Rudder/Speed Brake Actuator
Probabilistic Fatigue Life and Reliability Analysis -
A Case Study

Fred B. Oswald
NASA Glenn Research Center

Michael Savage
Univ. of Akron

Erwin V. Zaretsky
NASA Glenn Research Center

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Rudder/Speed Brake Actuator

Four actuators, each with two gear trains
Rudder/Speed Brake Actuator

- Overall Reduction 474 : 1
- Two Stage Spur Gear Input 19.75 : 1
- Differential Planetary 24 : 1

Actuator has 2 gear trains — each with 17 gears & 10 bearings
Background

• Space Shuttle designed for 100 missions in 10 years

• First Shuttle “Columbia” entered service Apr. 1981

• Actuators from Columbia were inspected during vehicle modification after sixth flight in 1984.

• No other actuator inspections or maintenance was planned during shuttle program
Background

Post-Shuttle Columbia crash Feb 2003 . . . .

- “Flight leader” Discovery had 30 flights in 19 years
- Corrosion & cracks were found on Discovery body flap actuator shaft spline
- Wear, damage & red-colored grease were found in rudder/speed brake actuator
Issues

1. What are the risks from fatigue, wear, lubrication breakdown for 20 or more flights per actuator?

2. What is an acceptable risk for single actuator and four actuators as a system?
Method of Analysis

1. Load analysis & cycle counts on bearings & gears

2. Probabilistic analysis for life & reliability using Lundberg-Palmgren theory applied to bearings & gears

3. Component life for operating profile in each actuator based on linear damage rule (Palmgren-Langer-Miner rule)

4. System life for single actuator & four actuators based on strict series reliability and Weibull theory
Analysis Tasks

- Relate Shuttle mission spectrum to cycles of actuator components
- Compute gear surface Hertzian fatigue life
- Compute gear tooth bending fatigue life
- Life analysis for gearbox bearings (supplied by actuator manufacturer)
- Combine lives to calculate life of actuator system(s)
Weibull Distribution Function

Linear Coordinates

Weibull Coordinates

\[ \ln \ln \left( \frac{1}{S} \right) = m \ln \left( \frac{L}{L_\beta} \right) \]
2-parameter Weibull Distribution Function

\[ \ln \ln \left( \frac{1}{S} \right) = m \ln \left( \frac{L}{L_{\beta}} \right) \]

where:

- \( S \) = probability of survival or reliability
- \( m \) = Weibull slope (shape factor)
- \( L \) = life @ reliability \( S \)
- \( L_{\beta} \) = Characteristic life (life @ 63.2% failure rate)
Lundberg-Palmgren Theory
System Life for Multiple Components

Strict Series Reliability

\[ S_S = S_1 \times S_2 \times S_3 \times ... S_n \]

where all survival probabilities are at the same time increment

Lundberg-Palmgren Theory (1947)
based on Weibull Distribution Function (1939)

Gives:

\[ \frac{1}{L_S^m} = \left( \frac{1}{L_1} \right)^{m_1} + \left( \frac{1}{L_2} \right)^{m_2} + \left( \frac{1}{L_3} \right)^{m_3} + ... + \left( \frac{1}{L_n} \right)^{m_n} \]

where all lives are at the same probability of survival
Linear Damage Rule

- A. Palmgren — 1924
- B. Langer — 1937
- M. Miner — 1945

\[
\frac{1}{L} = \frac{X_1}{L_1} + \frac{X_2}{L_2} + \frac{X_3}{L_3} + \ldots + \frac{X_n}{L_n}
\]

\[
X_1 + X_2 + X_3 + \ldots + X_n = 1
\]

\( L \sim \text{Life} \)

\( X \sim \text{time fraction} \)
System $L_{10}$ Life

$$L_{10} = \left[ \frac{\text{System Dynamic Capacity}}{\text{Equivalent Output Torque}} \right]^p$$

$L_{10} = \text{Output cycles @ 90\% probability of survival}$

From Linear Damage Rule:

Equivalent Output Torque =

$$T = \left[ \frac{X_1 T_1^p + X_2 T_2^p + X_3 T_3^p + \ldots + X_n T_n^p}{X_1 + X_2 + X_3 + \ldots + X_n} \right]^{1/p}$$

$X_n = \text{time @ condition } n$

$T_n = \text{torque @ condition } n$
Gear Tooth Surface Fatigue Life

\[ L_{10tc} = C_{10} W_N^{-4.3} F_e^{3.9} \rho^{-5.0} \rho^{-0.4} \]

where:

- \( L_{10tc} \): Contact fatigue 10% life for a single tooth
- \( C_{10} \): Gear tooth dynamic load capacity
- \( W_N \): Normal load on tooth
- \( F_e \): Effective tooth width
- \( \rho \): Curvature sum
- \( \ell \): Length of loaded tooth profile
Gear Surface Fatigue Life

\[ L_{10tc} = C_{10} W^{-4.3} F_e^{3.9} \rho^{-5.0} n^{-0.4} \]

\[ L_{10gc} = L_{10tc} n^{-1/m} \]

where:

