Adapting Guidance Methodologies for Trajectory Generation in Entry Shape Optimization

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Motivation

Flight Feasible Trajectories will Model **Realistic In-Flight Thermal States:**

- Allow for increased accuracy in Thermal Protection System sizing (potential mass savings)
- Reduce the number of design cycles required to close an entry spacecraft design (potential cost savings)
Novel Research Objective

Develop a planetary guidance algorithm that is adaptable to:
- Mission Profiles
- Vehicle Shapes
for integration into vehicle optimization.
Sample Concept of Spaceflight Operations

* Adapted graphic from NASA Johnson Space Center

Launch to:
- Earth Orbit
- Planetary Body

Exploration:
Vehicle completes mission over several day or weeks

De-Orbit

Separation

Atmospheric Entry

Descent

EDL

Landing
Planetary Entry Spacecraft Design (cont’d)

Mid - Low L/D Spacecraft

$\sigma$ – variable bank angle
$\alpha$ – fixed angle of attack

High L/D Spacecraft

$\sigma$ – variable bank angle
$\alpha$ – variable angle of attack

* Orion Capsule
  Prakash et al., NASA JPL

* MSL Capsule
  AIAA 2006-659

* Ellipsled
  Garcia et al., AIAA Conf. Paper

* Space Shuttle
  AIAA 2006-8013

* HL-20
  AIAA 2006-239

* NASP
  AIAA 2006-659

www.nasa.gov
Multi-Disciplinary Design, Analysis, and Optimization (MDAO)

**Vehicle Optimization**
- Computer Generated Spacecraft Models
- Available Descent Technologies
- Un/manned
- Planetary Models
- Mission Profile

**Entry Trajectory Modeling**
- **Coupled**

**Thermal Protection System (TPS) Sizing**
- Decoupled Iterations

**Structures**
- Decoupled Iterations

**Minimize:**
- Heat Rate (Trajectory/Shape)
- Ballistic Coefficient (Shape)

**Aerodynamic (C_D, C_L) & Aerothermodynamic (\dot{q}) Databases**

**Flight Feasible Trajectory Database**
- *(replace Traj. Opt.)*

**Guidance, Navigation, & Control**

**Mission Profile**
## Trajectory Optimization vs. Guidance

<table>
<thead>
<tr>
<th></th>
<th>Trajectory Optimization</th>
<th>Guidance</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Constraints</strong></td>
<td>Multiple included</td>
<td>Minimal included</td>
</tr>
<tr>
<td><strong>Objective</strong></td>
<td>Any variable of interest</td>
<td>Target specific</td>
</tr>
<tr>
<td><strong>Solution</strong></td>
<td>Purely numerical</td>
<td>Combination of numerical and analytical</td>
</tr>
<tr>
<td><strong>Time to Solution</strong></td>
<td>Minutes to hours</td>
<td>Seconds</td>
</tr>
<tr>
<td><strong>Guaranteed Solution</strong></td>
<td>No</td>
<td>Must enforce that a solution is found</td>
</tr>
<tr>
<td><strong>Parameter Changes</strong></td>
<td>Handles large parameter changes</td>
<td>Handles parameter changes that are relatively small</td>
</tr>
<tr>
<td><strong>Result</strong></td>
<td>Nominal Trajectory – not always realistic control</td>
<td>Flight Feasible Trajectory with realistic controls</td>
</tr>
</tbody>
</table>
Guidance Development Trade-Offs

**Adaptability**
Numerical formulation for adaptability to different vehicles and missions without significant changes

**Rapid Trajectory Generation**
Analytical driving function keep time to a solution low

**Minimize Range Error & Heatload**
Optimal Control theory to introduce heat load as an additional objective
Guidance Development Criteria

Guidance Specific (In-Flight)

- Determine flight feasible control vectors (control rate/acceleration constraints)
- Be highly robust to dispersions and perturbations
- Include a minimal number of mission dependent guidance parameters

Vehicle Design Specific

- Be applicable to multiple mission scenarios and vehicle dispersions
- Manage the entry heat load in addition to achieving a precision landing
Types of Guidance Techniques

*Reference Tracking Only* – follow a pre-defined track

*In-flight Reference Generation & Tracking* – Generate a real-time reference trajectory and follow that track

*In-flight Controls Search* – One dimensional search, usually solving equations of motion numerically

*In-flight Optimal Control* – Requires numerical methods to meet some cost function
## Types of Guidance Formulations

<table>
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<tr>
<th></th>
<th>Analytical Guidance</th>
<th>Numerical Guidance</th>
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</thead>
<tbody>
<tr>
<td><strong>Advantages</strong></td>
<td>• Simple to Implement</td>
<td>• Accurate trajectory solutions</td>
</tr>
<tr>
<td></td>
<td>• Computation time minimal</td>
<td>• No simplifying assumptions (possibility of multiple entry cases to be simulated with few modifications)</td>
</tr>
<tr>
<td></td>
<td>• Solution Guaranteed</td>
<td></td>
</tr>
<tr>
<td><strong>Disadvantages</strong></td>
<td>• Simplifications reduce accuracy of the trajectory solution</td>
<td>• Convergence is not assured</td>
</tr>
<tr>
<td></td>
<td>• Formulation tied to a specific entry case</td>
<td>• Convergence is not timely</td>
</tr>
</tbody>
</table>
Novel Approach to Guidance for MDAO

