A Model for Jet-Surface Interaction Noise Using Physically Realizable Upstream Turbulence Conditions

Mohammed Z. Afsar
*Imperial College London, 180 Queen’s Gate, London, SW7, UK*

S.J. Leib
*Ohio Aerospace Institute, 22800 Cedar Point Road, Cleveland, Ohio 44142, USA*

and

Richard F. Bozak
*National Aeronautics and Space Administration, Glenn Research Center, Cleveland, Ohio 44135, USA*
Motivations and Objectives

• Jet flows of technological interest are often close enough to solid boundaries so that the surface plays a direct role in the generation of sound as well as its propagation
• Proposed next-generation aircraft configurations may have exhaust systems tightly integrated with the airframe
• One problem of interest:
  – Exhaust jet interacting with wing or other nearby edge
• Experiments show that the presence of an external surface enhances the noise produced by the jet alone
• The aim of this paper is to further develop a prediction method for the noise generated by the interaction of a turbulent jet with the trailing-edge of a flat plate
Technical Approach

• The prediction method is based on application of the non-homogeneous Rapid-distortion Theory (RDT) introduced recently by Goldstein, Afsar and Leib (2013) (GAL)
  – Initial application to prediction of noise from interaction of large-aspect ratio rectangular jets with flat plate
• Improved source model for transverse velocity correlations and relation to ‘gust’ spectrum
• Use results from Reynolds-averaged Navier-Stokes (RANS) solutions to obtain the mean flow and inform the source model
Outline

• Brief review of GAL formulation
• New source model and effect on low-frequency roll-off
• RANS solutions
• RANS-based edge-noise predictions
• Conclusions
Review of GAL Formulation

• Rapid Distortion Theory
  – Linear analysis to study the interaction of turbulence with solid surfaces

• Assumptions and Approximations:
  – The turbulence intensity is small
  – The time scale for interaction is short compared with those over which non-linearity and viscous dissipation take place

• Problem is governed by the compressible Rayleigh equation

\[
\frac{D_0 u_i}{D} + \bar{u}_j \frac{U}{y_j} + \frac{p}{y_i} = 0
\]

\[
\frac{D_0 p}{D} + \frac{c^2 u_j}{y_j} = 0
\]

\[
U = U(y_T) \quad ; \quad c^2 = c^2(y_T) \quad ; \quad y = (y_1, y_2, y_3) = (y_1, y_T) \quad ; \quad \frac{D_0}{D} = \frac{1}{y_1} + U(y_T)
\]
Review of GAL Formulation: General Solution

Pressure in terms of scalar function
\[ p (y, t) = \frac{D_0^3}{D} (y, t), \]

Momentum flux in terms of scalar and arbitrary convected quantity
\[
\begin{align*}
\mathbf{u}_i (y, t) &= \left( \frac{D_0^3}{D} i \right) \left( \frac{\partial U}{\partial y_j} \right) \left[ \frac{\partial}{\partial y_j} \frac{D_0^3}{D} + 2 \frac{\partial U}{\partial y_j} \frac{\partial}{\partial y_k} \right] + \frac{1}{c^2} \frac{\partial U}{\partial y_j} \frac{\partial}{\partial y_k} \left( \frac{y_1}{U(y_T)}, y_T \right),
\end{align*}
\]

Scalar satisfies inhomogeneous adjoint Rayleigh equation
\[
L_a \phi \equiv \left( \frac{D_0^3}{D^3 \tau} - \frac{\partial}{\partial y_i} c^2 \left( \frac{\partial}{\partial y_i} \frac{D_0^3}{D \tau} + 2 \frac{\partial U}{\partial y_i} \frac{\partial}{\partial y_k} \right) \right) \phi = -\tilde{\omega}_c \left( \tau - y_1/U(y_T), y_T \right),
\]

Solution in terms of Rayleigh equation Green's function
\[
L g(y, x| x, t) = \left( y \ x \right) \left( \begin{array}{c} \frac{D_0^3}{D^3 \tau} \ D_0^3 g(y, \tau| x, t) \\
\frac{\partial U}{\partial y_i} \frac{\partial}{\partial y_j} \frac{\partial U}{\partial y_k} \left( \frac{y_1}{U(y_T)}, y_T \right) \end{array} \right)
\]

\[
p'(x,t) = \int_{-T}^{T} \int_{-T}^{T} \int_{-T}^{T} \frac{D_0^3 g(y, \tau| x, t)}{D^3 \tau} \tilde{\omega}_c \left( \tau - y_1/U(y_T), y_T \right) dy d\tau
\]

\[
\rho v'(x,t) \equiv u_i(x,t) \frac{\partial U}{\partial x_i} / |\nabla U| = -\frac{\partial U / \partial x_i}{|\nabla U|} \int_{-T}^{T} \int_{-T}^{T} g_i(y, \tau| x, t) \tilde{\omega}_c \left( \tau - y_1/U(y_T), y_T \right) dy d\tau
\]

Arbitrary, purely convected quantity
\[
\tilde{\omega}_c \left( \tau - y_1/U(y_T), y_T \right)
\]
Review of GAL Formulation
The ‘Gust’ Solution

