SEMI-ANNUAL PROGRESS REPORT

SCATTER THEORIES AND THEIR APPLICATION TO LUNAR RADAR RETURN

By

H. S. HAYRE

PR-33

September 1961

Work Conducted Under
National Aeronautics and Space Administration
Grant NsG 129-61
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The research work being done under this NASA grant is divided into the following three categories:

(1) An estimate of the radar return for the NASA Aerobee rocket shot at White Sands Missile Range. (WSMR)

(2) Development of new scatter theories, modification and correlation of existing scatter theories, and application of the theories to moon-echo data for estimation of the surface features of the moon.

(3) Acoustic modeling of the lunar surface and correlation of the theoretical with both full scale and acoustical experimental results.

**AEROBEE ROCKET SHOT AT WSMR**

Dr. W. W. Koepsel and Mr. H. S. Hayre made a trip to JPL, Pasadena, to obtain the available details of the Aerobee rocket radar, and other system parameters including information on the mode of receiving and recording data. Moreover the plan of work and the progress made by that time were also discussed with Mr. Walter E. Brown, Jr.

A theoretical estimate of radar scattering cross-section, pulse to pulse fading, range of fading (5-95%), pulse stretching, and mean peak power returned has been made. It was based on the data on the surface roughness obtained from the area maps and information on the radar and receivers obtained from JPL. An acoustic model of the area of impact (WSMR) of the rocket was prepared. The acoustic experiment will be performed upon receipt of wide beam sonic transducers around the first week of September. These transducers were ordered when all other efforts to obtain a wide beam pattern with various reflectors and collimeters proved unsuccessful.
Further work on the experimental design of wide beam transducers will be limited to experimenting with very small hemispheres of barium titanate. An experimental (acoustic) verification of the theoretical estimate will then be made and a revised estimate will be submitted.

**SCATTER THEORY AND ESTIMATION OF LUNAR SURFACE FEATURES**

The Davies-Moore-Hayre model for rough terrain was used to derive a theoretical expression for radar scattering cross-section. This was applied to the moon echo power data reported by Hughes (1960) and Pettengill (1960). This resulted in a paper entitled "Radar Cross Section - Applied to Moon Return," which has been accepted for publication in the IRE Proceedings. Reprints of this article have not as yet been received.

Another paper dealing with the determination of the quantized order of roughness for a surface, and its subsequent application to an estimate of the lunar surface near its center has been prepared. A copy of this paper is enclosed and it will be submitted for possible publication in a technical journal.

The moon's surface was modeled acoustically using spun aluminum spheres of 16.1 centimeter outside diameter. The surface roughness was modeled using various sizes of sand, acrylic plastic spray and wrinkle varnish. The spheres were rotated around their axis, while the transmitter-receiver transducers were kept fixed at a position. The distance (range) of the spheres from the transducers was also varied from 1.47 meters to 4.26 meters in four steps. The results of this experiment are summarized in the enclosed abstract, which is being sent to URSI for approval for presentation at IRE/URSI meeting in October 1961 at Austin, Texas. Further experimental work (acoustic) will be carried on using 30.5 centimeter aluminum spheres as well as on 7.6 centimeter solid wooden balls.
Folding of terrain elevation into amplitudes less than or equal to a wavelength has also been studied and a report or a paper will be prepared soon.

**FUTURE RESEARCH**

The following phases of research will be continued in the future.

(i) Estimation of the Lunar Surface
   a) Surface Roughness
      (a.1) Application of scatter theories to the available moon echo data
      (a.2) Acoustic simulation of the lunar surface
   b) Electrical Properties
      Reliable existing lunar echo data will be used to infer some probable estimate of the electrical properties of the lunar surface material

(ii) White Sands Missile Range - Aerobee Rocket Shot
    Acoustic simulation of Aerobee radar shot and submittal of the University of New Mexico estimate prior to the actual shot.