*Real-Time Trajectory Generation and Tracking*

**Adaptability**
- Numerically solve entry equations of motion
- Use generalized analytical functions to represent the reference

**Rapid Trajectory Generation**
- Use analytical driving function to keep time to a solution low
- Use Single Optimal Control Point with Blending

**Minimize Range Error & Heatload**
- Optimal Control theory used to introduce heat load objective

- Adaptation of Shuttle Entry Guidance Techniques
- Adaptation of Energy State Approximation Techniques
Skip Entry Critical Points

Test Case: Orion Capsule, L/D 0.4

Control: Bank Angle only

Begin with 1st Entry portion of the trajectory and gradually includes remaining phases.
Trajectory Simulation Validation

Truth Model

*Simulation of Rocket Trajectories (SORT)*

Developed by NASA Johnson Space Center for Space Shuttle Launch/Entry Simulations

![Graph showing the comparison of simulation outputs with different methods and time measurements.](image)
Flight Dynamics

- \( \phi \) - latitude
- \( \gamma \) - flight path angle
- \( \psi \) - azimuth
- \( \sigma \) - bank angle
- \( b \) - body fixed coordinate
- \( \theta \) - longitude

ECF – Earth Centered Fixed
Trajectory Modeling

\[
\begin{align*}
\dot{r} &= V \sin \gamma \\
\dot{\theta} &= \frac{V \cos \gamma \sin \psi}{r \cos \phi} \\
\dot{\phi} &= \frac{V \cos \gamma \cos \psi}{r}
\end{align*}
\]

\[
\begin{align*}
\dot{V} &= -D - g \sin \gamma + \Omega^2 r \cos \phi (\sin \gamma \cos \phi - \cos \gamma \sin \phi \cos \psi) \\
\dot{\gamma} &= \frac{1}{V} \left[ L \cos \sigma + \cos \gamma \left( \frac{V^2}{r} - g \right) + 2 \Omega V \cos \phi \sin \psi + \Omega^2 r \cos \phi (\cos \gamma \cos \phi + \sin \gamma \cos \psi \sin \phi) \right] \\
\dot{\psi} &= \frac{1}{V} \left[ L \sin \sigma \cos \gamma + \frac{V^2}{r} \cos \gamma \sin \psi \tan \phi - 2 \Omega V (\tan \gamma \cos \psi \cos \phi - \sin \phi) + \frac{r \Omega^2}{\cos \gamma} \sin \psi \sin \phi \cos \phi \right]
\end{align*}
\]

State Variables
- \(r\) - radial distance
- \(V\) - relative velocity
- \(\theta\) - longitude
- \(\phi\) - latitude
- \(\gamma\) - flight path angle
- \(\psi\) - azimuth

Control Variables
- \(\sigma\) - bank angle
- \(\alpha\) - angle of attack

Vehicle and Planet Variables
- \(L, D\) - Lift, Drag Acceleration
- \(g\) - gravity
- \(\Omega\) - Earth’s Rotation
- \(\rho\) - atmospheric density
General Entry Guidance Block Diagram

**Trajectory Solver**

**Reference Trajectory:** Analytical functions adapted from Shuttle Entry Guidance

**Bank Schedule Solution:** $\tilde{\sigma}_{cmd}$

**Range Prediction:** numerically solve equations of motion, range calculation

**Dispersed State:** $\tilde{y}_{disp}$

- Send $\tilde{\sigma}_{cmd}$ to flight simulation
- $R_{err} \approx 0$
- Yes
- $\tilde{\sigma}_{new}$
- No

**Targeting Algorithm**

**Solver:** Single Point Optimal Control

**Solution from Energy State Approximation**

**Purpose:** Targeting for precision landing and minimizing heatload
Control Solution: Shuttle Entry Guidance Adaptation

Shuttle Entry Guidance (SEG) Concept: Temperature Phase

- Reference Tracking Algorithm, Closed Form Solution

\[
\frac{d}{dt} \left( D = \frac{\rho V_r^2 C_D A}{2m} \right)
\]

\[
\dot{\rho} = \frac{d}{dt} \left( \rho_o e^{-\frac{h}{h_s}} \right) \Rightarrow \frac{\dot{\rho}}{\rho} = -\frac{\dot{h}}{h_s}
\]

\[
D_{\text{ref}} = C_2 V^2 + C_1 V + C_0
\]

\[
\gamma_{\text{ref}} = \text{constant}
\]
Control Solution: Shuttle Entry Guidance Adaptation

Improvements on Shuttle Entry Guidance “Drag Based Approach”

- Increase # of segments
- Increase order of polynomial
- Change Atmospheric Model representation
- Modify flight path angle representation

Challenges with Drag Based Approach

- Discontinuities between segments
- Increasing # of coefficients for storage with increasing segments and/or order
- Effect of small flight path angle assumption unknown
- Formulations are derived from 2DOF Longitudinal EOMs
Control Module: Shuttle Entry Guidance Adaptation

