Shape Evolution of Detached Bridgman Crystals Grown in Microgravity
Research Motivation

- What are the conditions for detachment in microgravity and how do they depend on the governing parameters?
  - Growth angle
  - Contact angle
  - Pressure differential
  - Bond number (ratio of gravity to capillarity)
- Which detached growth solutions are dynamically stable?
- How does an initial crystal radius evolve to one of the following states?
  - Stable detached gap
  - Attachment to the crucible wall
  - Meniscus collapse
- What are the effects of angular dependence on the crystal shape (faceting effects)?
Detached Crystal Growth
**Schematic Diagram of Detached Solidification**

- $\alpha$: growth angle
- $\theta$: contact or wetting angle
- $\Delta P = P_H - P_C$: Dimensionless pressure differential across the meniscus
- $z(r)$: meniscus shape
- $Z(r)$: crystal shape

The equation for the pressure differential is:

$$\Delta P = \Delta P_{\text{external}} + \rho gh + 2 \frac{\gamma}{r_{tm}}$$

where
- $\Delta P_{\text{external}}$: external gas pressure differential
- $\rho gh$: weight of melt (pressure head)
- $2 \frac{\gamma}{r_{tm}}$: capillary pressure from top meniscus
\[ \frac{\partial z}{\partial r} = \pm \frac{\Delta P(r^2 - 1) - 2 \cos \theta}{\sqrt{4r^2 - (\Delta P(r^2 - 1) - 2 \cos \theta)^2}} \]

Meniscus shape equation, \( z(r) \): 2 possible solutions for \( g = 0, B = 0 \)

\[ \frac{dZ}{dr} = \tan(\alpha + \beta) \]

Crystal shape equation, \( Z(r) \): 2 possible solutions in zero gravity

\[ \frac{dZ^\pm}{dr} = \frac{\sqrt{4r^2 - y^2} \tan \alpha \pm y}{\sqrt{4r^2 - y^2} \mp y \tan \alpha}, \quad y = \Delta P(r^2 - 1) - 2 \cos \theta \]
Crystal Evolution for $\theta + \alpha > 180^\circ; \ Z^+$ solution

Ge
$\alpha = 14.3^\circ$
$\theta = 172^\circ$

The graph shows the crystal evolution with varying parameters $P$. The x-axis represents the crystal radius ($r_0$), and the y-axis represents the crystal height ($Z^+$). The different curves correspond to different values of $P$: $P = 2.5$, $P = 1.9$, $P = 1.4$, $P = 1.3$, and $P = 1.2$. The graph illustrates how the crystal height changes with the crystal radius for different values of $P$.
Ge Crystal Evolution for $\theta + \alpha > 180^\circ$; $Z^+$ solution

Material: Ge
Growth Angle: 14.3°
Contact Angle: 172°
$\Delta P = 1.4$

(a) Radius decreases until meniscus collapses

(b, c) Radius increases or decreases until stable growth with a constant radius is reached
Ge Crystal Evolution for $\theta + \alpha > 180^\circ$; $Z^-$ solution

Material: Ge  
Growth Angle: 14.3°  
Contact Angle: 172°

For $Z^-$ solutions, all crystals decrease in radius until the menisci collapse.
Existence Region for $\theta + \alpha > 180^\circ$
Crystal Evolution for $\theta + \alpha < 180^\circ$

$Z^+$ solutions

$Z^-$ solutions

$\alpha = 14.3^\circ; \theta = 140^\circ$
Crystal Evolution for $\theta + \alpha < 180^\circ$

Material: Ge  
Growth Angle: 14.3°  
Contact Angle: 140°  
$\Delta P = 1.0$

(a, c) Radius decreases until meniscus collapses

(b) Radius increases or decreases until attachment at the crucible wall
Existence Region for $\theta + \alpha < 180°$

- $\frac{dr}{dZ} = 0$; unstable
- $\frac{dr}{dZ} \neq 0$
- no solution

$\theta = 140°$
$\alpha = 14.3°$
• “Influence of Containment on the Growth of Silicon-Germanium” (ICESAGE) is a collaborative investigation between NASA and the European Space Agency (ESA)

• The ICESAGE experiments will be conducted in the Low Gradient Furnace (LGF) in the Materials Science Laboratory on the International Space Station (ISS)

• ICESAGE will test the theories of crystal shape evolution in detached Bridgman growth

• Dependence on the parameters $\Delta P$, $\theta$, and the crystal starting position $r_0$ will be examined

• Launch is currently planned on a SpaceX flight in 2016
Summary

- A theory describing the shape evolution of detached Bridgman crystals in microgravity has been developed.
- A starting crystal of initial radius \( r_0 \) will evolve to one of the following states:
  - Stable detached gap
  - Attachment to the crucible wall
  - Meniscus collapse
- Only crystals where \( \alpha + \theta > 180^\circ \) will achieve stable detached growth in microgravity.
- Results of the crystal shape evolution theory are consistent with predictions of the dynamic stability of crystallization (Tatarchenko, *Shaped Crystal Growth*, Kluwer, 1993).
- Tests of transient crystal evolution are planned for ICESAGE, a series of Ge and GeSi crystal growth experiments planned to be conducted on the ISS.