Acceleration and Velocity Sensing from Measured Strain

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Overview

- What the technology does (Slide 3)
- Previous technologies (Slide 4)
- Technical features of two-step approach: Deflection (Slides 5-7)
- Technical features of new technology: Acceleration & Velocity (Slides 8-9)
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What the technology does

Problem Statement

- Improving fuel efficiency for an aircraft
  - Reducing weight or drag
    - Similar effect on fuel savings
    - Multidisciplinary design optimization (design phase) or active control (during flight)

- Real-time measurement of deflection, slope, and loads in flight are a valuable tool.
  - Active flexible motion control
  - Active induced drag control

- Wing deflection and slope (complete degrees of freedom) are essential quantities for load computations during flight.
  - Loads can be computed from the following governing equations of motion.
    \[
    [M]\ddot{q}(t) + [G]\dot{q}(t) + [K]q(t) = Q_a(Mach, q(t))
    \]
    - Internal Loads: using finite element structure model
    - External Load: using unsteady aerodynamic model

- Traditionally, strain over the wing are measured using strain gages.
  - Cabling would create weight and space limitation issues.
  - A new innovation is needed. Fiber optic strain sensor (FOSS) is an ideal choice for aerospace applications.

Wing deflection & slope at time t will be computed from measured strain.
Previous technologies

  - **NASA LRC**; Application is limited for "beam"; **static deflection & aerodynamic loads**

  - **University of Arizona** and **NASA LRC**; “Full 3D” application; **strain matching optimization; static deflection & loads**

  - **KAIST**; displacement-strain-transformation (DST) matrix; Use **strain mode shape**; Application was based on **beam structure**; **dynamic deflection**

  - **JAXA**; using inverse analysis. “Beam” application only; **static deflection & loads**

  - **NASA AFRC**; **closed-form equations** (based on beam theory); **static deflection**

  - **NASA AFRC**; “sectional” bending moment, torsional moment, and shear force along the “beam”.

  - **NASA LRC**; **curve-fitting; static deflection**

  - **Harvard University**, **Stanford University**, and **Howard Hughes Medical Institute**; Uses **beam theory**; **static deflection & loads**

  - **Oregon State University**; **Aerodynamic loads** are estimated from measured strain using virtual strain sensor technique.

- Pak, C.-g., “Wing Shape Sensing from Measured Strain,” AIAA 2015-1427, AIAA Infotech @ Aerospace, Kissimmee, Florida, January 5-9, 2015; accepted for publication on *AIAA Journal* (June 29, 2015); U.S. Patent Pending: Patent App No. 14/482784
  - **NASA AFRC**; “Full 3D” application; based on **System Equivalent Reduction Expansion Process**; **static deflection**
Proposed solutions:
- The method for obtaining the deflection over a flexible full 3D aircraft structure was based on the following two steps.
  - First Step: Compute wing deflection along fibers using measure strain data
    - Wing deflection will be computed along the fiber optic sensor line.
    - Strains at selected locations will be "fitted".
    - These fitted strains will be integrated twice to have deflection information. (Relative deflection w.r.t. the reference point)
    - This is a finite element model independent method.
  - Second Step: Compute wing slope and deflection of entire structures
    - Slope computation will be based on a finite element model dependent technique.
    - Wing deflection and slope will be computed at all the finite element grid points.

\[
\{q(t)\} = \begin{pmatrix} \delta_x(t) \\ \delta_y(t) \\ \delta_z(t) \end{pmatrix}
\]

\[
\{\dot{q}(t)\} = \begin{pmatrix} \delta_x(t) \\ \delta_y(t) \\ \delta_z(t) \\ \theta_x(t) \\ \theta_y(t) \\ \theta_z(t) \end{pmatrix}
\]

\[
\{Q_a(Mach, q(t))\}
\]
Technical features of two-step approach: Deflection Computation (continued)

- **First Step**
  - Use piecewise least-squares method to minimize noise in the measured strain data (strain/offset)
  - Obtain cubic spline (Akima spline) function using re-generated strain data points (assume small motion):
    \[
    \frac{d^2 \delta_k}{ds^2} = -\epsilon_k(s)/c(s)
    \]
  - Integrate fitted spline function to get slope data:
    \[
    \frac{d \delta_k}{ds} = \theta_k(s)
    \]
  - Obtain cubic spline (Akima spline) function using computed slope data
  - Integrate fitted spline function to get deflection data: \(\delta_k(s)\)

A measured strain is fitted using a piecewise least-squares curve fitting method together with the cubic spline technique.
Technical features of two-step approach : Deflection Computation (continued)

