FEAMAC/CARES Software Coupling Development Effort for CMC Stochastic-Strength-Based Damage Simulation

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Multiscale & Multiphysics Branch

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CARES: Ceramics Analysis and Reliability Evaluation of Structures
MAC/GMC: Micromechanics Analysis Code/Generalized Method of Cells

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Scope and Technical Challenge

- Predict the strength and service life of ceramic & composite structures

- Need to account for:
  - Wide variability in the strength of individual components (probabilistic/stochastic strength)
  - How strength changes with different types of loading (strength vs: multiaxial loading) and size of the structure (size-effect)
  - How strength degrades with time and fluctuating load
  - How strength/damage response of monolithic, anisotropic and composite material (architectures) differ
Approach / Outline

1. Overview: Describe the MAC and CARES codes
   - MAC/GMC: composite *micromechanics* model
   - CARES Unit Sphere: multiaxial *stochastic strength* model
     *(isotropy & anisotropy)*

2. Applying CARES to the MAC code to simulate stochastic damage progression in a ceramic matrix composite (CMC)
   - **Cellular Automaton**: Encouraging failure of adjacent elements - mimics crack-like growth
   - Visualization of element-by-element failure propagation for fiber, matrix, and interface

- **Status & Capability**: Current progress of code integration effort
  - **Examples**:
    1. Stress-strain response of a SiC-RBSN laminate (circa 1990)
    2. Time-dependent degradation notional example
Micromechanics links the size scales & provides the composite response based on the composite constituent materials

**FEAMAC:** MAC/GMC embedded in FEA as constitutive material

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**GMC (1990s)**
- 1st order displacement field in subcells
- Stresses and strains piecewise constant
- Number of linear algebraic equations function of number of subcells
- Local inelasticity/damage
- **No shear coupling**
- No “subcell mesh” sensitivity

**HFGMC (2000s)**
- 2nd order displacement field in subcells
- Elastic stresses and strains piecewise linear
- Number of linear algebraic equations is rather large
- Local inelasticity/damage
- Has shear coupling
- Has “subcell mesh” sensitivity

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RUC made subcells
Multiscale capability

We currently only use GMC in FEAMAC/CARES

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CARES: **Ceramics Analysis and Reliability Evaluation of Structures**

Life Prediction & Component Design Code For Advanced Ceramics

- Developed to predict the probability of failure of ceramic components under complex thermomechanical loading
- Combines Weibull & Weakest Link theory with concepts of linear elastic fracture mechanics (the *Unit Sphere* model)

**Component Reliability Analysis Capability:**
- Transient loads and temperatures
- Fast-Fracture Rupture
- Time-dependent \((da/dt)\) crack growth
- Cycle-dependent \((da/dn)\) crack growth
- Multiaxial stress failure models (*PIA & Unit Sphere & Tsai-Wu & Tsai-Hill*)
- Proof test

Approach for Life Prediction & Component Design of Composites

- Combine CARES, MAC & FEA codes

Move CARES from the macroscopic scale of the structure to the microscale of the individual RUC material constituents

FEAMAC/CARES Capability:
- Individual constituent and component level probability of failure tracked (for failure initiation)
- Individual & concurrent failure modes
- Laminate level analysis capability
- Progressive damage capability/simulation
- Subcells killed at random failure thresholds

Debonding/crack path physics at constituent level not explicitly included

FEAMAC/CARES

Abaqus UMAT “User Material”

Structural-Scale FEA

Element/Integration Point

Micromechanics Analysis

(MAC/GMC)

(CARES) Subroutine

Reliability analysis at the RUC level

Fiber Interface Matrix
**Progressive Damage Criterion**

Calculate failure probability, $P_f$, **for each material constituent of the RUC** associated with an element integration point.

- **CARES** calculated $P_f(CARES)$ of RUC
- Random number generated $P_f(Random)$ of RUC

$P_f(CARES) \geq P_f(Random)$

**Encourages more rapid damage propagation than failing individual subcells**

- Fail all material constituent subcells
  - Kill elastic modulus
- Don’t fail subcells

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**GRC**

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A cellular automaton is a collection of "colored" cells on a grid that evolves through discrete time steps according to a set of rules based on the states of neighboring cells.