Sensitivity to atmospheric non-linearity is significant during initial and final segments. **Need an Alternative Analytical Equation!**
Automated Selection of Transition Events

**Framework:**
- Allows for adaptability
- Automated generation of Reference Trajectory
- Open loop

**Study Objective:** Define bank profile for trajectory phases

<table>
<thead>
<tr>
<th>Phase</th>
<th>Bank Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>Entry Interface to Guidance Start</em></td>
<td>Constant Bank</td>
</tr>
<tr>
<td><em>Guidance Start to Guidance End</em></td>
<td>Trajectory Solver</td>
</tr>
<tr>
<td><em>Guidance End to Exit</em></td>
<td>Linear Transition to Meet 2\textsuperscript{nd} Entry Bank</td>
</tr>
<tr>
<td><em>Exit to 2\textsuperscript{nd} Entry</em></td>
<td>Attitude Hold</td>
</tr>
</tbody>
</table>
Automated Selection of Transition Events

- Metric to determine best trajectory: lowest range error, lowest heat load from EI to 2nd Entry, and bank transitions
## Automated Selection of Transition Events

### Study Results:

<table>
<thead>
<tr>
<th>Phase</th>
<th>Bank Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Entry Interface to Guidance Start</td>
<td>Constant Bank = (57.95^\circ)</td>
</tr>
<tr>
<td>Guidance Start to Guidance End</td>
<td>Trajectory Solver ({0.12 \quad 0.11} \text{ G'} \text{' s})</td>
</tr>
<tr>
<td>Guidance End to Exit</td>
<td>Linear Transition to Meet 2(^{nd}) Entry Bank</td>
</tr>
<tr>
<td></td>
<td>Linear Transition Velocity: (23,784.65 \text{ ft/s})</td>
</tr>
<tr>
<td>Exit to 2(^{nd}) Entry</td>
<td>Bank Attitude Hold = (70^\circ)</td>
</tr>
</tbody>
</table>

![Diagram showing flight path and transition events](image)

- **Guidance Start**: Entry Interface to Guidance Start
- **Guidance End**: Exit to 2\(^{nd}\) Entry
- **2\(^{nd}\) Entry Bank**: Transition events marked with red points.
General Entry Guidance Block Diagram

**Trajectory Solver**

**Reference Trajectory**: Analytical functions adapted from Shuttle Entry Guidance

**Bank Schedule Solution**: $\vec{\sigma}_{cmd}$

**Range Prediction**: numerically solve equations of motion, range calculation

**Dispersed State**: $\vec{y}_{disp}$

**Send $\vec{\sigma}_{cmd}$ to flight simulation**

Yes

$R_{err} \approx 0$

No

**Targeting Algorithm**

**Solver**: Single Point Optimal Control Solution from Energy State Approximation

**Purpose**: Targeting for precision landing and minimizing heatload
Targeting Algorithm Development

When is Targeting Activated?

1. Overshoot – Vehicle is predicted to fly way past target
2. Undershoot – Vehicle is predicted to fly short of the target

How to find a set of controls to Correct Over/Underhoot?

Adapt Energy State Approximation Methods:
Optimal control method that replaces altitude and velocity with specific energy height \( e \)

\[
e = \frac{V_r^2}{2g_o} + h
\]

Advantages: Allows for a compact set of analytical equations
Add heat load to the range error objective function

Disadvantage: Optimal control formulations may not converge to a solution

Solution: Derive a localized optimal control point instead and blend back reference trajectory
Targeting Algorithm Development

Must Relate Euler-Lagrange Equation

\[ \bar{\lambda} = \frac{\lambda_\psi}{\lambda_\gamma} = \tan \sigma^* \cos \gamma \]

\[ \lambda_\gamma \leq 0 \]

To Reference Trajectory Variables

\[ \frac{L}{D_{total}} \cos \sigma = \frac{1}{\rho \Phi_{ref}} \left[ V_r \dot{\gamma}_{ref} - \cos \gamma \left( \frac{V_r^2}{r} - g \right) - C_\gamma (y) \right] \]

Using trigonometry and other manipulations, the control equation is found

\[ \frac{L}{D_{total}} \sqrt{1+ \left( \frac{\lambda_\gamma}{\cos \gamma} \right)^2} = \frac{\left[ V \gamma_{ref} - \cos \gamma \left( \frac{V_r^2}{r} - g \right) - 2\Omega V \cos \phi \sin \psi - \Omega^2 r \cos \phi (\cos \gamma \cos \phi + \sin \gamma \cos \psi \sin \phi) \right]}{D_{approx}} \]
Targeting Algorithm Development

\[ \Phi_{\text{ref}} = \frac{C_D A}{2m} \left| V_r^2 \right| \]

Least Squares Curve Fitting:
3 Interpolation Points

\[ \Phi_{\text{blnd}} = Bb_2 V^2 + Bb_1 V + Bb_0 \]
Targeting Algorithm Development

