GLOBAL OPTIMIZATION OF $n$-MANEUVER, HIGH-THRUST TRAJECTORIES USING DIRECT MULTIPLE SHOOTING

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• Optimal number of maneuvers not known a priori
  – Deep-space maneuvers (DSMs) frequently improve performance
  – Two (or more) DSMs can be optimal for tightly/uniqely constrained trajectory legs

• Missions interested in best performance possible, i.e., the global optimum
  – Can enable cost- or time-constrained missions
  – Often interested in more than 1 objective, e.g., max. delivered mass & min. TOF

• Problem challenges:
  – Grid searches can be intractable for any number & variety of DSMs
  – Optimization requires initial guess
  – Maneuvers and gravity assists can create highly sensitive optimization problems
Objective

Globally optimize chemical propulsion based trajectories with an arbitrary number of maneuvers & gravity assists

Method should be:
• Automated
• No requirement of a user-defined initial guess
• Able to search broad design space
• Efficient (medium-fidelity is appropriate)
• Capable of handling multiple objectives
Prior Approaches to Global Search

- **Grid search**
  - Lambert scans over range of event dates & flyby bodies
  - Strategies developed to include one maneuver per leg
    - Patel and Longuski → STOUR (Purdue, JPL)
    - Lantukh and Russell (UT-Austin)
    - Maneuvers limited to specific type v-infinity leveraging or broken plane maneuver

- **Stochastic search**
  - Strategically sample design space
  - Typically use a direct, Lambert-based trajectory formulation:
    - Vary departure date, time to DSM, & DSM components
    - Propagate forward to DSM point
    - Solve Lambert problem to subsequent body, repeat for all legs
    - Guaranteed feasibility for unconstrained problems
  - Not limited in maneuver type, but problems are frequently very sensitive to initial guess
  - Most approaches not capable of more than 1 DSM
  - Global-local hybrid scheme can improve efficacy

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Example STOUR plot (TOF vs. LD)
Petropoulos et al., “Trajectories to Jupiter…” JSR, 2000
Multiple gravity assist with $n$ deep space maneuvers using shooting scheme $s$

$\rightarrow$ MGA$^n$DSMs

- Aim for robustness & efficiency with an arbitrary number of DSMs with a direct formulation
- Nonlinear programming problem
- Employs forward/backward shooting similar to Byrnes & Bright (CATO)
- Nominally Kepler propagation between maneuvers
- Analytic match point (MP) constraints
- User specifies $n$, optimizer reduces 1 or more DSM magnitude to zero if $<n$ optimal

$$c_{mp} = [r_x^+ - r_x^-, r_y^+ - r_y^-, r_z^+ - r_z^-, v_x^+ - v_x^-, v_y^+ - v_y^-, v_z^+ - v_z^-] = e^T$$
MGAnDSMs Mission Formulation

- Multiple gravity assists accommodated in mission structure
- Allows for variation of flyby bodies at control nodes of transcription
  - Journeys start & end a user-required bodies/states
  - Journeys can be composed of multiple phases with variable flyby bodies to improve performance (“null gene” approach)
- Zero sphere of influence patched conics
  - Flyby constraints ensure physically realizable gravity assist
- Flexible to numerous mission constraints
MGAnDSMs Global Optimization

- Combine monotonic basin hopping (MBH) & sequential quadratic programming (SQP) → MBH+SQP
- Stochastic, global search scheme
- No initial guess required
- Adept at multi-modal problems w/ clustered local minima
- Stochastic “hops” evaluated from base solution
Multi-objective Optimization

