Towards an Understanding of Atmospheric Balance

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Why is the extra-tropical atmosphere approx. quasi-geostrophic?

The stability of quasi-geostrophic flow with respect to ageostrophic perturbations
Derivation of (2-layer, f_{plane}) PE in terms of Normal Modes
Errico JAS 1981

\[ \frac{d}{dt} b_K = \sum_{L,M} \left[ C_1 b_L^* b_M^* + C_2 g_L^* g_M^* + C_3 g_L^* a_M^* + C_3^* g_M^* d_M^* + C_4 a_L^* a_M^* + C_4^* d_L^* d_M^* + C_5 a_L^* d_M^* \right], \]

\[ \frac{d}{dt} g_K = \sum_{L,M} \left[ C_6 b_L^* g_M^* + C_7 b_L^* a_M^* + C_7^* b_L^* d_M^* \right], \]

\[ \frac{d}{dt} a_K = i \omega_K a_K + \sum_{L,M} \left[ C_8 b_L^* g_M^* + C_9 b_L^* a_M^* + C_{10} b_L^* d_M^* \right], \]

\[ d_K = a_{-K}^*. \]

\[ C_1 = -\frac{1}{2} L \times M(L^{-2} - M^{-2}), \]
\[ C_2 = -\frac{1}{2} L \times M(\omega_L^{-2} - \omega_M^{-2}), \]
\[ C_3 = -(M^2 \omega_M^2 \omega_L^2)^{-1}[L \times M(M^2 - L^2) - i \omega_M(L^2 M \cdot K + M^2 L \cdot K)], \]
\[ C_4 = \frac{1}{2} \omega_M^{-2} \omega_L^{-2}[L \times M(M^{-2} - L^{-2})(1 + \omega_M \omega + i(\omega_L + \omega_M)(M^{-2} M \cdot K + L^{-2} L \cdot K)], \]
\[ C_5 = \omega_M^{-2} \omega_L^{-2}[L \times M(M^{-2} - L^{-2})(1 - \omega_M \omega_L) + i(\omega_L - \omega_M)(M^{-2} M \cdot K + L^{-2} L \cdot K)], \]
\[ C_6 = -L \times M L^{-2} \omega_M^{-2}(\omega_M^2 - \omega_L^2 + 1), \]
\[ C_7 = \omega_M^{-2} M^{-2}(L \times M + i \omega_M K \cdot M), \]
\[ C_8 = -\omega_M^{-2} L^{-2} L \times M(K \cdot L - i \omega_K L \times M), \]
\[ C_9 = (2 \omega_M^2 L^2 M^2)^{-1}[L \times M[L^2(1 - \omega_K \omega_M) - \omega_K(\omega_K M^2 - \omega_M K^2)] + i[2 \omega_K(L \times M)^2 + \omega_M L^2 K \cdot M]}, \]
\[ C_{10} = (2 \omega_M^2 L^2 M^2)^{-1}[L \times M[L^2(1 + \omega_K \omega_M) - \omega_K(\omega_K M^2 + \omega_M K^2)] + i[2 \omega_K(L \times M)^2 - \omega_M L^2 K \cdot M}].
Demonstration of Equipartition
Errico Tellus 1984
Examination of Balance
\[ \frac{dc_j}{dt} = -i\omega_j c_j + A(r,r) + B(r,g) + C(g,g) + D \]

Balance of Modes in a Climate Model

(a sophisticated scale analysis)

Normalized Sizes of Terms

Errico 1984, 1990 MWR; Errico et al. 1988 MWR
Diabatic Balance ?
The interplay of analysis and initialization
Errico et al. *MWR* 1993
Gravitational modes considered as forced and damped harmonic oscillators

Define $g(t)$ as the complex amplitude of a gravity-wave like mode at each time $t$, and let $R$ and $G$ be the sets of Rossby- and gravity-wave like modes. Then

$$\frac{dg}{dt} = -i\lambda g + N(R) + N(R,G) + N(G) + D(R,G) - \nu g$$

Consider $N(R) = F(t)$ as the dominant nonlinear term. Approximately then

$$\frac{dg}{dt} = -i\lambda g + F(t) - \nu g$$

Consider $F(t) = F(0) \exp(-i\mu t)$. Then

$$g(t) = \left[ g(0) - \frac{F(0)}{i\lambda - i\mu + \nu} \right] \exp(-(i\lambda + \nu)t) + \frac{F}{i\lambda - i\mu + \nu}$$

Errico 1981 JAS, 1984 MWR, 1997 J Japan MS
Harmonic Dial for External m=4 Mode, Period=3.7h