**Rule:** When failure of an element is encountered, the random failure threshold of the neighboring elements are adjusted to that of the failed element. *Load state determines which elements have highest probability of failure*.

**Failed element**

**Adjacent element**

**Adjacent 2 elements with highest**

$P_f$ (CARES) has

$P_f$ (Random) adjusted

**Random element failure**

Example: 0° Ply uniaxial ramp load 25x25 FEA mesh

Adjusted element $P_f$ (Random) more likely to be lower than original $P_f$ (Random) and fail sooner as load increases – enhancing damage propagation.
0º single ply tensile specimen (Load parallel to fiber axis)

Shown are two trial executions; one using the automaton adjusted element feature and one with the feature inactive.

Stress-strain response in matrix cracking region

- **50% Fail. Prob.**
- **Adjusted**
- **Not Adjusted**

CARES calculated 50% matrix failure probability prior to any damage initiation

Close-up of region where matrix damage initiates

Strain (MPa)

<table>
<thead>
<tr>
<th>Stress (MPa)</th>
<th>150</th>
<th>170</th>
<th>190</th>
<th>210</th>
<th>230</th>
<th>250</th>
<th>270</th>
<th>290</th>
<th>310</th>
<th>330</th>
<th>350</th>
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<tbody>
<tr>
<td>Load</td>
<td></td>
<td></td>
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<td></td>
<td></td>
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<td></td>
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</table>

Strain

<table>
<thead>
<tr>
<th>Strain</th>
<th>0.0005</th>
<th>0.001</th>
<th>0.0015</th>
<th>0.002</th>
<th>0.0025</th>
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<tr>
<td>Stress</td>
<td></td>
<td></td>
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<td></td>
<td></td>
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</tbody>
</table>

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0° single ply tensile specimen

Progression of damage in FE model of a unidirectional ply under longitudinal loading

Not Adjusted

(a) Early diffuse damage
(b) Damage clustering
(c) Final fiber failure

Automaton Adjusted

(d) Early crack-like damage
(e) Matrix damage more organized
(f) Final fiber failure

Legend:
- **Yellow**: Matrix failure
- **Green**: Adjacent to failed matrix
- **Red**: Fiber failure
- **Orange**: Adjacent to failed fiber
- **Blue**: No failure

(a) and (d): early matrix damage
(b) and (e): progression to substantial matrix damage
(c) and (f): final composite failure (fiber failure)
90° single ply tensile specimen \((Load \text{ transverse to fiber axis})\)

Shown are two trial executions; one using the automaton adjusted element feature and one with the feature inactive.

The graph illustrates the stress-strain behavior of the specimen. The red line represents the stress-strain curve when the feature is inactive, showing a more abrupt and brittle-like behavior. The blue dashed line represents the adjusted case, which exhibits a smoother transition.

Stress (MPa)

Strain
90° single ply tensile specimen

Progression of damage in FE model of a unidirectional ply under \textit{transverse} loading

Not Adjusted

(a) Early diffuse damage
(b) Final matrix failure

Automaton Adjusted

(c) Adjacent elements encouraged to fail in early damage stages
(d) Final matrix failure

- Yellow: Matrix failure
- Green: Adjacent to failed matrix
- Blue: No failure

(a) and (c); early matrix damage
(b) and (d); final composite failure (matrix failure)
Example: SiC/RBSN Laminated Composite in *On-Axis* & *Off-Axis* Loading

- Tested by Bhatt & Phillips (1990)

*displays key mechanisms/features for model material*

- SCS-6 fiber/Reaction Bonded Silicon Nitride matrix composite examined in detail by NASA
  - several papers published
- Laminated CMCs of interest to industry and less complex than woven composites
  - failure modes are not conflicted with complex fiber architecture
- [0] & [0/90] laminates display nonlinearity due to matrix failure, followed by fiber failure.
- Remaining ply orientations display sudden brittle failure.

