On small disturbance ascent vent behavior

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Outline

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• Math Model Development
  ○ Mass Conservation
  ○ Conductance
  ○ Small Disturbance Expressions
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JWST Observatory
Introduction

- James Webb Space Telescope (JWST)
  - Contains four large instruments within an enclosure incorporating some blanketed walls
    - Only vents through small aperture during ascent
    - Other venting would risk light leaks
    - Blankets limit allowable overpressure
  - Electronics compartment on shadowed side also requires limited overpressure due to multi-layer insulation blankets
- Useful to develop expressions in this limit to help design process
Objective

- Develop expressions in small disturbance limit for simple venting systems
  - Common conductance elements
    - Orifices
    - Ducts
  - Mass conservation statement
- Explore some limits for practical application
Mass Conservation Statement

- Mass accumulation rate
  - Mass generation rate within volume rigid $V$
  - Net rate vented across bounding surface $S$

$$\frac{d}{dt} \iiint_{V} \rho \, dV = \dot{m}_{\text{gen}} - \iint_{S} \rho \mathbf{u} \cdot dS$$
Mass Conservation Development

• Assume isothermal, ideal gas with constant properties throughout \( V \)
  ○ Recast statement in terms of gas load \( Q \)
    \[
    V \frac{dp}{dt} = \dot{m}_{\text{gen}} RT - \iint_S p u \cdot dS
    \]

• In this case, can say venting occurs across a discrete set of elements \( K \), define conductance \( F \)
  \[
  F_{1-2} \equiv \frac{Q}{p_1 - p_2} = \frac{\dot{m} RT}{p_1 - p_2} \quad \quad V \frac{dp_1}{dt} = \dot{m}_{\text{gen}} RT - \sum_{k=1}^{K} F_k (p_1 - p_2)
  \]
Orifice Conductance

- For a calorically perfect gas in continuum flow

\[
F_{\text{orifice}} = \sqrt{\frac{2\gamma RT}{(\gamma - 1)}} \frac{A}{1 - \frac{p_2}{p_1}} \left( \frac{p_2}{p_1} \right)^{\frac{1}{\gamma}} \sqrt{1 - \left( \frac{p_2}{p_1} \right)^{\frac{\gamma - 1}{\gamma}}}
\]

- Rewrite in terms of fairing pressure \( p_2 \) and pressure differential

\[
F_{\text{orifice}} = \sqrt{\frac{2\gamma RT}{(\gamma - 1)}} \frac{A}{1 - \frac{1}{1 + \frac{\Delta p}{p_2}}} \left( \frac{1}{1 + \frac{\Delta p}{p_2}} \right)^{\frac{1}{\gamma}} \sqrt{1 - \left( \frac{1}{1 + \frac{\Delta p}{p_2}} \right)^{\frac{\gamma - 1}{\gamma}}}
\]

- For small pressure differentials:

\[
F_{\text{ori, sm}} \equiv F_{\text{orifice}} (\Delta p \ll p_2) \approx A \sqrt{2RT \frac{p_2}{\Delta p}} \left( 1 - \frac{1}{\gamma} \frac{\Delta p}{p_2} \right) \approx A \sqrt{2RT \frac{p_2}{\Delta p}}
\]
Circular Duct Conductance

- Begin with Hagen-Poiseuille solution for average velocity in fully-developed, laminar flow

\[ \overline{u} = \frac{A}{8\pi\mu} \left( \frac{p_1 - p_2}{\ell} \right) \]

\[ F_{\text{duct}} = \frac{\dot{m}RT}{(p_1 - p_2)} = \frac{\overline{\rho}\overline{u}ART}{(p_1 - p_2)} = \frac{\overline{p}\overline{u}A}{(p_1 - p_2)} = \frac{A^2}{16\pi\mu\ell} (p_1 + p_2) \]

\[ F_{\text{duct, sm}} = \frac{A^2}{16\pi\mu\ell} \frac{p_2}{p_2} \left( 2 + \frac{\Delta p}{p_2} \right) \approx \frac{A^2}{8\pi\mu\ell} p_2 \]
Conductance Comparisons

\[ F_{\text{ori, sm}} \approx A \sqrt{2RT \frac{p_2}{\Delta p}} \]

\[ F_{\text{duct, sm}} \approx \frac{A^2}{8\pi\mu \ell} p_2 \]
Small Disturbance Solution

- For a single venting element

\[ V \frac{dp_1}{dt} = -F_{1-2} (p_1 - p_2(t)) \]

- If conductance not a function of pressure, would identify a time constant \( \tau = \frac{V}{F} \)

- In terms of \( \Delta p, p_2 \)

\[ \frac{dp_2}{dt} + \frac{d\Delta p}{dt} = -\frac{F_{1-2}}{V} \Delta p. \]

- Since \( p_1 \) is close to \( p_2 \) over all time, can neglect second time derivative
Small Disturbance Solution

- Grouping known quantities together

\[ F_{1-2} \Delta p \approx -V \frac{dp_2}{dt}. \]

- What was a first-order differential equation has been reduced to an algebraic expression!

- Find solutions for \( \Delta p \) using orifice and duct behavior
Limiting Orifice Behavior

• Substitute small disturbance equation for orifice
• Solving for $\Delta p$

$$\Delta p_{ori, sm}(t) = \frac{1}{2RT} \left( \frac{V}{A} \right)^2 \left( \frac{dp_2}{dt} \right)^2 \frac{p_2}{p_2}.$$

• Identical to solution developed intuitively by Scialdone!
• Highest value occurs where last term is maximized
  ○ Usually occurs during transonic disturbance period
Comments on Scialdone Formula

- Scialdone modified the orifice area by using a discharge coefficient
  - Based on what was originally an ASTM description for orifice plates within pipes
  - This author has not found this coefficient to be necessary when comparing against test data
- Original development recognized use of small disturbance assumptions, but created a time constant based on molecular flow and sonic conditions
  - Sonic conditions definitely violate small disturbance limit
  - Assumptions lead to minimum critical areas that are too large
    - Run up against thermal, optical, high-voltage restrictions
Limiting Duct Behavior

- Circular duct solution

\[ \Delta p_{\text{duct, sm}}(t) \approx -8\pi\mu \frac{V\ell}{A^2} \frac{dp_2}{dt} \approx -8\pi\mu \frac{V\ell}{A^2} \frac{d\ln p_2}{dt}. \]

- Note differences from orifice solution
  - Difference dependence on fairing depressurization rate
  - Lower dependence on volume
  - Presence of dynamic viscosity, duct length emphasize viscous rather than gasdynamic effects
Critical Reynolds Number

- Laminar flow condition violated when duct Re > 2000 – 4000
- Work with definition of Reynolds number and duct mass flow expression in small pressure disturbance limit to find

\[ \Delta p_{\text{laminar}} < \frac{32 \text{Re}_{\text{crit}} \mu^2 \ell RT}{d^3 p_2}. \]

○ Stubby ducts allow higher flow rates at constant diameter, but may also lead to turbulent conditions
  - If aspect ratio is stubby enough, the element may behave more like an orifice instead!
Concluding Remarks

- Overpressure model developed for isothermal, constant temperature venting of an ideal calorically perfect gas for a rigid volume in the presence of an external driving pressure, in the limit of small $\Delta p$
- Limiting expressions for $\Delta p$ were developed for venting across orifices and circular ducts in fully developed, laminar flow
  - Orifice equation identical to Scialdone’s, discussed how limits should be understood
  - Duct equation exhibits viscous effects, different dependence on driving pressure profile, found limit on validity based on critical Reynolds number