DEVELOPMENT OF A COUPLED AIR AND PARTICLE THERMAL MODEL FOR ENGINE ICING TEST FACILITIES

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Introduction
Icing Observations

• Many engine power-loss events reported since the 1990’s
• Mason et al.\textsuperscript{1} hypothesized how power-loss events can result from ice crystals entering the engine core
• Ingestion of ice into engine is studied
• NASA conducted tests at NRC’s RATFac
• Observed environmental conditions changed with cloud activation
  – Air temperature change
  – Air humidity change
  – Particle water content change
• Hypothesis: Thermal interaction between air and cloud particles

\textsuperscript{1}Mason, J., Strap, J., and Chow, P., “The Ice particle Threat to Engines in Flight,” presented at 44\textsuperscript{th} AIAA Aerospace Sciences Meeting and Exhibit 2006, USA, January 9-12, doi: 10.2514/6.2006-206
Thermal Model General Description

- Model couples air and cloud particle conservation equations
  - Mass, energy and momentum
- Simulates icing tunnel (applicable to engines too)
- Ice, water, and humid air mass broken into fundamental CV units
  - Uniform particle size (MVD representation)
  - Full particle size distribution
Assumptions

- Air and water vapor are treated as ideal gases
- Air is continually well mixed
- No supersaturation
- 1-D air and particle flow
- Dilute system (no particle interaction)
- Particles are spherical
- Discrete particle size distribution (bins)
- Uniform temperature within the particle
- Supercooling can occur
- Mixed phase particles are not spatially resolved
- Phase change occurs at particle surface at particle temperature
- Adiabatic tunnel walls
- The flow of particles and air is a continuous stream
- The fundamental CV is adiabatic and mass is conserved
  - Provisions for heat and mass transfer added using source terms
Particle Conservation of Energy
Single Particle Formulation (1/2)

• Change in the particle’s enthalpy, is due to the convective heat transfer and latent energy exchange due to mass transfer

\[
\frac{\partial H_p}{\partial t} = q_{\text{conv}} + q_{\text{latent}}
\]

• Rate of mass change with a water surface is proportional to the difference in vapor pressure between the particle surface and the ambient air

Ice: \( \rho_p C_p \frac{\pi d^3}{6} \frac{\partial T_p}{\partial t} = \pi d^2 h(T_{\text{air}} - T_p) + \pi d^2 h_m \rho_{\text{air}} L_{\text{subl}} (\omega_{\text{air}} - \omega_p) \)

Mix: \( \rho_p L_{\text{melt}} \frac{\pi d^3}{6} \frac{\partial \eta_p}{\partial t} = \pi d^2 h(T_{\text{air}} - T_p) + \pi d^2 h_m \rho_{\text{air}} L_{\text{subl/evap}} (\omega_{\text{air}} - \omega_p) \)

Water: \( \rho_p C_p \frac{\pi d^3}{6} \frac{\partial T_p}{\partial t} = \pi d^2 h(T_{\text{air}} - T_p) + \pi d^2 h_m \rho_{\text{air}} L_{\text{evap}} (\omega_{\text{air}} - \omega_p) \)
Empirical heat and mass transfer expressions for flow over a sphere\(^2\)

\[ Nu = \frac{hd}{k_{\text{air}}} = 2 + 0.6Re^{1/2}Pr^{1/3} \]

\[ Sh = \frac{h_m d}{D_{ab}} = 2 + 0.6Re^{1/2}Sc^{1/3} \]

\[ Re = \frac{\rho_{\text{air}}|v_{\text{air}} - v_p|d}{\mu_{\text{air}}} \]

\[ Pr = \frac{c_{\text{air}} \mu_{\text{air}}}{k_{\text{air}}} \]

\[ Sc = \frac{\mu_{\text{air}}}{\rho_{\text{air}} D_{ab}} \]

\[ \omega_{\text{air}} = \frac{MW_{\text{water}} P_{wv,\text{air}}}{MW_{\text{air}} P_{\text{air}}} \]

\[ \omega_{\text{surf}} = \frac{MW_{\text{water}} P_{wv,\text{surf}}}{MW_{\text{air}} P_{\text{air}}} \]

