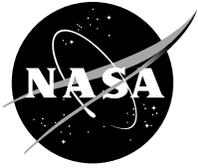


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# Requirements for Large Eddy Simulation Computations of Variable-Speed Power Turbine Flows

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March 2016

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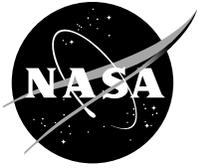
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# Requirements for Large Eddy Simulation Computations of Variable-Speed Power Turbine Flows

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## Abstract

Variable-speed power turbines (VSPTs) operate at low Reynolds numbers and with a wide range of incidence angles. Transition, separation, and the relevant physics leading to them are important to VSPT flow. Higher fidelity tools such as large eddy simulation (LES) may be needed to resolve the flow features necessary for accurate predictive capability and design of such turbines. A survey conducted for this report explores the requirements for such computations. The survey is limited to the simulation of two-dimensional flow cases and endwalls are not included. It suggests that a grid resolution necessary for this type of simulation to accurately represent the physics may be of the order of  $\Delta x^+ = 45$ ,  $\Delta x^+ = 2$  and  $\Delta z^+ = 17$ . Various subgrid-scale (SGS) models have been used and except for the Smagorinsky model, all seem to perform well and in some instances the simulations worked well without SGS modeling. A method of specifying the inlet conditions such as synthetic eddy modeling (SEM) is necessary to correctly represent the inlet conditions.

## Introduction and Motivation

Variable-speed power turbines (VSPTs) for rotorcraft applications operate at low Reynolds numbers and a wide range in incidence resulting from rotational speed variation (Welch, 2010). At all speeds, the blades operate in conditions like those in highly-loaded low pressure turbine (LPT) blades that have been the subject of research in recent years. The flow physics that are important to LPT operation resulting from the operating conditions, that is, separation, transition, and the resulting losses, are fully expected to apply to VSPTs. Blade designs are such that the on-design flow on the suction side experiences a vigorous acceleration and then goes through a region of rapid deceleration. The acceleration parameter  $K$  defined as  $(v/U^2) dU/dx$  typically ranges  $\pm 5.0 \times 10^6$  for a Reynolds number of  $10^5$  which is large compared to, for example, high pressure turbine since  $K$  is scaled by the Reynolds number (Mayle, 1991). Because of the presence of free-stream turbulence (FST) in the turbines and passing wakes from upstream blade rows and their associated turbulence, the process of transition is not natural. Background turbulence, even at low levels, discourages natural transition. The boundary layer on the suction side of the blades is initially laminar due to acceleration, but it can transition given high enough turbulence intensity. If the boundary layer does not transition in the initial accelerating, or mildly decelerating region on the suction side, the layer is prone to separation in the strongly decelerating flow regime and transition will take place upon reattachment. Efficiency lapses observed in LPTs at cruise conditions (low Reynolds number condition) is caused by separated flow in the LPT blades. Reattachment before the trailing edge would ensure lower losses while an open separation bubble leads to larger losses (Medic and Sharma, 2012). Reattachment length itself is a strong function of FST (Mayle, 1991). At sufficiently high operating rotational speeds with the associated negative incidence, the VSPT flow separates from the pressure surface and a separation and turbulent reattachment takes place on the pressure surface. It is obvious that development of the boundary layer with the contributing factors such as the effects of turbulence and

acceleration parameter is quite complex. A proper numerical simulation would account for all of the relevant phenomena.

As a consequence of FST, streaky structures called “Klebanoff streaks” are generated on the LPT blade surface upstream (Coull and Hodson, 2011 and McAuliffe and Yaras, 2009). These are the result of low frequency disturbances penetrating the boundary layer through a process called “shear sheltering,” which filters the high frequency disturbances. As shown computationally by Jacobs and Durbin (2001), the disturbances are able to penetrate the boundary layer better as the frequency of disturbances is reduced and as the local Reynolds number is reduced. As such, the leading edge of a blade, where the Reynolds number is low, is a receptive site for the Klebanoff streaks. These streaks are precursors to transition. Coull and Hodson reported from Marxen et al. (2009) that according to their experiments on a flat plate subject to favorable pressure gradient and adverse pressure gradient, in the adverse pressure gradient regions instability is also helped by Görtler-type vortices induced by concave streamlines close to laminar separation. The Klebanoff streaks convect and spread downstream, slower in speed than the free-stream velocity. The speed of leading edge and trailing edge of these streaks are 0.88 and 0.50 times free-stream velocity. The streaks grow and lift up from the blade surface downstream as they roll up into hairpin vortices, which subsequently break down.

For separated boundary layers, the process of transition in the bubble is caused by the Kelvin-Helmholtz instability and inflectional instability as summed up by Marty (2014). The former refers to the transition caused by the interface of the bubble and the free stream, while the latter refers to the inflection in the velocity profile within the bubble. As the upstream blade wake periodically passes over the blade surface, the Klebanoff streaks are intensified because of the elevated wake turbulence. Once the wake passes past the trailing end of the bubble, the transition moves further downstream. The cumulative effect of the wake passing according to McAuliffe and Yaras (2009) is to reduce the size of the separation bubble downstream, which contributes to a reduction in separation loss, while increasing the region for which the boundary layer is turbulent leading to an increase in loss.

The above description explains why proper and effective modeling of this flow is difficult to achieve. Strides have been made in turbulence modeling, which account for the various mechanisms of transition and turbulence in the context of Reynolds-averaged Navier-Stokes (RANS) methods. Still these models do not consistently produce physically correct results. For example, Ameri et al. (2014) used the laminar kinetic energy-type model of Walters and Cokljat (2008) for the simulation of VSPT blade flow of Flegel et al. (2014). While comparison to the experimental data was mostly favorable, the model produced bubble reattachment that was not fully turbulent and led to a second separation downstream under cruise conditions. Marty (2014) presented RANS and unsteady Reynolds-averaged Navier-Stokes (URANS) computations with Menter’s model (Menter et al. 2006 and Langtry et al. 2006) over a range of Reynolds numbers at a single turbulence intensity. The pressure distribution results presented were good, however, obtaining physically consistent results with the appropriate FST conditions was difficult to achieve.

