Acceleration and Velocity Sensing from Measured Strain

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Overview

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- Previous technologies (Slide 4)
- Technical features of two-step approach: Deflection (Slides 5-7)
- Technical features of new technology: Acceleration & Velocity (Slides 8-9)
- Computational Validation (Slides 10-22)
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- Conclusions (Slide 24)
What the technology does

**Problem Statement**

- Improving fuel efficiency for an aircraft
  - Reducing **weight** or **drag**
    - Similar effect on fuel savings
  - Multidisciplinary design optimization (design phase) or active control (during flight)

- Real-time measurement of deflection, slope, and loads in flight are a valuable tool.
  - Active flexible motion control
  - Active induced drag control

- Wing deflection and slope (complete degrees of freedom) are essential quantities for load computations during flight.
  - Loads can be computed from the following governing equations of motion.

\[
[M]q''(t) + [G]q'(t) + [K]q(t) = \{Q_a(Mach, q(t))\}
\]

- Internal Loads: using finite element structure model
  - \([M]q''(t), [G]q'(t), [K]q(t)\): Inertia, damping, and elastic loads
- External Load: using unsteady aerodynamic model
  - \(\{Q_a(Mach, q(t))\}\): Aerodynamic load

- Traditionally, strain over the wing are measured using strain gages.
  - Cabling would create **weight and space limitation** issues.
  - A **new innovation** is needed. **Fiber optic strain sensor** (FOSS) is an ideal choice for **aerospace** applications.

\[
q(t) = \begin{vmatrix}
\delta_x \\
\delta_y \\
\delta_z \\
\theta_x \\
\theta_y \\
\theta_z
\end{vmatrix}
\]

**Wing deflection & slope at time t will be computed from measured strain.**

Deflection

Slope (angle)
## Previous technologies

  - NASA LRC; Application is limited for "beam"; **static deflection & aerodynamic loads**
  - University of Arizona and NASA LRC; “Full 3D” application; **strain matching optimization; static deflection & loads**
  - KAIST; displacement-strain-transformation (DST) matrix; Use **strain mode shape**; Application was based on **beam structure**; **dynamic deflection**
  - JAXA; using inverse analysis. **“Beam” application only; static deflection & loads**
  - NASA AFRC; **closed-form equations** (based on **beam theory**); **static deflection**
  - NASA AFRC; **“sectional” bending moment, torsional moment, and shear force** along the “beam”.
  - NASA LRC; **curve-fitting; static deflection**
  - Harvard University, Stanford University, and Howard Hughes Medical Institute; Uses **beam theory; static deflection & loads**
  - Oregon State University; **Aerodynamic loads** are estimated from measured strain using virtual strain sensor technique.
- Pak, C.-g., “Wing Shape Sensing from Measured Strain,” AIAA 2015-1427, AIAA Infotech @ Aerospace, Kissimme, Florida, January 5-9, 2015; accepted for publication on *AIAA Journal* (June 29, 2015); U.S. Patent Pending: Patent App No. 14/482784
  - NASA AFRC; “Full 3D” application; based on **System Equivalent Reduction Expansion Process; static deflection**
Technical features of two-step approach: Deflection Computation

Proposed solutions:
- The method for obtaining the deflection over a flexible full 3D aircraft structure was based on the following two steps.
  - First Step: Compute wing deflection along fibers using measure strain data
    - Wing deflection will be computed along the fiber optic sensor line.
    - Strains at selected locations will be “fitted”.
    - These fitted strain will be integrated twice to have deflection information. (Relative deflection w.r.t. the reference point)
    - This is a finite element model independent method.
  - Second Step: Compute wing slope and deflection of entire structures
    - Slope computation will be based on a finite element model dependent technique.
    - Wing deflection and slope will be computed at all the finite element grid points.

\[
\begin{align*}
\{q(t)\} & = \begin{bmatrix} \delta_x(t) \end{bmatrix} \\
\{\varepsilon_x(t)\} & = \begin{bmatrix} \delta_y(t) \\
\delta_z(t) \\
\theta_x(t) \\
\theta_y(t) \\
\theta_z(t) \end{bmatrix}
\end{align*}
\]
Technical features of two-step approach: Deflection Computation (continued)

