Adapting Guidance Methodologies for Trajectory Generation in Entry Shape Optimization

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Motivation

Flight Feasible Trajectories will
Model Realistic In-Flight Thermal States:

• Allow for increased accuracy in Thermal Protection System sizing
  (potential mass savings)

• Reduce the number of design cycles required to close an entry
  spacecraft design (potential cost savings)
Novel Research Objective

Develop a planetary guidance algorithm that is adaptable to:
- Mission Profiles
- Vehicle Shapes
for integration into vehicle optimization.
Sample Concept of Spaceflight Operations

Launch to:
- Earth Orbit
- Planetary Body

Exploration:
Vehicle completes mission over several days or weeks

De-Orbit

Separation

Atmospheric Entry

EDL

Descent

Landing

* Adapted graphic from NASA Johnson Space Center
Planetary Entry Spacecraft Design (cont’d)

Mid - Low L/D Spacecraft

\[ \sigma \text{ – variable bank angle} \]
\[ \alpha \text{ – fixed angle of attack} \]

High L/D Spacecraft

\[ \sigma \text{ – variable bank angle} \]
\[ \alpha \text{ – variable angle of attack} \]

* Orion Capsule
  Prakash et al., NASA JPL
  www.nasa.gov

* Space Shuttle
  AIAA 2006-659

* MSL Capsule
  AIAA 2006-8013

* Ellipsled
  Garcia et al., AIAA Conf. Paper

* NASP
  AIAA 2006-239

* HL-20
  AIAA 2006-239

* Orion Capsule
  AIAA 2006-239

* Ellipsled
  Garcia et al., AIAA Conf. Paper
Multi-Disciplinary Design, Analysis, and Optimization (MDAO)

Minimize:
- Heat Rate (Trajectory/Shape)
- Ballistic Coefficient (Shape)

Available Descent Technologies
Un/manned
Planetary Models
Mission Profile

Vehicle Optimization
Entry Trajectory Modeling
Thermal Protection System (TPS) Sizing
Structures

Computer Generated Spacecraft Models

Flight Feasible Trajectory Database
(replace Traj. Opt.)

Aerodynamic ($C_D$, $C_L$) & Aerothermodynamic ($\dot{q}$) Databases

Guidance, Navigation, & Control

Decoupled Iterations

Coupled

Mission Profile

Decoupled Iterations
## Trajectory Optimization vs. Guidance

<table>
<thead>
<tr>
<th></th>
<th>Trajectory Optimization</th>
<th>Guidance</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Constraints</strong></td>
<td>Multiple included</td>
<td>Minimal included</td>
</tr>
<tr>
<td><strong>Objective</strong></td>
<td>Any variable of interest</td>
<td>Target specific</td>
</tr>
<tr>
<td><strong>Solution</strong></td>
<td>Purely numerical</td>
<td>Combination of numerical and analytical</td>
</tr>
<tr>
<td><strong>Time to Solution</strong></td>
<td>Minutes to hours</td>
<td>Seconds</td>
</tr>
<tr>
<td><strong>Guaranteed Solution</strong></td>
<td>No</td>
<td>Must enforce that a solution is found</td>
</tr>
<tr>
<td><strong>Parameter Changes</strong></td>
<td>Handles large parameter changes</td>
<td>Handles parameter changes that are relatively small</td>
</tr>
<tr>
<td><strong>Result</strong></td>
<td>Nominal Trajectory – not always realistic control</td>
<td>Flight Feasible Trajectory with realistic controls</td>
</tr>
</tbody>
</table>
Guidance Development Trade-Offs

**Adaptability**
Numerical formulation for adaptability to different vehicles and missions without significant changes

**Rapid Trajectory Generation**
Analytical driving function keep time to a solution low

**Minimize Range Error & Heatload**
Optimal Control theory to introduce heat load as an additional objective
Guidance Development Criteria

*Guidance Specific (In-Flight)*
- Determine flight feasible control vectors (control rate/acceleration constraints)
- Be highly robust to dispersions and perturbations
- Include a minimal number of mission dependent guidance parameters

*Vehicle Design Specific*
- Be applicable to multiple mission scenarios and vehicle dispersions
- Manage the entry heat load in addition to achieving a precision landing
Types of Guidance Techniques

*Reference Tracking Only* – follow a pre-defined track

*In-flight Reference Generation & Tracking* – Generate a real-time reference trajectory and follow that track

*In-flight Controls Search* – One dimensional search, usually solving equations of motion numerically

*In-flight Optimal Control* – Requires numerical methods to meet some cost function
## Types of Guidance Formulations

<table>
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<tr>
<th></th>
<th>Analytical Guidance</th>
<th>Numerical Guidance</th>
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</thead>
<tbody>
<tr>
<td><strong>Advantages</strong></td>
<td>• Simple to Implement&lt;br&gt;• Computation time minimal&lt;br&gt;• Solution Guaranteed</td>
<td>• Accurate trajectory solutions</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• No simplifying assumptions (possibility of multiple entry cases to be simulated with few modifications)</td>
</tr>
<tr>
<td><strong>Disadvantages</strong></td>
<td>• Simplifications reduce accuracy of the trajectory solution&lt;br&gt;• Formulation tied to a specific entry case</td>
<td>• Convergence is not assured&lt;br&gt;• Convergence is not timely</td>
</tr>
</tbody>
</table>
Adaptability
Numerically solve entry equations of motion
Use generalized analytical functions to represent the reference