- Second Step: Based on General Transformation
  - Definition of the generalized coordinates vector \( \{q\}_k \) and the othonormalized coordinates vector \( \{\eta\}_k \) at discrete time \( k \)
    \[
    \{q\}_k = \{q_M\}_k = [\Phi]\{\eta\}_k = [\Phi_M]\{\eta\}_k
    \]
  - For all model reduction/expansion techniques, there is a relationship between the master (measured or tested) degrees of freedom and the slave (deleted or omitted) degrees of freedom which can be written in general terms as
    \[
    \{q_M\}_k = [\Phi_M]\{\eta\}_k \quad (\text{master DOF})
    \]
    \[
    \{q_S\}_k = [\Phi_S]\{\eta\}_k \quad (\text{slave DOF})
    \]
  - Changing master DOF at discrete time \( k \) \( \{q_M\}_k \) to the corresponding measured values \( \{\bar{q}_M\}_k \)
    \[
    \{\bar{q}_M\}_k = [\Phi_M]\{\eta\}_k
    \]
    \[
    [\Phi_M]^T \{\bar{q}_M\}_k = [\Phi_M]^T [\Phi_M]\{\eta\}_k
    \]
    \[
    \{\eta\}_k = ([\Phi_M]^T [\Phi_M])^{-1} [\Phi_M]^T \{\bar{q}_M\}_k \quad \{q\}_k = [\Phi_M]\{\Phi_M\}^{-1} [\Phi_M]^T \{\bar{q}_M\}_k
    \]

- Expansion of displacement using SEREP: kinds of least-squares surface fitting; most accurate reduction-expansion technique
  - \( \{q_{Mk}\} \): master DOF at discrete time \( k \); \textit{deflection} along the fiber “computed from the first step”
  - \( \{q_{Sk}\} = [\Phi_S][[\Phi_M]^T [\Phi_M]]^{-1} [\Phi_M]^T \{\bar{q}_{Mk}\} \): deflection and slope all over the structure
  - \( \{q_{Mk}\} = [\Phi_M][[\Phi_M]^T [\Phi_M]]^{-1} [\Phi_M]^T \{\bar{q}_{Mk}\} \): smoothed master DOF
Technical features of new technology: Acceleration Computation

- From \( \{q\}_k = \{q_M\}_k = \Phi_M \{ \eta \}_k \) \( \{ \dot{q} \}_k = \{ \dot{q}_M \}_k = \Phi_M \{ \dot{\eta} \}_k \)

- Assume simple harmonic motion for normalized coordinates. \( \ddot{\eta}_i(k) = -\omega_i^2 \eta_i(k) \) \( i = 1, 2, ..., n \)

- Acceleration at discrete time \( k \) can be expressed

\[
\{ \ddot{\eta} \}_k = -\begin{bmatrix}
\omega_1^2 & 0 & \cdots & 0 \\
0 & \omega_2^2 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \omega_n^2
\end{bmatrix} \{ \eta \}_k
\]

\[
\{ \dot{q} \}_k = -\Phi_M \{ \omega_i^2 \} \{ \eta \}_k \quad \text{Eq. (9)}
\]

- Substituting Eq. (6) into (9) gives

\[
\{ \ddot{q} \}_k = -\Phi_M \{ \omega_i^2 \} \{ \ddot{\eta} \}_k \quad \text{Eq. (6)}
\]

Computed from unsteady strain distribution at a selected point using an on-line parameter estimation technique together with an AutoRegressive Moving Average (ARMA) model

\[
\{ \ddot{q} \}_k = -\begin{bmatrix}
\Phi_M \{ \omega_i^2 \} \\
\Phi_S \{ \omega_i^2 \}
\end{bmatrix} \left( \Phi_M^T \Phi_M \right)^{-1} \Phi_M^T \{ \ddot{q}_M \}_k
\]

Master DOF at discrete time \( k \); deflection along the fiber “computed from the first step”

Basis function for least squares surface fitting: eigen function, comparison function, etc.
Technical features of New Technology: Velocity Computation

- From $\{q\}_k = \{\dot{q}^M\} = \begin{bmatrix} \Phi_M \end{bmatrix} \{\eta\}_k$ 
  $\{\dot{q}\}_k = \{\dot{q}^M\} = \begin{bmatrix} \Phi_M \end{bmatrix} \{\dot{\eta}\}_k$ 
  $\{\ddot{q}\}_k = - \begin{bmatrix} \Phi_M \omega_i^2 \end{bmatrix} \begin{bmatrix} \Phi_M \end{bmatrix}^T \begin{bmatrix} \Phi_M \end{bmatrix}^{-1} \begin{bmatrix} \Phi_M \end{bmatrix}^T \{\ddot{\eta}_M\}_k$

- Consider
  - Backward difference: $\{\dot{\eta}\}_k = \frac{\{\eta\}_k - \{\eta\}_{k-1}}{\Delta t}$ has “phase shift” issue
  - Central difference: $\{\dot{\eta}\}_k = \frac{\{\eta\}_{k+1} - \{\eta\}_{k-1}}{2\Delta t}$ needs future response at time $k$

