Image Navigation and Registration (INR) Performance Assessment Tool Set (IPATS) for the GOES-R Advanced Baseline Imager and Geostationary Lightning Mapper

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Topics

• Introduction
• Algorithmic approach
• Implementation
• Preliminary testing results
• Summary
Introduction

• In March 2014 the GOES-R flight project initiated an effort to develop tools for independent evaluation of on-orbit Image Navigation and Registration (INR) performance of the Advanced Baseline Imager (ABI) and Geostationary Lightning Mapper (GLM)
  – *The Product Monitor (PM), developed by the ground project, provides heritage capability for INR performance assessment*
  – *Program desired an independently developed capability for INR performance assessment using different techniques for risk reduction*

• The INR Performance Assessment Tool Set (IPATS) has been developed to:
  – *Measure INR performance characteristics*
  – *Generate image-level and multi-image-level statistics*
  – *Provide data visualization capability*
  – *Archive results*
## ABI and GLM image characteristics

**Simulated ABI image of hurricane Katrina**  
Credit: University of Wisconsin/Cooperative Institute for Meteorological Satellite Studies

**GLM field of view (thin line for GOES West, thick line for GOES East)**  
http://www.goes-r.gov/products/ATBDs/baseline/Lightning_v2.0_no_color.pdf

<table>
<thead>
<tr>
<th>Spectral</th>
<th>ABI – Level 1B</th>
<th>GLM – Level 1β</th>
</tr>
</thead>
<tbody>
<tr>
<td>• 16 bands, 0.4 μm to 14 μm</td>
<td>• Single band (777 nm)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Spatial Resolution</th>
<th>ABI – Level 1B</th>
<th>GLM – Level 1β</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Fixed Grid (FG) coordinate system with sample spacing of 14, 28, or 56 μrad (0.5, 1, or 2 km at nadir)</td>
<td>• 8 km at nadir, 14 km at edge of field</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Coverage</th>
<th>ABI – Level 1B</th>
<th>GLM – Level 1β</th>
</tr>
</thead>
</table>
| • Full Disk (FD): 17.4 deg diameter centered at nadir  
• CONUS: Rectangular, 5000 km EW x 3000 km NS  
• Mesoscale: Rectangular, 1000 km EW x 1000 km NS | • Near full disk | |

<table>
<thead>
<tr>
<th>Temporal</th>
<th>ABI – Level 1B</th>
<th>GLM – Level 1β</th>
</tr>
</thead>
<tbody>
<tr>
<td>• FD: 5 or 15 min; CONUS: 5 min; Mesoscale: 30 sec</td>
<td>• 150 sec</td>
<td></td>
</tr>
</tbody>
</table>
INR metrics of interest

- **Navigation (NAV) error (ABI & GLM)**
  - *Difference between location of pixel in data product and true location*

- **Frame-to-frame registration (FFR) error (ABI)**
  - *Relative navigation error of corresponding pixels of same band in consecutive images*

- **Within-frame registration (WIFR) error (ABI)**
  - *Difference between radial separation of two pixels on the FG and their true angular separation*

- **Swath-to-swath registration (SSR) error (ABI)**
  - *Relative navigation error of two neighboring pixels on opposite sides of image swath boundary*

- **Channel-to-channel registration (CCR) error (ABI)**
  - *Relative navigation error of corresponding pixels of different bands in the same frame*

*Key metric for any type of error is “3-sigma error”, 99.73rd percentile of distribution of error magnitudes over a 24 hour data collection period.*
Concept for INR metric measurement using image registration techniques

- For NAV, shifted sub-image is cropped from ABI or GLM image, stationary sub-image is truth map:
  - High contrast Landsat 8 derived chip projected to FG for ABI NAV
  - ABI image for GLM NAV, with GLM background image resampled to FG
- Both sub-images cropped from pair of images for ABI FFR, CCR, SSR
- ABI WIFR is derived from pair of ABI measured NAV errors

Common concept for measuring all metrics except ABI WIFR
IPATs algorithm design is modular

- Algorithm for processing each metric is decomposed into chain of algorithmic steps or modules
- Alternative sub-algorithms are selected for some modules, as appropriate, to provide flexibility and support performance trades
- Isolation of sub-algorithms common to all metrics results in a simpler, more efficient implementation by eliminating redundancies
- IPATS user selects the appropriate processing path that will generate each metric and the preferred sub-algorithms to optimize performance

Modular algorithm design provides flexibility to select optimum chain of algorithm modules for each metric within unified framework
Image preprocessing

• Sub-image pair identification
  – For ABI NAV, IPATS cycles through a library of Landsat chips
  – For other ABI metrics and GLM NAV, a predetermined array of evaluation windows is provided as an input to IPATS