- **First Step**
  - Use piecewise least-squares method to minimize noise in the measured strain data (strain/offset)
  - Obtain cubic spline (Akima spline) function using re-generated strain data points (assume small motion):
    \[
    \frac{d^2 \delta_k}{ds^2} = -\varepsilon_k(s)/c(s)
    \]
  - Integrate fitted spline function to get slope data:
    \[
    \frac{d\delta_k}{ds} = \theta_k(s)
    \]
  - Obtain cubic spline (Akima spline) function using computed slope data
  - Integrate fitted spline function to get deflection data: \(\delta_k(s)\)

A measured strain is fitted using a piecewise least-squares curve fitting method together with the cubic spline technique.
Technical features of two-step approach: Deflection Computation (continued)

- Second Step: Based on General Transformation
  - Definition of the generalized coordinates vector \( \{q\}_k \) and the othonormalized coordinates vector \( \{\eta\}_k \) at discrete time \( k \):
    \[
    \{q\}_k = \{q_M\} = [\Phi]\{\eta\}_k = [\Phi_M]\{\eta\}_k
    \]
  - For all model reduction/expansion techniques, there is a relationship between the **master (measured or tested)** degrees of freedom and the **slave (deleted or omitted)** degrees of freedom which can be written in general terms as:
    \[
    \{q_M\}_k = [\Phi_M]\{\eta\}_k
    \]
    \[
    \{q_S\}_k = [\Phi_S]\{\eta\}_k
    \]
  - Changing master DOF at discrete time \( k \) \( \{q_M\}_k \) to the corresponding measured values \( \{\tilde{q}_M\}_k \)
    \[
    \{\tilde{q}_M\}_k = [\Phi_M]\{\eta\}_k
    \]
    \[
    [\Phi_M]^T\{\tilde{q}_M\}_k = [\Phi_M]^T[\Phi_M]\{\eta\}_k
    \]
    \[
    \{\eta\}_k = ([\Phi_M]^T[\Phi_M])^{-1}[\Phi_M]^T\{\tilde{q}_M\}_k
    \]
    \[
    \{q\}_k = [\Phi_M]^T([\Phi_M]^T[\Phi_M])^{-1}[\Phi_M]^T\{\tilde{q}_M\}_k
    \]

- Expansion of displacement using SEREP: kinds of least-squares surface fitting; most accurate reduction-expansion technique
  - \( \{q_Mk\} \): master DOF at discrete time \( k \); **deflection** along the fiber **computed from the first step**
  - \( \{q_Sk\} = [\Phi_S][\Phi_M]^T[\Phi_M])^{-1}[\Phi_M]^T\{q_M\}_k \): deflection and slope all over the structure
  - \( \{q_Mk\} = [\Phi_M][\Phi_M]^T[\Phi_M])^{-1}[\Phi_M]^T\{q_M\}_k \): smoothed master DOF
Technical features of new technology: Acceleration Computation

- From \( \{q\}_k = \{q_M\} = [\Phi_M] \{\eta\}_k \) \( \{\ddot{q}\}_k = \{\ddot{q}_M\} = [\Phi_M] \{\ddot{\eta}\}_k \)

- Assume simple harmonic motion for normalized coordinates. \( \ddot{\eta}_i(k) = -\omega_i^2 \eta_i(k) \) \( i = 1,2,...,n \)

- Acceleration at discrete time \( k \) can be expressed

  \[
  \begin{bmatrix}
  \ddot{\eta}_1(k) \\
  \ddot{\eta}_2(k) \\
  \vdots \\
  \ddot{\eta}_n(k)
  \end{bmatrix}
  = -
  \begin{bmatrix}
  \omega_1^2 & 0 & \cdots & 0 \\
  0 & \omega_2^2 & \cdots & 0 \\
  \vdots & \vdots & \ddots & \vdots \\
  0 & 0 & \cdots & \omega_n^2
  \end{bmatrix}
  \begin{bmatrix}
  \eta_1(k) \\
  \eta_2(k) \\
  \vdots \\
  \eta_n(k)
  \end{bmatrix}
  = -[\omega_i^2] \{\eta\}_k
  \]