• Change in the air enthalpy is due to the convective heat transfer and the sensible energy change of the water vapor mass that has changed phase

\[ - \frac{\partial H_{air}}{\partial t} = q_{\text{conv}} + q_{\text{wv,sens}} \]

• \( q_{\text{wv,sens}} \) is thermal mixing

\[ - m_{air} C_{air} \frac{\partial T_{air}}{\partial t} = \pi d^2 h (T_p - T_{air}) + \frac{\partial m_{wv}}{\partial t} \int_{T_{air}}^{T_p} C_{wv} \, \partial T \]

• \( m_{wv} \) can be mass from evaporation or mass to condense
Change in particle mass due to vapor phase change

\[- \frac{\partial m_p}{\partial t} = \rho_p \frac{\partial}{\partial t} \left( \frac{\pi d^3}{6} \right) = \pi d^2 h_m \rho_{air} (\omega_{air} - \omega_p)\]

Change in air mass is opposite the change in particle mass

\[- \frac{\partial m_{air}}{\partial t} = - \frac{\partial m_p}{\partial t} = \pi d^2 h_m \rho_{air} (\omega_p - \omega_{air})\]
• MVD vs particle distribution
  − Smaller particles have greater SA/Vol ratio \( \rightarrow \) faster transfer response
  − Cumulative differences will add up to a different final result
  − Fraction of the total water mass calculated for every particle size bin

• An energy balance equation for every particle size \( i \)

\[
\rho_{p,i} C_{p,i} \frac{\pi d_i^3}{6} \frac{\partial T_{p,i}}{\partial t} = \pi d_i^2 h_i (T_{air} - T_{p,i}) + \pi d_i^2 h_{m,i} \rho_{air} L_i (\omega_{air} - \omega_{p,i})
\]

• One air energy equation contains the sum of all the particle heat transfers and vapor sensible energy transfers.

\[
m_{air} C_{air} \frac{\partial T_{air}}{\partial t} = \sum_{i=1}^{n} \left[ \pi d_i^2 h_i (T_{p,i} - T_{air}) + \frac{\partial m_{wv,i}}{\partial t} \int_{T_{air}}^{T_{p,i}} C_{wv,i} \partial T \right] (#_i)
\]

  • \( n \) = number of particle size bins
  • \(#_i\) = number of particles in the \( i^{th} \) bin
A mass balance equation for every particle size $i$

$$- \frac{\partial m_{p,i}}{\partial t} = \pi d_i^2 h_{m,i} \rho_{air}(\omega_{air} - \omega_{p,i})$$

One air mass equation contains the sum of all the particle mass transfers.

$$- \frac{\partial m_{air}}{\partial t} = - \sum_{i=1}^{n} \left[ \frac{\partial m_{p,i}}{\partial t} \right] (#_i)$$
Conservation of Momentum

- Conservation of momentum equation is solved in reference to particle

\[
F = F_{drag} + F_g = m_p a = \rho_p \frac{\pi d^3}{6} \frac{\partial v_p}{\partial t}
\]

- \( F_g = 0 \)
- \( F_{drag} = \frac{1}{2} \rho_{air} U^2 A C_D \)
- \( U = v_{air} - v_p \)
- \( A = \pi d^2 \)

\[
C_D = \frac{24}{Re} + \frac{2.6(\frac{Re}{5.0})^{1.52}}{1+(\frac{Re}{5.0})} + \frac{0.411(\frac{Re}{263,000})^{-7.94}}{1+(\frac{Re}{263,000})^{-8.00}} + \frac{Re^{0.80}}{461,000} \quad \text{(Morrison correlation)}
\]

- \( \frac{\partial v_p}{\partial t} = \frac{3}{4} \rho_{air} C_D (v_{air} - v_p)^2 \)

Supercooled Freezing Formulation

- Four stages in supercooled freezing
  - 1. Sensible liquid cooling (below 273.15K)
  - 2. Latent heat release
  - 3. Latent freezing (at 273.15K)
  - 4. Sensible ice cooling

- Homogeneous crystallization temperature as a function of diameter
  \[ T_{hc} = 7.2015 \ln(d) + 214.64 \]