It may be desirable to perform high fidelity methods such as direct numerical simulation (DNS), large eddy simulation (LES), or hybrid methods to simulate this type of flow with better accuracy. As the DNS is extremely expensive, we would like to explore the possibility of using various levels of approximation such as using LES instead of DNS or further modeling to realize savings in computational cost by using wall-modeled LES (WMLES) or detached eddy simulation (DES) instead of LES where the near-wall modeling is done via RANS modeling.

For LES to be applied successfully to the LPT or VSPT problem, it appears that the mechanisms responsible for transition need to be well resolved or if they are too small scaled to be resolved they need to be modeled to get the appropriate effect. Improvements in accuracy of the predictions should be substantial or LES may not be justifiable given the advancements in modeling of transitional flows using

RANS models. LES should be able to successfully represent the free-stream turbulence (turbulence decay), the aforementioned leading edge receptivity, and other mechanisms that may trigger the process of transition including the effect of FST on separation. The flow structures responsible for the exchange between the free-stream and near-wall flows which contribute to transition should be properly resolved. Separation bubble transition, which may depend on more than one mode of transition should be well represented. The fully turbulent wake region is also crucial to the success of LES in prediction of losses.

A literature survey of the computations relevant to the low Reynolds number LPT flows was performed. The intent was to establish a minimum requirement for grid resolution (DNS) and the necessary requirements for LES for the various parts of the flow field from LPT computations or from computations of equivalent physics. That survey and a brief discussion of the relevance of hybrid methods to VSPT flows is presented herein.

## Direct Numerical Simulation Studies

DNS studies should help establish the most stringent grid requirements. Some of the surveys in this report look at the whole LPT flow and a few look at flows that contain relevant physics. A summary of the surveyed literature with the utilized grid resolution is presented in Table I.

McAuliffe and Yaras (2009) aimed at understanding transition mechanism inside a separation bubble on a flat plate with an imposed free-stream velocity and FST. They used a contoured outer wall over a flat plate flow to simulate the appropriate pressure gradient. They also modeled the grid turbulence and its decay upstream of the leading edge of the flat plate. An incompressible solver with second order central difference in space and second order backward difference in time was utilized. For the low FST (0.1 percent) simulation a grid with  $1.1 \times 10^6$  grid elements was used, and for the elevated (1.4 percent) FST simulation,  $4.2 \times 10^6$  grid elements were used. For the cases of flow at the high and low FST the Reynolds numbers were 440,000 and 390,000, respectively, based on the location of separation. The grid resolution measures were  $\Delta x^+ = 24$ ,  $\Delta y_{\min}^+ = 0.6$ , and  $\Delta z^+ = 24$  at the high free-stream turbulence intensity (FSTI) and  $\Delta x^+ = 19$ ,  $\Delta y_{\min}^+ = 0.5$ , and  $\Delta z^+ = 19$  at low FST based on shear velocity in the turbulent boundary layer downstream of reattachment. The spatial and temporal resolutions were chosen such that the turbulence scales in the turbulent boundary layer downstream of the separation bubble were resolved down to about 35 times the Kolmogorov scale. The Kolmogorov length scale can be computed from a relationship, which yields the ratio of the largest scales to the smallest scales of turbulence namely,  $L/\eta = Re^{3/4}$  where  $Re$  signifies the Reynolds number based on large-scale flow features and  $L$  represents the largest scale eddy size. McAuliffe and Yaras found this resolution to be adequate for capturing the turbulence associated with transition and reattachment of separated shear layers. The selected spatial and temporal resolutions were reported to be consistent with DNSs involving separation bubbles, performed by other research groups. The number of grid points from a low FSTI (0.1 percent) to a high FSTI (1.4 percent) case (approximately a factor of 4) was changed to accommodate the upstream simulation of grid turbulence for the high FSTI case in a region of approximately twice the flat-plate length upstream. The simulations, although unsteady, did not include the effect of any wakes which is an important ingredient in the transition process is turbines. Also, the span of the plate for the high FST case was 20 percent wider than the low FST case to allow for increased three-dimensionality of the bypass transition process.

Wu et al. (1999) performed a DNS study (incompressible) on an LPT blade including accounting for upstream wake. Blade was the T106 (Weiss and Fottner, 1995) at  $Re = 1.48 \times 10^5$ . The domain was from  $x = -0.5$  axial chord to  $x = 2 \times$  axial chord and the spanwise dimension was 0.15 axial chords. The  $2.5 \times 10^7$  and  $5.7 \times 10^7$  grid points were utilized for the computations without and with upstream wakes. Preliminary simulation was performed first, on a 25 million (M) point (pt) grid containing 769, 257, and

129 pt in the streamwise, wall-normal, and spanwise directions, respectively; and then on a 57 M pt grid containing 1153, 385, and 129 pt. In the fine grid, 288 streamwise points were distributed upstream of the leading edge and 576 pt were used on the blade. Near the trailing edge, 40 grid points were placed in the boundary layer. As mentioned there were 385 pt in the transverse direction, as such most of the points are resolving the free-stream in order to resolve the wake distorting and passing through. Agreement with pressure distribution, though good, was not one to one because the underlying experiments were run at different free stream turbulence intensity levels and did not include a wake. The DNS study serves to highlight the need for resolving wake but did not include the effect of free stream turbulence.

Jacobs and Durbin (2001) have shown, through highly resolved DNS of flow over a flat plate with FST, large streaky structures that produce what they refer to as “negative jets,” which are located at the upper part of the boundary layer and are precursors to the process of bypass transition. These structures are different, they argue, from the Klebanoff modes, which they estimate have only a tangential role in the transition process. Table I contains information about the grid they used. Ham et al. (2000) used the existence of this precursor, as we will discuss later in this report, to gauge if the transition modeling in the context of LES was working.

