Kalman Filter for Mass Property and Thrust Identification (MMS)

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Multiplicative Extended Kalman Filter (MEKF) for MMS On-board Attitude and Rate Determination

- MEKF formulation (points-of-interest)
- Measurement Model (star sensor)
- Flight Performance

MEKF for Identification of System Properties
(center-of-mass, moments of inertia, and thrust)

- State Augmentation
- Measurement Model (accelerometer)
- Thruster Warm-Up Model
- Simulated Test-Case Performance
- Simulated Monte Carlo Performance
- Flight Performance
Brief MEKF Review

**Multiplicative Extended Kalman Filter**
- Extended Kalman Filter (EKF) variant
- 1st flight use SPARS (1969)
- rigorous formation by Lefferts, Markley, and Shuster (1982)
- continued refinement and advocacy by Markley (2003)

**EKF with Additive Update**

\[ \mathbf{q}_{\text{true}} = \Delta \mathbf{q} + \hat{\mathbf{q}} \]

- error quaternion state used in filter
- full (global) state estimate

**MEKF Multiplicative Update**

\[ \mathbf{q}_{\text{true}} = \delta \mathbf{q} \otimes \hat{\mathbf{q}} \]

- error quaternion not directly used in filter
- full (global) state estimate

**Quaternion Left-Multiple Operator**

\[ \mathbf{q} \otimes \equiv \left[ \begin{array}{c} q_1 \\ q_2 \\ q_3 \\ q_4 \end{array} \right] \otimes = \left[ \begin{array}{c} q_1 \\ q_2 \\ q_3 \\ q_4 \end{array} \right] = q_4 \mathbb{I} + \left[ \begin{array}{c} -q_1^\times \\ q_1^\times \end{array} \right] = \left[ \begin{array}{ccc} q_4 \mathbb{I} & -q_1^\times & q_1^\times \\ -q_1^\times & 0 \end{array} \right] \]

**Skew-Symmetric “Cross-Product” Operator**

\[ \mathbf{\omega}^\times \equiv \left[ \begin{array}{c} \omega_x \\ \omega_y \\ \omega_z \end{array} \right] ^\times = \left[ \begin{array}{ccc} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{array} \right] \]
The MEKF uses a reduced three component attitude parameterization as the error-state inside the filter.

- Could use any three-component attitude representation (e.g. Euler rotation axis/angle, Gibbs vector, Modified Rodrigues parameters, Tait-Bryan angles, etc.)

- MMS chose (twice) the Gibbs vector parameterization:
  - free of singularities up to ±180°
  - largest possible 180° map to infinity (compatible with Gaussian “tails”)
  - avoids accumulation of numerical errors in full-state quaternion norm through explicit normalization in reset operation that is neither ad hoc or require transcendental evaluations
  - observation model insensitive to sign ambiguity in star camera’s output quaternion
  - diagonals of error covariance matrix (P) map directly to attitude error variance (σ²)

Mathematical mappings:

\[ \delta \theta \equiv 2 \delta g \equiv 2 \frac{\delta q_{1:3}}{\delta q_4} \]

Error state used in filter
\( \delta g \) is attitude error
\( \delta q \) relationship to error quaternion

Inverse mappings:

\[ \delta q = \pm \frac{1}{\sqrt{1 + \| \delta g \|^2}} \begin{bmatrix} \delta g \\ 1 \end{bmatrix} = \frac{1}{\sqrt{4 + \| \delta \theta \|^2}} \begin{bmatrix} \delta \theta \\ 2 \end{bmatrix} \approx \begin{bmatrix} \delta \theta \\ 2 \\ 1 \end{bmatrix} \]