- Expressions to determine melt fraction after latent heat release
  \[ H_{sens,super} = m_p \int_{T_{hc}}^{273.15} C_p(T) dT \]
  \[ \eta_p = 1 - \frac{m_p L_{melt} - H_{sens,super}}{m_p L_{melt}} \]

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• Written in MATLAB version R2014a
• Solves conservation equations using built-in ODE45 solver
• Relative and absolute convergence tolerance of $10^{-12}$
• Mass transferred between the air and particle(s) balanced to $10^{-15}$
• Energy transferred between the air and particle(s) balanced to $10^{-4}$
  – Model accuracy dependent on accuracy of property values ($C_p$, $L_{heat}$, etc.)
### Baseline 1 Tests

<table>
<thead>
<tr>
<th>Test #</th>
<th>Parameter Changed</th>
<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline 1</td>
<td>Slip Velocity</td>
<td>m/s</td>
<td>25</td>
</tr>
<tr>
<td></td>
<td>Particle Temperature</td>
<td>K</td>
<td>273.15</td>
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<tr>
<td></td>
<td>Pressure</td>
<td>Pa</td>
<td>28,000</td>
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</tbody>
</table>

**Takeaway:** Wet-bulb temperature determines state of particle
## Parametric Analysis
### Baseline 2 Tests

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Value</th>
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<tbody>
<tr>
<td>Particle Diameter</td>
<td>µm</td>
<td>10</td>
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<tr>
<td>IWC</td>
<td>g/m³</td>
<td>1</td>
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<tr>
<td>Pressure</td>
<td>Pa</td>
<td>88,000</td>
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<tr>
<td>Relative Humidity</td>
<td>%</td>
<td>50</td>
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<tr>
<td>Air Temperature</td>
<td>K</td>
<td>280.15</td>
</tr>
<tr>
<td>Particle Temperature</td>
<td>K</td>
<td>271.15</td>
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<tr>
<td>Slip Velocity</td>
<td>m/s</td>
<td>5</td>
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<table>
<thead>
<tr>
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<th>Parameter Changed</th>
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<th>Value</th>
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</thead>
<tbody>
<tr>
<td>Baseline 2</td>
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<td></td>
</tr>
<tr>
<td>4</td>
<td>IWC</td>
<td>g/m³</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>Relative Humidity</td>
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<td>80</td>
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<tr>
<td>6</td>
<td>Air Temperature</td>
<td>K</td>
<td>271.15</td>
</tr>
</tbody>
</table>

**Takeaway:** Air thermal mass >> Particle thermal mass
• Air velocity, air temperature, and vapor content vary from injection to test section at RATFac (no icing cloud)
• CFD model previously written to approximate $v_{\text{air}}$ and $T_{\text{air}}$
• Centerline values normalized
• Water vapor profile analogous to normalized air temperature profile
• Approximate changing values as sources in expressions (no cloud)
  - Valid for tests Mach ~ 0.2 and $P_{\text{total}}$ ~ 44000 Pa at test section
Model – Experiment Comparison