Rapid Trajectory Generation
Use analytical driving function keep time to a solution low
Use Single Optimal Control Point with Blending

Minimize Range Error & Heatload
Optimal Control theory used to introduce heat load objective

Adaptation of Shuttle Entry Guidance Techniques
Adaptation of Energy State Approximation Techniques
Skip Entry Critical Points

Test Case: Orion Capsule, L/D 0.4

Control: Bank Angle only

Begin with 1st Entry portion of the trajectory and gradually includes remaining phases.
Trajectory Simulation Validation

Simulation of Rocket Trajectories (SORT)
Developed by NASA Johnson Space Center for Space Shuttle Launch/Entry Simulations

Truth Model

Graph showing altitude over time for different simulations:
- Open Loop Simulation (MATLAB)
- Open Loop Reference (SORT)
- Closed Loop Simulation (MATLAB)
- Closed Loop Reference (SORT)
Flight Dynamics

- **ECF** – Earth Centered Fixed
  - $\phi$ – latitude
  - $\gamma$ – flight path angle
  - $\psi$ – azimuth
- $\sigma$ – bank angle
- $\theta$ – longitude

**Horizontal Plane Diagrams**
- $V$ – velocity
- $V_{proj}$ – projected velocity
- $x_b$, $y_b$, $z_b$ – body fixed coordinate
- $D$, $L$ – horizon

$b$ – body fixed coordinate
Trajectory Modeling

\[
\begin{align*}
\dot{r} &= V \sin \gamma \\
\dot{\theta} &= \frac{V \cos \gamma \sin \psi}{r \cos \phi} \\
\dot{\phi} &= \frac{V \cos \gamma \cos \psi}{r}
\end{align*}
\]

\[
\begin{align*}
\dot{V} &= -D - g \sin \gamma + \Omega^2 r \cos \phi (\sin \gamma \cos \phi - \cos \gamma \sin \phi \cos \psi) \\
\dot{\gamma} &= \frac{1}{V} \left[ L \cos \sigma + \cos \gamma \left( \frac{V^2}{r} - g \right) + 2\Omega V \cos \phi \sin \psi + \Omega^2 r \cos \phi (\cos \gamma \cos \phi + \sin \gamma \cos \psi \sin \phi) \right] \\
\dot{\psi} &= \frac{1}{V} \left[ L \sin \sigma + \frac{V^2}{r} \cos \gamma \sin \psi \tan \phi - 2\Omega V (\tan \gamma \cos \psi \cos \phi - \sin \phi) + \frac{r \Omega^2}{\cos \gamma} \sin \psi \sin \phi \cos \phi \right]
\end{align*}
\]

State Variables
- \( r \) - radial distance
- \( V \) - relative velocity
- \( \theta \) - longitude
- \( \phi \) - latitude
- \( \gamma \) - flight path angle
- \( \psi \) - azimuth

Control Variables
- \( \sigma \) - bank angle
- \( \alpha \) - angle of attack

Vehicle and Planet Variables
- \( L, D \) - Lift, Drag Acceleration
- \( g \) - gravity
- \( \Omega \) - Earth’s Rotation
- \( \rho \) - atmospheric density
General Entry Guidance Block Diagram

**Trajectory Solver**

**Reference Trajectory:** Analytical functions adapted from Shuttle Entry Guidance

**Bank Schedule Solution:** $\tilde{\sigma}_{cmd}$

**Range Prediction:** numerically solve equations of motion, range calculation

**Dispersed State:** $\tilde{y}_{disp}$

---

**Targeting Algorithm**

**Solver:** Single Point Optimal Control Solution from Energy State Approximation

**Purpose:** Targeting for precision landing and minimizing heatload

---

Send $\tilde{\sigma}_{cmd}$ to flight simulation

$R_{err} \approx 0$

Yes

$\tilde{\sigma}_{new}$

No
Control Solution: Shuttle Entry Guidance Adaptation

Shuttle Entry Guidance (SEG) Concept: Temperature Phase

- Reference Tracking Algorithm, Closed Form Solution

\[ \frac{d}{dt} \left( D = \rho V_r^2 C_D A \right) \]

Reference Trajectory

\[ D_{ref} = C_2 V^2 + C_1 V + C_0 \]
\[ \gamma_{ref} = \text{constant} \]

Bank Schedule

\[ \dot{\rho} = \frac{d}{dt} \left( \rho_0 e^{-\frac{h}{h_s}} \right) \Rightarrow \frac{\dot{h}}{h} = -\frac{\dot{\rho}}{\rho} \]
Control Solution: Shuttle Entry Guidance Adaptation

Improvements on Shuttle Entry Guidance “Drag Based Approach”