Rai and Moin (1991) performed an early DNS of flat plate flow with a broadband inlet disturbance and observed turbulence-induced transition. They found that the  $x$ -dependence of friction coefficient,  $C_f$ , was quite sensitive to grid resolution. In their most resolved computation using  $\Delta x^+ = 28$  and  $\Delta z^+ = 10$ ,  $C_f$  rose too abruptly, to a level well above the turbulent correlation. Rai and Moin estimated that a grid spacing finer by a factor of 1.5 to 2 in the streamwise and spanwise direction may be appropriate. Based on their simulations, Jacobs and Durbin also speculate that the erroneous  $C_f(x)$  curve computed by Rai and Moin was a consequence of inadequate streamwise resolution.

Wu and Moin (2009) studied bypass transition on a zero-pressure gradient flat plate where they were interested in obtaining experimental quality data. They found that the grid density requirement was more stringent than previously determined for flows without FST. They suggest that was caused by the additional requirement for resolving FST. This agrees with the work of Wu et al. (1999) discussed above. A coarse streamwise grid, they state, has “noticeable” effect on flow statistics. They also state that while for the fully turbulent regime and preturbulent regime the computation results of a fine 210 million and a coarser 105 million point grids are very close, the transition point for the coarse grid is significantly earlier than for the fine grid. This illustrates the criticality of transition process in the success of transitional flow simulations.

Kalitzin et al. (2003) extended the work of Wu and Durbin (2001) for the T106 blade by adding a turbulence-free and a grid-turbulence condition to the wake passing turbulence. They increased the number of grid points in the grid turbulence case by an additional 50 percent. Their work shed light on the mechanisms of transition including a description of the distribution of turbulence in the passage and amplification of turbulence near the leading edge and in the trailing edge of the blade on the pressure side. Their results showed time-averaged friction coefficient over the blade, which suggested no separation on the suction side for the three cases considered where in fact the experiment showed separation for the clean case. The Reynolds number was  $1.5 \times 10^5$  based on axial chord and inlet conditions. Natural transition for the clean blade and bypass for the other two cases was observed. In general, agreement with data was not achieved as it should have because of the use of an incompressible solver (for a compressible flow) and lack of smoothness in the grid.

Wissink (2003) also performed a DNS of the case described above, but at a much higher inlet angle ( $45^\circ$  instead of  $34.4^\circ$ ). Various grid dimensions were used with the largest at approximately  $17 \times 10^6$  pt. The grid spacing in wall units was not provided (thus not shown in Table I) but they report that the pressure distribution was not sensitive to the grid densities at their chosen densities. Notably, there was

disagreement with the data which because of the uncertainty resulting from the inlet angle and incompressibility assumption.

Sandberg et al. (2012) also using DNS (grid size is given in Table I) were able to match very well both pressure and wake profile from the experimental results of Stadtmüller (2001). Three cases of “clean,” 1.1 percent FST and 3.8 percent FST were investigated. They also added the effect of reduced frequency of wake passing to their computations in a subsequent paper by Chen et al. (2013). The Reynolds number was 60,000 and exit Mach number was 0.4. Fourth-order Runge–Kutta scheme for time and compact scheme for space ( $\xi$ – $\eta$ ) and FFTW3 for spanwise direction was used. The compressible method used was able to match the data and describe the effect of the free stream turbulence upon the pressure distribution downstream of the suction side separation location. The DNS simulations described above were carried out on a spanwise domain that consisted of 15 to 20 percent of chord in width. A multiblock grid with smooth and well-shaped cells were used.

## Large Eddy Simulation

LES has the promise of being an alternative approach for simulation of transitional and turbulent flows. Unlike DNS, only the large scales and not all scales of the flow are resolved, and it is assumed that these smaller scales are homogeneous and isotropic. Modeling the smaller scales (subgrid) in place of resolving them, allow relaxation of grid resolution as well as reduction of CPU requirements compared to DNS. The question we are interested in answering is whether or not LES is capable of correctly describing the LPT transition process and if so, what are the requirements for the simulation in terms of both grid resolution and modeling of subgrid scales (SGSs). More fundamental simulations, such as those of flows in simple channels or over flat plates have been considered as isolated phenomena, which we will survey before we look at the problem of transition and the whole LPT blade passage. The grid resolution requirements and SGS modeling requirements for each case are summarized in Table I.

Subgrid models are used in areas where the grid resolution does not account for the effect of small scales. Such models, as have been suggested, should not overwhelm and dissipate the disturbances in the near-wall region. For example, the often used Smagorinsky (1963) model is unable to achieve this and other models are suggested to ensure that the generated eddy viscosity approaches zero near the walls and is not over-dissipative. In some of the cases listed in Table I, wall-adapting local eddy-viscosity (WALE) model of Nicoud and Ducros (1999) or high-pass filtered (HPF) Smagorinsky model of Stolz, et al. (2005) or similarly suitable models are used.

Although not directly applicable to VSPT, the savings may be appreciated by considering that Sayadi and Moin (2011) worked the problem of natural transitional for flow over a flat plate. The simulation results using a  $9.8 \times 10^6$  grid point were compared to a separate DNS with approximately  $10^9$  pt. The results indicated that LES is capable of producing the transitional behavior of the boundary layer with an appropriate SGS model. Sayadi and Moin report that while the Smagorinsky model did not produce the desired effects, the Dynamic Smagorinsky model (Germano, 1991) did. The reason for Smagorinsky model not being able to produce the transition effect was reported to be the production of eddy viscosity upstream of transition which damped out the disturbances responsible for transition. In an earlier work Sayadi and Moin (2010) reported that a no-SGS model case also worked, presumably because there is no problem of artificially increased dissipation of disturbances in the laminar region of the flow. The dimensionless wall grid resolution based on the wall shear was approximately  $\Delta x^+ = 45$ ,  $\Delta y_{\min}^+ = 1$ , and  $\Delta z^+ = 11$  and the Reynolds number was  $Re_x = 8 \times 10^5$ . Grid spacing used a shear stress value that was computed in the initial turbulent regime and was well past the transition location (and thus conservative). The method used for numerical simulation was a sixth-order compact finite difference for space and

implicit-explicit time integration applied for time. For the explicit time advancement, a third-order Runge–Kutta scheme was employed for the time integration and an A-stable scheme was used for the implicit portion. The number of grid points used was  $960 \times 160 \times 64$  totaling 9.8 M pt. Sagaut and Deck (2009) suggest in their paper that the classically retained  $\Delta x^+$ ,  $\Delta y_{\min}^+$  and  $\Delta z^+$  in LES to capture the near wall turbulent structures are 50, 1 and 15, and did not make any distinction for the accuracy of schemes used. Sayadi and Moin's grid resolution agrees with the general rule provided by Sagaut and Deck. The computations show that LES may be used to compute the transitional boundary layer and suggest that in the absence of FST (natural transition) LES without a SGS model may be reasonably accurate in predicting the transition location and level of shear stress.