- MGAnDSMs structured within multi-objective hybrid optimal control algorithm (discrete & continuous variables)
- Multi-objective genetic algorithm (GA) serves as outer loop systems optimizer around direct-method inner loop trajectory optimizer
  - Outer loop: non-dominated Sorting Genetic Algorithm II (NSGA-II) searches over discrete mission parameters, defining trajectory problem for inner loop
    - Variables include: Flyby body, target body, launch vehicle, launch C3, launch epoch
  - Inner loop: MBH+SQP solves trajectory problem & establishes obj. func. values
  - Generates representation of Pareto front (optimal tradeoff between objectives)
Established global optimum serves a test case for comparison of MGAN\text{DSMs} & a Lambert-based transcription (MGADSM\text{k}); one DSM allowed

MGAN\text{DSMs} identified global optimum in 10 out of 10 cases

MGADSM\text{k} identified solution w/in 50 m/s of global optimum, but not the global optimum (1.004 km/s) after an 8 hour run time

Order of magnitude improvement in time to best solution

\begin{itemize}
  \item Median time is 25x faster
\end{itemize}

### Cassini Test Case Comparison

<table>
<thead>
<tr>
<th>Run Number</th>
<th>MGAN DSMs Time to Identify Global Optimum (minutes)</th>
<th>MGADSMk Time to Identify Best Solution (minutes)</th>
<th>MGADSMk Time Identify Solution within 50 m/s of Global Optimum (minutes)</th>
<th>MGADSMk Best ΔV (km/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.3</td>
<td>223.8</td>
<td>220.3</td>
<td>1.025</td>
</tr>
<tr>
<td>2</td>
<td>7.9</td>
<td>140.9</td>
<td>32.9</td>
<td>1.043</td>
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<td>8.8</td>
<td>260.8</td>
<td>149.3</td>
<td>1.026</td>
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<td>4</td>
<td>12.8</td>
<td>74.3</td>
<td>74.3</td>
<td>1.014</td>
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<td>5</td>
<td>63.1</td>
<td>178.1</td>
<td>122.0</td>
<td>1.020</td>
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<td>6</td>
<td>4.3</td>
<td>295.4</td>
<td>112.0</td>
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<td>465.7</td>
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<td>80.6</td>
<td>58.4</td>
<td>1.033</td>
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<td>10</td>
<td>38.4</td>
<td>63.6</td>
<td>63.6</td>
<td>1.038</td>
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<tr>
<td>Mean</td>
<td>16.1</td>
<td>217.2</td>
<td>162.7</td>
<td>1.032</td>
</tr>
<tr>
<td>Median</td>
<td>8.4</td>
<td>200.9</td>
<td>117.0</td>
<td>1.033</td>
</tr>
</tbody>
</table>
Example Problem: Multi-objective Optimization to Jupiter

- Evaluated MGAnDSMs multi-objective capability on a multiple gravity assist to Jupiter with up to 2 DSMs per phase
- Flyby bodies are varied with any combination up to 5 bodies
- Three objectives: maximize $\log_{10}(\text{final mass})$, minimize TOF, minimize Jupiter arrival C3
  - Pareto front: 3D surface of equally optimal solutions
- Population size of 256 and the MBH+SQP inner loop is allowed to run for 40 minutes

### Outer-loop Design Variables

<table>
<thead>
<tr>
<th>Design Variable</th>
<th>Value</th>
<th>Resolution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Launch window open epoch</td>
<td>{1/1/2021, 1/1/2022, 1/1/2023, 1/1/2024}</td>
<td>1 year</td>
</tr>
<tr>
<td>Flyby body</td>
<td>{Venus, Earth, Mars, null, null, null}</td>
<td>n/a</td>
</tr>
<tr>
<td>Flight time</td>
<td>[730, 3467.5] days</td>
<td>182.5 days</td>
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</tbody>
</table>