- Split Rayleigh equation Green’s function into two components:

\[ g(y, |x,t) = g^{(0)}(y, |x,t) + g^{(s)}(y, |x,t) \]

- \( g^{(0)}(y, |x,t) \) satisfies the inhomogeneous Rayleigh equation with

\[ \hat{n}_i \partial \left[ D_0^3 g^{(0)}(y, |x,t) / Dt^3 \right] / \partial y_i = 0 \quad \text{for} \ y_T \in S, \quad \infty < y_1 < \infty \]

- \( g^{(s)}(y, |x,t) \) is the ‘scattered’ solution which satisfies the homogeneous Rayleigh equation subject to:

\[ - \hat{n}_i \partial \left[ D_0^3 g^{(s)}(y, |x,t) / Dt^3 \right] / \partial y_i = 0 \quad \text{for} \ y_T \in S, \quad \infty < y_1 < 0 \]

- Jump conditions across plate’s downstream extension \( 0 < y_1 < \)

- The streamwise-homogeneous solution, \( g^{(0)}(y, |x,t) \), is referred to as the ‘gust’ solution and is the input to the interaction problem

\[ \left[ \rho v'_{\perp}(x,t) \right]^{(0)} = - \frac{\partial U / \partial x_i}{|\nabla U|} \int_{-T}^{T} \int_{y} g^{(0)}(y, \tau |x,t) \tilde{\omega}_c \left( \tau - y_1/U(y_T), y_T \right) dy d\tau \]
Review of GAL Formulation
Relation between $\tilde{\omega}_c$ Spectrum and Measurable Turbulence Statistics

• Assume: Relation between $\tilde{\omega}_c$ and $\nu$ is the same as that in a streamwise-homogeneous flow where surface is doubly infinite

• Invert relation between $\tilde{\omega}_c$ and $\nu$ in the ‘gust’ solution for a two-dimensional mean flow (spanwise homogeneous)

• Relate:
  – Spectrum of $\tilde{\omega}_c$:
    \[
    S(y_2, y_2; k_3, \omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i(\omega \tau - k_3 \eta_3)} \langle \tilde{\omega}_c(t, y_2, y_3) \tilde{\omega}_c(t + \tau, y_2, y_3 + \eta_3) \rangle d\tau d\eta_3
    \]
    To

  – Measurable turbulence quantities:
    \[
    \langle v'_\perp(x, t) v'_\perp(x_1, \tilde{x}_2, x_3 + \eta_3, t + \tau) \rangle = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} \int_{-\infty}^{\infty} v'_\perp(x, t) v'_\perp(x_1, \tilde{x}_2, x_3 + \eta_3, t + \tau) dt dx_3
    \]
Review of GAL Formulation

Relation between $\tilde{\omega}_c$ Spectrum and Measurable Turbulence Statistics

$S(y_2, \tilde{y}_2; k_3, \omega) = \frac{U'(y_2)U'(\tilde{y}_2)}{U(y_2)U(\tilde{y}_2)} \left(1 + \frac{y_2 - y_d}{y_d} b_0 \right) \left(1 + \frac{\tilde{y}_2 - y_d}{y_d} b_0 \right) \frac{F_\perp(y_d, y_d | y_2, \tilde{y}_2, \omega, k_3) F(\tilde{y}_2; k_3, \omega)}{E(y_2; k_3, \omega) E(\tilde{y}_2; k_3, \omega)}$

where

$E(y_2; k_3, \omega) = \frac{U(y_d) U(y_2)}{c^2(y_d)} \left(1 + \frac{y_2}{y_d} b_0 \right) + \frac{\frac{2}{U^2(y_2) + k_3^2}}{\sqrt{\frac{2}{U^2(y_2) + k_3^2} + k_0^2 \frac{U^2(y_2) b_0}{c_x U'' y_d}}}$

$f_\perp(x_2, \tilde{x}_2 | k, \tilde{k}, \eta, \tau) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-i(x_2-k_1 \tilde{x}_2)} \left( \rho v^{(0)}_{\perp}(x, t) \rho v^{(0)}_{\perp}(\tilde{x}_1, \tilde{x}_2, x_3 + \eta, t + \tau) \right) dx_2 d\tilde{x}_1$,

$F_\perp(x_2, \tilde{x}_2 | y_2, \tilde{y}_2, \omega, k_3) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-i(\omega - k_1 \eta)} f_\perp(x_2, \tilde{x}_2 | \omega / U(y_2), \omega / U(\tilde{y}_2), \eta, \tau) d\eta d\tau$

- ‘Gust’ solution provides the input (upstream boundary condition) to the jet-surface interaction problem in terms of measurable turbulence quantities
- Model needed for transverse momentum fluctuation space-time correlations.
Review of GAL Formulation
Scattered Solution