(1st order only)
## On-board MEKF Models

### State Dynamics

<table>
<thead>
<tr>
<th>Nonlinear Full-State Model</th>
<th>Linearized Error-State Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \dot{x}(t) = f(q(t), \omega(t), u(t), w(t)) ]</td>
<td>[ \delta \dot{x} = f(\delta \theta(t), \delta \omega(t), u(t), w(t)) ]</td>
</tr>
<tr>
<td>[ = \left{ \begin{array}{l} f_q(q, \omega) \ f_\omega(\omega, u) \end{array} \right} + Gw ]</td>
<td>[ = \left{ \begin{array}{l} f_\theta(\delta \theta, \delta \omega) \ f_\omega(\delta \omega, u) \end{array} \right} + G(t)w ]</td>
</tr>
<tr>
<td>[ \begin{bmatrix} \dot{q} \ \dot{\omega} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \begin{bmatrix} -\omega \times &amp; \omega \ -\omega^T &amp; 0 \end{bmatrix} q \ I^{-1} [\tau(u) - \omega \times I \omega] \end{bmatrix} + Gw ]</td>
<td>[ \begin{bmatrix} \dot{\theta} \ \dot{\omega} \end{bmatrix} \approx \begin{bmatrix} -\omega \times &amp; I_3 \ 0_{3 \times 3} &amp; I^{-1} [I \omega \times -\dot{\omega} \times I] \end{bmatrix} \begin{bmatrix} \delta \theta \ \delta \omega \end{bmatrix} + \begin{bmatrix} w_\theta \ w_\omega \end{bmatrix} \approx F(t) \delta x + w ]</td>
</tr>
</tbody>
</table>

Process Noise: \( w(t) \sim N(0, Q(t)) \)

- MMS has no gyros. Inertia matrix knowledge is required.
- Derivation of attitude error dynamics derivation a bit more involved than for non-additive states.

### Measurement Updates

- **Measurement residual**: \( p_k \)
- **Expected measurement** (based on full-state est.)
- **Expected measurement** (linearize) measurement sensitivity matrix
- **Zero due to reset op**

\[
\begin{bmatrix} \delta \theta_k^+ \\ \delta \omega_k^+ \end{bmatrix} = \begin{bmatrix} \delta \theta_k^- \\ \delta \omega_k^- \end{bmatrix} + K_k \begin{bmatrix} y_k - h(\hat{q}_k^-, \hat{\omega}_k^-) - H_k (\hat{a}_k^-, \hat{\omega}_k^-) \begin{bmatrix} \delta \theta_k^- \\ \delta \omega_k^- \end{bmatrix} \end{bmatrix}
\]

\[
H_k \equiv \left[ \frac{\partial h}{\partial q} \cdot \frac{\partial q}{\partial (\delta \theta)} \frac{\partial h}{\partial \omega} \right] \hat{a}_k, \hat{\omega}
\]

Even though \( H_k \) is not used here (due to reset op) it is needed for covariance propagation. NOTE: partial derivatives are with respect to **error** states (but result only differs for non-additive states).
μASC Star Tracker System (STS) provided by the Technical University of Denmark (DTU)

- Four camera head units (CHUs)
- Redundant centralized electronics
- 4 Hz update rate
- Measurements combined as a pre-processing step in to single measurement update for computationally simpler on-board MEKF processing
- Spec performance levels (4 heads combined):

<table>
<thead>
<tr>
<th>Axis</th>
<th>Accuracy (1σ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transverse</td>
<td>20 arcsec</td>
</tr>
<tr>
<td>Boresite</td>
<td>60 arcsec</td>
</tr>
</tbody>
</table>

**Measurement Model**

\[
(y_{chu})_k = (h_{chu})_k = \delta \theta_k + (v_{chu})_k = (\delta \theta_{chu})_k = 2 \left( \frac{\delta q_{1:3}}{\delta q_{4}} \right)_k = 2 \left( \frac{(q_{chu})_k \hat{q}_{k}^{-1}}{(q_{chu})_k \hat{q}_{k}^{-1}} \right)_{1:3}
\]

