Determination of the Contact Angle Based on the Casimir Effect

Konstantin Mazuruk, UAH and Martin P. Volz, MSFC/NASA

Introduction

On a macroscopic scale, a nonreactive liquid partially covering a homogeneous solid surface will intersect the solid at an angle called the contact angle. For molten metals and semiconductors, the contact angle is materially dependent upon both the solid and liquid and typical values fall in the range 80°-170°, depending on the crucible material. On a microscopic scale, the glass does not exist in such a sharp contact angle but rather the liquid and solid surfaces merge smoothly and continuously. Consider the example of the so-called detached Bridgman crystal growth process. In this technique, a small gap is formed between the growing crystal and the crucible wall. The angle is then described as being 

$\sigma = \frac{\cos \theta}{r}$

where $r$ is the radius of curvature of the surface, we obtain the Laplace equation for the microscopic meniscus shape:

$\sigma = 2 \kappa \cos \theta$

Equating the two forces: the disjoining pressure and the capillary force due to the curvature of the surface, we obtain the Laplace equation for the microscopic meniscus shape

$\sigma = 2 \kappa \cos \theta$

Contact angle theory

The basic model of the contact angle is due to Derjaguin and Frumkin (1938) [2]. It relies on the concept of the disjoining pressure (Derjaguin, 1936). Below, we will outline this theory. The Fig. [1] depicts the drop of fluid on the surface for a semi-wetting case. When the gap between the drop and the solid is large, the disjoining pressure is zero, and the horizontal surface tension is $\sigma$. At the drop position on the solid as depicted in Fig. 1, the horizontal surface tension is $\sigma$. The difference is due to the potential energy difference, which can be expressed through the drop length $P$ as

$\sigma - \sigma' = \int \gamma(h) dh = \cos \theta = \frac{1}{\Gamma} \int \gamma(h) dh$

Role of zero-point energy

The "physical vacuum" consists primarily of quantum fluctuations of electromagnetic fields. The average energy density of these fluctuations is enormous, and usually it is referred to as the zero-point energy, signifying its existence at zero temperature. One of the effects of these fluctuations is the so-called van der Waals attraction between the macroscopic bodies. The theory of this effect between two flat bodies has been developed by Lifshitz in 1936 and an elegant formalism based on photon Green functions has been given by Dzyaloshinsky, Lifshitz, and Pitaevskii. This theory provides the value for the disjoining pressure. The disjoining pressure is the force normal to the unit surface element, derived from the Maxwell electromagnetic stress tensor. Below is displayed a set of equations to be solved:

$T(r,r') = \int_0^\infty \left[ \frac{\partial \bar{\Omega}(r,r')}{\partial \bar{\varepsilon}} - \frac{1}{2} \frac{\partial \bar{\Omega}(r,r')}{\partial \bar{\varepsilon}} \left( 1 + \varepsilon^2 \right)^{-1} \right] \bar{\varepsilon} d\varepsilon$

Equation for the contact angle

$\cos \alpha = 1 - \frac{1}{\Gamma} \int \gamma(h) dh$

Conclusion

The slope correction for the contact angle is small for small angles. For larger angles, the slope correction has to be taken into account. Therefore, the macroscopic concept of balancing forces as applied to the interface interactions is not accurate, and a microscopic picture should be implemented. The available models of Casimir force can be implemented for the discussed issue, are not well justified. An accurate numerical evaluation of the van der Waals force is needed at this point to reach more conclusive results. The presented idea can further be extended to include effects of electrostatic fields, or other forces. In the forthcoming paper, a more elaborate evaluation of the shape effect on the Casimir force will be presented. The discussed idea can also be used to study capillary surface waves, close to the contact line, induced by fluctuating electromagnetic fields.