- \( L_{10tc} \) = Contact fatigue 10% life for a single tooth
- \( L_{10gc} \) = Surface fatigue 10% life for gear
- \( n \) = Number of teeth
- \( m \) = Weibull slope = 2.5
Gear Surface Fatigue Life

Lowest-life gear: 16 tooth input pinion
Max Hertz stress 1818 MPa (263.7 ksi)
Pinion $L_{10} = 1.807 \times 10^{12}$ cycles
based on weighted output torque 29.8 Nm (263.7 in-lb)

Gear system life (contact):

$$L_{10gc} = L_{10tc} n^{-1/m}$$

$$\frac{1}{L_{Sc}^m} = \left(\frac{1}{L_{1c}}\right)^{m_1} + \left(\frac{1}{L_{2c}}\right)^{m_2} + \left(\frac{1}{L_{3c}}\right)^{m_3} + \ldots + \left(\frac{1}{L_{nc}}\right)^{m_n}$$
Gear Tooth Bending Fatigue Life

AGMA Bending Stress Number

\[ s_t = W_t K_o K_v K_s \frac{P_d K_m K_B}{F J} \]

where:

- \( s_t \) = Bending stress number, lb/in\(^2\)
- \( W_t \) = Tangential load, lb
- \( K_o \) = Overload factor
- \( K_v \) = Dynamic factor
- \( K_s \) = Size factor
- \( P_d \) = Diametral pitch, in\(^{-1}\)
- \( F \) = Face width, in
- \( K_m \) = Load distribution factor
- \( K_B \) = Rim thickness factor
- \( J \) = Geometry factor
Gear Tooth Bending Fatigue Life

\[ L_{10gb} = \left( \frac{S}{S_t} \right)^p \]

where:

- \( L_{10gb} \) = Bending stress life for a single tooth
- \( S \) = Gear tooth strength (database)
- \( s_t \) = Bending stress
- \( p \) = Stress-life exponent
Gear Tooth Bending Fatigue Life

Lowest-life gear for tooth bending 16 t input pinion

Bending stress 1070 MPa (155 ksi),

$L_{10b}$ life = $13 \times 10^{12}$ cycles

Gear system life (bending):

$$L_{10gb} = \left( \frac{S}{S_t} \right)^p$$

$$\frac{1}{L_{mb}} = \left( \frac{1}{L_{1b}} \right)^{m_1} + \left( \frac{1}{L_{2b}} \right)^{m_2} + \left( \frac{1}{L_{3b}} \right)^{m_3} + ... + \left( \frac{1}{L_{nb}} \right)^{m_n}$$
# Gear Tooth Load Cycles (for 1 tooth @ Output)

<table>
<thead>
<tr>
<th>Gear</th>
<th>No Teeth</th>
<th>Teeth – load cycles</th>
<th>Mesh Cycles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output Ring</td>
<td>81</td>
<td>9 – 1</td>
<td>9</td>
</tr>
<tr>
<td>Output Planet</td>
<td>18</td>
<td>9 – 1</td>
<td>9</td>
</tr>
<tr>
<td>Fixed Ring</td>
<td>90</td>
<td>9 – 1</td>
<td>9</td>
</tr>
<tr>
<td>Fixed Planets</td>
<td>18</td>
<td>9 – 1</td>
<td>9</td>
</tr>
<tr>
<td>Sun</td>
<td>54</td>
<td>36-1</td>
<td>36</td>
</tr>
<tr>
<td>Sun-Planets</td>
<td>18</td>
<td>36-1</td>
<td>36</td>
</tr>
<tr>
<td>Output Spur</td>
<td>87</td>
<td>26-1</td>
<td>26</td>
</tr>
<tr>
<td>Intermed. Output</td>
<td>19</td>
<td>12-1 + 7-2</td>
<td>26</td>
</tr>
<tr>
<td>Intermed. Input</td>
<td>69</td>
<td>44-1 + 25-2</td>
<td>94</td>
</tr>
<tr>
<td>Input Spur</td>
<td>16</td>
<td>2-5 + 14-6</td>
<td>94</td>
</tr>
</tbody>
</table>
## Load Spectrum (for 100 flights)

<table>
<thead>
<tr>
<th>Event</th>
<th>Moment N-m (in-lb)</th>
<th>Duration (Minutes)</th>
<th>Output gear teeth loading cycles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ferry</td>
<td>560 (5,000)</td>
<td>420</td>
<td>32,392,514</td>
</tr>
<tr>
<td>Ferry</td>
<td>24,000 (212,000)</td>
<td>0.25</td>
<td>19,281</td>
</tr>
<tr>
<td>Ascent</td>
<td>9,000 (80,000)</td>
<td>0.2</td>
<td>15,425</td>
</tr>
<tr>
<td>Ascent</td>
<td>5,200 (46,000)</td>
<td>1.7</td>
<td>131,113</td>
</tr>
<tr>
<td>Ascent</td>
<td>11,000 (98,000)</td>
<td>0.1</td>
<td>7,713</td>
</tr>
<tr>
<td>Descent</td>
<td>16,000 (140,000)</td>
<td>33.0</td>
<td>2,545,126</td>
</tr>
<tr>
<td>Descent</td>
<td>27,000 (240,000)</td>
<td>1.0</td>
<td>77,125</td>
</tr>
</tbody>
</table>
Life vs. Reliability – 1 Actuator