**Measurement Residual**

\[
(r_{chu})_k = (y_{chu})_k - h (\hat{q}_{k}, \hat{\omega}_{k}) - H_{chu} \left[ \frac{\delta \theta_{chu}}{\delta \omega_{k}} \right] = (\delta \theta_{chu})_k
\]

\[
H_{chu} = \left[ \frac{\partial h_{chu}}{\partial (\delta \theta)} \frac{\partial h_{chu}}{\partial \omega} \right]_{\hat{q}_{k}, \hat{\omega}} = \begin{bmatrix} I_3 & 0_{3 \times 3} \end{bmatrix}
\]

Image from MMS-3, CHU-B

Insensitive to STS quaternion sign!
Examining the performance of the MEKF rate estimation for the first two thruster-pulses of a calibration maneuver (EA019) executed on 1 April 2015.
Full EA019 maneuver rate profile

- exercises all twelve thrusters individually, in matched pulse-pairs, 1/2 nutation cycle apart
- thrusters #1 (radial) and #12 (axial) exercised in double pairs to characterize warm-up
Star Tracker per Camera Head Unit (CHU) Measurement Residuals

Axis  Spec\(^{\dagger}\) (1σ)
---  ---
Transverse  20 asec
Boresite  60 asec

\(^{\dagger}\)expected ensemble solution performance with all four heads measurements
Augmented MEKF for Ground-based System Identification
MMS maneuvering performance requires accurate knowledge of

- **(Fuel) Inertia Tensor**—lacking gyroscopes, the second mass moment of inertia knowledge directly affects the accuracy of the rate estimate. Angular rate errors (along with center-of-mass knowledge) affect the centripetal compensation algorithms used in closed-loop orbital maneuvers. *Since the dry system properties were well known prior to launch, only the fuel’s contribution to inertia is estimated.*

- **(Fuel) Center-of-Mass**—knowledge of the lever-arm from the CM to the accelerometer sensor heads affects the ability to remove gyro-dynamic biases from the incremental velocity output of the AMS.

- **Steady-State Thrust**—closed-loop incremental velocity feedback removes the majority of the maneuvering system’s sensitivity to knowledge errors in thruster. However, in order to achieve 1% (3σ) maneuvering accuracy, it was shown via Monte Carlo simulations that 3% (3σ) steady-state thrust knowledge was necessary due to the corruption of rate-propagation with an incorrect torque (gyro rate-substitution would alleviate).

- **Warm-up Knock-down Factor**—warm-up effects of the cat-bed in the hydrazine thruster’s thrust-chamber can degrade initial thrust by as much as 15%. In order to account for this in the system dynamics, a simplified thermal-model of the thruster was added to the MEKF. Two thermal-states per thruster are required. Thermal coefficients of the model were determined from pre-flight test data.

- **Accelerometer Intrinsic Biases**—in order to use the accelerometer measures for thrust-determination, the intrinsic thermo-electrical biases of the AMS sensor heads must also be estimated.

**Augmented State Vector**

\[
\delta x = \begin{bmatrix} \delta \theta & \delta \omega & \delta b & \delta r_f & \delta I_f & \delta f_{ss} & \delta T_c & \delta T_x \end{bmatrix}^T
\]

original FSW states
AMS biases
fuel CM
fuel inertia
steady-state thrust
thrust chamber temp
structure temp

6 states vector
Acceleration Measurement System (AMS), manufactured by ZIN Technologies

- three orthogonal Honeywell QA3000 accelerometers
- 100 kHz analog-to-digital sampling
- dynamic range of greater than ±25,000 μg
- resolution of less than 1 μg
- short-term (1σ) bias stability over a twelve hour period of better than 1 μg
- effective bandwidth of 250 Hz
- 1 KHz (down-sampled) acceleration integrated (corrected and summed) to produce an incremental velocity-change output at 4 Hz
- low-pass bias estimation filter
Accelerometer Measurement Model

Modeled as a proof-mass connected to a rigid-body by tri-axial springs, the device acceleration relative to a body-fixed origin is

\[ a_d = -\frac{k_d}{m_p} \xi \equiv A \left( \dot{V}_o - a_{grav} \right) + \dot{\omega} \times r_d + \omega \times \omega \times r_d \]

Introducing the base-body’s center-of mass \((r_c)\) yields a truth model

\[ a_d = \frac{f_t}{m} + \dot{\omega} \times (r_d - r_c) + \omega \times \omega \times (r_d - r_c) - \left( 2 \cdot \omega \times \dot{r}_c + \ddot{r}_c \right) \]

where \(f_t\) is the acceleration due to body-fixed thrusters.