- Substituting Eq. (6) into (9) gives

  \[
  \{\ddot{q}\}_k = -[\Phi_M [\omega_i^2]] [\Phi_M]^{-1} [\Phi_M]^T \{\dddot{q}_M\}_k \quad \text{Eq. (9)}
  \]

  \[
  \{q\}_k = [\Phi_M] [\Phi_M]^{-1} [\Phi_M]^T \{\dddot{q}_M\}_k \quad \text{Eq. (6)}
  \]

Computed from unsteady strain distribution at a selected point using an on-line parameter estimation technique together with an AutoRegressive Moving Average (ARMA) model

\( \{\dddot{q}\}_k \) = \( (\Phi_M)^T [\Phi_M]^{-1} [\Phi_M]^T \{\dddot{q}_M\}_k \)

- Master DOF at discrete time \( k \); deflection along the fiber “computed from the first step”

Basis function for least squares surface fitting: eigen function, comparison function, etc.
From \( \{q\}_k = \{\dot{q}\}_k = [\Phi_M] \{\eta\}_k \Rightarrow [\Phi_S] \{\dot{\eta}\}_k \), the velocity \( \{\dot{q}\}_k \) can be computed as:

\[
\{\dot{q}\}_k = - \begin{bmatrix} \Phi_M \omega_i^2 \\ \Phi_S \omega_i^2 \end{bmatrix} [\Phi_M]^T [\Phi_M]^{-1} [\Phi_M]^T \{\tilde{q}_M\}_k
\]

Consider:
- Backward difference: \( \{\dot{\eta}\}_k = \frac{\{\eta\}_k - \{\eta\}_{k-1}}{\Delta t} \) has "phase shift" issue
- Central difference: \( \{\dot{\eta}\}_k = \frac{\{\eta\}_{k+1} - \{\eta\}_{k-1}}{2\Delta t} \) needs future response at time \( k \)

From linear AR model for the \( i \)-th orthonormalized coordinate:

\( \eta_i(k) = a_{1i} \eta_i(k-1) + a_{2i} \eta_i(k-2) \)

- Future prediction: \( \eta_i(k+1) \) at time \( k \): \( \eta_i(k+1) = a_{1i} \eta_i(k) + a_{2i} \eta_i(k-1) \)
- Central difference becomes:

\[
\dot{\eta}_i(k) = \frac{a_{1i} \eta_i(k) + (a_{2i} - 1) \eta_i(k-1)}{2\Delta t}
\]

AR coefficients \( a_{1i} \) & \( a_{2i} \) for the \( i \)-th mode are computed from the \( i \)-th frequency \( \omega_i \) which are estimated from the parameter estimation.

Computed from estimated frequencies:

\[
\dot{\eta}_i(k) = \frac{a_{1i} \eta_i(k) + (a_{2i} - 1) \eta_i(k-1)}{2\Delta t}
\]

Master DOF at discrete time \( k \): deflection along the fiber "computed from the first step"

Basis function for least squares surface fitting: eigen function, comparison function, etc.
Computational Validation

Cantilevered rectangular wing model
Cantilevered Rectangular Wing Model

- Configuration of a wind tunnel test article
  - Has aluminum insert (thickness = 0.065 in) covered with 6% circular arc cross-sectional shape (plastic foam)
  - Impulsive load is applied at the leading-edge of wing tip section
- MSC/NASTRAN sol 112: Modal transient response analysis
  - Compute strain
  - Compute deflection & acceleration (target)
- Two-step approach
  - Compute deflection and acceleration from computed strain
  - Compare computed deflection and acceleration with respect to target values

22 Simulated FOSS locations
Fiber optic strain sensors: 11(upper) + 11(lower)
Model Tuning

- Idealization of the plastic foam weight
  - Case 1: equally smeared in aluminum plate.
  - Case 2: lumped mass weight are computed based on 6% circular-arc cross sectional shape.
    - Use structural dynamic model tuning technique