Targeting Technique 1 – Design Space Interrogation

\[ C_\Phi \] - drag/density ratio coefficient

\[ d\lambda \] - change in Lagrange multiplier

\[ dV \] - change in relative velocity at next point

Targeting Technique 2 – Design Space Interrogation

\[ d\lambda \] - change in Lagrange multiplier

\[ dV_1 \] - change in relative velocity halfway to curve fit end point

\[ \Delta(dE) \] - second order change in energy
Targeting Algorithm Development

Targeting Technique 1 – Design Space Interrogation

<table>
<thead>
<tr>
<th>Case</th>
<th>Dispersion</th>
<th>Target Miss</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Increase Entry Flight Path Angle</td>
<td>Undershoot</td>
</tr>
<tr>
<td>2</td>
<td>Decrease Entry Flight Path Angle</td>
<td>Overshoot</td>
</tr>
<tr>
<td>3</td>
<td>L/D Dispersion</td>
<td>Overshoot</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Lower Limit</th>
<th>Upper Limit</th>
<th>Incr.</th>
<th>units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_\Phi$</td>
<td>0</td>
<td>1</td>
<td>ND</td>
<td></td>
</tr>
<tr>
<td>$d\lambda$</td>
<td>0</td>
<td>1</td>
<td>0.01</td>
<td>ND</td>
</tr>
<tr>
<td>$dV$</td>
<td>100</td>
<td>1000</td>
<td>100</td>
<td>ft/s</td>
</tr>
</tbody>
</table>

Undershoot Trajectory Correction, ESA Quadratic (blue line), $dV = 500$ ft/s
Targeting Algorithm Development

FPA Dispersion - Undershoot
Targeting Algorithm Development

FPA Dispersion - Overshoot
Targeting Algorithm Development

Aerodynamic Dispersion - Overshoot
Shape Optimization Analog

Current Guidance Algorithms – Robust to ~20% aerodynamic dispersions

Must exceed 20% to demonstrate potential for integration into MDAO

**ANALOG**: Changing angle of attack disperses $C_L$ and $C_D$
Targeting Algorithm Development

Guidance Algorithm for Comparison – Apollo Derived Final Phase Guidance

Reference Tracking to a stored trajectory database, function of relative velocity

Performance Results – Threshold Miss Distance, 1 nmi
Targeting Algorithm Development

Targeting Technique 1 – Targeting Procedure

1. Guess a value for $d\lambda$
2. Iterate on $dV$ using secant method to converge on a zero range error trajectory
3. If no solution is found, $d\lambda$ is incremented and the iteration is repeated
4. Solution is then flown in flight simulation
Targeting Algorithm Development

Targeting Implementation, 1st and 2nd Phase - Results

\[
\begin{align*}
\alpha &= 152^\circ, \quad L/D = 0.418 \\
\alpha &= 153^\circ, \quad L/D = 0.402 \\
\alpha &= 154^\circ, \quad L/D = 0.386 \\
\alpha &= 155^\circ, \quad L/D = 0.371 \\
\alpha &= 156^\circ, \quad L/D = 0.357 \\
\alpha &= 157^\circ, \quad L/D = 0.343 \\
\alpha &= 158^\circ, \quad L/D = 0.328 \\
\alpha &= 159^\circ, \quad L/D = 0.313 \\
\alpha &= 160^\circ, \quad L/D = 0.299
\end{align*}
\]
Targeting Algorithm Development

Targeting Technique 2

Use Energy Height \( e = \frac{V^2}{2g_o} + h \) to determine Control Point \([V_{new}, \Phi_{new}]\)

Undershoot \(\rightarrow\) energy dissipating \((de/dt)\) too fast

Overshoot \(\rightarrow\) energy dissipating \((de/dt)\) too slow

Since Velocity is an independent variable
and a pseudo control \(de/dV\) is examined
Targeting Algorithm Development

Targeting Technique 2

Recall the equation for the ratio of drag acceleration to density: \( \frac{D}{\rho} = \frac{C_D A V_r^2}{2m} \)

-Extract altitude and velocity from \([dV_1, \Delta(dE)]\) to find \(\Phi_{\text{new}}\)
Targeting Algorithm Development

Targeting Technique 2 – Design Space Interrogation

<table>
<thead>
<tr>
<th></th>
<th>Lower Limit</th>
<th>Upper Limit</th>
<th>Incr.</th>
<th>units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d\lambda$</td>
<td>0</td>
<td>$d\lambda_{\text{limit}}$</td>
<td>ND</td>
<td></td>
</tr>
<tr>
<td>$dV_1$</td>
<td>0</td>
<td>1524</td>
<td>Predict</td>
<td>m/s</td>
</tr>
<tr>
<td>$\Delta(dE)$</td>
<td>0</td>
<td>$\Delta(dE)_{\text{limit}}$</td>
<td>Predict</td>
<td>m</td>
</tr>
</tbody>
</table>

Limit are trajectory dependent and control system dependent

$$\lambda_{\text{min/max}} = \tan \sigma_{\text{min/max}} \cos \gamma_i$$

$$d\lambda_{\text{limit}} = \mp (\bar{\lambda}_{\text{min/max}} - \bar{\lambda}_{\text{old}})$$