SiC/RBSN
Bhatt & Phillips
(1990)

Rectangular specimens under uniaxial tensile loading
25.4 mm x 12.7 mm x 1.2 mm
30% fiber volume fraction

Exact stress-strain curves not available!

[0]_8

Unreinforced RBSN
[45]_8
[90]_8

[10]_8

[0]_8

[+45_2/-45_2]_8

[0_2/90_2]_8

Matrix cracking

Strain, %

Experimental Results

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SiC/RBSN Example Procedure & Setup

Abaqus FEA
S4 Shell elements
Fixed-displacement ramp load

Stochastic strength analysis:
(from individual trials / simulations / realizations)

- Use CARES Unit Sphere failure criterion
  - assume Isotropic material constituent strength
    - for simplicity and initial testing

1) Cool down from stress-free temperature of 550° to room temperature 23°
2) apply fixed-displacement ramp load

Residual stresses in constituents

MAC/GMC RUC

(10x20 mesh)

Interfacial failure modes and sliding resistance not considered

- Weibull parameters correlated to experimental results for 0° tensile specimen
- Interface strength made large:
  - Encourage matrix to fail before interface
## Constituent properties of SiC/RBSN with anisotropic thermal expansion coefficients

<table>
<thead>
<tr>
<th>Constituent</th>
<th>Modulus, GPa</th>
<th>Poisson ratio</th>
<th>Longitudinal coefficient of thermal expansion, $\alpha_L$ (m/m/°C)</th>
<th>Transverse coefficient of thermal expansion, $\alpha_T$ (m/m/°C)</th>
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</thead>
<tbody>
<tr>
<td>Fiber</td>
<td>390</td>
<td>0.17</td>
<td>$4.1 \times 10^{-6}$</td>
<td>$1.84 \times 10^{-6}$</td>
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<tr>
<td>Matrix</td>
<td>110</td>
<td>0.22</td>
<td>$2.2 \times 10^{-6}$</td>
<td>$2.2 \times 10^{-6}$</td>
</tr>
<tr>
<td>Interface</td>
<td>1.8</td>
<td>0.22</td>
<td>$2.0 \times 10^{-6}$</td>
<td>$2.0 \times 10^{-6}$</td>
</tr>
</tbody>
</table>

### Assumed Weibull Parameters:

- **Fiber**
  - $m_V = 20$
  - $\sigma_{oV} = 2875$ Mpa $\cdot m^{3/20}$
- **Matrix**
  - $m_V = 5.0$
  - $\sigma_{oV} = 150$ Mpa $\cdot m^{3/5}$
- **Interface**
  - $m_V = 5.0$
  - $\sigma_{oV} = 80$ Mpa $\cdot m^{3/5}$
“The fiber is orthotropic, with different coefficients of thermal expansion along and perpendicular to the fiber axis.”;

Residual matrix stresses after cool-down from temperature
- Effect of anisotropic fiber-thermal-expansion-coefficient, $\alpha_f$ on RUC
Effect of matrix Weibull modulus on stress-strain response

Calibrating to experimental data

0° single ply tensile specimen

Matrix damage

$m_V$
- 5
- 10
- 20
- 60

Strain

Stress (MPa)
FEAMAC/CARES Calibrated to 550° C stress-free temperature & 24% fiber volume fraction, $V_f$

**Prediction (30% $V_f$)**

**Calibration (24% $V_f$)**

Actual stress-strain curve from:
Chulya, A., Gyekenyesi, J. P., and Bhatt, R. (1991);
“Mechanical Behavior of Fiber Reinforced SiC/RBSN Ceramic Matrix Composites: Theory and Experiment. NASA TM 103688;

Effect of matrix fragments On fiber response not modeled

CARES calculated 50% matrix failure probability prior to any damage initiation

Exp. 24% fiber vol.

Exp. 30% fiber vol.

