Hybrid Differential Dynamic Programming with Stochastic Search

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Motivation

Research Objective:
“… to efficiently and robustly optimize high-dimensional spacecraft trajectories”
(e.g. low-thrust, many revolutions)

- Large number of decision variables
- Many local optima
- Hard to form an appropriate initial guess

Popular direct methods: discretize trajectory into $M$ stages of $N$ decision variables (e.g. thrust vector) and solve the nonlinear programming problem (NLP) of size $M*N$

Differential Dynamic Programming (DDP) instead solves many subproblems, i.e. DDP solves $M$ NLPs of size $N$
AND
without the need for ‘black box’ NLP solvers (SNOPT, IPOPT, fmincon, etc.)
Alphabet Soup

This study implements HDDP to compute spacecraft trajectories and uses MBH as a stochastic search step to find better solutions.

DDP – Differential Dynamic Programming
  ■ a trajectory optimization algorithm

HDDP – Hybrid Differential Dynamic Programming
  ■ a recent variant of DDP by Lantoine and Russell

MBH – monotonic basin hopping
  ■ multi-start algorithm to search many local optima

EMTG – Evolutionary Mission Trajectory Generator
  ■ NASA Goddard open source mission design tool

FBLT – finite burn low-thrust
  ■ two-body equations with continuous low-thrust

ALM – augmented Lagrangian method
  ■ Constrained optimization by adding penalty term to Lagrangian cost function
Dynamic Programming

- Solve a complex problem by breaking it down into smaller subproblems

Differential Dynamic Programming

- Iteratively perform backward sweep on the trajectory to update the control sequence
  - Minimizing quadratic expansion of objective function yields feedback control law

Hybrid Differential Dynamic Programming

- Use state transition matrix (Φ) and state transition tensor to map sensitivities
- NLP techniques
  - Augmented Lagrangian Method
  - Trust-region Methods
**Forward pass** on nominal control sequence $\bar{u}$:

**Backward sweep** recursively solves subproblems for control update $\delta u$:

**HDDP Iteration**

**Forward pass** on nominal control sequence $\bar{u}$:

- $\bar{u}_1$ to $\bar{u}_2$ via $\Phi_1$
- $\bar{u}_2$ to $\psi$ via $\Phi_2$

**Backward sweep** recursively solves subproblems for control update $\delta u$:

- $\delta u_1$ to $\delta u_2$ via $\Phi_1$
- $\delta u_2$ to $\psi$ via $\Phi_2$

**trust-region subproblems**

**Derivatives of $\psi$**
Augmented Lagrangian Method

Terminal constraints are enforced by added penalty to cost function:

\[ J = h + \lambda^T \psi + \sigma \psi^T \psi \]

- \( h \): original objective
- \( \lambda \): Lagrange multipliers
- \( \psi \): constraint violations
- \( \sigma \): penalty parameter

An initial \( \sigma \) must be sufficiently large, and continually increased to drive \( \psi \to 0 \).

Tuning \( \sigma \) and its update factor \( \kappa_\sigma \) can be tedious.
HDDP is a gradient based method that will converge to a solution nearby an initial guess.

- motivates a stochastic search across many local optima

MBH has previously been implemented for controlled random search in spacecraft trajectory design.

- ESA Advanced Concept Team and EMTG
  - applied to ‘black box’ NLP solvers

HDDP now fills the role of the NLP solver.
1. HDDP computes nominal solution from initial guess

2. Until MBH stopping criteria (compute time or $N_{hop}$):
   a. Introduce perturbations to decision variables
      - random draw from Pareto distribution
   b. If $rand(0,1) < \rho_{time-hop}$
      - Shift time variables forward or backward 1 synodic period
   c. Reinitialize HDDP with perturbed solution as initial guess
   d. Accept iterate if:
      - feasible and cost improves
      - infeasible and violations reduced
Computed Examples

- Example Earth-Mars rendezvous transfer from Lantoine and Russell is used for validation and test case for variable time of flight with MBH.