Dispersion Cases:

<table>
<thead>
<tr>
<th>$\alpha$ [deg]</th>
<th>$L/D$ Dispersion</th>
<th>Target Miss</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st Phase Only</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nominal</td>
<td>0.4 (0%)</td>
<td></td>
</tr>
<tr>
<td>152</td>
<td>0.42 (+5%)</td>
<td>Undershoot</td>
</tr>
<tr>
<td>162</td>
<td>0.28 (-30%)</td>
<td>Overshoot</td>
</tr>
<tr>
<td>165</td>
<td>0.23 (-43%)</td>
<td>Overshoot</td>
</tr>
<tr>
<td>167</td>
<td>0.2 (-50%)</td>
<td>Undershoot</td>
</tr>
</tbody>
</table>
Targeting Algorithm Development

*Design Space Interrogation, Results: Range Error [%]*

\( \alpha = 152^\circ, \text{ Undershoot} \)

\( \alpha = 162^\circ, \text{ Overshoot} \)

\( \alpha = 165^\circ, \text{ Overshoot} \)

\( \alpha = 167^\circ, \text{ Undershoot} \)
Targeting Algorithm Development

Design Space Interrogation, Results: Heatload [J/cm^2]

\( \alpha = 152^\circ, \text{ Undershoot} \)

\( \alpha = 162^\circ, \text{ Overshoot} \)

\( \alpha = 165^\circ, \text{ Overshoot} \)

\( \alpha = 167^\circ, \text{ Undershoot} \)
Targeting Algorithm Development

*Design Space Interrogation, Results:* Bank Rate $[\text{deg/s}]$

$\alpha = 152^\circ$, Undershoot

$\alpha = 162^\circ$, Overshoot

$\alpha = 165^\circ$, Overshoot

$\alpha = 167^\circ$, Undershoot
Targeting Algorithm Development Results

Dispersions –
Apollo Derived Guidance = -20% dispersion
MDAO Algorithm = -43% dispersion

Managing heatload may be a challenge for dispersions greater than 20%
Conclusions

Guidance Specific (In-Flight)
- Determine flight feasible control vectors (control rate/acceleration constraints)
  - Be highly robust to dispersions and perturbations
- Include a minimal number of mission dependent guidance parameters

Vehicle Design Specific
- Be applicable to multiple mission scenarios
  - vehicle dispersions
- Manage the entry heat load in addition to achieving a precision landing
Acknowledgements

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Supervisor

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Thank You !!!
Questions?
Additional Slides
(optional)
## Overview

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<td>Dissertation Research Plan and Status</td>
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<td></td>
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<td>Key Results #3</td>
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<td></td>
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<td>Dissertation Findings and Status</td>
</tr>
</tbody>
</table>
Big Picture:
Spacecraft Design Process
Vehicle Optimization and TPS Sizing

Example Objective Function: \( \dot{q}_{\text{conv}} = 1.83 \times 10^{-4} \sqrt{\rho R_n(1 - h_W/H_s)} V^3 \)

Results

What is Flight Feasible?

- Reaches Target @ Landing Speeds
- Control does not exceed system limits

- Used for all geometries within optimization to find heat rate

- Some studies use new trajectories, but there is no accounting for bank constraints or target accuracy

- None of these studies incorporated flight feasible trajectories
Proposed Approach to MDAO for Spacecraft Design

- Vehicle Optimization
- Planetary Entry Guidance
- Thermal Protection System (TPS) Sizing
- Structures

- Aerodynamic ($C_d$, $C_L$) & Aerothermodynamic ($\dot{q}$) Databases
- Flight Feasible Trajectory Database
- Guidance, Navigation, & Control

Key Components:
- Computer Generated Spacecraft Models
- Available Descent Technologies
- Un/manned Planetary Models
- Mission Profile

Coupled and Decoupled Iterations
Trajectory Modeling for Design vs. In-Flight Trajectory Modeling
Planetary Entry Guidance Literature Review

• **High L/D, Earth**: Space Shuttle, X-33, X40A
  • Most Robust: In Flight Trajectory Shaping with Reference Tracking
  • Least Robust: Reference Tracking Only
• **Low L/D, Earth**: Apollo, Orion
  • Most Robust: In-Flight Controls Search
  • Least Robust: Reference Tracking Only
• **Other Planetary Entry Vehicles**: MSR, MSL, Biconic
  • Flight Tested algorithms preferred
Planetary Entry Guidance Literature Review (cont’d)

Key Results

Modern guidance algorithms: optimal control is potential framework, but
Robust guidance algorithms: combination of numerical and analytical approaches
Least robust algorithms: purely analytical solutions
Adaptability of guidance algorithms: very limited among all algorithms
Heat load management: not included
Convergence still an issue
Trajectory Optimization Literature Review

*Trajectory Optimization*

*Traj* - Nonlinear constrained optimization

*Mission* - Sequential Quadratic Programming

*Energy State Method* – Reduced Order Modeling, one dimensional parameter search

*Pseudospectral Methods* – Combination indirect and direct method, mapping and discretization of domain
Trajectory Optimization Literature Review (cont’d)