(iii) Estimation of Surface Roughness in General from Radar Data
    a) Statistical description of roughness
    b) Fading as an indicator of surface roughness
    c) Effect of range on fading
    d) Folding of terrain elevations into amplitudes less than or equal to a wavelength
    e) Roughness determination from return power fading and variance spectrum
    f) Specular-scatter component variation with various orders of roughness.
It will be proposed that the radar data and television pictures of the moon taken by Ranger/s be correlated to improve the estimates of the roughness of the lunar surface. This work shall be proposed to be covered by supplementary appropriations for an extension of this grant.
PUBLICATIONS


TRAVEL

The following trips were made by the research personnel for purposes of discussion of the research work, attending NASA sponsored conferences and exchanging research notes with other people in this or allied fields:

1. Dr. R. K. Moore attended the conference of the Advisory Group for Radio Experiments in Space at the University of Texas, Austin, Texas on February, 1961.

2. Dr. W. W. Koepsel and Mr. H. S. Hayre travelled to JPL Pasadena, for discussions with Mr. W. E. Brown, Jr.

3. Dr. W. W. Koepsel attended AGRES meeting at Cambridge, Mass. in June, 1961 and visited the Naval Research Laboratories in Washington, D. C., for discussion of lunar research being done there.

4. Dr. R. K. Moore visited Ohio State University with Dr. W. W. Koepsel to discuss certain work on scattering conducted by their personnel.
SURFACE ROUGHNESS OF THE MOON*

By H. S. Hayre

Electrical Engineering Department

University of New Mexico

ABSTRACT

The radar scattering cross-section obtained for the Davies-Moore-Hayre model for rough terrain is shown to fit all the existing reliable lunar radar scattering cross-section data as analyzed by this and other authors. It is also suggested that the angles of incidence for study of radar return from the moon's surface should be divided into three approximate ranges namely zero to three degrees, three to fifteen degrees and fifteen to ninety degrees. The radar cross-section is then analyzed in each range of angles to estimate the order of roughness of the moon's surface.

Moore's separation technique for specular and scatter components is used to obtain a quantized description of almost smooth and rough terrains. The results so obtained are then compared with Evans' definition of a rough surface.

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*This work is sponsored by National Aeronautics and Space Administration under grant NsG 129-61.
In recent years considerable interest has been aroused in the radar determination of the order of surface roughness of terrestrial bodies. In particular, quite a few (Evans 1960, Hayre 1961, Hughes 1960, Leadabrand 1960, Pettengill 1960) attempts have been made recently using the results of monostatic and bistatic radar reception of lunar echoes, to determine the roughness of the visible lunar surface. This paper attempts to quantize the word "rough" in terms of statistical properties of a surface. This definition is applied to the moon return.

Surfaces are often referred to as quasi-smooth, smooth, almost smooth, rough and very rough, etc., but rarely is the roughness specified quantitatively. This is closely related to separation of specular and scatter return of electromagnetic waves from a surface, where specular return is defined as the return from a mirror-like perfectly smooth surface, and the scatter as that from a non-smooth surface. In practice, the return from a rough surface can be said to be made up of both specular and scatter components. It is from this that some inference (Moore 1957, Senior 1960) can be made about the nature of roughness. Any relative motion of the radar and the target area causes fading of the returned signal. The fading spectrum has also been used to detect the order of roughness, as fading statistics of signal return from a particular terrain are determined by the statistical description of the surface, the relative motion, the carrier frequency and other radar parameters.

The radar scattering cross-section ($\sigma_o$) is also employed in the prediction of surface roughness of the target terrain. The plot of radar scattering cross-section versus the angle of incidence (9)
tends to be flat for very rough surfaces. It will be shown later that this curve approaches \( \cos^2 \theta \) variation in the limit for rough surfaces. For most surfaces, the \( \sigma_0 \) vs. \( \theta \) curve becomes steep as the angle of incidence approaches zero (vertical incidence) as shown by various authors (Dye 1959, Hayre and Moore 1961).

Some authors (Evans 1960, 1961, Leadabrand 1960, Senior 1960) have suggested that the center portion of the visible surface of the moon is quasi-smooth, because approximately fifty percent of the echo power is returned from a central region, approximately 210 miles in diameter.