<table>
<thead>
<tr>
<th>Properties</th>
<th>Case 1 Model</th>
<th>Case 2 Model</th>
</tr>
</thead>
<tbody>
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<td>E</td>
<td>9847900</td>
<td>9207766</td>
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<tr>
<td>G</td>
<td>3639672</td>
<td>3836570</td>
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<tr>
<td>density</td>
<td>0.11166</td>
<td>0.1</td>
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<tr>
<td>Foam weight</td>
<td>Smeared</td>
<td>Lumped mass</td>
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<tr>
<td>Total weight</td>
<td>0.3806 lb</td>
<td>0.3806 lb</td>
</tr>
<tr>
<td>Xcg</td>
<td>2.28 inch</td>
<td>2.28 inch</td>
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<tr>
<td>Ycg</td>
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</tr>
<tr>
<td>thickness</td>
<td>0.065 inch</td>
<td>0.065 inch</td>
</tr>
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</table>

### Measured vs. Computed Frequencies

<table>
<thead>
<tr>
<th>Mode</th>
<th>Measured (Hz)</th>
<th>Case 1 (Hz)</th>
<th>% Error</th>
<th>Case 2 (Hz)</th>
<th>% Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>14.29</td>
<td>15.09</td>
<td>5.6</td>
<td>14.29</td>
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<td>2</td>
<td>80.41</td>
<td>77.40</td>
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<td>-0.8</td>
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<td>248.76</td>
<td>N/A</td>
</tr>
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<td>N/A</td>
<td>262.02</td>
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<td>N/A</td>
</tr>
<tr>
<td>6</td>
<td>N/A</td>
<td>455.22</td>
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<td>459.34</td>
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</tr>
<tr>
<td>7</td>
<td>N/A</td>
<td>511.27</td>
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<td>485.61</td>
<td>N/A</td>
</tr>
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<td>8</td>
<td>N/A</td>
<td>642.72</td>
<td>N/A</td>
<td>606.65</td>
<td>N/A</td>
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<tr>
<td>9</td>
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<td>718.59</td>
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<tr>
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<td>773.93</td>
<td>N/A</td>
<td>747.65</td>
<td>N/A</td>
</tr>
</tbody>
</table>

Objective function: frequency error

Design variables

- Flexible plastic foam
- 0.065” aluminum insert
- 6% Circular arc

Structural Dynamics Group
Chan-gi Pak-12/21
Mode Shapes

Mode 1: 14.29 Hz
Mode 2: 80.17 Hz
Mode 3: 89.04 Hz
Mode 4: 248.76 Hz
Mode 5: 252.41 Hz
Two Sample Cases

- **Case 1 computations**
  - Case 1 properties are used to make the target responses.
    - Use NASTRAN modal transient response analysis (sol112)
    - 1200 time steps
  - Mode shapes from Case 1 are used to calculate transformation matrices.
    - Mode shapes are **eigen function**.
  - Frequencies are estimated from strain data computed using Case 1 model.

\[
\{q\}_k = \left[ \Phi_M \right] \left( [\Phi_M]^T [\Phi_M] \right)^{-1} [\Phi_M]^T \{\bar{q}_M\}_k \\
\{\ddot{q}\}_k = -\left[ \Phi_M \left[ \omega_i^2 \right] \right] \left( [\Phi_M]^T [\Phi_M] \right)^{-1} [\Phi_M]^T \{\bar{q}_M\}_k
\]

- **Case 2 computations**
  - Case 2 properties are used to make the target responses.
    - Use NASTRAN modal transient response analysis (sol112)
    - 1200 time steps
  - Mode shapes from Case 1 are used to calculate transformation matrices.
    - Mode shapes are **comparison function**.
  - Frequencies are estimated from strain data computed using Case 2 model.

<table>
<thead>
<tr>
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\[
\{\ddot{q}\}_k = \left[ \Phi_M \right] \left( [\Phi_M]^T [\Phi_M] \right)^{-1} [\Phi_M]^T \{\bar{q}_M\}_k
\]

Comparison functions are used for Case 2
Estimated System Frequencies: Case 1

- Use Bierman's U-D Factorization Algorithm
- Number of AR Coefficients = 20
- Covariance matrix resetting interval = 80 time steps
- Forgetting factor = 0.98
- Sampling time = 0.00062667 sec
- Nyquist frequency = 797.9 Hz
- Target frequencies & Time histories of strain: obtained from NASTRAN run
  - Strain values are obtained from the first element of the leading-edge fiber element located at the lower surface.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Target (Hz)</th>
<th>Estimated (Hz)</th>
<th>% Error</th>
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<tr>
<td>1</td>
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Strain distribution @ T=0.188001 sec
Use eigen functions for the transformation matrices