Without NNMI

With NNMI

Errico 1997 J Japan Met Soc
Behavior of gravitational modes in a climate model:
Time series (harmonic dials) of complex mode amplitudes
Errico MWR 1989

16 days shown
Behavior of gravitational modes in a climate model:
Power spectra of complex mode amplitudes
Errico MWR 1989

Solid: Westward propagating      Dashed: Eastward propagating
Behavior of gravitational modes in a climate model:
Power spectra of convective heating
Errico MWR 1989
Diabatic balance vs appropriate cutoff
Errico and Rasch *Tellus* 1988

![Diabatic balance vs appropriate cutoff](image-url)
Higher-order Machenhauer schemes
Errico *MWR* 1989

Vertical Mode Index
Other Issues
Vertical modes in discrete models

10 level MAMS
Modes 1, 2, 7 (H=10,000, 2050, 13 m)

72 level GEOS-5
Mode 29 (H=13m)
(Notice only plotted up to $\sigma$=0.1)

23 zero crossings above for $\sigma$<0.1
High amplitude modes in the upper atmosphere

72 level GEOS-5 model with top at 0.01 hPa
Global mean squared divergence tendency

Structures of 3 largest scale vertical normal modes

GMAO-GSI 3DVAR
72 level model
Derivation of (2-layer) PE in terms of Normal Modes
Errico JAS 1981
Figure 4. The kinetic energy (K), available potential energy (A), rotational-mode energy (R), gravitational-mode energy (G), and total energy (E) contributed by vertical modes of indicated equivalent depths at \( t = 0 \). The integers on the right-hand side indicate corresponding vertical-mode indices \( \ell \).
Derivation of (2-layer) PE in terms of Normal Modes
Errico JAS 1981

Figure 6. The (a) and (b) $R$ and (c) and (d) its complement components of the (a) and (c) $u'$ and (b) and (d) $T'$ fields on $\sigma = 0.55$ at $t = 0$ for SV1 determined using the $E$ norm applied to the dry form of the linearized model. Contour intervals are (a) and (c) $1 \text{ m s}^{-1}$, (b) $1 \text{ K}$, and (d) $0.5 \text{ K}$, with zero-contours omitted and negative values shown dashed. See text for further explanation.
Derivation of (2-layer) PE in terms of Normal Modes
Errico JAS 1981

Figure 8. The (a) and (c) $u'$ and (b) and (d) $T'$ fields on $\sigma = 0.55$ at $t = 24$ h determined from the linearized evolutions begun from (a) and (b) R-mode components of SV1 and (c) and (d) their complement of SV1. Contour intervals are (a) 10 m s$^{-1}$, (b) 2 K, (c) 5 m s$^{-1}$, and (d) 1 K, with zero-contours omitted and negative values shown dashed. See text for further explanation.
Partitioning of analysis error energy in terms of normal modes:
(as inferred from an OSSE)

<table>
<thead>
<tr>
<th>Vert mode index</th>
<th>$k$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
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<tbody>
<tr>
<td>Equiv Depth</td>
<td>$H$(m)</td>
<td>10943</td>
<td>4444</td>
<td>1538</td>
<td>628</td>
<td>311</td>
<td>175</td>
<td>109</td>
<td>71</td>
<td>49</td>
</tr>
<tr>
<td>G-mode Energy</td>
<td>$G$(J/kg)</td>
<td>.18</td>
<td>.16</td>
<td>.22</td>
<td>.32</td>
<td>.31</td>
<td>.32</td>
<td>.29</td>
<td>.28</td>
<td>.25</td>
</tr>
<tr>
<td>R-Mode Energy</td>
<td>$R$(J/kg)</td>
<td>.82</td>
<td>.47</td>
<td>.38</td>
<td>.51</td>
<td>.52</td>
<td>.58</td>
<td>.56</td>
<td>.45</td>
<td>.33</td>
</tr>
<tr>
<td>Ratio G/TE</td>
<td>$f_g$</td>
<td>.18</td>
<td>.25</td>
<td>.37</td>
<td>.39</td>
<td>.37</td>
<td>.36</td>
<td>.34</td>
<td>.38</td>
<td>.43</td>
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</tbody>
</table>
Summary

1. Much can be learned from some old works
2. The standard Normal Modes provide useful concepts and tools
3. The standard Normal Modes have limitations
   a. the universality of vertical modes
   b. internal modes (when \( C \approx U \))
   c. more realistic basic states (e.g. as for SVs)
4. Is Initialization still an issue?
5. There is more to understand
   a. time scales of moist diabatic processes
   b. effects of top boundary conditions, non-hydrostatic behavior
   c. SV behavior