critical invariant is $I_1$ (brittle) not $J_2$ (metals)
   - Failure mechanisms capture via local continuum damage model

2. Non-uniform distribution of voids/porosities must be incorporate within the RUC - accurate deformation and failure response

3. Variations in Weave Parameters (micro, meso, and macro) appear to contribute to variation in measured material macrolevel response.
   a) Primary Variables appear to be
      - Constituent material constants (micro)
      - Spatial distribution of void locations (meso); shape is sheet like
   b) Secondary Variables appear to be
      i. Tow void content (meso)
      ii. Tow Aspect Ratio (meso)
      iii. Tow volume fraction (macro)

4. Assuming Normal Probability Distributions → showed that only the ultimate failure stress/strain (statistically speaking) is influenced at the structural level by lower scale features.
Future Work

1. Examine the influence of these parameters on the time-dependent material response and corresponding life.
2. Incorporation of constituent property distribution in the analysis
3. Incorporate environmental degradation (due to oxidation / moisture)
4. Multivariate statistics and stochastic processes for coupled architectural/material parameters
5. Incorporate MSGMC into ImMAC 5.0

Acknowledgement

S.M. Arnold - work supported by the Supersonics Project within the Fundamental Aeronautics Program
K.C. Liu - Partially funded under NASA GSRP.
THANK YOU

QUESTIONS

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NASA Multiscale Analysis Center of Excellence (MACE) Established in 2010 at GRC
Average Values of Four Key Composite Response Attributes: E, PLS, H and $\sigma_{UTS}$

**Avg. Modulus**

<table>
<thead>
<tr>
<th>RUCs</th>
<th>Avg. Modulus (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>205</td>
</tr>
<tr>
<td>2</td>
<td>210</td>
</tr>
<tr>
<td>3</td>
<td>215</td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>

**Avg. Second Modulus**

<table>
<thead>
<tr>
<th>RUCs</th>
<th>Avg. Modulus (GPa)</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>440</td>
</tr>
<tr>
<td>2</td>
<td>460</td>
</tr>
<tr>
<td>3</td>
<td>480</td>
</tr>
<tr>
<td>4</td>
<td></td>
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<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>

**Avg. First Matrix Cracking Stress**

<table>
<thead>
<tr>
<th>RUCs</th>
<th>Avg. FMC Stress (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>105</td>
</tr>
<tr>
<td>2</td>
<td>110</td>
</tr>
<tr>
<td>3</td>
<td>115</td>
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<td>4</td>
<td></td>
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<tr>
<td>5</td>
<td></td>
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<tr>
<td>6</td>
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</table>

**Avg. Fail Stress**

<table>
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<th>Avg. Fail Stress (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
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<tr>
<td>2</td>
<td>71</td>
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<td>3</td>
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<tr>
<td>5</td>
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<tr>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>
Average Values of Four Key Composite Response Attributes: $E$, PLS, $H$ and $\sigma_{UTS}$

Remember 5x5 has lowest DoF