<table>
<thead>
<tr>
<th>Spacecraft</th>
<th>Mission</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_0$</td>
<td>$t_0$</td>
</tr>
<tr>
<td>1000.0 kg</td>
<td>April 12, 2007</td>
</tr>
<tr>
<td>$T_{max}$</td>
<td>$TOF$</td>
</tr>
<tr>
<td>.5 N</td>
<td>348.79 days</td>
</tr>
<tr>
<td>$I_{sp}$</td>
<td></td>
</tr>
<tr>
<td>2000 s</td>
<td></td>
</tr>
</tbody>
</table>

- Objective is to maximize final mass with penalty on position and velocity errors at Mars arrival.

\[
h = -m_f \\
\psi = \begin{bmatrix} r_f - r_M \\ v_f - v_M \end{bmatrix}\]
**Fixed TOF Validation**

Table 1: Comparison of Spacecraft Final Mass

<table>
<thead>
<tr>
<th>Model</th>
<th>Final Mass (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HDDP standard</td>
<td>$m_f = 598.66$ kg</td>
</tr>
<tr>
<td>HDDP - FBLT</td>
<td>$m_f = 603.29$ kg</td>
</tr>
<tr>
<td>EMTG - FBLT</td>
<td>$m_f = 603.45$ kg</td>
</tr>
</tbody>
</table>

Table 2: Comparison of Lagrange multipliers

<table>
<thead>
<tr>
<th>Model</th>
<th>Lagrange Multipliers</th>
</tr>
</thead>
<tbody>
<tr>
<td>HDDP standard</td>
<td>$\lambda = [0.5095, -1.2700, -0.2665, 0.1178, 2.0701, 0.13404]^T$</td>
</tr>
<tr>
<td>HDDP - FBLT</td>
<td>$\lambda = [1.0793, -2.3127, -0.5920, -0.1125, 2.9337, 0.0463]^T$</td>
</tr>
</tbody>
</table>
ALM Tuning

MBH can be used to cover for a poorly tuned HDDP

Table 3: Survey of ALM tuning and improvements with MBH

<table>
<thead>
<tr>
<th>$\sigma_0$</th>
<th>$\kappa_s$</th>
<th>Iterations</th>
<th>$m_f$ (kg)</th>
<th>$m_f, N_{hop} = 30$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1.1</td>
<td>123</td>
<td>failed</td>
<td>603.29*</td>
</tr>
<tr>
<td></td>
<td>1.5</td>
<td>541</td>
<td>failed</td>
<td>603.04</td>
</tr>
<tr>
<td></td>
<td>2.0</td>
<td>643</td>
<td>601.87</td>
<td>602.80</td>
</tr>
<tr>
<td>100</td>
<td>1.1</td>
<td>231</td>
<td>failed</td>
<td>603.07</td>
</tr>
<tr>
<td></td>
<td>1.5</td>
<td>181</td>
<td>602.93</td>
<td>603.17</td>
</tr>
<tr>
<td></td>
<td>2.0</td>
<td>892</td>
<td>601.67</td>
<td>602.04</td>
</tr>
<tr>
<td>1000</td>
<td>1.1</td>
<td>352</td>
<td>failed</td>
<td>602.95</td>
</tr>
<tr>
<td></td>
<td>1.5</td>
<td>1119</td>
<td>602.50</td>
<td>603.14</td>
</tr>
<tr>
<td></td>
<td>2.0</td>
<td>940</td>
<td>601.75</td>
<td>602.21</td>
</tr>
</tbody>
</table>
\( \delta t_0 \) and \( \delta t_f \) are now decision variables.

### Table 4: Variable Time of Flight Results

<table>
<thead>
<tr>
<th>( \delta t_0 )</th>
<th>-23.89 days</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \delta t_f )</td>
<td>43.78 days</td>
</tr>
<tr>
<td>( m_f )</td>
<td>674.95 kg</td>
</tr>
</tbody>
</table>
MBH is now applied to the variable time problem in HDDP

- Perturb time variables only
- 20% chance of hopping between 2007 and 2009 opportunities
- Stopping criteria: \( N_{hop} = 100 \)
- 1\(^{st}\) order state transition matrix only to quickly generate results
- Reset \( \sigma \) with each hop

**Table 5: Results for \( N_{hop} = 100 \)**

<table>
<thead>
<tr>
<th>( \delta t_0 )</th>
<th>836.85 days</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \delta t_f )</td>
<td>894.95 days</td>
</tr>
<tr>
<td>( m_f )</td>
<td>745.57 kg</td>
</tr>
</tbody>
</table>
- Transfer in next synodic period
- Departure/arrival in different phases of Earth/Mars orbit
- Reduced to a 2-burn solution
- Compare 745.57 kg final mass to 674.95 kg before hopping
Summary

- Implementation based on HDDP by Lantoine and Russell validated with EMTG
- Successfully introduced time variables to an Earth-Mars rendezvous example
- Employed MBH as stochastic search step with HDDP computing the spacecraft trajectory
- MBH shown to guide HDDP through large steps in time variables and across synodic period
- Additional benefit found in MBH helping HDDP overcome poor tuning of algorithm parameters

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References