50% Fail Prob 24%

50% Fail. Prob. 30%

FEAMAC/CARES 24%

FEAMAC/CARES 30%
0° Single Ply $[0]_8$

Non-linear (graceful) failure behavior

Calibrated to experimental data for 24% fiber volume fraction

FEAMAC/CARES analysis was for a single ply to speed computation

Experimental Data:
Matrix Frac. Str.: $227 \pm 41$ MPa
Ultimate Str: $682 \pm 150$ MPa

Note: very few specimens were tested which means the range of uncertainty (the confidence bounds) for $m$ is large!

Predicted response for 30% fiber volume fraction

Individual simulation

- Fiber $m_V = 20$ $\sigma_{ov} = 2875$ Mpa $\cdot m^{3/20}$
- Matrix $m_V = 5.0$ $\sigma_{ov} = 150$ Mpa $\cdot m^{3/5}$
- Interface $m_V = 5.0$ $\sigma_{ov} = 80$ Mpa $\cdot m^{3/5}$
**0° Single Ply**

*Strength scatter from proportional limit strain offset*

- 0.005% strain offset
- 0.01% strain offset

*Linear elastic*

**Intersection of simulation trial with strain offset**

- 1-of-10 Individual simulation trials

**FEAMAC/CARES:**
- Mean = 238.2 MPa
- Std. Dev. = 10.28 MPa

**Experimental Data:**
- Matrix Frac. Str.: 227 ±41 Mpa

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*PLS is defined as the stress at 0.005% strain offset:*

Damage progression of $0^\circ$ tensile specimen - two trials (undeformed plot)
Prediction for Ten Trials for 90° Fiber Orientation
Assuming matrix and interface are isotropic strength materials

FEAMAC/CARES analysis was for a single ply to speed computation

\([90]_8\)

1-of-10 Individual simulation trials

Experimental Data:
Frac. Str.: 27 ±3 MPa

Brittle behavior

Note: very few specimens were tested which means the range of uncertainty (the confidence bounds) for \(m\) is large!
90° Tensile specimen at final failure for 10 trials – Undeformed plots

Final specimen failure from matrix damage

Loading Direction
Prediction for Ten Trials for 10° Fiber Orientation
Assuming matrix and interface are isotropic strength materials

FEAMAC/CARES analysis was for a single ply to speed computation

[10]_8

Brittle behavior

Experimental Data:
Frac. Str.: 162 MPa

Note: very few specimens were tested which means the range of uncertainty (the confidence bounds) for m is large!
10° off-axis tensile specimen; 10 trials at final (matrix) failure; deformed plots

- Edges are allowed to freely deform (warp) on cool-down
- After cool-down; bottom edge fixed in loading direction when displacement load applied
- After cool-down; single node along top edge (middle) fixed in direction perpendicular to displacement direct.
Prediction for Ten Trials for 45° Fiber Orientation
Assuming matrix and interface are isotropic strength materials

Experimental Data:
Frac. Str.: 43 MPa

Note: very few specimens were tested which means the range of uncertainty (the confidence bounds) for m is large!

FEAMAC/CARES analysis was for a single ply to speed computation

CARES calculated 50% matrix failure probability prior to any damage initiation

Brittle behavior

1-of-10 Individual simulation trials

\[ [45]_8 \]
Prediction for Ten Trials for $[+45_2/-45_2]_8$ Fiber Orientation
Assuming matrix and interface are isotropic strength materials

FEAMAC/CARES analysis was for four plys (+45/-45/-45/+45) to speed computation

$[+45_2/-45_2]_8$

Matrix cracks approx. normal to loading direction

1-of-10 Individual simulation trials

Experimental Data:
Matrix Frac. Str.: $75 \pm 10$ MPa; $m \approx 8.8$
Ultimate Str: $88 \pm 16$ MPa; $m \approx 6.3$

Note: very few specimens were tested which means the range of uncertainty (the confidence bounds) for $m$ is large!
For \([+45_2/-45_2]_8\) Fiber Orientation; 10 trials at final (matrix) failure; deformed plots

FEAMAC/CARES analysis was for four plies \((+45/-45/-45/+45)\) to speed computation
FEAMAC/CARES analysis was for four plies (0/90/90/0) to speed computation.