• Increase # of segments
• Increase order of polynomial
• Change Atmospheric Model representation
• Modify flight path angle representation

Challenges with Drag Based Approach

• Discontinuities between segments
• Increasing # of coefficients for storage with increasing segments and/or order
• Effect of small flight path angle assumption unknown
• Formulations are derived from 2DOF Longitudinal EOMs
Control Module: Shuttle Entry Guidance Adaptation

Sensitivity to atmospheric non-linearity is significant during initial and final segments. Need an Alternative Analytical Equation!
Automated Selection of Transition Events

Framework:
- Allows for adaptability
- Automated generation of Reference Trajectory
- Open loop

Study Objective: Define bank profile for trajectory phases

<table>
<thead>
<tr>
<th>Phase</th>
<th>Bank Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>Entry Interface to Guidance Start</em></td>
<td>Constant Bank</td>
</tr>
<tr>
<td><em>Guidance Start to Guidance End</em></td>
<td>Trajectory Solver</td>
</tr>
<tr>
<td><em>Guidance End to Exit</em></td>
<td>Linear Transition to Meet 2\textsuperscript{nd} Entry Bank</td>
</tr>
<tr>
<td><em>Exit to 2\textsuperscript{nd} Entry</em></td>
<td>Attitude Hold</td>
</tr>
</tbody>
</table>
Automated Selection of Transition Events

- Metric to determine best trajectory: lowest range error, lowest heat load from EI to 2nd Entry, and bank transitions
Automated Selection of Transition Events

Study Results:

<table>
<thead>
<tr>
<th>Phase</th>
<th>Bank Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>Entry Interface to Guidance Start</em></td>
<td>Constant Bank = 57.95°</td>
</tr>
<tr>
<td><em>Guidance Start to Guidance End</em></td>
<td>Trajectory Solver</td>
</tr>
<tr>
<td></td>
<td>{0.12 0.11} G’s</td>
</tr>
<tr>
<td><em>Guidance End to Exit</em></td>
<td>Linear Transition to Meet 2nd Entry Bank</td>
</tr>
<tr>
<td></td>
<td>Linear Transition Velocity: 23,784.65 ft/s</td>
</tr>
<tr>
<td><em>Exit to 2nd Entry</em></td>
<td>Bank Attitude Hold = 70°</td>
</tr>
</tbody>
</table>

![Diagram showing phases and events with labels](image)
General Entry Guidance Block Diagram

**Trajectory Solver**

**Reference Trajectory:** Analytical functions adapted from Shuttle Entry Guidance

**Bank Schedule Solution:** \( \vec{\sigma}_{cmd} \)

**Range Prediction:** numerically solve equations of motion, range calculation

**Dispersed State:** \( \vec{y}_{disp} \)

**Targeting Algorithm**

**Solver:** Single Point Optimal Control Solution from Energy State Approximation

**Purpose:** Targeting for precision landing and minimizing heatload

Send \( \vec{\sigma}_{cmd} \) to flight simulation

\( R_{err} \approx 0 \)

Yes

\( \vec{\sigma}_{new} \)

No
Targeting Algorithm Development

When is Targeting Activated?

1. Overshoot – Vehicle is predicted to fly **way past target**
2. Undershoot – Vehicle is predicted to fly **short of the target**

How to find a set of controls to Correct Over/Underhoot?

*Adapt Energy State Approximation Methods*: Optimal control method that replaces altitude and velocity with specific energy height ($e$)

$$ e = \frac{V_r^2}{2g_o} + h $$

**Advantages**: Allows for a compact set of analytical equations

- Add heat load to the range error objective function

**Disadvantage**: Optimal control formulations may not converge to a solution

**Solution**: Derive a **localized** optimal control point instead and blend back reference trajectory
Targeting Algorithm Development

Must Relate Euler-Lagrange Equation

\[ \bar{\lambda} = \frac{\lambda_\psi}{\lambda_\gamma} = \tan \sigma^* \cos \gamma \]
\[ \lambda_\gamma \leq 0 \]

To Reference Trajectory Variables

\[ \frac{L}{D_{\text{total}}} \cos \sigma = \frac{1}{\rho \Phi_{\text{ref}}} \left[ V_r \dot{\gamma}_{\text{ref}} \cos \gamma \left( \frac{V_r^2}{r^2} - g \right) - C_\gamma(y) \right] \]

Using trigonometry and other manipulations, the control equation is found

\[ \frac{L}{D_{\text{total}}} \sqrt{\frac{1}{1 + \left( \frac{\lambda}{\cos \gamma} \right)^2}} = \]
\[ \left[ V \gamma_{\text{ref}} \cos \gamma \left( \frac{V_r^2}{r^2} - g \right) - 2\Omega V \cos \phi \sin \psi - \Omega^2 r \cos \phi \left( \cos \gamma \cos \phi + \sin \gamma \cos \psi \sin \phi \right) \right] \]
\[ D_{\text{approx}} \]
Targeting Algorithm Development

\[ \Phi_{\text{ref}} = \frac{C_D A}{2m} \left| V_r^2 \right|_{\text{ref}} \]

