TURBULENT CONCENTRATION OF MM-SIZE PARTICLES IN THE PROTOPLANETARY NEBULA: 
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**Introduction:** The initial accretion of primitive bodies (here, asteroids in particular) from freely-floating nebula particles remains problematic. Traditional growth-by-sticking models encounter a formidable “meter-size barrier” [1] (or even a mm-to-cm-size barrier [2]) in turbulent nebulae, making the preconditions for so-called “streaming instabilities” difficult to achieve even for so-called “lucky” particles [3]. Even if growth by sticking could somehow breach the meter size barrier, turbulent nebulae present further obstacles through the 1-10km size range [4]. On the other hand, nonturbulent nebulae form large asteroids too quickly to explain long spreads in formation times, or the dearth of melted asteroids [5]. Theoretical understanding of nebula turbulence is itself in flux; recent models of MRI (magnetically-driven) turbulence favor low-or-no-turbulence environments [6], but purely hydrodynamic turbulence is making a comeback, with two recently discovered mechanisms generating robust turbulence which do not rely on magnetic fields at all [7,8].

An important clue regarding planetesimal formation is an apparent 100km diameter peak in the pre-depletion, pre-erosion mass distribution of asteroids [9]; scenarios leading directly from independent nebula particulates to large objects of this size, which avoid the problematic m-km size range, could be called “leapfrog” scenarios [10-12]. The leapfrog scenario we have studied in detail involves formation of dense clumps of aerodynamically selected, typically mm-size particles in turbulence, which can under certain conditions shrink inexorably on 100-1000 orbit timescales and form 10-100km diameter sandpile planetesimals. There is evidence that at least the ordinary chondrite parent bodies were initially composed entirely of a homogeneous mix of such particles [4]. Thus, while they are arcane, turbulent concentration models acting directly on chondrule size particles are worthy of deeper study. The typical sizes of planetesimals and the rate of their formation [11,12] can be estimated using a statistical model with properties inferred from large numerical simulations of turbulence [13]. Nebula turbulence is described by its Reynolds number $Re = (L/\eta)^{4/3}$, where $L = H/\alpha^{1/2}$ is the largest eddy scale, $H$ is the nebula gas vertical scale height, $\alpha$ the turbulent viscosity parameter, and $\eta$ is the Kolmogorov lengthscale (typically about 1km) with eddy turnover time $t_{\eta}$. In the nebula, $Re$ is far larger than any numerical simulation can handle, so some physical arguments are needed to extend the results of numerical simulations to nebula conditions. In this paper, we report new physics to be incorporated into our statistical models.

**Cascade model and multiplier distributions:** The spatial distribution of particle concentration can be captured statistically by a cascade model [11,13] which predicts the probability distribution functions (PDFs) for dense particle clumps; these PDFs are essentially the volume fractions of the nebula which have the necessary properties (solids mass and local vorticity) for planetesimal formation. A cascade model presumes that, as energy flows from large eddies to smaller ones, particles and fluid properties are partitioned unequally at each “level” of the cascade from “parent” into “daughter” eddies. 

**Results:** Figure 2 shows our results for the widths of the multiplier PDFs [18], binned on a wide range of lengthscales $r$ given in units of $\eta$. In this figure, the particle stopping time $t_r$ has been normalized to a scale-dependent value $St_r$ using the inertial range timescale ratio $(r/\eta)^{-2/3}$, and the constant parameter $St = t_r/t_\eta$ where $t_\eta$ is the Kolmogorov eddy time. In this plot, a value of $St_r = 1$ represents a particle having stopping time equal to the eddy time for the spatial scale $r$. Except at the largest binning scales ($r \geq 128\eta$), this scaling...
collapses our previous results [18] into an invariant family of curves, allowing us to predict the value of \( \beta \) for any combination of lengthscale and particle stopping time. The minimum near \( St_r = 0.2 \) suggests that particles with stopping time somewhat shorter than the eddy time on lengthscale \( r \) are most strongly concentrated within spatial averages over lengthscale \( r \). This scaling collapses the multipliers much better than an alternate scaling found by [19] which collapses the normalized concentration pdfs themselves; the concentration pdfs can be thought of as the cumulative result of multipliers acting sequentially over all larger scales. After a number of eddy bifurcations to smaller scale, particles of a small but finite range of sizes might thus end up concentrated within a single clump, which may be consistent with new determinations of chondrule size distributions which find them to be broader than previously believed [20]. Our model will ultimately make testable predictions along these lines.

However, the curves for the largest length-scales (\( r > L/10 \) where \( L \sim 1000 \eta \)) diverge from this scale-invariant curve, probably because, in the top decade of length-scales, turbulent stretching and vortex tube formation have not yet reached their fully developed state (see [8] and references therein). For these largest scales, multiplier statistics are better characterized by their scale as a fraction of \( L \) rather than as a multiple of \( \eta \). Moreover, at the largest scales, where nominal multiplier \( \beta \) values are large, the \( \beta \)-function itself does not provide a complete description of the cascade properties (figure 3). The scale at which the \( \beta \)-function becomes a suitable fit to the actual multiplier pdf is close to the scale at which the \( \beta(St_r) \) curves of figure 2 become universal.

The cascade model for planetesimal formation by turbulent concentration [11] can now be modified to allow for the observed level-dependence in the particle concentration multiplier pdfs (figures 2 and 3), as well as in the multiplier pdfs for gas enstrophy (vorticity squared) which also determine the cascade results and, we have found, also differ from values used

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\rho(r) = (r/\eta)^{-2/3} \rho_{St}
\]

in [11,12]. However, the asymptotic \( \beta \) value we determine for dissipation does not quite match the scale-free value determined by [14] in atmospheric flows. Thus, some effort must be dedicated to understanding how the results presented here may change with \( Re \) itself, and we are pursuing this. Because nebula properties such as turbulent intensity and gas density are unknown to an order of magnitude [11,12], these findings do not, on the face of it, qualitatively change the outlook for the scenario, but will lead to quantitative changes in predictions of planetesimal IMF and chondrule size distributions.