\([0_2/90_2]_s\) Fiber Orientation

Assuming matrix and interface are isotropic strength materials.

Experimental Data:
Matrix Frac. Str.: 127 ± 26 MPa
Ultimate Str: 294 ± 87 MPa

Non-linear (graceful) failure behavior

Note: very few specimens were tested which means the range of uncertainty (the confidence bounds) for \(m\) is large!
Prediction for Ten Trials for $[0_2/90_2]_s$ Fiber Orientation

Strength scatter from proportional limit strain offset

0.005% strain offset

0.01% strain offset

Linear elastic

Intersection of simulation trial with strain offset

1-of-10 Individual simulation trials

Experimental Data:
Matrix Frac. Str.: 127 ±26 MPa

FEAMAC/CARES:
Mean = 133.3 MPa
Std. Dev. = 8.11 MPa
0° Double-Notched Tensile Specimen

Failure mode showed axial splitting of matrix

FEAMAC/CARES analysis was for a single ply to speed computation

Experiment

[0]₈

Axial splitting

Fiber Direction

Loading Direction
$0^\circ$ Double-Notched vs: Central-Hole Tensile Specimen

Early matrix damage

Matrix damage progression
Time-dependent Failure Example: Static Loading
(Matrix Damage Accumulation From Slow Crack Growth)

Strain response for applied static tensile load over time

![Graph showing strain response over time for different applied static loads.]

- **Applied Static Load (MPa):** 170, 180, 190, 200
- **Damage increases with time and load**
- **Effect of N**
- **CARES 50%**
- **P_f for matrix failure**
- **Note: Parameter “B” is related to parameter “A”**

Service life prediction
Longitudinal stress applied to a 0° SiC/RBSN ply

10 time increments per time magnitude

Slow Crack Growth
Power Law:

\[
\frac{da}{dt} = AK_{\text{eq}}^N
\]

Weibull Parameters
- \( m = 7 \) (Weibull slope)
- \( \sigma_0 = 106 \text{ Mpa} \bullet \text{mm}^{3/7} \)

Fatigue Parameters
- \( N = 20 \) (fatigue slope)
- \( B = 1.0 \times 10^9 \text{ MPa}^2 \bullet \text{sec} \)
Conclusions

• Progressive damage simulation of composite structures incorporating probabilistic material strength models is possible with the FEAMAC/CARES code

• The Unit Sphere multiaxial model was used predict the strength response of a SiC-RBSN composite for various fiber orientations under uniaxial tension

• Reasonable correlation to matrix cracking strength experimental data was achieved assuming the matrix was an isotropic material with $m \approx 5$, and assuming residual stresses from thermal processing were present

• Brittle behavior vs: non-brittle failure (*graceful failure*) demonstrated

• Localized damage modes at stress concentration features shown

**Acknowledgement**

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Extra Material
Abstract:
Reported here is a coupling of two NASA developed codes: CARES (Ceramics Analysis and Reliability Evaluation of Structures) with the MAC/GMC (Micromechanics Analysis Code/ Generalized Method of Cells) composite material analysis code. The resulting code is called FEAMAC/CARES and is constructed as an Abaqus finite element analysis UMAT (user defined material). Here we describe the FEAMAC/CARES code and an example problem (taken from the open literature) of a laminated CMC in off-axis loading is shown. FEAMAC/CARES performs stochastic-strength-based damage simulation response of a CMC under multiaxial loading using elastic stiffness reduction of the failed elements.
Mesh effect & time step sensitivity

CARES calculated 50% matrix failure probability prior to any damage initiation

Trade off: Mesh sensitivity vs: localization of damage

0° single ply tensile specimen (Load parallel to fiber axis)
Time-Dependent Life Prediction Theory - Slow Crack Growth and Cyclic Fatigue Crack Growth Laws