Least Squares Curve Fitting:
3 Interpolation Points
\[ \Phi_{\text{blnd}} = Bb_2 V^2 + Bb_1 V + Bb_0 \]
Targeting Algorithm Development

Targeting Technique 1 – Design Space Interrogation

\( C_\phi \) - drag/density ratio coefficient

\( d\lambda \) - change in Lagrange multiplier

\( dV \) - change in relative velocity at next point

Targeting Technique 2 – Design Space Interrogation

\( d\lambda \) - change in Lagrange multiplier

\( dV_1 \) - change in relative velocity halfway to curve fit end point

\( \Delta(dE) \) - second order change in energy
Targeting Algorithm Development

Targeting Technique 1 – Design Space Interrogation

<table>
<thead>
<tr>
<th>Case</th>
<th>Dispersion</th>
<th>Target Miss</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Increase Entry Flight Path Angle</td>
<td>Undershoot</td>
</tr>
<tr>
<td>2</td>
<td>Decrease Entry Flight Path Angle</td>
<td>Overshoot</td>
</tr>
<tr>
<td>3</td>
<td>L/D Dispersion</td>
<td>Overshoot</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Lower Limit</th>
<th>Upper Limit</th>
<th>Incr.</th>
<th>units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_\Phi$</td>
<td>0</td>
<td>1</td>
<td>ND</td>
<td></td>
</tr>
<tr>
<td>$d\lambda$</td>
<td>0</td>
<td>1</td>
<td>0.01</td>
<td>ND</td>
</tr>
<tr>
<td>$dV$</td>
<td>100</td>
<td>1000</td>
<td>100</td>
<td>ft/s</td>
</tr>
</tbody>
</table>
Targeting Algorithm Development

FPA Dispersion - Undershoot
Targeting Algorithm Development

FPA Dispersion - Overshoot
Targeting Algorithm Development

Aerodynamic Dispersion - Overshoot
Shape Optimization Analog

Current Guidance Algorithms – Robust to ~20% aerodynamic dispersions

Must exceed 20% to demonstrate potential for integration into MDAO

**ANALOG:** Changing angle of attack disperses $C_L$ and $C_D$
Targeting Algorithm Development

Guidance Algorithm for Comparison – Apollo Derived Final Phase Guidance
Reference Tracking to a stored trajectory database, function of relative velocity

Performance Results – Threshold Miss Distance, 1 nmi
Targeting Algorithm Development

Targeting Technique 1 – Targeting Procedure

1. Guess a value for $d\lambda$
2. Iterate on $dV$ using secant method to converge on a zero range error trajectory
3. If no solution is found, $d\lambda$ is incremented and the iteration is repeated
4. Solution is then flown in flight simulation
Targeting Algorithm Development

Targeting Implementation, 1st and 2nd Phase - Results

\[
\begin{align*}
\alpha &= 152^\circ, \quad \text{L/D} = 0.418 \\
\alpha &= 153^\circ, \quad \text{L/D} = 0.402 \\
\alpha &= 154^\circ, \quad \text{L/D} = 0.386 \\
\alpha &= 155^\circ, \quad \text{L/D} = 0.371 \\
\alpha &= 156^\circ, \quad \text{L/D} = 0.357 \\
\alpha &= 157^\circ, \quad \text{L/D} = 0.343 \\
\alpha &= 158^\circ, \quad \text{L/D} = 0.328 \\
\alpha &= 159^\circ, \quad \text{L/D} = 0.313 \\
\alpha &= 160^\circ, \quad \text{L/D} = 0.299
\end{align*}
\]
Targeting Algorithm Development

Targeting Technique 2

Use Energy Height \( e = \frac{V_r^2}{2g_o} + h \) to determine Control Point \([V_{new}, \Phi_{new}]\)

Undershoot → energy dissipating (\(\frac{de}{dt}\)) too fast

Overshoot → energy dissipating (\(\frac{de}{dt}\)) too slow

Since Velocity is an independent variable and a pseudo control \(\frac{de}{dV}\) is examined
Targeting Algorithm Development

Targeting Technique 2

Recall the equation for the ratio of drag acceleration to density:

\[
\frac{D}{\rho} = \frac{C_D A}{2m} V_r^2
\]

-Extract altitude and velocity from \([dV_1, \Delta(dE)]\) to find \(\Phi_{new}\).
## Targeting Algorithm Development

**Targeting Technique 2 – Design Space Interrogation**

<table>
<thead>
<tr>
<th></th>
<th>Lower Limit</th>
<th>Upper Limit</th>
<th>Incr.</th>
<th>units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d\lambda$</td>
<td>0</td>
<td>$d\lambda_{\text{limit}}$</td>
<td>ND</td>
<td>ND</td>
</tr>
<tr>
<td>$dV_1$</td>
<td>0</td>
<td>1524</td>
<td>Predict</td>
<td>m/s</td>
</tr>
<tr>
<td>$\Delta(dE)$</td>
<td>0</td>
<td>$\Delta(dE)_{\text{limit}}$</td>
<td>Predict</td>
<td>m</td>
</tr>
</tbody>
</table>