- The ‘scattered’ solution, $g^{(s)}(y, |x,t)$, represents the effects of the presence of the trailing edge
- Satisfies homogeneous Rayleigh equation
- Streamwise-discontinuous boundary conditions on the plate and its downstream extension
- Solve by Wiener-Hopf technique
- Low-frequency asymptotic solution
Acoustic Spectrum for Jet-Surface Interaction in Planar Flow

\[ I_\omega(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{i\omega \tau} P_s(x,t) P_s(x,t+\tau) d\tau \approx \left( \frac{k_\infty}{4\pi|x|} \right) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} D(\theta,\psi,M(y_2)) \delta(y_2,\tilde{y}_2;k_3^{(s)},\omega) dy_2 d\tilde{y}_2 \]

Directivity Factor

\[ D(\theta,\psi,M(y_2)) = \frac{\left[ M(y_2)M(\tilde{y}_2) \right]^{3/2} (\beta - \cos \theta)}{\left[ 1 - M(y_2)\cos \theta \right]\left[ 1 - M(\tilde{y}_2)\cos \theta \right]\sqrt{1 - \beta M(y_2)\beta M(\tilde{y}_2)}} \]

\[ M(y_2) = \frac{U(y_2)}{c} \left( 1 - \sin^2 \cos^2 \right)^{1/2} \]

\[ k_3^{(s)} = \frac{-c}{\sin \cos} \]

Polar Directivity

Azimuthal Directivity
Upstream Turbulence Model

\[ \left\langle v'_\perp(x,t)v'_\perp(x_1,\tilde{x}_2,x_3+\eta_3,t+\tau) \right\rangle \equiv \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} \int \left\langle v'_\perp(x,t)v'_\perp(x_1,\tilde{x}_2,x_3+\eta_3,t+\tau) \right\rangle dt dx_3 \]

\[ \left\langle \rho v'_\perp(x_1,y_d,x_3,t)\rho v'_\perp(\tilde{x}_1,y_d,x_3+\eta_3,t+\tau) \right\rangle = L_3 \Psi(\tilde{x}_1) \left[ a_0 + a_1 \tau \frac{\partial}{\partial \tau} + a_2 \eta_1 \frac{\partial}{\partial \eta_1} + \ldots \right] e^{-X(\tau,\eta_1,\eta_3)} \]

\[ X(\eta_1,\eta_3) = \sqrt{\left( \frac{1}{l_1} \right)^2 + \left( \frac{U_c}{l_0} \right)^2 / l_0^2 + \left( \frac{1}{l_3} \right)^2} \]
Application to Interaction of a Large-Aspect Ratio Rectangular Jet with a Semi-Infinite Flat Plate

Cases:
- Trailing edge distance from nozzle exit plane: $x_d/D = 5.7$
- Standoff distance: $y_d/D = 1.2$
- Aspect Ratios: AR = 4 and AR = 8
- Jet exit acoustic Mach numbers: $Ma = 0.7$ and 0.9
Effect of Source Model on Low-Frequency Roll-off of Edge-Noise Spectrum

Analytic Mean Profile

\[ U(y_2) = U_d \left[ e^{2(y_2 - y_d)^2} e^{2(t_d/2)^2} \right] / \left[ 1 + e^{2(t_d/2)^2} \right] \]

\[ a_0 = 0.04 \left( \infty U_d \right)^2 ; \left( l_0, l_1, l_3 \right)/D_J = \left( 0.53, 0.01, 0.01 \right) ; \left( L_2, L_3 \right)/D_J = \left( 0.5, 20 \right) \]

\[ U_c = 0.68 U_d ; b_0 = 0.52 \]

\[ (a_0 = 1 ; a_2 = 0) \]

\[ (a_0, a_1, a_2) = (0.82, 0.88, 0.05) \]
RANS Solutions
Comparisons with Experiment

- SolidWorks® Flow Simulation
  - Cartesian meshing
  - Immersed boundary approach
  - Solution-adaptive refinement
  - Two-scale wall functions
  - Modified k-e (Lam-Bremhorst)

![Transverse Profiles of Normalized Mean Velocity and Turbulent kinetic energy at edge of plate](Image)

Contours of Normalized Mean Velocity and Turbulent kinetic energy at edge of plate
RANS Solutions
Turbulence Quantities for Noise Predictions

- Variation of $Tke$ and Turbulent Length Scales at the edge of the plate with Jet Exit Velocity and Nozzle Aspect Ratio

$$0 = C \cdot k$$

$$L_{RANS} = k^{3/2}$$

- Little variation with exit Mach number
- Significant variation with aspect ratio
RANS-Based Edge-Noise Predictions

\[ Y_0 = 0.04 \left( U_d \right)^2; \left( l_0, l_1, l_3 \right)/D_J = (0.53, 0.01, 0.01); \left( L_2, L_3 \right)/D_J = (0.5, 20) \]

\[ U_c = 0.68 U_d; b_0 = 0.52 \]

\[ Ma = 0.9 \]

\[ Ma = 0.7 \]
Summary and Conclusions

• Extended the GAL model to include a finite de-correlation region in the upstream turbulence correlation function
• Showed that the presence of a de-correlation region directly affects the low-frequency algebraic decay of the jet-surface interaction noise spectrum
• Implemented a RANS-based RDT prediction method that takes into account the reduction in length scales and turbulent kinetic energy with nozzle aspect ratio predicted by these flow solutions.
• This approach generally gives reasonably good predictions even for moderate aspect ratio jets.