A discussion on fundamental studies using LES for the simulation of FST-induced transition of Jacobs and Durbin (2001) and separation-induced bypass transition of Yang and Voke (2001) follows.

Earlier we discussed the concept of a negative jets as a precursor to bypass transition as described by Jacobs and Durbin (2001). Ham et al. (2000) suggest that since negative jets are hundreds of wall units long and 60 wall units wide, it may be possible to use LES to model such structures. They proposed and implemented LES modeling of wake-induced (bypass) transition over a flat plate with specific emphasis on the resolution of turbulent spots and the negative jets, which are precursors to bypass transition. The process of transition as described by the streamwise distribution of shear stress on the wall was compared to DNS results. An initial grid with a  $\Delta x^+ = 95$  and  $\Delta z^+ = 35$  was able to correctly predict the beginning of transition. It, however, was not able to produce the transition process nor were the turbulent spots seen in the computational domain. Ham et al. speculated that the streamwise grid was under-resolved and concluded from their work that an appropriate choice is  $\Delta x^+ = 45$ ,  $\Delta y^+ = 2$ , and  $\Delta z^+ = 17$  which generated the turbulent spots and negative jets, although they underpredicted the length of transition. The values quoted are based on maximum shear velocity. Note that the resolution recommended agrees with the resolution recommended suggested by Sagaut and Deck (2009). Ham et al. also made LES and RANS ( $v^2$ - $f$ ) computations of flat plate flow with wake-induced transition. URANS ( $v^2$ - $f$  model) was able to match the DNS results better than LES. They concluded from their results that the URANS was a good choice as it produced better results for  $C_f$  considering the relative economy of the computations if only the shear stress distribution is desired. As to the discrepancy in predicting the length of transition, they speculated that a model other than their chosen Dynamic Smagorinsky model may have done a better job.

Regarding separation-induced turbulent flows, Yang and Voke (2001) studied separation from a rounded leading edge (diameter  $d$ ) of a flat plate using LES and they used a Dynamic Smagorinsky model. Their simulations did not include the effect of FST. Their results were compared to relevant experimental data for the reattachment process and transition. The spanwise dimension of the domain was  $2d$  (a change to  $4d$  showed no appreciable change in behavior of the flow). They describe the process of transition as via a free shear layer developing and forming spanwise vortices in the first half of the bubble. Spanwise vortices grow with an amplification rate, which is larger than that in the case of viscous instabilities. These initial spanwise vortices are further distorted and deformed leading to streamwise vorticity formation associated with significant three-dimensional motions, eventually breaking down at about the reattachment point and rapidly becoming a turbulent boundary layer. For the modeling, grid resolution details are expressed in terms of wall units based on the shear layer downstream of reattachment at  $x/l = 2.5$  ( $l$  is the mean separation bubble length). The streamwise mesh sizes varied from  $\Delta x^+ = 10$  to 30.5, while  $\Delta z^+$  was equal to 9 and at the wall  $\Delta y^+$  was unity. The time step used in the simulations was  $0.005d/U_0$  ( $U_0$  being the free stream flow speed). They reported a resulting averaged subgrid eddy viscosity of about 3.5 times the molecular viscosity as the mesh resolutions were quite fine in most of the important flow regions and not far away from what would be recognized as DNS. The eddy

viscosity was zero in the laminar regime and picked up inside the bubble to around a value of about 8 times the viscosity near the reattachment point.

A discussion on simulations performed on LPT blades follows. In their LPT simulation of wake (from a bar wake generator) passing across the Weiss and Fottner (1995) T106 blade cascade, Sarkar and Voke (2005) used the classical Smagorinsky model with a reduced value of model constant of 0.125 (from 0.2 to 0.22), justified by the computation economy and the nature of the problem. They used this simulation to describe the transition process on the suction side. Sarkar and Voke suggest that because the Smagorinsky model is absolutely dissipative, it is incapable of predicting the reverse cascade regions and overestimates the SGS dissipation. As described earlier, the problem is due, in part, to the model being only based on local large-scale quantities and yielding nonzero residual stresses, even in the laminar region. They suggest that this limitation can be overcome by using the low-Reynolds number model of Voke (1996). The Voke model uses the Smagorinsky model and wall damping akin to the van-Driest model. Earlier, Huai et al. (1997) compared the performance of a localized dynamic model with that of the low-Reynolds number Smagorinsky model and with the Smagorinsky model with intermittency modification to simulate the transitional flow over a flat plate. They concluded that simple corrections to the Smagorinsky model were effective although they did not include FST in their simulations. Sarkar and Voke (2005) reported the grid size that they used on the surface and in the midpassage (Table 1). They used a slice equal to 10 percent of the chord. On the blade, the range of  $\Delta x^+$  was 10 to 45; lowest  $y^+$  was 1.3, and  $\Delta z^+$  was 16. They also report the midpassage grid resolution, which is critical to the success of such a simulation as ( $\Delta x^+=40$ ,  $\Delta y^+=40$ , and  $\Delta z^+=16$ ) for a total of  $2.36 \times 10^6$  pt.