A study of some recent moon echo results (Evans 1960, 1961, Pettengil 1960) seems to suggest that it may be necessary to analyze the variation of radar cross-section with the angle of incidence in three different ranges. This is necessitated because small changes in \( \theta \) correspond to very large changes in distance from the center of the moon surface. For example, \( \theta = \alpha + B \), and \( \alpha_{\text{max}} = 17 \) minutes where \( 2\alpha \) and \( 2B \) are the angles subtended by an incremental ring of the illuminated area on the moon surface at the observation point on the earth and the center of the moon respectively. The angle of incidence (\( \theta \)) varies from zero to ninety degrees as the angle \( \alpha \) varies from zero to 17 minutes, and the corresponding radius from the center of the visible surface to the illuminated area changes from zero to the radius of the moon. This would then imply that one would associate different roughness characteristics with each range of angles as discussed later in this article.

If the range is far greater than the size of the largest perturbation of the target terrain, the change in the magnitude of the field produced at the receiving antenna due to variation with
range of the amplitude of individual facet returns is negligible, while the corresponding effect on the phase variation between returns from separate facets will be predominant. In theoretical analysis of pulse radar return, it is sometimes assumed (Brown 1960) that the surface perturbations can be "folded" into irregularities, whose magnitudes are of the order of a wavelength or less. In case of moon study, with radar frequency of 1 KMC, the returned signal from the top of a crater rim, 300 meters above the average lunar surface, would be essentially similar to that returned from the average surface. This seems to suggest that the dividing line between rough and very rough surface lies around irregularities of the order of one half wavelength. The discussion of this subject in the following paragraphs shows that this transition occurs in the range of values of $\sigma/\lambda$ from approximately 0.1125 to 0.125.

Various models have so far been used to predict the roughness of the target terrain from the radar return data. The Davies-Moore-Hayre model (Hayre and Moore 1961) uses the experimental autocovariance of the elevations and their probability density function, in applying the Kirchhoff-Huygens' principle. Certain standard assumptions about perfect conductivity, isotropic scatterers, near-vertical incidence and no part of the terrain being shadowed by any other part, etc., were employed. Contour maps were used to calculate the autocovariance functions. This resulted in the following expression for the radar scattering cross-section

$$
\sigma_o = \left(\frac{4k \pi k^2}{\lambda^2}\right)\left(\frac{\theta \cos^2 \phi}{\sin \theta}\right) \exp\left(-4 k^2 \frac{e^{-2} \cos^2 \theta}{\sin \theta}\right) \sum_{n=1}^{\infty} \left[\left(\frac{4 k^2 e^{-2} \cos^2 \theta}{(n-1)!}\right)\right] \left[2 k^2 e^{-2} \sin^2 \theta + n^2\right]^{-3/2}
$$

(1)
where \( \sigma \) = standard deviation of elevation of points on the surface above the mean surface,

\( \lambda \) = wavelength

\( k = \frac{2\pi}{\lambda} \), wave number,

\( \Theta = \) angle of incidence

\( B = \) roughness characteristic constant (in autocovariance function)

In case of an area with large value for the standard deviation, the scattering cross-section \( \sigma_0 \), in db will be essentially proportional to \( \cos^2\Theta \), as is evident from Equation (1), because 

\[
\log \left( \frac{4\sqrt{2} \pi B^2/\lambda^2}{9\cos^2\Theta/\sin\Theta} \right) (9\cos^2\Theta/\sin\Theta)
\]

will be negligible as compared to 

\[ -4k^2 \sigma^{-2} \cos^2\Theta. \]

Similarly, the contribution from the log of the infinite sum, will also be negligible.

Some other authors (Leadabrand 1960, Pettengill 1960) have asserted that the resulting \( \cos^2\Theta \) variation is a good fit for the lunar radar scattering cross-section (db) for incident angles greater than 13-15 degrees.