### Common Mission Parameters

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Launch window</td>
<td>365.24 days</td>
</tr>
<tr>
<td>Launch declination</td>
<td>[-28.5, 28.5] deg</td>
</tr>
<tr>
<td>Launch vehicle curve</td>
<td>Atlas V, 551</td>
</tr>
<tr>
<td>Chemical $I_{sp}$</td>
<td>320 s</td>
</tr>
<tr>
<td>Jupiter arrival date</td>
<td>Determined by optimizer</td>
</tr>
<tr>
<td>Jupiter insertion orbit semi-major axis</td>
<td>10,054,900 km $(140.6 R_J)$</td>
</tr>
<tr>
<td>Jupiter insertion orbit eccentricity</td>
<td>0.911</td>
</tr>
<tr>
<td>Maximum number of DSMs</td>
<td>2</td>
</tr>
<tr>
<td>Inner-loop objective function</td>
<td>Max: $\log_{10}(\text{final mass})$</td>
</tr>
<tr>
<td>Inner-loop run time</td>
<td>40 minutes</td>
</tr>
</tbody>
</table>
Best Non-Dominated Front

- Representation of 3D Pareto front generated after 100 generations
- 3.5 days of run time on 64-core processor
Example Optimal Jupiter Trajectories

Highest Delivered Mass Trajectory

• EVEEJ sequence
• Delivered mass: 3192 kg
• TOF: 7.9 years
• Arrival C3: 34.1 km/s

Shortest TOF Trajectory

• Direct EJ (2 DSMs optimal)
• Delivered mass: 642 kg
• TOF: 2.0 years
• Arrival C3: 45.1 km/s
Optimal Jupiter Trajectory with 2 DSMs in 1 Phase

EVEJ sequence

Event # 1: launch Earth
7/7/2022
C3 = 10.387 km²/s²
DLA = -22.5°
m = 5023 kg

Event # 3: chemical burn deep-space 1/22/2024
Δv = 0.188 km/s
m = 4300 kg

Event # 4: unpowered flyby Venus
5/15/2024
vc = 7.219 km/s
DEC = -7.8°
altitude = 300 km
m = 4300 kg

Event # 6: chemical burn deep-space 9/4/2025
Δv = 0.535 km/s
m = 3626 kg

Event # 7: insertion Jupiter
7/3/2028
vc = 5.608 km/s
DEC = -0.5°
Δv = 1.289 km/s
m = 2405 kg

Event # 2: chemical burn deep-space 8/28/2023
Δv = 0.300 km/s
m = 4565 kg
Conclusions

- Developed high-thrust trajectory optimization transcription for any number of maneuvers between flybys
- Multiple shooting framework & analytic derivatives provide robustness and enable outer-loop efficiency
- Formulation allows for an automated, global search without a user-supplied initial guess
- Capability to generate Pareto-optimal solutions using multi-objective hybrid optimal control algorithm
- General applicability to almost any variety of interplanetary mission
  - Flexible to unique trajectory/mission constraints
- Large problems can become computationally tractable
Backup
Multi-objective Optimization

- Want to optimize any number of mission design metrics
  - e.g., payload mass, TOF, arrival C3
  - Often coupled & competing
  - Fully map mission trade-offs between optimal solutions

- Optimize multiple objectives simultaneously
  - Entire set of optimal solutions
  - Goal: generate representation of Pareto front
  - Traditionally use repetitions of single objective technique
• Develops globally-optimal Pareto solutions using non-dominated sorting
  – Conducts stochastic, global search with population of designs

• Fitness assignment based on “nearness” to Pareto front
  – \( x_1 \) dominates \( x_2 \) if:

\[
\forall p : f_p(x_1) \leq f_p(x_2) \quad p = 1,2,\ldots,n_{obj}
\]

and

\[
\exists p : f_p(x_1) < f_p(x_2) \quad p = 1,2,\ldots,n_{obj}
\]

• If neither design dominates other, they are non-dominant

• Non-dominated sorting:
  – Assign fitness based on design’s non-dominated front
  – Designs closer to Pareto front get more mating opportunities
Genetic Algorithm

- Models Darwinian evolution
  - Mimic natural selection & reproduction
- Searches with population of designs
- Globally search design space
- No initial guess required