- From linear AR model for the $i$-th orthonormalized coordinate $\eta_i(k) = a_{1i} \eta_i(k-1) + a_{2i} \eta_i(k-2)$
  - Future prediction $\eta_i(k+1)$ at time $k$ $\eta_i(k+1) = a_{1i} \eta_i(k) + a_{2i} \eta_i(k-1)$
  - Central difference becomes $\dot{\eta}_i(k) = \frac{a_{1i} \eta_i(k) + (a_{2i} - 1) \eta_i(k-1)}{2\Delta t}$
  - AR coefficients $a_{1i}$ & $a_{2i}$ for the $i$-th mode are computed from the $i$-th frequency $\omega_i$ which are estimated from the parameter estimation

- $\{\dot{q}\}_k = \{\dot{q}^M\} = \begin{bmatrix} \Phi_M \end{bmatrix} \{\dot{\eta}\}_k$
  - $\dot{\eta}_i(k) = \frac{a_{1i} \eta_i(k) + (a_{2i} - 1) \eta_i(k-1)}{2\Delta t}$
  - $\{\eta\}_k = \begin{bmatrix} \dot{\eta}_1 \cr \dot{\eta}_2 \cr \vdots \cr \dot{\eta}_i \cr \vdots \cr \dot{\eta}_l \end{bmatrix}$

- Master DOF at discrete time $k$; deflection along the fiber “computed from the first step”

- Basis function for least squares surface fitting: eigen function, comparison function, etc.
Computational Validation

Cantilevered rectangular wing model
Cantilevered Rectangular Wing Model

- Configuration of a wind tunnel test article
  - Has aluminum insert (thickness = 0.065 in ) covered with 6% circular arc cross-sectional shape (plastic foam)
  - Impulsive load is applied at the leading-edge of wing tip section
- MSC/NASTRAN sol 112: Modal transient response analysis
  - Compute strain
  - Compute deflection & acceleration (target)
- Two-step approach
  - Compute deflection and acceleration from computed strain
  - Compare computed deflection and acceleration with respect to target values

- Two-step approach
  - Compute deflection and acceleration from computed strain
  - Compare computed deflection and acceleration with respect to target values

22 Simulated FOSS locations
Fiber optic strain sensors: 11(upper) + 11(lower)

Fiber optic strain sensors: 11(upper) + 11(lower)

A-A

6% Circular arc

0.065” aluminum insert

Flexible plastic foam

Applied load

Strain plot element

Plate elements

Rigid element

X

Y

Z
Model Tuning

- Idealization of the plastic foam weight
  - Case 1: equally smeared in aluminum plate.
  - Case 2: lumped mass weight are computed based on 6% circular-arc cross sectional shape.
  - Use structural dynamic model tuning technique

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<th>Case 1 Model</th>
<th>Case 2 Model</th>
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<td>G</td>
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<td>0.3806 lb</td>
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<tr>
<td>thickness</td>
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<td>0.065 inch</td>
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Design variables

Measured vs. Computed Frequencies

<table>
<thead>
<tr>
<th>Mode</th>
<th>Measured (Hz)</th>
<th>Case 1 (Hz)</th>
<th>% Error</th>
<th>Case 2 (Hz)</th>
<th>% Error</th>
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<tbody>
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<td>262.02</td>
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</tr>
<tr>
<td>6</td>
<td>N/A</td>
<td>455.22</td>
<td>N/A</td>
<td>459.34</td>
<td>N/A</td>
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<tr>
<td>7</td>
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<td>8</td>
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<td>642.72</td>
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<td>606.65</td>
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<td>773.93</td>
<td>N/A</td>
<td>747.65</td>
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</tr>
</tbody>
</table>
Mode Shapes

Mode 1: 14.29 Hz

Mode 2: 80.17 Hz

Mode 3: 89.04 Hz

Mode 4: 248.76 Hz

Mode 5: 252.41 Hz
Two Sample Cases

Case 1 computations

- Case 1 properties are used to make the target responses.
  - Use NASTRAN modal transient response analysis (sol112)
  - 1200 time steps
- Mode shapes from Case 1 are used to calculate transformation matrices.
  - Mode shapes are *eigen function*.
- Frequencies are estimated from strain data computed using Case 1 model.

\[
\{\dot{q}\}_k = \left[ \Phi_M \right] \left( \left[ \Phi_M \right]^T \left[ \Phi_M \right] \right)^{-1} \left[ \Phi_M \right]^T \{\ddot{q}_M\}_k \quad \{q\}_k = - \left[ \Phi_M \left[ \omega_i^2 \right] \right] \left( \left[ \Phi_M \right]^T \left[ \Phi_M \right] \right)^{-1} \left[ \Phi_M \right]^T \{\ddot{q}_M\}_k
\]

Case 2 computations

- Case 2 properties are used to make the target responses.
  - Use NASTRAN modal transient response analysis (sol112)
  - 1200 time steps
- Mode shapes from Case 1 are used to calculate transformation matrices.
  - Mode shapes are *comparison function*.
  - Case 1 model: Not validated model
  - Case 2 model: Validated model
- Frequencies are estimated from strain data computed using Case 2 model.