Limit are trajectory dependent and control system dependent

$$\dot{\lambda}_{\text{min/max}} = \tan \sigma_{\text{min/max}} \cos \gamma_i$$

$$d\lambda_{\text{limit}} = \mp (\bar{\lambda}_{\text{min/max}} - \bar{\lambda}_{\text{old}})$$

### Dispersion Cases:

<table>
<thead>
<tr>
<th>$\alpha$ [deg]</th>
<th>L/D Dispersion</th>
<th>Target Miss</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal</td>
<td>0.4 (0%)</td>
<td></td>
</tr>
<tr>
<td>152</td>
<td>0.42 (+5%)</td>
<td>Undershoot</td>
</tr>
<tr>
<td>162</td>
<td>0.28 (-30%)</td>
<td>Overshoot</td>
</tr>
<tr>
<td>165</td>
<td>0.23 (-43%)</td>
<td>Overshoot</td>
</tr>
<tr>
<td>167</td>
<td>0.2 (-50%)</td>
<td>Undershoot</td>
</tr>
</tbody>
</table>
Targeting Algorithm Development

*Design Space Interrogation, Results: Range Error [%]*

\( \alpha = 152^\circ, \text{Undershoot} \)

\( \alpha = 162^\circ, \text{Overshoot} \)

\( \alpha = 165^\circ, \text{Overshoot} \)

\( \alpha = 167^\circ, \text{Undershoot} \)
Targeting Algorithm Development

**Design Space Interrogation, Results:** Heatload $[\text{J/cm}^2]$

- $\alpha = 152^\circ$, Undershoot
- $\alpha = 162^\circ$, Overshoot
- $\alpha = 165^\circ$, Overshoot
- $\alpha = 167^\circ$, Undershoot
Targeting Algorithm Development

*Design Space Interrogation, Results: Bank Rate [deg/s]*

\[ \alpha = 152^\circ, \text{Undershoot} \]

\[ \alpha = 162^\circ, \text{Overshoot} \]

\[ \alpha = 165^\circ, \text{Overshoot} \]

\[ \alpha = 167^\circ, \text{Undershoot} \]
Targeting Algorithm Development Results

Dispersions –
Apollo Derived Guidance = -20% dispersion
MDAO Algorithm = -43% dispersion

Managing heatload may be a challenge for dispersions greater than 20%
Conclusions

**Guidance Specific (In-Flight)**
- ✔️ Determine flight feasible control vectors (control rate/acceleration constraints)
  - Be highly robust to dispersions and perturbations
- ✔️ Include a minimal number of mission dependent guidance parameters

**Vehicle Design Specific**
- ✔️ Be applicable to multiple mission scenarios and vehicle dispersions
- ✔️ Manage the entry heat load in addition to achieving a precision landing
Acknowledgements

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UC Davis Mechanical and Aerospace Engineering Staff

Thank You !!!
Questions?
Additional Slides (optional)
## Overview

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<td>Introduction to Planetary Entry Guidance</td>
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<td>Dissertation Research Plan and Status</td>
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<td>MAPGUID Development</td>
<td>MAPGUID Proposed Approach</td>
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<td></td>
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<td></td>
<td>Key Results #2</td>
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<td>Key Results #3</td>
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<td>Key Results #4</td>
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<td>Proposed Approach</td>
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<td>During Guidance</td>
<td>Key Results #1</td>
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<td>Key Results #2</td>
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<td>Guidance/COBRA</td>
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<td>Integration</td>
<td>Key Results #1</td>
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<td>Key Results #2</td>
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<td>Key Results #3</td>
</tr>
<tr>
<td>Closing Remarks</td>
<td>Dissertation Findings and Status</td>
</tr>
</tbody>
</table>
Big Picture:
Spacecraft Design Process
MDAO Literature Review

Vehicle Optimization and TPS Sizing

Example Objective Function: \[ \dot{q}_{conv} = 1.83 \times 10^{-4} \sqrt{\rho R_n (1 - h_w / H_s)} V^3 \]

Results

- Most studies use a single trajectory to find altitude-velocity corresponding to maximum heat rate.
- Used for all geometries within optimization to find heat rate.
- Some studies use new trajectories, but there is no accounting for bank constraints or target accuracy.
- None of these studies incorporated flight feasible trajectories.

What is Flight Feasible?