Michelassi et al. (2003) applied LES to the DNS case of Wu and Durbin (2001). They applied the same wake inlet conditions and computational domain as Wu and Durbin. The effect of FST outside of the wake was not included in the DNS or in the present LES. It extended  $0.5 \times C_{ax}$  upstream of the leading edge of the blades to  $1.0 \times C_{ax}$  downstream of the trailing edge. The spanwise width was  $0.15 \times C_{ax}$ . The grid was an H grid of dimensions,  $646 \times 256 \times 64$  nodes in the streamwise, pitchwise, and spanwise directions, respectively. There were 15 to 20 pt placed in the turbulent boundary layer on the suction side near the trailing edge. The cell sizes in wall units, in the  $x$ ,  $y$ , and  $z$  directions, were below 70, 0.2 to 3, and 15 to 20, respectively. These values indicated a poor resolution of the boundary layer to the authors, although a finer resolution was not attempted. A second-order incompressible method was used for the simulations. The SGS model used the dynamic model by Germano et al. (1991) with the modification by Lilly (1992) in the LES. The procedure employed an eddy viscosity model together with a procedure for reducing the model constant whenever the flow is well resolved. The SGS model employed filtering and averaging in the homogeneous spanwise direction. Michelassi et al. concluded from comparison to DNS that the analysis was not able to fully reproduce the complex mechanism that triggers the transition to turbulence of the suction-side boundary layer and that LES predicted a transition point delayed by approximately 10 percent  $C_{ax}$ . They concluded that the resolution of the suction-side boundary layer was probably not sufficient to describe the fine-scale fluctuations and the turbulent spots. Moreover, some of the differences in reproducing the skin friction predicted by DNS were attributable partly to the SGS model and partly to the coarseness of the grid in the flow core, which does not allow resolution of all of the fine-scale activity convected by the wake. We have previously seen (Wu and Durbin, 2001) the importance of resolving the core flow to the success of transition modeling.

Medic and Sharma performed LES computations with various FST levels using a set of blades (Pak blades) with different pressure distributions designed for experimental and numerical studies (Sharma, 2011). The WALE SGS model of Nicoud and Ducros (1999) was used. The numerical scheme used was second order in both space and time. The computational domain was from  $X/C_x = -1$  to  $X/C_x = 2$  with a span of  $L_z/C_x = 0.2$ . The base Reynolds number (based on axial chord and exit velocity) was 100,000 and

other cases were also run with Reynolds numbers ranging from 30,000 to 150,000 and different pressure distributions. The goal was to assess the effect of Reynolds numbers and FSTs on the losses. The baseline grid consisted of approximately 35 million cells. The grid type was O–H, with the O-block containing  $768 \times 192 \times 128$  computational cells and the H-block containing  $640 \times 192 \times 128$  cells. As such, there were 768 cells around the blade and 128 cells in the spanwise direction. The grid resolution in nondimensional units, based on a time-averaged friction coefficient, at midchord on the suction side, was given as  $\Delta x^+ = 30$ ,  $\Delta y_{\min}^+ = 0.25$ , and,  $\Delta z^+ = 10$ . The wall shear-stress distribution was not provided. The grid resolution could be coarser than these values suggest depending on the shear stress value at the location. Two other grid resolutions were considered where the grid was separately doubled in the streamwise and spanwise directions for the purpose of better resolving the laminar-to-turbulent transition. The streamwise refinement resulted in 1536 cells around the airfoil and the spanwise resolution was doubled to 256 computational cells. Medic and Sharma concluded that their baseline grid was sufficiently refined. The chosen timestep,  $\Delta t^+$ , was approximately equal to 0.2 (for Reynolds number of 100,000 based on the exit velocity and axial chord.) and resulted in roughly 6,000 timesteps for one flow-through. Approximately 10 flow-through times were run and the statistics were compiled over the last five. This LES framework (and the WALE SGS model in particular), it is claimed, has shown ability to predict laminar-to-turbulent transition. The discrepancies that were observed prompted the authors to undertake DNS analyses for some of the cases where the differences were more significant. They did not show those cases but assert that the results were fairly “similar.” The most important takeaway was that the role of the SGS model on this grid resolution was mainly in the wake of the blade—that is the only region where  $\nu/v_{\text{sgs}}$  (kinematic viscosity/eddy kinematic viscosity) exceeded unity. The reason may be that the grid resolution is so fine that the subgrid model is not invoked.

Medic and Sharma state in their paper that the LES analyses performed captured the main trends and the fundamental characteristics of the airfoils they studied. They suggest that LES can be useful for assessing new airfoil designs and the flow physics involved. Another major conclusion from this work was that, compared to DNS as baseline, LES performs significantly better than RANS across the range of Reynolds numbers and FST levels attempted.

## Hybrid Models

Hybrid models are used to relieve the LES grid resolution requirements where the turbulent flow in the near wall region is treated using URANS. A review provided by Tyacke et al. (2013) describes the various methods of simulation and the associated grid requirements. There is an increasing benefit from using the hybrid schemes in terms of reduced grid requirements, compared to LES, as the Reynolds number increases. At Reynolds numbers over 100,000 there appear to be an increasing benefit and below very little benefit is accrued from using hybrid schemes. The Reynolds number of 100,000 is appropriate to the rear-stage LPTs and thus not much benefit is gained from using hybrid schemes for those LPT blades. The author is not aware of studies showing the effectiveness of hybrid models in prediction of transition.

## Summary and Conclusions

The proposition by Ham et al. that the “negative jet” structures are a precursor to bypass transition, leads to the conclusion that the resolution of these structures should be a goal of the simulations for low pressure turbines (LPTs) in the pretransition regime. The size of these structures is reported to be in the hundreds in the streamwise direction and about 60 in the spanwise direction as expressed in wall units. A set of grid size values of 10, 1, and 10 in wall units for  $\Delta x^+$ ,  $\Delta y_{min}^+$  and  $\Delta z^+$  may be surmised to be necessary from a survey of DNS of bypass transition and separation-induced transition in the literature. As for large eddy simulation (LES), Ham et al. were able to observe these negative jets, at least in the initial stages, in their LES simulation. Ham et al. used a fine grid of 45, 2, 17 based on maximum friction velocity. They suggest that a coarser grid may not be able to produce the turbulent spots, which appear downstream of the negative jets.