**Power Law:** - Slow Crack Growth (SCG)

\[
\frac{da}{dt} = A K_{leq}^N
\]

**Combined Power Law & Walker Law:** SCG and Cyclic Fatigue

\[
\frac{da}{dt} = A_1 g K_{leq}^N + A_2 f_c (1 - R)^Q K_{leq}^N
\]
Modeling individual time steps in the life prediction methodology enables simulating transient events such as turbine start-up/shut-down or atmospheric re-entry. A computationally efficient methodology has been developed that can extrapolate the reliability calculation for an arbitrary number of $Z$ cycles – where each cycle is described by $k$ number of time steps. This conceivably allows the coupling of other effects such as stiffness degradation and oxidation effects on the individual time steps and this can be accounted for interactively within the transient finite element and micromechanics analysis.
**Transient Life Prediction Theory - Power Law SCG**

**Reliability formula for** $k$ **discrete time steps over** $Z$ **cycles:**

\[
P_{SV}(t_k) = \exp\left\{- \sum_{i=1}^{n} \frac{V_i}{4\pi} \left[ \prod_{j} \left( \frac{\sigma_{Ieq,k,T_{max}}}{\sigma_{0BV,k}} \right)^{N_{V,k} - 2} + \right] \right\}
\]

\[
\frac{\sigma_{Ieq,k} Z \Delta t_k}{N_{V,k} - 2} \left[ m_{V,k} \left( N_{V,k} - 2 \right) \right]_k \frac{\sigma_{Ieq,j} Z \Delta t_j}{N_{V,j} - 2} \left[ m_{V,j} \left( N_{V,j} - 2 \right) \right]_j + \ldots
\]

\[
\ldots + \frac{\sigma_{Ieq,2} Z \Delta t_2}{N_{V,2} - 2} \left[ m_{V,2} \left( N_{V,2} - 2 \right) \right]_2 + \frac{\sigma_{Ieq,1} Z \Delta t_1}{N_{V,1} - 2} \left[ m_{V,1} \left( N_{V,1} - 2 \right) \right]_1
\]

**Individual time step:** Each time step can have different loading, Weibull, and fatigue parameters. Compatibility of failure probability is maintained between the individual time steps.
SiC/RBSN Notional Example for SCG

0° Degree tensile specimen under a static load over time

- Use same 10x20 mesh, RUC, and material properties as previous SiC/RBSN off-axis loading example

Weibull and Slow Crack Growth (SCG) Parameters

<table>
<thead>
<tr>
<th>Constituent</th>
<th>Weibull modulus, $m_V$</th>
<th>Weibull scale parameter, $\sigma_{0V}$, MPa $\cdot$ mm$^{3}$/m$^{V}$</th>
<th>Fatigue exponent, $N_v$ (Equation (11))</th>
<th>Fatigue constant, $B_V$, MPa$^2$ $\cdot$ sec (Equation (17))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fiber</td>
<td>20.0</td>
<td>2875.0</td>
<td>100.0</td>
<td>$1.0 \times 10^{10}$</td>
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<tr>
<td>Matrix</td>
<td>7.0</td>
<td>106.0</td>
<td>20.0</td>
<td>$1.0 \times 10^9$</td>
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<tr>
<td>Interface</td>
<td>7.0</td>
<td>60.0</td>
<td>100.0</td>
<td>$1.0 \times 10^{10}$</td>
</tr>
</tbody>
</table>
Unit Sphere Multiaxial (Batdorf) Model:
Puts linear elastic fracture mechanics into Weibull weakest-link theory

● Incremental failure probability is the product of two probabilities:

\[ \Delta P_f = P_1 \cdot P_2 \]

\[ P_1 = \text{Probability of the existence of a crack having a critical strength between } \sigma_c \text{ and } \sigma_c + \Delta \sigma_c \text{ in the incremental volume } \Delta V \]

\[ P_2 = \text{Probability a crack having a critical strength of } \sigma_c \text{ will be oriented in a direction such that it will fail under the applied multiaxial stress state} \]