- Reaches Target @ Landing Speeds
- Control does not exceed system limits
- Used for all geometries within optimization to find heat rate.
Proposed Approach to MDAO for Spacecraft Design

Vehicle Optimization

Planetary Entry Guidance

Thermal Protection System (TPS) Sizing

Structures

Aerodynamic ($C_d$, $C_l$) & Aerothermodynamic ($\dot{q}$) Databases

Flight Feasible Trajectory Database

Guidance, Navigation, & Control

Reduced Decoupled Iterations

Coupled

Available Descent Technologies

Un/manned

Planetary Models

Mission Profile

Computer Generated Spacecraft Models
Trajectory Modeling for Design vs. In-Flight Trajectory Modeling
Planetary Entry Guidance Literature Review

- **High L/D, Earth**: Space Shuttle, X-33, X40A
  - Most Robust: In Flight Trajectory Shaping with Reference Tracking
  - Least Robust: Reference Tracking Only
- **Low L/D, Earth**: Apollo, Orion
  - Most Robust: In-Flight Controls Search
  - Least Robust: Reference Tracking Only
- **Other Planetary Entry Vehicles**: MSR, MSL, Biconic
  - Flight Tested algorithms preferred
Planetary Entry Guidance Literature Review (cont’d)

**Key Results**

Modern guidance algorithms: optimal control is potential framework, but convergence still an issue
Trajectory Optimization Literature Review

*Trajectory Optimization*

*Traj* - Nonlinear constrained optimization

*Mission* - Sequential Quadratic Programming

*Energy State Method* – Reduced Order Modeling, one dimensional parameter search

*Pseudospectral Methods* – Combination indirect and direct method, mapping and discretization of domain
Trajectory Optimization Literature Review (cont’d)

Key Results

- **Curse of dimensionality:** Convergence time increases with dimensionality.
- **No convergence to a solution:** Fidelity of modeling may be compromised.
Introduction to Planetary Entry Guidance
Guidance Development Process

*Guidance must be robust to many dispersions: (Atmospheric properties, Aerodynamics properties, Navigational Inputs, Entry Interface Conditions, Mass, Control System performance, and many others)
Baseline Vehicle & Mission
## Case Study Parameters

<table>
<thead>
<tr>
<th>Vehicle</th>
<th>Orion Capsule, L/D = 0.4</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Trajectory</strong></td>
<td>Skip Entry for Lunar Return</td>
</tr>
<tr>
<td><strong>Control</strong></td>
<td>Bank Angle only</td>
</tr>
<tr>
<td><strong>Atmospheric Model</strong></td>
<td>1976 Standard Atmosphere</td>
</tr>
<tr>
<td><strong>Gravity Model</strong></td>
<td>Central Force + Zonal Harmonics</td>
</tr>
<tr>
<td><strong>Aerodynamics</strong></td>
<td>$C_L$, $C_D$ corresponding to Mach #</td>
</tr>
<tr>
<td></td>
<td>CBAERO Databases, function of Mach #, Dynamic Pressure, and Angle of Attack</td>
</tr>
<tr>
<td><strong>Trajectory Simulation</strong></td>
<td>MATLAB Simulation validated against SORT Trajectories</td>
</tr>
</tbody>
</table>
Trajectory Simulations Developed

Open Loop Numerical Predictor- Corrector (NPC) Simulation
Used to test guidance formulations

3DOF Rotating Spherical Planet

\[
\begin{align*}
\dot{r} &= V \sin \gamma \\
\dot{\theta} &= \frac{V \cos \gamma \sin \psi}{r \cos \phi} \\
\dot{\phi} &= \frac{V \cos \gamma \cos \psi}{r} \\
\dot{V} &= -D - g \sin \gamma + \Omega^2 r \cos \phi (\sin \gamma \cos \phi - \cos \gamma \sin \phi \cos \psi) \\
\dot{\psi} &= \frac{1}{V} \left[ L \cos \sigma + \cos \gamma \left( \frac{V^2}{r} - g \right) + 2 \Omega V \cos \phi \sin \psi + \Omega^2 r \cos \phi (\cos \gamma \cos \phi + \sin \gamma \cos \psi \sin \phi) \right] \\
\end{align*}
\]

Flight Simulation - Closed Loop Guidance Testing
Using equations derived from Newton’s 2nd Law, dynamics of relative motion, and Earth Centered Inertial (ECI) coordinate system
Trajectory Solver Development
## Control Solution: Shuttle Entry Guidance Adaptation

<table>
<thead>
<tr>
<th>Segments</th>
<th>Order</th>
<th># of stored coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>7 (3)</td>
<td>Irrational</td>
<td>168</td>
</tr>
<tr>
<td>7 (5)</td>
<td>Irrational</td>
<td>105</td>
</tr>
<tr>
<td>14</td>
<td>5</td>
<td>84</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>21</td>
</tr>
</tbody>
</table>

\[ D_{ref} = C_2 V^{x_2} + C_1 V^{x_1} + C_0 V^{x_0} \]

![Graph showing drag curve fit accuracy with plotted data points.](image)
Control Solution: Shuttle Entry Guidance Adaptation

Would Cubic Spline Interpolation work?

\[ h_s = \left( \frac{1}{P} \frac{dP}{dh} - \frac{1}{T} \frac{dT}{dh} \right)^{-1} \]
Targeting Algorithm Development
Targeting Algorithm Development

Targeting Technique 1 – Trajectory Behavior to Full Set of Aerodynamic Dispersion

Can Technique 1 find a trajectory that points toward correcting the range error?
General Conclusions
Targeting Algorithm Development