Acceleration measurement model assumes \(n\) uni-axial measurements (along \(u_n\)) corrupted by bias, noise and scale factor errors

\[
\begin{pmatrix}
a_x \\
a_y \\
a_z
\end{pmatrix}
_{t_k} = \left( O^T O \right)^{-1} O^T \begin{bmatrix}
(1 + \delta \kappa_1) \hat{u}_1^T \\
(1 + \delta \kappa_2) \hat{u}_2^T \\
\vdots \\
(1 + \delta \kappa_n) \hat{u}_n^T
\end{bmatrix} a_d + \begin{bmatrix}
b_1 \\
b_2 \\
\vdots \\
b_n
\end{bmatrix} + \begin{bmatrix}
\eta_1 \\
\eta_2 \\
\vdots \\
\eta_n
\end{bmatrix}
\]

\( \left( h_{\text{ams}} \right)_k \)
Augmented Measurement Models

STS measurement sensitivity matrix

\[
(H_{\text{chu}})_k = \begin{bmatrix}
I_3 & 0_{3\times 3} & 0_{3\times 3} & 0_{3\times 3} & 0_{3\times 3} & 0_{3\times 3} & 0_{3\times 3}
\end{bmatrix} \hat{x}_k
\]

AMS measurement sensitivity matrix

\[
(H_{\text{ams}})_k = \begin{bmatrix}
0_{3\times 3} & \frac{\partial h_{\text{ams}}}{\partial \omega} & I_3 & \frac{\partial h_{\text{ams}}}{\partial r_f} & \frac{\partial h_{\text{ams}}}{\partial I_f} & \frac{\partial h_{\text{ams}}}{\partial I_{ss}} & \frac{\partial h_{\text{ams}}}{\partial T_c} & 0_{3\times 3}
\end{bmatrix} \hat{x}_k
\]

AMS measurement noise

\[
A_{0-10 \text{ Hz}} \leq (a_{\text{rms}})^2 = \frac{8^2 \mu g^2}{10 \text{ Hz}} = 6.4 \frac{\mu g^2}{\text{Hz}} = 615.9 \left(\frac{\mu m}{s^2}\right)^2
\]
Introduced knowledge errors into a simulated test-case system

- pair of pulses from thruster #1 at 20 and 36 secs

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>accelerometer biases</td>
<td>+20 µg</td>
</tr>
<tr>
<td>fuel center-of-mass</td>
<td>+50 mm</td>
</tr>
<tr>
<td>fuel moments of inertia</td>
<td>+10 kg-m²</td>
</tr>
<tr>
<td>steady-state thrust magnitude</td>
<td>+5%</td>
</tr>
</tbody>
</table>

Steady-State Force Estimation Test-Case Results

(actual force produced by thruster (includes warm-up effect))
System ID: 2-Pulse Test-Case

- Center-of-Mass Estimation Test-Case Results

- initial error = 15.2 mm
  final error = 0.2 mm

- initial error = 15.2 mm
  final error = 2.1 mm

- initial error = 15.3 mm
  final error = 2.9 mm
System ID: 2-Pulse Test-Case

Composite Inertia Tensor Estimation Test-Case Results
System ID: 2-Pulse Test-Case

AMS Bias Estimation Test-Case Results

![Graph showing bias estimation and true bias over time.](image-url)
Hundreds of parameters in the high-fidelity simulation of the MMS spacecraft were randomly perturbed within the expected distributions and (conservative) uncertainty limits of ground-based knowledge.