Roughness may be specified quantitatively in terms of standard deviation (\( \sigma \)) and autocovariance constant (\( B \)), maximum and minimum size of the surface perturbations or a roughness coefficient contained in the probability density function for the areas at a particular elevation (Brown, 60). A search of the literature on the subject shows that an order of roughness separating an 'almost smooth' and a rough terrain has not been specified. For instance, Evans (1960) assumes that a surface is rough if the size of its perturbations are greater than one eighth of a wavelength. The
distribution of such perturbation above and below this size is not discussed. Moore (1957) uses an interesting technique to separate the specular and scatter radar return components. The reduction of the specular component with increasing roughness may be used to indicate a transitional order of roughness between an almost smooth surface and a rough surface. Moore lists a separation factor 'a' which shows reduction in specular due to energy going into scatter, equal to \( \exp\left(\frac{-2k^2 \sigma^2}{\lambda^2}\right) \). When the exponent \( 2k^2 \sigma^2 \) becomes unity, the factor 'a' reduces to 0.368. The value of \( \sigma^2/\lambda \) corresponding to this reduction in specular component is 0.1125, and it is comparable with the assumed value given by Evans.

The scattering cross-section versus the angle of incidence curve may also be studied for some indication of a roughness transition zone between 'almost smooth' and rough surfaces. Davies-Moore-Hayre's model for roughness terrain, (approximately described by Equation (1) for scattering cross-section) passes in the limit as \( \sigma \to 0 \) to an impulse function \( \delta (\theta = 0, \sigma = 0) \). The scattering cross-section curve is very steep near zero angle of incidence for a slightly rough surface. Now as the roughness is increased, this curve becomes flatter but never reaches a horizontal line. For given dielectric properties, some of these curves of \( \sigma_0 \) vs. \( \theta \) for varying degrees of roughness, cross each other in the range of angles between zero and about ten degrees. The constants \( \sigma^2 \) and B are indicative of the order of roughness of a surface. A small \( \sigma^2 \) and a large B represent a surface that appears smooth to the eye and "relatively smooth" to the radar while a comparatively large \( \sigma^2 \) and a small B would represent a "relatively rough" surface.
The above discussion suggests that the specification of standard deviation and the autocovariance constant, $B$, in terms of the wavelength used, is necessary for qualifying the target terrain as "rough" or "very rough." All other such adjectives as quasi-smooth etc., would thus be eliminated.

As previously suggested (Hayre 1961), the ratio of standard deviation to the wavelength ($\sigma/\lambda$) of approximately 0.1 and $B/\lambda$ of 1.0 correspond to the lunar radar cross-section calculated from radar data taken at wavelength of 10 cm, using a pulse length of 5 micro seconds (Hughes 1960).

The suggestion of analysis of lunar radar return data by ranges of angle of incidence, may now be applied to the moon echo data collected and analyzed by various authors so far (Evans 1960, 1961, Hugfors 1960, Hughes 1960, Leadebrand 1960, Pettengill 1960). The general expression for radar scattering cross-section (1) not only fits the approximate expressions calculated for various ranges of angles such as listed below, but also gives expected results in the range of angle from zero to three degrees:

$$\sigma_0 \propto \exp (-10 \theta) \quad \text{or} \quad \exp (-10 \sin \theta) \quad \text{for} \quad 3^\circ \leq \theta \leq 14^\circ \quad (2)$$

$$\sigma_0 \propto \cos^2 \theta \quad \text{for} \quad 14^\circ \leq \theta \leq 90^\circ \quad (3)$$

Equation (1) can be approximated by the following expression

$$\sigma_0 \propto (B/\lambda)^2 \exp \left(-4 \frac{\sigma^2 \epsilon^2}{\lambda^2}\right) \sum_{n=1}^{\infty} \left[\left(\frac{4 \epsilon^2 \sigma^2}{\lambda^2}\right)^n\right] \left[2 B^2 \epsilon^2 \sigma^2 + \epsilon^2 \right]^{-3/2} \quad \text{for} \quad 0^\circ \leq \theta \leq 3^\circ \quad (4)$$

as $\cos \theta$ and $\sin \theta$ are essentially unity and $\theta$ respectively for this range of angles.
Conclusions