Pontryagin’s Principle in Optimal Control

Find Optimal Control \( \vec{u}^* \) and \( \sigma^*(t) \) and \( V^*(t) \)

for dynamic system \( \dot{x} = f(\vec{x}, \vec{u}, t) \)

The optimal control satisfies the constraints including the Euler-Lagrange Equation:

\[
\dot{\phi} = \frac{V \cos \gamma}{r} \quad \lambda = \frac{\lambda_\psi}{\lambda_\gamma} = \tan \sigma^* \cos \gamma
\]

\[
\gamma = \frac{1}{V} \left[ L \cos \sigma + \cos \gamma \left( \frac{1}{r} - g \right) + 2\Omega \psi \cos \phi \sin \psi + \Omega^2 r \cos \phi (\cos \gamma \cos \phi + \sin \gamma \cos \psi \sin \phi) \right]
\]

\[
\dot{\psi} = \frac{1}{V} \left[ \frac{L \sin \sigma}{\cos \gamma} + \frac{V^2}{r} \cos \gamma \sin \psi \tan \phi - 2\Omega V (\tan \gamma \cos \psi \cos \phi - \sin \phi) + \frac{r \Omega^2}{\cos \gamma} \sin \psi \sin \phi \cos \phi \right]
\]
Targeting Algorithm Development

Targeting Technique 1

\[ \dot{\lambda}_{\text{new}} = \lambda_{\text{old}} \pm d\lambda \]

Determines new bank angle at current time step

\[ \dot{\gamma} = \frac{1}{V} \left[ L \cos \sigma + \cos \gamma \left( \frac{V^2}{r} - g \right) + 2\Omega V \cos \phi \sin \psi 
+ \Omega^2 r \cos \phi (\cos \gamma \cos \phi + \sin \gamma \cos \psi \sin \phi) \right] \]

Calibrated for Each Dispersed Case

\[ \Phi_{\text{new bound}} = \sqrt{1 + \left( \frac{\dot{\lambda}}{\cos \gamma} \right)^2 \left[ \dot{\gamma}_{\text{ref}} - \cos \gamma \left( \frac{V^2}{r} - g \right) - 2\Omega V \cos \phi \sin \psi - \Omega^2 r \cos \phi (\cos \gamma \cos \phi + \sin \gamma \cos \psi \sin \phi) \right]} \]

\[ \Phi_{\text{new}} = \Phi_{\text{old}} \pm C_\Phi |\Phi_{\text{old}} - \Phi_{\text{new bound}}| \]

Determines Blended Trajectory that nulls range error

\[ V_{\text{initial}} = V_{\text{current}} + 0.01 dV \]

\[ V_{\text{ref}, f} = V_{\text{current}} - (1 - 0.01) dV \]
Targeting Algorithm Development
Targeting Technique 1 – Design Space Interrogation

• The blending technique exhibits potential to find new bank profiles that null the range error

• The design space is constrained by control system limitations

• There is a zero range error solution for each change in \( d\lambda \)
Targeting Algorithm Development

Targeting Technique 1 – Trajectory Behavior to Full Set of Aerodynamic Dispersion

Why did this not follow the Expected Behavior?

The reference bank profile over-corrects with respect to the dispersion of L/D
Targeting Algorithm Development

Targeting Technique 2

Now that the blended function is fully defined \( \Phi_{\text{blnd}} = Bb_2V^2 + Bb_1V + Bb_0 \)

The following equation can be used to solve for:

\[
\dot{\gamma}_{i,\text{new}} = \frac{1}{V_i} \left[ \frac{L}{D}_{\text{total},i} \sqrt{\frac{1}{1 + \left( \frac{1}{\lambda} \frac{1}{\cos \gamma_i} \right)^2 \rho_i \Phi_{\text{blnd},i} + 2 \Omega V_i \cos \phi_i \sin \psi_i +} \right. \\
\cos \gamma_i \left( \frac{V_i^2}{r_i} - g_i \right) + \Omega^2 r_i \cos \phi_i (\cos \gamma_i \cos \phi_i + \sin \gamma_i \cos \psi_i \sin \phi_i) \right]
\]

The FPA rate table is shifted accordingly.
Targeting Algorithm Development

Design Space Interrogation, Results: Bank Acceleration [deg/s^2]

\( \alpha = 152^\circ, \text{ Undershoot} \)

\( \alpha = 162^\circ, \text{ Overshoot} \)

\( \alpha = 165^\circ, \text{ Overshoot} \)

\( \alpha = 167^\circ, \text{ Undershoot} \)
Targeting Algorithm Development

Targeting Technique 1 – Targeting Implementation, 1st and 2nd Phase

1. Guess a value for $d\lambda$

2. Iterate on $dV$ using secant method to converge on a zero range error trajectory

3. If no solution is found $d\lambda$ is incremented and the iteration is repeated

4. Solution is then flown in flight simulation

Performance Metric –

Compare range of aerodynamic dispersions this algorithm can handle to the range of aerodynamic dispersions a heritage algorithm can handle.
Trajectory Solver Research Questions

Can a simplification in the equations of motion be made without loss of accuracy?