- Target
- Current Method

22 fibers
At grid 51
Acceleration Time Histories: Case 1

Use eigen functions for the transformation matrices

- Target
- Current Method

- 22 fibers
- At grid 51
Velocity Time Histories: Case 1

-200 -150 -100 -50 0 50 100 150 200

Z velocity (inch/sec)

Time (sec)

-200 -100 -50 0 50 100

Pitch rate (inch/sec)

Time (sec)

-200 -150 -100 -50 0 50 100 150 200

Z velocity (inch/sec)

Time (sec)

-200 -100 -50 0 50 100

Pitch rate (inch/sec)

Time (sec)

-200 -150 -100 -50 0 50 100 150 200

Z velocity (inch/sec)

Time (sec)

-200 -100 -50 0 50 100

Pitch rate (inch/sec)

Time (sec)
Estimated System Frequencies: Case 2

- Use Bierman’s U-D Factorization Algorithm
- Number of AR Coefficients = 20
- Covariance matrix resetting interval = 80 time steps
- Forgetting factor = 0.98
- Sampling time = 0.0006487 sec
- Nyquist frequency = 770.8 Hz
- Target frequencies & Time histories of strain: obtained from NASTRAN run
  - Strain values are obtained from the first element of the leading-edge fiber element located at the lower surface.

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</tbody>
</table>

Strain distribution @ T=0.19461 sec
Deflection Time Histories: Case 2

- Use comparison functions for the transformation matrices.
- Target: Current Method
- 6, 10, & 22 fibers
- At grid 2601

Graphs show:
- Pitch angle (radian) vs. Time (sec)
- Z deflection (inch) vs. Time (sec)
Acceleration Time Histories: Case 2

Use comparison functions for the transformation matrices.
Velocity Time Histories: Case 2

- Target:
- Current Method:

- 6, 10, & 22 fibers
- At grid 2601

Z velocity (inch/sec) vs. Time (sec)

Pitch rate (radian/sec) vs. Time (sec)
Summary of Computation Error

- % Error \( \equiv \frac{\sum_{k=0}^{n} |\text{Current approach}(k) - \text{Target}(k)|}{\sum_{k=0}^{n} |\text{Target}(k)|} \)

<table>
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<tr>
<th>Model</th>
<th>Grid (# of fiber)</th>
<th>Deflection</th>
<th>Velocity</th>
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<tr>
<td></td>
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<td>Z</td>
<td>Pitch</td>
</tr>
<tr>
<td>Case 1</td>
<td>51(22)</td>
<td>1.55</td>
<td>5.36</td>
</tr>
<tr>
<td></td>
<td>2601(22)</td>
<td>1.38</td>
<td>5.76</td>
</tr>
<tr>
<td>Case 2</td>
<td>2601(10)</td>
<td>1.67</td>
<td>5.99</td>
</tr>
<tr>
<td></td>
<td>2601(6)</td>
<td>1.79</td>
<td>6.35</td>
</tr>
</tbody>
</table>

- Z deflection errors are the smallest
  - Z deflections are input for the second step.
    - Z deflections along the leading-edge fiber (grid 51) are input for the second step. (master DOF)
    - Pitch angle at grid 51 as well as Z deflection and pitch angle at grid 2601 are output from the second step. (slave DOF)
    - Therefore, it’s less accurate than master DOFs.

- Acceleration and velocity errors are bigger than the displacement errors.
- Even six fibers also give good answer.
  - No big difference between 6, 10, & 22 fibers.
Conclusions

- **Acceleration** and **velocity** of the cantilevered rectangular wing is successively obtained using the proposed approach.
  - Simple harmonic motion was assumed for the acceleration computations.
    - System frequencies are estimated from the time histories of strain measured at the leading-edge of the root section through the use of the parameter estimation technique together with the ARMA model.
  - The central difference equation with a linear AR model is used for the computations of velocity.
    - AR coefficients are computed using the estimated system frequencies.
    - Phase shift issue associated with the backward difference equation are overcome with the proposed approach.
  - The total of six fibers provides the good results.
    - Quality of results based on 6, 10, and 22 fibers are similar.
Questions ?