It was recently shown (Hayre 1961) that the radar scattering cross-section expression (1) derived from the Davies-Moore-Hayre model (1961) fits the experimental moon echo data taken by Hughes (1960) for $\sigma / \lambda = 0.1$ and $B / \lambda = 1.0$ in the range of angles of incidence from $3^\circ$ to $14^\circ$. Later on Pettengill (1960), approximated his experimental lunar scattering cross-section variation by $\exp(-10.1 \sin \theta)$. Other results obtained by Evans (1960) at 2.5 meter wavelength, Leadabrand (1960) at 0.75 meter wavelength, and Trexler (1958) at 1.5 meter wavelength indicate that the above results concerning the shape and distribution of received power versus the lag time are reasonable. The wavelengths corresponding to the frequencies used by Hughes and Pettengill are 0.1 and 0.682 meter respectively. The same power distribution for more than one frequency can only be explained by the fact that a certain given rough surface may give the same reflected power because the scattered power vs. the order of roughness has maxima and minima in it (Born et al 1959). In view of this, let us be rather conservative and select the largest wavelength (2.5 meter) from this group. Now if the criterion mentioned in the beginning of this paper and paragraph is used, it would seem that the central portion of the visible lunar surface is indeed rough. The standard deviation of this surface may be of the order of $0.1 \times 2.5 = 0.25$ meters. The autocovariance constant $B$ may be approximately 2.5 or so. These numbers may be changed, if one wants to be further conservative and employ a safety factor.

Nothing may be inferred about the presence or absence of dust layers, if any, from the above analysis. The above estimate
of the standard deviation of the central lunar surface, included in about one-tenth radius of the moon, does seem to give an indication that crevices deeper than two times the standard deviation (0.5 meters) may only exist in small number. The probability of existence of such crevices would be less than approximately 4.55%, if the distribution of surface perturbations above and below the average lunar surface is assumed to be normal. Further work on the order of roughness of the moon's surface is very essential for the successful instrumentation of the soft lunar landing surveyor.
REFERENCES


Hayre, H. S., "Radar Scattering Cross-Section Applied to Moon Return," (Accepted for publication in I.R.E. Proc.).


ABSTRACT

ACOUSTIC SIMULATION OF MOON-ECHOES*

By
H. S. Hayre, W. W. Koepsel, R. J. Tillery and D. W. Boone

Linear and non-linear acoustic modeling of radar return from terrain has been successfully done for the last two years at the University of New Mexico. Statistical information such as range of fading, mean power, rate of fading etc., are parameters determined by these experiments in addition to the radar scattering cross section. These techniques were employed to model the moon surface on a set of 16.1 centimeter diameter aluminum spheres. The roughness of surface features was modeled on a scale relative to that found by telescope. The order of roughness was varied from smooth polished surface to that with sand particles of sizes up to 1.168 millimeters glued on it. Four values for the range of the sphere from the transmitting-receiving transducers were used. These were 18.24, 30.64, 43.10 and 52.94 radii of the sphere representing the moon. The range scale differed from the wavelength scale as was required by the size of the acoustic tank, the transducer and the power limitation of the system. The transducers were kept stationary, while the spheres were rotated around their axes, in order to simulate the relative rotation of the earth with respect to the moon.

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1Electrical Engineering Department, University of New Mexico.

2Secondary School Teacher, Participant, National Science Foundation, Engineering Research Program.
Incident pulses of 4 $\mu$ seconds duration at 1.02 mc with 20 PPS were used and the return pulse was amplified for scope presentation. A strip camera was used to record this presentation of data. The reduction and analysis of this data resulted in the following observations:

(i) The average range of fading seems to increase with an increased order of surface roughness. There is some indication of its gradual increase with range as well.

(ii) The average range of fading for the return signals from the entire sphere at a fixed range is approximately 15 db, which was reported* as a typical average fading range for a wide variety of terrains.

(iii) More than 50% of the power return is contained in the return from the region bounded by 0.3-0.4 radius in the case of spheres with acrylic plastic layer (0.0127-0.0254 mm) and wrinkle paint (0.126-0.203 mm).

(iv) The range of fading seems to increase from 4.0 ± 0.45 db at maximum surface roughness of 0.0127 mm to 11 ± 4.6 db at an order of maximum surface roughness of 1.168 mm.

Further work on smaller and larger size spheres with increased order of roughness is in progress at the University of New Mexico.