**Key Results**

- Convergence time increases with dimensionality.
- Fidelity of modeling may be compromised.
Introduction to Planetary Entry Guidance
Guidance Development Process

Trajectory Design
- Trajectory Optimization
  - classical optimization
  - genetic optimization
- Bank Profile Scan
  - Determine constraint boundaries to identify flight corridors

Reference Trajectory Found
- Constant Bank
- Modulated Bank
  - linear ramp, user-defined scans at each time step

Guidance Development
- Identify Target:
  - Orbit
  - Landing Site
  - Descent Device
  - Activation State
  - Multiple

Trajectory Modeling in Real-Time to Predict Target Acquisition:
- Profiles Stored in Tables
- Analytical Equations
- Numerical Integration of EOMs
- Hybrid Approach

Targeting Algorithm:
- Parameterization
- One-Dimensional Bank Search
- Optimal Control Formulation
- Analytical Equation Blending
- Search Family of Trajectories
- Hybrid Approach

Apply Dispersions

Guided Trajectories Found

*Guidance must be robust to many dispersions: (Atmospheric properties, Aerodynamics properties, Navigational Inputs, Entry Interface Conditions, Mass, Control System performance, and many others)
Baseline Vehicle & Mission
# Case Study Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Vehicle</strong></td>
<td>Orion Capsule, L/D = 0.4</td>
</tr>
<tr>
<td><strong>Trajectory</strong></td>
<td>Skip Entry for Lunar Return</td>
</tr>
<tr>
<td><strong>Control</strong></td>
<td>Bank Angle only</td>
</tr>
<tr>
<td><strong>Atmospheric Model</strong></td>
<td>1976 Standard Atmosphere</td>
</tr>
<tr>
<td><strong>Gravity Model</strong></td>
<td>Central Force + Zonal Harmonics</td>
</tr>
<tr>
<td><strong>Aerodynamics</strong></td>
<td>$C_L$, $C_D$ corresponding to Mach #</td>
</tr>
<tr>
<td></td>
<td>CBAERO Databases, function of Mach #, Dynamic Pressure, and Angle of Attack</td>
</tr>
<tr>
<td><strong>Trajectory Simulation</strong></td>
<td>MATLAB Simulation validated against SORT Trajectories</td>
</tr>
</tbody>
</table>
Trajectory Simulations Developed

Open Loop Numerical Predictor- Corrector (NPC) Simulation
Used to test guidance formulations

3DOF Rotating Spherical Planet

\[
\begin{align*}
\dot{r} &= V \sin \gamma \\
\dot{\theta} &= \frac{V \cos \gamma \sin \psi}{r \cos \phi} \\
\dot{\phi} &= \frac{V \cos \gamma \cos \psi}{r}
\end{align*}
\]

\[
\dot{V} = -D - g \sin \gamma + \Omega^2 r \cos \phi (\sin \gamma \cos \phi - \cos \gamma \sin \phi \cos \psi)
\]

\[
\dot{\psi} = \frac{1}{V} \left[ L \cos \sigma + \cos \gamma \left( \frac{V^2}{r} - g \right) + 2\Omega V \cos \phi \sin \psi + \Omega^2 r \cos \phi (\cos \gamma \cos \phi + \sin \gamma \cos \psi \sin \phi) \right]
\]

\[
\dot{\psi} = \frac{1}{V} \left[ \frac{L \sin \sigma}{\cos \gamma} + \frac{V^2}{r} \cos \gamma \sin \psi \tan \phi - 2\Omega V (\tan \gamma \cos \psi \cos \phi - \sin \phi) + \frac{r \Omega^2}{\cos \gamma} \sin \psi \sin \phi \cos \phi \right]
\]

Flight Simulation - Closed Loop Guidance Testing
Using equations derived from Newton’s 2nd Law, dynamics of relative motion, and Earth Centered Inertial (ECI) coordinate system
Trajectory Solver Development
Control Solution: Shuttle Entry Guidance Adaptation

Drag Curve Fit Accuracy

<table>
<thead>
<tr>
<th>Segments</th>
<th>Order</th>
<th># of stored coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>7 (3)</td>
<td>Irrational</td>
<td>168</td>
</tr>
<tr>
<td>7 (5)</td>
<td>Irrational</td>
<td>105</td>
</tr>
<tr>
<td>14</td>
<td>5</td>
<td>84</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>21</td>
</tr>
</tbody>
</table>

\[ D_{\text{ref}} = C_2 V^{x_2} + C_1 V^{x_1} + C_0 V^{x_0} \]
Control Solution: Shuttle Entry Guidance Adaptation

Would Cubic Spline Interpolation work?

\[ h_s = \left( \frac{1}{P} \frac{dP}{dh} - \frac{1}{T} \frac{dT}{dh} \right)^{-1} \]
Targeting Algorithm Development
Targeting Algorithm Development

Targeting Technique 1 – Trajectory Behavior to Full Set of Aerodynamic Dispersion

Can Technique 1 find a trajectory that points toward correcting the range error?
General Conclusions
Targeting Algorithm Development

