Design and Stability of an On-Orbit Attitude Control System Using Reaction Control Thrusters

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Overview

- NASA is providing preliminary design and requirements for the Space Launch System Exploration Upper Stage (EUS).
- The EUS will provide upper stage capability for vehicle ascent as well as on-orbit control capability.
- Requirements include performance of on-orbit burn to provide Orion vehicle with escape velocity.
- On-orbit attitude control is accommodated by a on-off Reaction Control System (RCS).
- Paper provides overview of approaches for design and stability of an attitude control system using a RCS.
  - Draws heavily from research and development in support of Space Shuttle and Space Station programs. Includes pitfalls and lesson’s learned from flight experience.
Paper Summary

- Vehicle Attitude Dynamics and Phase Plane Control
- Phase Plane Stability and Filter Design
- Jet Selection
- Maneuver/Steering Algorithms
- Thruster Hardware Specifications
Vehicle Attitude Dynamics and Phase Plane Control

- **Attitude Dynamics are summarized:**

\[
\begin{align*}
J_1 \dot{\omega}_1 &= (J_2 - J_3)\omega_2 \omega_3 + T_{\text{axl}} + u_1 \\
J_2 \dot{\omega}_2 &= (J_3 - J_1)\omega_1 \omega_3 + T_{\text{axl}} + u_2 \\
J_3 \dot{\omega}_3 &= (J_1 - J_2)\omega_1 \omega_2 + T_{\text{axl}} + u_3 \\
\eta &= \cos \left( \frac{\phi}{2} \right) \quad \dot{\eta} = -\frac{1}{2} \varepsilon^T \dot{\omega} \\
\dot{\varepsilon} &= \hat{\alpha} \sin \left( \frac{\phi}{2} \right) \quad \dot{\varepsilon} = \frac{1}{2} (\varepsilon^T + \eta I) \dot{\phi} \\
\Lambda_{12} &= \begin{bmatrix} \eta & \dot{\varepsilon} \end{bmatrix}
\end{align*}
\]

- **Phase Plane Design Examples are Provided:**

![Simple Switch Lines](image1)

![Hysteresis Added](image2)

![Ares I-X Design](image3)

![Shuttle Design](image4)
Phase Plane Stability and Filter Design

• Phase Plane control designs are nonlinear, hence traditional linear design approaches are generally not available.

• Paper presents RCS filter design and phase plane stability approaches based on research performed on the Space Shuttle and Space Station programs.

• Stability margin design goals are provided:
Paper describes approaches to derive a linear representation of the nonlinear system, concentrating on describing functions.

Phase plane is converted into an equivalent PD controller with a relay:

Relay is modeled by a describing function.

\[
N(A) = \begin{cases} 
0 & A < \delta \\
\frac{4}{\pi^2} \sqrt{1 - \left(\frac{\delta}{A}\right)^2} & A > \delta
\end{cases}
\]
Phase Plane Stability and Filter Design (continued)

- Describing function relay representation is still a nonlinear system as describing function gain is dependent on input amplitude ($A$):

- Linearize system by deriving value of $A$ which maximizes the describing function ($A^*$):

\[ A^* = \sqrt{2\delta} \quad \rightarrow \quad N(A^*) = \frac{2}{\pi\delta} \]

- Maximizing the describing function gain represents peak RCS control response to state error, which maximizes flex response to RCS firings (conservative approach).
Phase Plane Stability (continued)

• Given a System:

• Substitute the relay with a peak gain representation derived from the describing function:

\[ A^* = \sqrt{2}\delta \quad \Rightarrow \quad N(A^*) = \frac{2}{\pi\delta} \]

• The resulting derivation is a linear representation of phase control system.
Phase Plane Stability (continued)

- Example. Model rigid body control and ideal latency:

\[
\Psi(s) \rightarrow a_c \rightarrow \frac{1}{s^2} \rightarrow \phi \rightarrow \frac{1-T_D s}{2(1+T_D s/2)} \rightarrow 2/\pi \delta + \frac{2}{\pi (RL)} s \rightarrow \Psi_c(s)
\]

- The phase plane controller is a PD representation with the gains proportional to the phase plane deadzone (attitude and rate) limits.