Statistical percent of failures vs. Number of flights

- Bearing system
- One actuator system
- Gear system
- Gear teeth bending
- Gear surfaces
Life vs. Reliability – 4 Actuators

Statistical percent of failures vs. Number of flights

- Four actuator systems
- Gear systems
- Bearing systems
- Gear teeth bending
- Gear surfaces

Number of flights: 1, 10, 100, 1000, 2000
Statistical percent of failures: 1, 5, 10, 20, 50, 90, 95
Comparison of Life & Reliability for 1 & 4 Actuators

Diagram showing the statistical percent of failures vs. number of flights for one actuator system and four actuator systems. The graph indicates that as the number of flights increases, the percent of failures also increases, with four actuator systems experiencing higher failure rates than the one actuator system.
Predicted Lives of Rudder/Speed Brake Actuator Bearings

<table>
<thead>
<tr>
<th>Bearing No.</th>
<th>1a</th>
<th>1b</th>
<th>2</th>
<th>3a</th>
<th>3b</th>
<th>4</th>
<th>6a</th>
<th>6b</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bearing type</td>
<td>roller</td>
<td>roller</td>
<td>roller</td>
<td>roller</td>
<td>roller</td>
<td>ball</td>
<td>ball</td>
<td>ball</td>
</tr>
<tr>
<td>$L_{10B}$ life (hrs)</td>
<td>647,000</td>
<td>647,000</td>
<td>15,823</td>
<td>4,875</td>
<td>4,875</td>
<td>54,973</td>
<td>1,509</td>
<td>1,089</td>
</tr>
</tbody>
</table>

$L_{10B}$ life data provided for bearings by actuator manufacturer

No bearing failure expected before 5.3% of min. $L_{10B}$ life:

$L_{10B} = 0.053(1089 \text{ hrs}) = 58 \text{ hrs or 7 flights}$
### Rudder/Speed Brake Actuator Probability of Survival

<table>
<thead>
<tr>
<th>Number of flights</th>
<th>Number of actuators</th>
<th>Gear Reliability, percent</th>
<th>Actuator Bearing Reliability, percent</th>
<th>Total System Reliability, percent</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Tooth Bending Fatigue</td>
<td>Tooth Surface Fatigue</td>
<td>Combined Bending &amp; Surface</td>
</tr>
<tr>
<td>100</td>
<td>1</td>
<td>95.9</td>
<td>99.98</td>
<td>95.9</td>
</tr>
<tr>
<td>100</td>
<td>4</td>
<td>84.71</td>
<td>99.91</td>
<td>84.7</td>
</tr>
<tr>
<td>20</td>
<td>1</td>
<td>99.93</td>
<td>99.999+</td>
<td>99.93</td>
</tr>
<tr>
<td>20</td>
<td>4</td>
<td>99.7</td>
<td>99.998</td>
<td>99.7</td>
</tr>
<tr>
<td>12</td>
<td>1</td>
<td>99.98</td>
<td>99.999+</td>
<td>99.98</td>
</tr>
<tr>
<td>12</td>
<td>4</td>
<td>99.92</td>
<td>99.998</td>
<td>99.92</td>
</tr>
<tr>
<td>Number of Actuators</td>
<td>Number of Flights</td>
<td>Probability of Survival (failure), %</td>
<td></td>
<td></td>
</tr>
<tr>
<td>---------------------</td>
<td>-------------------</td>
<td>-------------------------------------</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>100</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>98.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>97.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>82</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>100</td>
<td>94</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>100</td>
<td>90</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bearing System</td>
<td>100</td>
<td>99.8</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>97.5</td>
<td>86</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Weakest Bearing</td>
<td>100</td>
<td>99.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>98.8</td>
<td>93</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

90% reliability (10% failure) for 20 flights unacceptable
New goal 12 flights, 94% reliability (6% failure)
No failures predicted for up to 7 flights
Summary of Results

• Rudder/Speed Brake Actuators limited to 12 flights

• Reliability for 12 flights:  98.6% for one actuator, 94% for four actuators

• No failures expected for up to 7 flights

• Four actuator system reliability:  90% for 20 flights and 46% for 100 flights

• Life & reliability of actuator system dominated by bearings
Epilogue

• Shuttle Discovery returned to flight, July 2005
• Shuttle Atlantis concluded program, July 2011
• Space Shuttle Program had a total of 135 missions from 1981 — 2011
• At retirement, flight leader, Discovery had 39 missions in 27 years (9 after return to flight)
• 22 missions for 3 remaining shuttles from return to flight until end of program