In their paper, Sagaut and Deck (2009) offer that the classically retained  $\Delta x^+$ ,  $\Delta y_{min}^+$ , and  $\Delta z^+$  in LES to capture the near wall turbulent structures are 50, 1, and 15. Many of the LES cases shown in Table I use mesh that is finer than Sagaut and Deck’s recommendations. Sarkar and Voke, simulating an LPT passage, use values of  $\Delta x^+=10$  to 45,  $\Delta y_{min}^+=1.3$ , and  $\Delta z^+=16$  on the blade and sizes of 40, 40, 16 in the free stream based on average wall shear stress. The latter should be more in line with second-order codes and finer than 60, 3, and 15, the values that were locally computed by Michelassi et al. and reported as under-resolving in their work.

As for grid topology, O–H grid appeared to be more successful than the H grid and multiblock used by Sandberg et al., which resulted in very successful simulations (not necessarily the cause). Smoothness appeared to be important as shown by Kalitzin et al., and finally the majority of modeling cases run with LPT blades have a span=0.15 to 0.2 of chord. Some of the cases, it appears, were run on too fine a grid in an attempt to improve results without much success, which suggests the underlying problem to be other than grid resolution and use of incompressible solver where inappropriate. We conclude in this review that the recommendation of Ham et al. of  $\Delta x^+=45$ ,  $\Delta y^+=2$ , and  $\Delta z^+=17$ , based on maximum shear velocity, may be a suitable choice and comport with the fully turbulent flow recommendation of Sagaut and Deck. This recommendation calls for larger values than those suitable for separation-induced flows, which are  $\Delta x^+=10-35$ ,  $\Delta y^+=1$ , and  $\Delta z^+=9$ . As such the grid may need to be further resolved in the event of a separation bubble. Note that the recommendation of Ham is conservative since the grid resolution criteria they recommend is based on the maximum shear velocity and for transitional flow the shear velocity, away from the leading edge, has a variation of about threefold.

LES by necessity needs a subgrid-scale (SGS) model; whether explicit or implicit. The quality of the results obtained from the LES simulation is thus subject to the modeling constraints as is Reynolds-averaged Navier-Stokes (RANS). This was shown to be an especially important issue in modeling of transition. Subgrid models as have been suggested should not overwhelm and dissipate the disturbances in the near wall region. For example, the often used Smagorinsky model is unable to achieve this and other models are suggested to ensure that the generated eddy viscosity approaches zero near the walls and is not over-dissipative. In some of the cases we have listed in Table I, the wall-adapting local eddy-viscosity (WALE) model of Nicoud and Ducros (1999) or high-pass filtered (HPF) Smagorinsky (Stolz et al., 2005) or other more suitable models are used. A model such as WALE appears to be easier to implement and not limited by the requirement of a homogeneous direction and as such may be a good candidate if the work is further extended to fully three-dimensional passage including endwalls. This model was used by Medic and Sharma and by Marty. Sayad and Moin showed success without any explicit SGS model. The SGS models used in all the works that were surveyed are noted in Table I.

Another necessary ingredient in LES or DNS simulations is a means of producing inlet turbulence. There are various means of inlet turbulence generation. Table I lists the methods used for inlet turbulence generation for the works that were surveyed herein. Some teams computed the inlet turbulence as an integral part of the simulation. These include placement of turbulence generating bars within the computational domain upstream of the flow as in the square jet modeling performed by MacAuliffe and Yaras. In general, turbulence generation at the inlet have used either precursor simulation methods or synthesis inlets. The periodic box method is a precursor method and uses a cyclic method to generate inlet turbulence. Turbulence is computed separately from the flow simulations and imposed at the inlet. Some of the teams used synthetic turbulence generation procedure such as the Smirnov (2001) method where some inlet length scale and turbulence energy level is specified and an inlet development length is required.

## References

- Ameri, Ali A.; Giel, Paul W.; and Flegel, Ashlie B., 2014: Simulation of VSPT Experimental Cascade Under High and Low Free-Stream Turbulence Conditions. AIAA 2014–3035.
- Chen, Li-Wei; Pichler, Richard; and Sandberg, Richard D., 2013: Compressible DNS of a Low Pressure Turbine Subjected to Inlet Disturbances. Workshop on Direct and Large-Eddy Simulation, Dresden, Germany.
- Coull, John D.; and Hodson, Howard P., 2011: Unsteady Boundary-Layer Transition in Low-Pressure Turbines. *J. Fluid Mech.*, vol. 681, pp. 370–410.
- Flegel, Ashlie B.; Giel, Paul W.; and Welch, Gerard E., 2014: Aerodynamic Effects of High Turbulence Intensity on a Variable-Speed Power-Turbine Blade With Large Incidence and Reynolds Number Variations, AIAA 2014–3933.
- Germano, Massimo, et al., 1991: A Dynamic Subgrid-Scale Eddy Viscosity Model. *Phys. Fluids A*, vol. 3, no. 7, pp. 1760–1765.
- Ham, F.E., et al., 2000: LES and Unsteady RANS of Boundary-Layer Transition Induced by Periodically Passing Wakes. Center for Turbulence Research Proceedings of the Summer Program 2000, pp. 249–260.
- Huai, Xiaoli; Joslin, Ronald D.; and Piomelli, Ugo, 1997: [Large-Eddy Simulation of Transition to Turbulence in Boundary Layers](#). *Theoret. Comput. Fluid Dynamics*, vol. 9, pp. 149–163.
- Jacobs, R.G.; and Durbin, P.A., 2001: Simulations of Bypass Transition. *J. Fluid Mech.*, vol. 428, pp. 185–212.
- Jagannathan, Shriram; Schwänen, Markus; and Duggleby, Andrew, 2012: Low Pressure Turbine Relaminarization Bubble Characterization Using Massively-Parallel Large Eddy Simulations. *J. Fluids Eng.*, vol. 134, no. 2, pp. 021102—021102–13.
- Kalitzin, Georgi; Wu, Xiaohua; and Durbin, Paul A., 2003: DNS of Fully Turbulent Flow in a LPT Passage. *Int. J. Heat Fluid Fl.*, vol. 24, pp. 636–644.
- Langtry, R.B.; Menter, F.R.; Likki, S.R.; Suzen, Y.B.; Huang, P.G.; and Völker, S.: A Correlation-Based Transition Model Using Local Variables Part II - Test Cases and Industrial Applications. *Trans. ASME, J. Turbomach.*, 128:423-434, July 2006
- Lilly, D.K., 1992: A Proposed Modification of the Germano Subgrid-Scale Closure Method. *Phys. Fluids A*, vol. 4, no. 3, pp. 633–635.
- Marty J., 2014: Numerical Investigations of Separation-Induced Transition on High-Lift Low-Pressure Turbine Using RANS and LES Methods. Proceedings of the Institution of Mechanical Engineers, Part A: Journal of Power and Energy, pp. 924–952.
- Marxen, Olaf, et al., 2009: Mechanisms for Spatial Steady Three-Dimensional Disturbance Growth in a Non-Parallel and Separating Boundary Layer. *J. Fluid Mech.*, vol. 634, pp. 165–189.
- Mayle, Robert Edward, 1991: Discussion: The 1991 IGTI Scholar Lecture: The Role of Laminar-Turbulent Transition in Gas Turbine Engines. *J. Turbomach.*, vol. 113, no. 4, pp. 509–536.