The full EA019 maneuver was simulated and statistics collected on the accuracy of the augmented MEKF system identification process. The results from 300 runs are shown.

### Steady-State Thrust Estimation Monte Carlo Statistics

**F$_{thr01}$ Error Histogram**

- pre ($\mu=-2.4\%, \sigma=4.4\%)$
- post ($\mu=-0.3\%, \sigma=0.8\%$)

**F$_{thr12}$ Error Histogram**

- pre ($\mu=-1.9\%, \sigma=4.6\%)$
- post ($\mu=-0.4\%, \sigma=0.8\%$)
System ID: Monte Carlo Results

Composite Mass Property Estimation Monte Carlo Statistics

- Center of Mass
- Moments of Inertia

X and Y axes results similar
Ground processing of the EA019 maneuver for MMS1 produced the following results:

**System ID: MMS1 EA019 Flight Results**

**Lateral Center-of-Mass Estimation**
- $c_{mx}$
- $c_{my}$

- Pre: -0.1, 0.1 mm
- Post: 3.3, 4.7 mm

**Axial Center-of-Mass Estimation**
- $c_{mz}$

- Pre: 605.3 mm
- Post: 604.6 mm
System ID: MMS1 EA019 Flight Results

**System ID: MMS1 EA019 Flight Results**

**I_{xx} [kg-m^2]**
- Pre: 991.5 kg-m^2
- Post: 968.8 kg-m^2

**I_{yy} [kg-m^2]**
- Pre: 966.2 kg-m^2
- Post: 935.7 kg-m^2

**I_{zz} [kg-m^2]**
- Pre: 1614.9 kg-m^2
- Post: 1598.3 kg-m^2

**Thruster 01 (radial)**
- Pre: 17.06 N
- Post: 18.38 N

**Thruster 12 (axial)**
- Pre: 4.27 N
- Post: 3.97 N
Comparison of pre-flight and post-calibration system identification for observatory MMS1

<table>
<thead>
<tr>
<th>State</th>
<th>Units</th>
<th>Pre-Cal</th>
<th>Post-Cal</th>
<th>Difference</th>
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</thead>
<tbody>
<tr>
<td>CM-x</td>
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<td>3.31</td>
<td>3.45</td>
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<tr>
<td>CM-y</td>
<td>mm</td>
<td>0.13</td>
<td>4.72</td>
<td>4.59</td>
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<tr>
<td>CM-z</td>
<td>mm</td>
<td>605.28</td>
<td>604.56</td>
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<td>kg-m²</td>
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<table>
<thead>
<tr>
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<th>Units</th>
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<th>Post-Cal</th>
<th>Difference</th>
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<td>N</td>
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<td>09</td>
<td>N</td>
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<td>N</td>
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<tr>
<td>12</td>
<td>N</td>
<td>4.27</td>
<td>3.97</td>
<td>-0.30</td>
</tr>
</tbody>
</table>
Additional Flight Validation

Supports the notion that the ground estimates of the radial thrust was roughly 8% above expectations.

Early maneuver performed without the AMS feedback used by the controller.

Mode Transition at 7200 sec

Inertial ΔVₓ [m/s]

- maneuver target
- closed-loop (AMS)
- open-loop (EVD)

open/closed-loop divergence

Δv trim region

velocity reset
Implementation details with regards to the MEKF formulation were discussed.

The MMS on-board attitude and rate estimation MEKF was documented, and flight results presented.

An augmented state MEKF for ground-based estimation of thruster output, center-of-mass, moments-of-inertia, and accelerometer biases was developed.

- A simple two-pulse test-case results were shown.
- Monte Carlo performance statistics were presented for a full calibration of the twelve MMS thrusters.
- Flight system identification results from the MMS EA019 calibration maneuver were shown and compared to pre-flight system knowledge.
Thank you for your attention.

Many thanks to our conference organizers for a wonderful event.

Any questions?