Pontryagin’s Principle in Optimal Control

Find Optimal Control $\tilde{u}^*$, $\sigma^*(t)$ and $V^*(t)$

for dynamic system $\dot{x} = f(x, \tilde{u}, t)$

The optimal control $\dot{e} = \frac{V}{g_0} + \frac{h_{geo}^2}{h^2} \dot{h}$

The original Euler-Lagrange Equation:

$$\left. \frac{\partial H}{\partial u} \right|_{u=u^*} = 0$$

$$\dot{\phi} = \frac{V \cos \gamma}{r}, \quad \lambda = \frac{\lambda_{\psi}}{\lambda_{\gamma}} = \tan \sigma^* \cos \gamma$$

$$\dot{\gamma} = \frac{1}{V} \left[ L \cos \sigma + \cos \gamma \left( \frac{1}{r} - g \right) + 2\nu \nu \cos \phi \sin \psi + \Omega^2 r \cos \phi (\cos \gamma \cos \phi + \sin \gamma \cos \psi \sin \phi) \right]$$

$$\dot{\psi} = \frac{1}{V} \left[ \frac{L \sin \sigma}{\cos \gamma} + \frac{V^2}{r} \cos \gamma \sin \psi \tan \phi - 2\Omega V (\tan \gamma \cos \psi \cos \phi - \sin \phi) + \frac{r \Omega^2}{\cos \gamma} \sin \psi \sin \phi \cos \phi \right]$$
Targeting Algorithm Development

Targeting Technique 1

\[ \tilde{\lambda}_{new} = \tilde{\lambda}_{old} \pm d\lambda \]

Determines new bank angle at current time step

\[ \dot{\gamma} = \frac{1}{V} \left[ L \cos \sigma + \cos \gamma \left( \frac{V^2}{r} - g \right) + 2\Omega V \cos \phi \sin \psi 
+ \Omega^2 r \cos \phi (\cos \gamma \cos \phi + \sin \gamma \cos \psi \sin \phi) \right] \]

Calibrated for Each Dispersed Case

Determines Blended Trajectory that nulls range error

\[ \Phi_{new, bound} = \frac{\sqrt{1+\left( \frac{\dot{x}}{\cos \gamma} \right)^2 [V\dot{\gamma}_{ref} - \cos \gamma \left( \frac{V^2}{r} - g \right) - 2\Omega V \cos \phi \sin \psi - \Omega^2 r \cos \phi (\cos \gamma \cos \phi + \sin \gamma \cos \psi \sin \phi)]}}{\rho_{D_{total}}} \]

\[ \Phi_{new} = \Phi_{old} \pm C_\Phi \Phi_{old} \Phi_{new, bound} \]

\[ V_{initial} = V_{current} + 0.01 dV \]

\[ V_{ref,f} = V_{current} - (1 - 0.01) dV \]
Targeting Algorithm Development
Targeting Technique 1 – Design Space Interrogation

• The blending technique exhibits potential to find new bank profiles that null the range error

• The design space is constrained by control system limitations

• There is a zero range error solution for each change in $d\lambda$
Targeting Algorithm Development

Targeting Technique 1 – Trajectory Behavior to Full Set of Aerodynamic Dispersion

Why did this not follow the Expected Behavior?

*The reference bank profile over-corrects with respect to the dispersion of L/D*
Targeting Algorithm Development

Targeting Technique 2

Now that the blended function is fully defined \( \Phi_{blnd} = Bb_2V^2 + Bb_1V + Bb_0 \)

The following equation can be used to solve for:

\[
\dot{\gamma}_{i,new} = \frac{1}{V_i} \left[ \frac{L}{D}_{total,i} \sqrt{1 + \left( \frac{1}{\lambda \cos \gamma_i} \right)^2 \rho_i \Phi_{blnd,i} + 2\Omega V_i \cos \phi_i \sin \psi_i + \cos \gamma_i \left( \frac{V_i^2}{r_i} - g_i \right) + \Omega^2 r_i \cos \phi_i \left( \cos \gamma_i \cos \phi_i + \sin \gamma_i \cos \psi_i \sin \phi_i \right) } \right]
\]

The FPA rate table is shifted accordingly
Targeting Algorithm Development

*Design Space Interrogation, Results:* Bank Acceleration [deg/s^2]

\[ \alpha = 152^\circ, \text{ Undershoot} \]

\[ \alpha = 162^\circ, \text{ Overshoot} \]

\[ \alpha = 165^\circ, \text{ Overshoot} \]

\[ \alpha = 167^\circ, \text{ Undershoot} \]
Targeting Algorithm Development

Targeting Technique 1 – Targeting Implementation, 1st and 2nd Phase

1. Guess a value for $d\lambda$

2. Iterate on $dV$ using secant method to converge on a zero range error trajectory

3. If no solution is found $d\lambda$ is incremented and the iteration is repeated

4. Solution is then flown in flight simulation

Performance Metric –

Compare range of aerodynamic dispersions this algorithm can handle to the range of aerodynamic dispersions a heritage algorithm can handle.
Trajectory Solver Research Questions

Can a simplification in the equations of motion be made without loss of accuracy?