- The closed loop transfer function is derived:

\[
\Psi_c(s) = \frac{2a_c}{\pi} \left[ - \frac{T_D}{2(RL)} s^2 + \left( \frac{1}{RL} - \frac{T_D}{2\delta} \right) s - \frac{1}{\delta} \right] \rightarrow \frac{T_D}{2} s^3 + \left( 1 - \frac{a_c T_D}{\pi (RL)} \right) s^2 + \frac{2a_c}{\pi} \left( \frac{1}{RL} - \frac{T_D}{2\delta} \right) s + \frac{2a_c}{\pi \delta}
\]

- And the necessary Condition for stability derived:

\[
\frac{1}{RL} - \frac{T_D}{\delta} - \frac{a_c T_D}{\pi (RL)^2} + \frac{a_c T_D^2}{2\pi (RL)\delta} > 0
\]
Phase Plane Stability (continued)

- Given the stability condition:

\[
\frac{1}{RL} - \frac{T_D}{\delta} - \frac{a_\epsilon T_D}{\pi (RL)^2} + \frac{a_\epsilon T_D^2}{2\pi (RL)\delta} > 0
\]

- Stability thresholds can be derived:

\[
\delta > \frac{T_D - \frac{a_\epsilon T_D^2}{2\pi (RL)}}{\frac{1}{RL} - \frac{a_\epsilon T_D}{\pi (RL)^2}}
\]

\[
T_D < \frac{a_\epsilon \delta - \left(a_\epsilon^2 \delta^2 + \pi^2 (RL)^4\right)^{1/2} + \pi (RL)^2}{a_\epsilon (RL)}
\]

Smallest Deadband

Allowable Latency
Phase Plane Stability (continued)

- Paper provides an example of how the stability condition maps to the RCS time domain simulation.
  \[ T_D < \frac{a_c \delta - \left( a_c^2 \delta^2 + \pi^2 (RL)^4 \right)^{\frac{1}{2}} + \pi (RL)^2}{a_c (RL)} \]

- Stable RCS Control:

- Unstable RCS Control:
Key RCS Filter Design Principles

- Paper provides key filter Design Principles for RCS:
  
  - **Key Filter Design 1:** Rigid body Stability
  
  - **Key Filter Design 2:** Flex Gain Margins
  
  - **Key Filter Design Principal 3:** Minimizing Filter Induced Lag
  
  - **Key Filter Design Principal 4:** Feed Forward during Thruster Firings
Key Filter Design 2: Flex Gain Margins

- Flex body dynamics can drive an RCS unstable.

Phase Plane Flex Instability

Phase Plane Stability Margins Using Linearized System

Frequency Domain Indicates High Amplitude for Low Frequency Flex

Forcing Function Approach Slows Good Flex Margins

Flex Response

Rate Limit

measured and filtered flex rate for 4 bi-polar flaxis: 1 jfc: -1 0 0
Key Filter Design Principal 3: Minimizing Filter Induced Lag

- Filter induced lag can result in a RCS limit cycle instability.

![Diagram showing vehicle rate during jet firing with actual and filtered responses, and phase plane with drift channels demonstrating lag and filtered signals.](image-url)
RCS Jet Selection

- Paper Addresses multiple RCS jet selection approaches:
  - Table look-up.
  - Algorithms that accommodate mass property changes.
  - Fuel Optimal Jet Select.
  - Command preshaping to avoid structural excitation.

Two Space Shuttle Jet Select Algorithms

- Dot Product: Would select jets 1 and 2
- Minimum Angle: Would select jets 2 and 4
RCS Maneuvering/Steering Algorithms

- Paper Addresses multiple RCS maneuvering/steering approaches:
  - Eigen Axis Maneuvers.
  - Torque-Free Maneuvers (Russian MIR).
  - Steering Formulation.
  - Fuel Optimal (Space Station “Zero Prop Maneuver”).
Thruster Hardware Specifications

- Discusses Shuttle RCS hardware design/control criteria:
  - Control authority must exceed all known disturbances by a factor of two.

\[ \hat{T}_c > 2 \max \left[ -\boldsymbol{\omega} \times \hat{\boldsymbol{J}} \hat{\boldsymbol{\omega}} \right] + \max \left( \hat{T}_{ext} \right) \]