- McAuliffe, Brian R.; and Yaras, Metin I., 2009: Transition Mechanisms in Separation Bubbles Under Low- and Elevated-Freestream Turbulence. *J. Turbomach.*, vol. 132, no. 1, pp. 011004—011004–10.
- Medic, Gorazd; and Sharma, Om, 2012: Large-Eddy Simulation of Flow in a Low-Pressure Turbine Cascade. *ASME GT2012–68878*.
- Menter, F.R., et al., 2004: A Correlation-Based Transition Model Using Local Variables—Part I: Model Formulation. *J. Turbomach.*, vol. 128, no. 3, pp. 413–422.
- Michálek, Jan; Monaldi, Michelangelo; and Arts, Tony, 2012: Aerodynamic Performance of a Very High Lift Low Pressure Turbine Airfoil (T106C) at Low Reynolds and High Mach Number With Effect of Free Stream Turbulence Intensity. *J. Turbomach.*, vol. 134, no. 5.
- Michelassi, V., et al., 2003: Large-Eddy Simulation of Flow Around Low-Pressure Turbine Blade With Incoming Wakes. *AIAA J.*, vol. 41, no. 11.
- Nicoud, F.; and Ducros, F. 1999: Subgrid-Scale Stress Modelling Based on the Square of the Velocity Gradient Tensor. *Flow Turbul. Combust.*, vol. 62, no. 3, pp. 183–200.
- Rai, Man Mohan; and Moin, Parviz, 1991: Direct Simulations of Turbulent Flow Using Finite-Difference Schemes. *J. Comp. Physics*, vol. 96, no. 1, pp. 15–53.
- Sagaut, Pierre; and Deck, Sebastien, 2009: Large Eddy Simulation for Aerodynamics: Status and Perspectives. *Phil. Trans. R. Soc.*, vol. 367, vol. 2849–2860.
- Sandberg, R.D.; Pichler, R.; and Chen, L., 2012: Assessing the Sensitivity of Turbine Cascade Flow to Inflow Disturbances Using Direct Numerical Simulation. Presented at the 13th International Symposium for Unsteady Aerodynamics, Aeroacoustics and Aeroelasticity in Turbomachinery, Tokyo, Japan.
- Sarkar, S.; and Voke, Peter, R., 2005: Large-Eddy Simulation of Unsteady Surface Pressure Over a Low-Pressure Turbine Blade due to Interactions of Passing Wakes and Inflexional Boundary Layer. *J. Turbomach.*, vol. 128, pp. 221–231.
- Sayadi, T.; and Moin, P., 2010: A Comparative Study of Subgrid Scale Models for the Prediction of Transition in Turbulent Boundary Layers. *Center for Turbulence Research Annual Research Briefs*, 2010, p. 237.
- Sayadi, T.; and Moin, P., 2011: Predicting Natural Transition Using Large Eddy Simulation. *Center for Turbulence Research Annual Research Briefs*, p. 97.
- Sharma, Om, 2011: The Role of Physical and Numerical Experiments in the Development of High Performance Axial Flow Turbines. *Proceedings of CHT–15*, CHT–15–254.
- Smagorinsky, J., 1963: General Circulation Experiments With the Primitive Equations. *Mon. Wea. Rev.*, vol. 91, pp. 99–164.
- Smirnov, A.; Shi, S.; and Celik, I., 2001: Random Flow Generation Technique for Large Eddy Simulations and Particle-Dynamics Modeling. *J. Fluids Eng.*, vol. 123, no. 2, pp. 359–371.
- Stadtmüller, P., 2001: Investigation of Wake-Induced Transition on the LP Turbine Cascade T106A-EIZ. *DFG-Verbundprojekt Fo 136/11, Version 1.0*, University of the Armed Forces Munich, Germany, 2001.
- Stolz, S.; Schlatter, P.; and Kleiser, L., 2005: High-Pass Filtered Eddy-Viscosity Models for Large-Eddy Simulations of Transitional and Turbulent Flow. *Phys. Fluids*, vol. 17, no. 6, 065103.
- Tyacke, James, et al., 2013: Large Eddy Simulation for Turbines: Methodologies, Cost and Future Outlooks. *J. Turbomach.*, vol. 136, no. 6, pp. 061009–1—061009–13.
- Voke, Peter R., 1996: Subgrid-Scale Modelling at Low Mesh Reynolds Number. *Theoret. Comput. Fluid Dynamics*, vol. 8, no. 2, pp. 131–143.
- Walters, D. K.; and Cokljat, D.: 2008: A Three-Equation Eddy-Viscosity Model for Reynolds-Averaged Navier-Stokes Simulation of Transitional Flow, *ASME, J. Turbomach.*, vol. 130, 121401-01 to 121401-13
- Weiss, A.P.; and Fottner, L., 1995: The Influence of Load Distribution on Secondary Flow in Straight Turbine Cascades. *J. Turbomach.*, vol. 117, no. 1, pp. 133–141.