Can a simplification on flight path angle be made without loss of accuracy?
Simplified Equations of Motion Study

3DOF Rotating, Spherical Earth (3RSP)

\[
\dot{\psi} = \frac{1}{V} \left[ \frac{L \sin \sigma}{\cos \gamma} + \frac{V^2}{r} \cos \gamma \sin \psi \tan \phi - 2\Omega V (\tan \gamma \cos \psi \cos \phi - \sin \phi) + \frac{r\Omega^2}{\cos \gamma} \sin \psi \sin \phi \cos \phi \right]
\]

3DOF Non-Rotating Spherical Planet

\[
\dot{\psi} = \frac{1}{V} \left[ \frac{L \sin \sigma}{\cos \gamma} + \frac{V^2}{r} \cos \gamma \sin \psi \tan \phi \right]
\]

3DOF Non-Rotating Flat Planet

\[
\dot{\psi} = \frac{1}{V} \left[ \frac{L \sin \sigma}{\cos \gamma} \right]
\]

2DOF Longitudinal Equations (2LON)

\[
\begin{align*}
\dot{h} &= V \sin \gamma \\
\dot{s} &= V \cos \gamma \\
\dot{V} &= -D - g \sin \gamma \\
\dot{\gamma} &= \frac{1}{V} \left[ L \cos \sigma + \cos \gamma \left( \frac{V^2}{r} - g \right) \right]
\end{align*}
\]
Simplified Equations of Motion Study (cont’d)
Simplified Equations of Motion Study (cont’d)

![Graph showing altitude vs. total range with specific ranges and values for SORT Nominal, 3RSP, and 2LON configurations.]
Trajectory Solver Research Questions

Can a simplification in the EOMs be made without loss of accuracy?
Not for a skip trajectory

Can a simplification on flight path angle be made without loss of accuracy?
Control Solution: Shuttle Entry Guidance Adaptation

\[ \dot{h}_{ref} = -h_s \left[ \frac{\dot{D}_{ref}}{D_{ref}} - \frac{2\dot{V}}{V} - \frac{\dot{C}_D}{C_D} \right] \]
Control Solution: Shuttle Entry Guidance Adaptation
Control Solution: Shuttle Entry Guidance Adaptation

\[ P = \rho RT \]

\[ \frac{\dot{\rho}}{\rho} = \frac{\dot{P}}{P} - \frac{\dot{T}}{T} \]

\[ h_s = \left( \frac{1}{P \frac{dP}{dh}} - \frac{1}{T \frac{dT}{dh}} \right)^{-1} \]

Atmospheric Temperature Slope Changes

Approximation
Reference
Control Solution: Shuttle Entry Guidance Adaptation

Need to Resolve 1st Segment to Capture Atmospheric Non-Linearity

IDEA: Curve fit drag with Mach Number

\[ D_{\text{ref}} = \sum_{i=1}^{n} C_i M^i \]
Control Solution: Shuttle Entry Guidance Adaptation

Check Altitude Acceleration Approximation
Trajectory Solver Research Questions

Can a simplification in the EOMs be made without loss of accuracy?  
**Not for a skip trajectory**

Can a simplification on flight path angle be made without loss of accuracy?
Range Prediction Sensitivity to Flight Path Angle Assumption

• Apollo and Shuttle Entry guidance formulations approximate flight path angle (FPA) to be small:

\[ \gamma \ll 1 \text{ rad} \quad \text{and/or} \quad \dot{\gamma} \ll 1 \text{ rad/s} \]

Why does this matter?

• If predicted range does not equal the range to landing site then targeting is erroneously active

• Are model reductions in the Trajectory Module and Control Module valid based on the nominal case?
Range Prediction Sensitivity to Flight Path Angle
Assumption

Case Studies:
A. Apply $\gamma < < 1 \text{ rad}$ to Trajectory Module only

B. Apply $\gamma < < 1 \text{ rad}$ to Controls Module only

C. Apply $\dot{\gamma} < < 1 \text{ rad} / s$ to bank equation only

\[
\frac{L}{D_{v,\text{ref}}} = \frac{1}{\rho \Phi_{\text{ref}}} \left[ V_r \dot{\gamma}_{\text{ref}} - \cos \gamma \left( \frac{V_r^2}{r} - g \right) - C_\gamma(y) \right]
\]
Range Prediction Sensitivity to Flight Path Angle Assumption

**Nominal** 661.73 [nmi]

<table>
<thead>
<tr>
<th>Case</th>
<th>Total Range [nmi]</th>
<th>% Range Error</th>
<th>Termination</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>662.39</td>
<td>0.099%</td>
<td>Drag Limit</td>
</tr>
<tr>
<td>B</td>
<td>649.74</td>
<td>1.813%</td>
<td>Drag Limit</td>
</tr>
<tr>
<td>C</td>
<td>632.13</td>
<td>4.474%</td>
<td>Velocity Limit</td>
</tr>
</tbody>
</table>

**Conclusion** *FPA* approximation can be applied to the **trajectory module**, but not to the **control module**