<table>
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<th>Mode</th>
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\[
\{\dot{q}\}_k = \left[ \Phi_M \right] \left[ \dot{q}_M \right] = \left[ \Phi_S \right] \{\ddot{q}\}_k \quad \{\ddot{q}\}_k = - \left[ \Phi_M \left[ \omega_i^2 \right] \right] \left( \left[ \Phi_M \right]^T \left[ \Phi_M \right] \right)^{-1} \left[ \Phi_M \right]^T \{\ddot{q}_M\}_k
\]

\[
\dot{\eta}_l(k) = \frac{a_{1i} \eta_l(k) + (a_{2i} - 1) \eta_l(k - 1)}{2\Delta t}
\]

\[
\{\eta\}_k = \left( \left[ \Phi_M \right]^T \left[ \Phi_M \right] \right)^{-1} \left[ \Phi_M \right]^T \{\ddot{q}_M\}_k
\]
Estimated System Frequencies: Case 1

- Use Bierman's U-D Factorization Algorithm
- Number of AR Coefficients = 20
- Covariance matrix resetting interval = 80 time steps
- Forgetting factor = 0.98
- Sampling time = 0.00062667 sec
- Nyquist frequency = 797.9 Hz
- Target frequencies & Time histories of strain: obtained from NASTRAN run
  - Strain values are obtained from the first element of the leading-edge fiber element located at the lower surface.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Target (Hz)</th>
<th>Estimated (Hz)</th>
<th>% Error</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>15.09</td>
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Chan-gi Pak - 16/21

Structural Dynamics Group

Deflection Time Histories: Case 1

Use eigen functions for the transformation matrices
Acceleration Time Histories: Case 1

Use eigen functions for the transformation matrices

22 fibers
At grid 51

Target
Current Method
Velocity Time Histories: Case 1

- Target
- Current Method

- 22 fibers
- At grid 51
Estimated System Frequencies: Case 2

- Use Bierman’s U-D Factorization Algorithm
- Number of AR Coefficients = 20
- Covariance matrix resetting interval = 80 time steps
- Forgetting factor = 0.98
- Sampling time = 0.0006487 sec
- Nyquist frequency = 770.8 Hz
- Target frequencies & Time histories of strain: obtained from NASTRAN run
  - Strain values are obtained from the first element of the leading-edge fiber element located at the lower surface.

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Strain distribution @ T=0.19461 sec
Deflection Time Histories: Case 2

- Use comparison functions for the transformation matrices

- 6, 10, & 22 fibers
- At grid 2601
Use comparison functions for the transformation matrices.
Velocity Time Histories: Case 2

- Target
- Current Method

- 6, 10, & 22 fibers
- At grid 2601
Summary of Computation Error

- % Error ≡ \( \frac{\sum_{k=0}^{n}|Current\, approach\, (k) - Target(k)|}{\sum_{k=0}^{n}|Target(k)|} \)

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<th>Model</th>
<th>Grid (# of fiber)</th>
<th>% Error</th>
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</tr>
<tr>
<td>Case 2</td>
<td>2601(22)</td>
<td>1.38</td>
</tr>
<tr>
<td></td>
<td>2601(10)</td>
<td>1.67</td>
</tr>
<tr>
<td></td>
<td>2601(6)</td>
<td>1.79</td>
</tr>
</tbody>
</table>

- Z deflection errors are the smallest
  - Z deflections are input for the second step.
    - Z deflections along the leading-edge fiber (grid 51) are input for the second step. (master DOF)
    - Pitch angle at grid 51 as well as Z deflection and pitch angle at grid 2601 are output from the second step. (slave DOF)

- Acceleration and velocity errors are bigger than the displacement errors.
- Even six fibers also give good answer.
  - No big difference between 6, 10, & 22 fibers.
Conclusions

- **Acceleration** and **velocity** of the cantilevered rectangular wing is successively obtained using the proposed approach.
  - **Simple harmonic motion** was assumed for the **acceleration** computations.
    - System frequencies are estimated from the time histories of strain measured at the leading-edge of the root section through the use of the **parameter estimation technique** together with the ARMA model.
  - The **central difference** equation with a **linear AR model** is used for the computations of **velocity**.
    - AR coefficients are computed using the estimated system frequencies.
    - Phase shift issue associated with the backward difference equation are overcome with the proposed approach.
  - The total of six fibers provides the good results.
    - Quality of results based on 6, 10, and 22 fibers are **similar**.
Questions ?