● Component failure probability:

\[ P_f = 1 - \exp \left\{ - \int_V \left[ \int_0^{\sigma_e} P_1(\sigma_c)P_2(\sigma_c) \, d\sigma_c \right] \, dV \right\} \]

\[ P_2 \text{ involves Integration of an equivalent stress } \sigma_e, \]
\[ \text{where } \sigma_e \geq \sigma_c, \text{ over the surface of a unit radius sphere (all possible flaw orientations) divided by the total surface area of the unit radius sphere} \]

Mixed-Mode Fracture Criteria:
- Normal stress (shear-insensitive cracks)
- Maximum tensile stress
- Total coplanar strain energy release rate
- Noncoplanar (Shetty)

Flaw Shapes:
- Griffith crack
- Penny-shaped crack
CARES Unit Sphere Multiaxial model has crack geometry & mixed-mode fracture criterion

- Two models for transverse isotropy

(1) Flaw / Fracture-Plane Orientation Anisotropy

(2) Strength Orientation Anisotropy

σ_{IC} or K_{IC} varies with orientation

Anisotropic Unit Sphere model defined in a material coordinate system reference frame

All possible vectors normal to the flaw plane of the fiber or tow or unidirectional lamina

Width of belt defines extent of misalignment

Similar to Puck’s composite failure criterion except in a probabilistic framework

When represented on a unit radius sphere of all possible flaw plane orientations the flaw plane normals define an equatorial belt distribution
**Multiaxial Performance:** biaxial response predicted from a MAC/GMC RUC for combined longitudinal (L) and transverse (T) loading on a unidirectional PMC vs: FEA.

50% probability of failure envelope.

**PMC from WWFE**

- Unit Sphere parameters adjusted so GMC results matched FEA results for uniaxial tension and compression.
- Intermediate points are predictions.
45° off-axis tensile specimen; 10 trials at final failure deformed plots

- Edges are allowed to freely deform (warp) on cool-down
- After cool-down; bottom edge fixed in loading direction when displacement load applied
- After cool-down; single node along top edge (middle) fixed in direction perpendicular to displacement direct.
Prediction for Ten Trials for $[0_2/90_2]_s$ Fiber Orientation

Closer view

**Experimental Data:**
Matrix Frac. Str.: $127 \pm 26$ MPa
Ultimate Str: $294 \pm 87$ MPa

CARES calculated 50% matrix failure probability prior to any damage initiation
For $[0_2/90_2]_s$ fiber orientation; four trials with deformed plots

**Progression from matrix failure to final fiber failure**

FEAMAC/CARES analysis was for four plys (0/90/90/0) to speed computation
Path Forward

• Continue demonstrate/benchmark capability on CMCs (using available literature)
  ➢ For uniaxial & multiaxial failure response  
    (orientation, lamination, stress concentration, flexural)
    - Fast-fracture
    - Time & cycle dependent
      - more detailed micromechanical models of failure modes
• Develop / incorporate enviromechanical degradation models
• Investigate applicability to predict EBC damage progression
• Develop / incorporate anisotropic elastic modulus degradation based on CARES critical fracture angle probability density distribution
• Improve software efficiency (memory, speed, multiprocessing)
  ❖ Demonstrate this capability on component/structure
Approach For Life Prediction & Component Design Of Composites

- Combine CARES, MAC & FEA codes where
  Micromechanics provides the link between structures & materials

**CARES:** monolithic ceramics

- Probabilistic strength
- Mechanistic-based multiaxial failure model
- Efficient life prediction algorithm
- Isotropic and transverse isotropy

**MAC/GMC:** composites analysis

- Micromechanics
- Accurate RUC stress fields
- Flexibility in RUC designs
- Progressive damage capability
- Computationally efficient

Move CARES from the macroscopic scale of the structure to the microscale of the individual material constituents & RUC with the FEA-MAC micromechanics code.