<table>
<thead>
<tr>
<th>Units</th>
<th>Scan 877</th>
<th>Scan 983</th>
<th>Scan 1003</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{\text{air, inj}}$</td>
<td>K</td>
<td>262.0</td>
<td>256.1</td>
</tr>
<tr>
<td>$T_{\text{air, target}}$</td>
<td>K</td>
<td>288.4</td>
<td>278.0</td>
</tr>
<tr>
<td>$\text{SH}<em>{\text{inj}}$ (RH$</em>{\text{inj}}$)</td>
<td>g$<em>{\text{vapor}}$/kg$</em>{\text{dry air}}$ (%)</td>
<td>0.07 (2)</td>
<td>0.07 (5)</td>
</tr>
<tr>
<td>$\text{SH}<em>{\text{target}}$ (RH$</em>{\text{target}}$)</td>
<td>g$<em>{\text{vapor}}$/kg$</em>{\text{dry air}}$ (%)</td>
<td>4.07 (16.1)</td>
<td>2.88 (35.4)</td>
</tr>
<tr>
<td>GWC$_{\text{inj/target}}$</td>
<td>g/m$^3$</td>
<td>2.1</td>
<td>2.4</td>
</tr>
<tr>
<td>LWC$_{\text{inj/target}}$</td>
<td>g/m$^3$</td>
<td>1.0</td>
<td>0</td>
</tr>
<tr>
<td>IWC$_{\text{inj/target}}$</td>
<td>g/m$^3$</td>
<td>0</td>
<td>8.4</td>
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<tr>
<td>$\eta_{\text{inj/target}}$</td>
<td>-</td>
<td>1.0</td>
<td>0.0</td>
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<tr>
<td>P</td>
<td>Pa</td>
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<tr>
<td>$V_{\text{air, inj/target}}$</td>
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<td>85.7</td>
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<td>$T_{\text{water, inj}}$</td>
<td>K</td>
<td>278.15</td>
<td>-</td>
</tr>
<tr>
<td>$T_{\text{ice, inj}}$</td>
<td>K</td>
<td>-</td>
<td>256.15</td>
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<tr>
<td>MVD$_{\text{water, inj}}$</td>
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<td>-</td>
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<td>MVD$_{\text{ice, inj}}$</td>
<td>µm</td>
<td>-</td>
<td>45.5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Units</th>
<th>Scan 877</th>
<th>Scan 983</th>
<th>Scan 1003</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta T_{\text{air, exp}}$</td>
<td>K</td>
<td>-2.6</td>
<td>-2.9</td>
</tr>
<tr>
<td>$\Delta T_{\text{air, sim}}$</td>
<td>K</td>
<td>-0.54</td>
<td>-0.75</td>
</tr>
<tr>
<td>$\Delta \text{GWC}_{\text{exp}}$</td>
<td>g/m$^3$</td>
<td>0.5</td>
<td>1.0</td>
</tr>
<tr>
<td>$\Delta \text{GWC}_{\text{sim}}$</td>
<td>g/m$^3$</td>
<td>0.13</td>
<td>0.16</td>
</tr>
<tr>
<td>$\Delta \text{LWC}_{\text{exp}}$</td>
<td>g/m$^3$</td>
<td>-0.5</td>
<td>0.5</td>
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<tr>
<td>$\Delta \text{LWC}_{\text{sim}}$</td>
<td>g/m$^3$</td>
<td>-0.13</td>
<td>0.0</td>
</tr>
<tr>
<td>$\Delta \text{IWC}_{\text{exp}}$</td>
<td>g/m$^3$</td>
<td>0.0</td>
<td>-1.5</td>
</tr>
<tr>
<td>$\Delta \text{IWC}_{\text{sim}}$</td>
<td>g/m$^3$</td>
<td>0.0</td>
<td>-0.16</td>
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<tr>
<td>$\Delta \eta_{\text{exp}}$</td>
<td>-</td>
<td>0.00</td>
<td>0.07</td>
</tr>
<tr>
<td>$\Delta \eta_{\text{sim}}$</td>
<td>-</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

- **Conditions**
  - Scan 887: Water sprayed into low P, low RH, very warm air
  - Scan 983: Ice sprayed into medium P, medium RH, warm air
  - Scan 1003: Water & Ice sprayed into medium P, medium RH, warm air

**Takeaway:** Model accounts for ~20% of experimentally observed changes
Sources for Model – Experiment Discrepancy

- Experimentally observed water/ice film on tunnel walls
  - Increased residence time for evaporation
- Poor approximation for the vapor mass source term
- Non-spherical particles increase heat and mass transfer
- Turbulence at spray nozzle may enhance transfer
- Uncertainties with experimentally measured values
  - Independent corroborating measurements minimize this possibility
Thermal model that couples particle and air conservation equations

- Simulates:
  - All phase change types
  - Supercooled and normal freezing
  - Single particle and full particle distribution sprays (ice, water, combined)
  - Complicated icing tunnels with energy and mass sources

- Air temperature, pressure (air mass), and RH dominate $T_{wb}$

- Model compared to experiments conducted at NRC
  - Simulated ~ 20% of the cloud and air changes observed experimentally
  - Reasons for discrepancy are offered

- Future work to determine sources for discrepancy
- Model can be modified to simulate other icing facilities (PSL)