Aerodynamic Shape Optimization with Goal-Oriented Error Estimation and Control

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Motivation

• Challenges of simulation-based design
  - High CFD expertise in mesh generation
    ‣ Long setup time
    ‣ High cost due to repeated flow solves on fine meshes or high uncertainty due to inappropriate meshes

• Success of error estimation and mesh adaptation in goal-oriented simulations
Objectives

Adaptive discretization of aerodynamic shape optimization problems

**Accuracy**
- Improve design confidence
  - Direct control over objective function discretization error

**Automation**
- Reduce level of CFD expertise
  - Eliminate the need to handcraft a mesh appropriate for all candidate designs
  - Shorten problem setup time

**Progress toward improved efficiency**
- Reduce cost by systematically increasing depth of refinement as designs improve
  - Progressive optimization strategy
Problem Formulation

\[
\min_X J(X, Q) \\
\text{subject to} \\
R(X, Q) = 0 \quad \forall X \in \Omega
\]

- Gradient-based optimization
  \[
  \frac{dJ}{dX} \rightarrow 0
  \]
- Steady Euler equations

Spatial Discretization: \( J_H(X, Q_H), R_H(X, Q_H) \)

- Second-order finite-volume method
- Cartesian mesh with embedded boundaries

✓ Complex geometry
✓ Automation
✓ \( h \)-refinement

Cut cells
Problem Formulation

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subject to

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Discretization Error

Design Space

Error Estimate (fixed X)

- Leverage adjoint method
  - Error estimates via the method of adjoint weighted residuals
  - Objective function gradient via the discrete adjoint method
Dual Role of Adjointsl 

**Gradients**

\[ J_H = f(X, Q_H) \]

e.g. \( C_D + (C_L - C_L^*)^2 \)

**Error Estimates**

\[ e = |J_h - J_H| \]
Dual Role of Adjoints

**Gradients**

\[
J_H = f(X, Q_H)
\]
e.g. \(C_D + (C_L - C_L^*)^2\)

\[
\frac{dJ}{dX} = \frac{\partial J}{\partial X} + \frac{\partial J}{\partial Q} \frac{dQ}{dX}
\]

\[
0 = \frac{\partial R}{\partial X} + \frac{\partial R}{\partial Q} \frac{dQ}{dX}
\]

\[
\left[ \begin{array}{c}
\frac{\partial R}{\partial Q}
\end{array} \right]^T \psi = \frac{\partial J}{\partial Q}
\]

\[
\frac{dJ}{dX} = \frac{\partial J}{\partial X} - \psi^T \frac{\partial R}{\partial X}
\]

**Error Estimates**

\[
e = |J_h - J_H|
\]

\[
J_h \approx J_h(Q_H) + \frac{\partial J(Q_H)}{\partial Q} \Delta Q
\]

\[
0 \approx R_h(Q_H) + \frac{\partial R(Q_H)}{\partial Q} \Delta Q
\]

\[
J_h \approx J_h(Q_H) - \psi^T R_h(Q_H)
\]
Error Estimation Details

\[ J_h(Q_h) \approx J_h(Q_H) - \psi_h^T R_h(Q_H) \]

\[ J_c = J_h(Q_H) - \psi_H^T R_h(Q_H) \]

**Error Estimate**

\[ \mathcal{E} = C |J_c - J_H| \]

**Refinement Indicator**

\[ \eta_H = \left( \tilde{\psi}_h - \psi_H \right)^T R_h(Q_H) \]

\[ \eta = \sum_{i=1}^{N} |\eta_i| \]

Number of Cells

Log$_{10}$ -3 -2 -1 0 1 2 3
Verification: Supersonic Vortex

\[ J = \int_{B_{ro}} p \, dl \]

True Error

Error Estimate

\[ \mathcal{E} = C \left| J_c - J_H \right| \]

- No limiter, \( \mathcal{O}(h^2) \)
- Effectivity close to 1

Uniform Refinement
Verification: Supersonic Vortex

\[ J = \int_{B_{r_o}} p \, dl \]

Uniform Refinement

Refinement Indicator

\[ \eta_H = \left| \left( \tilde{\psi}_h - \psi_H \right)^T R_h(Q_H) \right| \]

- Sharp estimate of remaining error
- Localization very conservative
Optimization with Mesh Adaptation

- Integration into existing, fixed mesh, optimization framework
  - Build sequence of adapted meshes
  - Pass values of objective and gradient from finest mesh to optimizer
Optimization with Mesh Adaptation

- Integration into existing, fixed mesh, optimization framework
  - Build sequence of adapted meshes
  - Pass values of objective and gradient from finest mesh to optimizer