- Welch, Gerald E., 2010: Assessment of Aerodynamic Challenges of a Variable-Speed Power Turbine for Large Civil Tilt-Rotor Application. Proceedings of the 66th Annual Forum and Technology Display Forum (NASA/TM—2010-216758), Phoenix, AZ. <http://ntrs.nasa.gov>
- Wissink, J.G., 2003: DNS of Separating, Low Reynolds Number Flow in a Turbine Cascade With Incoming Wakes. *Int. J. Heat Fluid Fl.*, vol. 24, pp. 626–635.
- Wu, Xiaohua; and Durbin, Paul A., 2001: Evidence of Longitudinal Vortices Evolved From Distorted Wakes in a Turbine Passage. *J. Fluid Mech.*, vol. 446, pp. 199–228.
- Wu, Xiaohua, et al., 1999: Simulation of Boundary Layer Transition Induced by Periodically Passing Wakes. *J. Fluid Mech.*, vol. 398, pp. 109–154.
- Wu, Xiaohua; and Moin, Parviz, 2009: Direct Numerical Simulation of Turbulence in a Nominally Zero-Pressure-Gradient Flat-Plate Boundary Layer. *J. Fluid Mech.*, vol. 630, pp. 5–41.
- Yang, Zhiyin; and Voke Peter R., 2001: Large-Eddy Simulation of Boundary-Layer Separation and Transition at a Change of Surface Curvature. *J. Fluid Mech.*, vol. 439, pp. 305–333.

TABLE I.—SUMMARY OF SOME PUBLISHED CASES AND GRID AND MODELING REQUIREMENTS

Works	Order-space/time	Grid dimensions, $\Delta x+$ , $\Delta y+$ , $\Delta z+$	Inlet turbulence	Turbulence model	Comment/conclusion
McAuliffe and Yaras (2009)	Second order in time and space	$\Delta x+=24$ , $\Delta y_{\min}+=0.6$ , and $\Delta z+=24$ at high FSTI and $\Delta x+=19$ , $\Delta y_{\min}+=0.5$ , and $\Delta z+=19$ at low FSTI	Calm inlet or square jets (4×4 array) 5.1% Tu decays to 1.4%	DNS	Contoured passage, flat plate flow, separation study. Physical interpretation.
Wu et al. (2001)	Second in space, second in time, and Fourier in Z	25 and 57 M points (pt)	Imposed from auxiliary simulation	DNS	No FST. Wake only. Showed that resolving the path of the wake is crucial. $Re_2=2.7 \times 10^5$ .
Jacobs and Durbin (2001)	Second order	11.7, 1, and 6 71 M (2048×180×192)	Analytic	DNS	Data and physics of bypass transition with FST.
Rai and Moin (1991)	High order	$\Delta x+=28$ , $\Delta z+=10$	Analytic perturbations	DNS	Flat plate, bypass transition. Too coarse as $C_f$ rose too high.
Wu and Moin (2009)	Second order	$\Delta x+=5.5$ , $\Delta z+=1.5$ , 10 pt below $y+=5$ 210 M pt	Separate spectral computation Imposed intermittently	DNS	Zero-pressure gradient flat plate with periodic inlet turbulence convected in.
Kalitzin et al. (2003)	Second in space, second in time and Fourier in Z	28/2.3/19 86 M pt	Periodic box	DNS	Same as Wu, 505 more grid point added to FST cases. Under-resolution near the exit? H grid.
Sandberg et al. (2012)	Compact scheme (fourth) and spectral in spanwise	10, 2.5, and 11. ( $18 \times 10^6$ pt)	Digital filtering technique	DNS	Computed excellent pressure/wake profile, same case as Wissink (2003) (T106), $Re=60K$ .
Sayadi and Moin (2011)	Sixth order space and third order time	Fine: 960×160×64 $\Delta x+$ , $\Delta y+$ $\Delta z+$ Fine: 45.46 0.93 10.75 Coarse: 82.75 4.3 1.3	None	LES	Natural transition Dynamic Smagorinsky model or no model produced transition.
Yang and Voke (2001)		$\Delta x+ = 10$ to 30.5, while $\Delta z+ = 9$ and at the wall $\Delta y+ = 1$	None	LES	Separation induced by leading-edge curvature. Dynamic Smagorinsky.
Ham et al. (2000)	Second order in space and time	$\Delta x+ = 45$ , $\Delta y+ = 2$ , $\Delta z+ = 17$ , based on maximum friction velocity	Same as Wu, et al. (1999)	LES, Dynamic Smagorinsky	Transition length too short compared to DNS of Jacobs and Durbin. Observed under-resolution in streamwise.
Michelassi et al. (2003)		60, 3, and 15 computed locally	Wu and Durbin wake	LES	Wake inlet, under-resolved, incompressible. Transition too late (10% $C_{ax}$ ).
Medic and Sharma (2012)	Second order in space and time	$\Delta x+=30$ , $\Delta y_{\min}+=0.25$ , and $\Delta z+=10$ , 35 million, $Lz/C_{ax}=0.2$	Periodic box	LES/WALE	Unable to accurately compute losses but results are better than RANS ( $k-\omega$ ),
Sarkar and Voke (2005)	Second order in space and time	On the blade (10 to 45, 1.3, and 16), computed locally, $Lz=0.1x$ chord Midpassage (40, 40, 16) $23.6 \times 10^6$ pt	Wake data extracted from precursor simulations of flow past a thin cylinder	LES/LRN Smagorinsky	Compared to Wu and Durbin (2001).
Marty (2014) (ONERA)	Third order in space and second in time	$\Delta x+=20$ , $\Delta y+=0.05$ , $\Delta z+=5$ , for $Re_2=80,000$ , $\Delta x+=35$ , $\Delta y+=0.08$ , $\Delta z+=20$ , for $Re_2=140,000$ $27 \times 10^6$ pt	Model of Smirnov et al.	LES/WALE	T106C, Michálek et al. (2012) Shows LES and RANS with transition model to give similar results for losses but both approximately 50% in error.
Jagannathan et al. (2012)	Spectral	$y+ \ll 1$ $2.5 \times 10^6$ pt	Isotropic+ Bars at $-5.0 C_x$ upstream	LES-HPF Smagorinsky (to eliminate eddy viscosity in laminar region)	Incompressible. 4% short of separation and profiles do not agree well in turbulent regime. Data Schobeiri et al. (2008).