Can a simplification on flight path angle be made without loss of accuracy?
Simplified Equations of Motion Study

3DOF Rotating, Spherical Earth (3RSP)

\[ \dot{\psi} = \frac{1}{V} \left[ \frac{L \sin \sigma}{\cos \gamma} + \frac{V^2}{r} \cos \gamma \sin \psi \tan \phi - 2\Omega V (\tan \gamma \cos \psi \cos \phi - \sin \phi) + \frac{r \Omega^2}{\cos \gamma} \sin \psi \sin \phi \cos \phi \right] \]

3DOF Non-Rotating Spherical Planet

\[ \dot{\psi} = \frac{1}{V} \left[ \frac{L \sin \sigma}{\cos \gamma} + \frac{V^2}{r} \cos \gamma \sin \psi \tan \phi \right] \]

3DOF Non-Rotating Flat Planet

\[ \dot{\psi} = \frac{1}{V} \left[ \frac{L \sin \sigma}{\cos \gamma} \right] \]

2DOF Longitudinal Equations (2LON)

\[ \dot{h} = V \sin \gamma \]
\[ \dot{s} = V \cos \gamma \]
\[ \dot{V} = -D - g \sin \gamma \]
\[ \dot{\gamma} = \frac{1}{V} \left[ L \cos \sigma + \cos \gamma \left( \frac{V^2}{r} - g \right) \right] \]
Simplified Equations of Motion Study (cont’ d)
Simplified Equations of Motion Study (cont’ d)
Trajectory Solver Research Questions

Can a simplification in the EOMs be made without loss of accuracy?
Not for a skip trajectory

Can a simplification on flight path angle be made without loss of accuracy?
Control Solution: Shuttle Entry Guidance Adaptation

\[
\dot{h}_{ref} = -h_s \left[ \frac{\dot{D}_{ref}}{D_{ref}} - \frac{2\dot{V}}{V} - \frac{\dot{C}_D}{C_D} \right]
\]

\( h_s = \frac{RT}{g} \)

Four Drag Segments (~30 points/segment) Quadratic Curve Fit for Drag

- Colored stars indicate the end of a segment.
- Number of segment goes in increasing order with time.
- Open circles correspond to the nominal profile.
Control Solution: Shuttle Entry Guidance Adaptation
Control Solution: Shuttle Entry Guidance Adaptation

\[ P = \rho RT \]

\[ \frac{\dot{\rho}}{\rho} = \frac{\dot{P}}{P} - \frac{\dot{T}}{T} \]

\[ h_s = \left( \frac{1}{\frac{dP}{dh}} - \frac{1}{\frac{dT}{dh}} \right)^{-1} \]

![Graph showing atmospheric temperature slope changes with altitude and altitude rate.](image-url)
Control Solution: Shuttle Entry Guidance Adaptation

Need to Resolve 1st Segment to Capture Atmospheric Non-Linearity

IDEA: Curve fit drag with Mach Number

\[ D_{ref} = \sum_{i=1}^{n} C_i Ma^i \]
Control Solution: Shuttle Entry Guidance Adaptation

Check Altitude Acceleration Approximation
Trajectory Solver Research Questions

Can a simplification in the EOMs be made without loss of accuracy?  
**Not for a skip trajectory**

Can a simplification on flight path angle be made without loss of accuracy?
Range Prediction Sensitivity to Flight Path Angle Assumption

- Apollo and Shuttle Entry guidance formulations approximate flight path angle (FPA) to be small: $\gamma \ll 1 \text{ rad}$ and/or $\dot{\gamma} \ll 1 \text{ rad/s}$

Why does this matter?
- If predicted range does not equal the range to landing site then targeting is erroneously active
- Are model reductions in the Trajectory Module and Control Module valid based on the nominal case?
Range Prediction Sensitivity to Flight Path Angle Assumption

Case Studies:

A. Apply $\gamma << 1\, \text{rad}$ to Trajectory Module only

B. Apply $\gamma << 1\, \text{rad}$ to Controls Module only

C. Apply $\dot{\gamma} << 1\, \text{rad/s}$ to bank equation only

\[
\frac{L}{D_{v,ref}} = \frac{1}{\rho \Phi_{ref}} \left[ V_r \dot{\gamma}_{ref} - \cos \gamma \left( \frac{V_r^2}{r} - g \right) - C_{\gamma}(y) \right]
\]
Range Prediction Sensitivity to Flight Path Angle Assumption

**Nominal** 661.73 [nmi]

<table>
<thead>
<tr>
<th>Case</th>
<th>Total Range [nmi]</th>
<th>% Range Error</th>
<th>Termination</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>662.39</td>
<td>0.099%</td>
<td>Drag Limit</td>
</tr>
<tr>
<td>B</td>
<td>649.74</td>
<td>1.813%</td>
<td>Drag Limit</td>
</tr>
<tr>
<td>C</td>
<td>632.13</td>
<td>4.474%</td>
<td>Velocity Limit</td>
</tr>
</tbody>
</table>

**Conclusion** *FPA* approximation can be applied to the *trajectory module*, but not to the *control module*