- In each design iteration, perform fixed (user specified) number of adaptations
  - Fixed depth strategy
  - Robust and precise control over computational resources
Optimization with Mesh Adaptation

- In each design iteration:
  - Start with same initial mesh
  - Adapt until prescribed refinement level is attained
- May be inefficient
Progressive Optimization

- Increase mesh refinement in each optimization subproblem
  - Converge a sequence of improving discretizations

\[ X \to X^* \text{ as } \mathcal{E} \to 0 \]
Progressive Optimization

- Increase mesh refinement in each optimization subproblem
  - Converge a sequence of improving discretizations

\[ X \to X^* \text{ as } E \to 0 \]

- N+1 Cycles
Progressive Optimization

- Increase mesh refinement in each optimization subproblem
  - Converge a sequence of improving discretizations

\[ X \rightarrow X^* \text{ as } \mathcal{E} \rightarrow 0 \]
Progressive Optimization

- Increase mesh refinement in each optimization subproblem
  - Converge a sequence of improving discretizations

- Stopping Criterion
  1. Gradient or KKT norms, or stall
  2. Specified number of search directions
  3. Diminishing changes in objective function
  4. Ratio of design improvement to error: refine when
     \[ J_{i-1} - J_i < \mathcal{E} \]
Sonic-Boom Mitigation Inverse Design

Optimize aircraft shape by prescribing quieter near-field signals

\[ J = \frac{1}{p_{\infty}^2} \int (p - p_{\text{target}})^2 dS \]

1. Pressure-signature analysis
2. Shape optimization on a fixed mesh
3. Progressive optimization
Pressure Signature of Delta-Wing Body

Determine pressure signature 3.6 body-lengths below the model

Freestream Conditions:
- $M_\infty = 1.68$
- $C_L = 0.15$
Mesh and Solution

Initial Mesh: 879 cells

12 Adaptations: 4.5M cells

Isobars

Refinement Indicator:

$$\eta_H = \left| \left( \tilde{\psi}_h - \psi_H \right)^T R_h(Q_H) \right|$$

Near-field on symmetry plane
Pressure Signature

Experiment

Initial mesh, 879 cells
10 adaptations, 590k cells
12 adaptations, 4.5M cells
13 adaptations, 12M cells

Distance along sensor

\[ \frac{\Delta p}{p_\infty} \]

0 5 10 15 20 25 30

0 0.01 0.02 0.03

-0.02 -0.01 0 0.01 0.02
Error Convergence

\[ J = \frac{1}{p_\infty^2} \int (p - p_\infty)^2 dS \]

- Error bars represent level of discretization error

\[ \mathcal{E} = 2 |J_c - J_H| \]

- Remaining error term is small and is \( O(h^2) \)
- Error indicator is \( O(h) \) (due to localization)
Inverse Design on Fixed Meshes

Approach: use adaptation to guide construction of a fixed mesh for shape optimization runs

$M_\infty = 1.6^\circ$
$\alpha = 0.612$
$h/L = 2.0$

9.3 M Cells

Full aircraft configuration: 180 design variables
Optimization Targets

\[ J = \frac{1}{p_\infty^2} \int (p - p_{\text{target}})^2 dS \]
Optimization Results

- 50 design iterations (SNOPT)
- Ground noise 76.7 PLdB, 9.6 dB reduction in perceived loudness

![Graph showing optimization results](image-url)

On-track, $\Phi = 0^\circ$

Off-track, $\Phi = 15^\circ$
Optimization with Adaptation

Model Problem Setup

- Prescribe a target signature from a known shape
- 10 design variables that control body radius
- \( M_\infty = 1.5 \) and \( \alpha = 0^\circ \)
Optimization with Adaptation

Model Problem Setup

- Prescribe a target signature from a known shape
- 10 design variables that control body radius
- $M_\infty = 1.5$ and $\alpha = 0^\circ$
Optimization with Adaptation

Consider two cases

1. **Fixed-depth strategy**: 7 refinements in each design iteration
2. **Progressive optimization**: Increment from 4 to 7 refinements (allow designs to advance as far as possible on each level)

7 Adaptations, ~650k cells
Optimization with Adaptation

Fixed-Depth Strategy
7 Refinements

Progressive Optimization

Progressive optimization is about a factor of two faster than fixed-depth strategy
Summary and Outlook

• Progress toward a gradient-based optimization framework with capability to perform adaptive meshing in each design iteration
  - Promising approach to enhance accuracy, efficiency and automation of simulation-based design

• Future work
  - Use of error estimates to limit oversolving
  - Transfer of Hessian matrix as the design moves from mesh to mesh
  - Dynamic error control and mesh re-use
Questions

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Cart3D website and publications:
http://people.nas.nasa.gov/aftosmis/cart3d/