• Sub-image resampling
  – Images ingested by IPATS may have different resolutions and must be resampled to common resolution prior to correlation

• Edge enhancement
  – Sobel filtering, Roberts filtering

• Sub-image pair screening
  – Since the correlation step is computationally expensive, judicious screening reduces IPATS execution time while maintaining tool performance

ABI pre-launch rehearsal proxy images provided by GOES-R ground project using AIPS test data
Image Pair Correlation and Post-Processing

• Two similarity metrics are available for image pair correlation
  – *Pearson correlation coefficient (CC)*
  – *Normalized mutual information (NMI)*

• Post correlation processing
  – *Refinement of CC or NMI peak location in correlation plane*
    • Centroiding
    • Maximum of parabola passing through peak value and two nearest neighbors
  – *Statistics of INR errors within an image* – mean, median, standard deviation, median absolute deviation from median (MAD), absolute value of mean + 3 times standard deviation (approximation of 99.73rd percentile)
  – *Statistics of INR errors over all images within 24 hour data collection period* – same statistics as for single image
Measurement uncertainty estimation

- INR errors measured by IPATS include both the intrinsic INR error of interest as well as measurement error associated with limitations of the IPATS algorithm and processing.

- IPATS applies an analytical estimate of measurement uncertainty to filter out measured errors most likely to be dominated by measurement error, rather than intrinsic INR error, prior to generation of error statistics.

Measurement error broadens the distribution of measured INR error magnitudes, causing inaccurate inference of 99.73rd percentile if not minimized.
IPATS implementation: IPSE

Image Pair Selector and Evaluator

- Coded in c++
- Catalogs specified input images
- Identifies pairs of images to be calculated
  - For each pair, identify regions within the overlapping portion to evaluate
- Perform evaluation of each identified region of each pair
- Save results into Image Pair Registration Record (IPRR) database
  - Analyst guided comparisons of IPRRs using ODAT
IPATS Output Data Analysis Tool (ODAT)
Viewer for IPSE results and performs data analysis

• Provides end-user with a graphical user interface (GUI) to analyze IPSE results
  – Converts complex IPSE SQL database into usable tabular form for end-users
  – Allows end-users to perform complex SQL queries without knowing SQL (visually string queries together)
  – Converts internal SQL database into a Python Pandas DataFrame for analysis
  – Performs data analysis on registration error using Pandas DataFrame objects and NumPy

• Development
  – Python 2.7+
  – SciPy (www.scipy.org)
    • NumPy and Pandas packages used for data analysis
    • Matplotlib package used for plotting
IPATS Output Data Analysis Tool (ODAT)

**IPRR Viewer**

- Reads SQL database and outputs IPSE results into a table
- Easy-to-use interface that allows end-user to export the results to a CSV and generate statistics and/or plots
Multispectral Landsat Chip Library

Overview

- Library created to assess NAV of spectrally similar ABI bands to Landsat 8 “Truth” chips
  - Remap spectral chips to match ABI imagery for correlation

<table>
<thead>
<tr>
<th>GOES-R ABI Band</th>
<th>GOES-R ABI Wavelength Range (µm)</th>
<th>GOES-R ABI Nominal IGFOV (km)</th>
<th>Landsat 8 Band</th>
<th>Landsat 8 Wavelength Range (µm)</th>
<th>Landsat 8 Nominal GSD (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.45-0.49</td>
<td>1</td>
<td>2 - Blue</td>
<td>0.45-0.51</td>
<td>30</td>
</tr>
<tr>
<td>2</td>
<td>0.59-0.69</td>
<td>0.5</td>
<td>4 - Red</td>
<td>0.64-0.67</td>
<td>30</td>
</tr>
<tr>
<td>3</td>
<td>0.846-0.885</td>
<td>1</td>
<td>5 - NIR</td>
<td>0.85-0.88</td>
<td>30</td>
</tr>
<tr>
<td>4***</td>
<td>1.371-1.386</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>1.58-1.64</td>
<td>1</td>
<td>6 - SWIR</td>
<td>1.57-1.65</td>
<td>30</td>
</tr>
<tr>
<td>6</td>
<td>2.225-2.275</td>
<td>2</td>
<td>7 - SWIR</td>
<td>2.11-2.29</td>
<td>30</td>
</tr>
<tr>
<td>7</td>
<td>3.80-4.00</td>
<td>2</td>
<td>7 – SWIR</td>
<td>2.11-2.29</td>
<td>30</td>
</tr>
<tr>
<td>8***</td>
<td>5.77-6.6</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9***</td>
<td>6.75-7.15</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10***</td>
<td>7.24-7.44</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>8.3-8.7</td>
<td>2</td>
<td>10 – TIR</td>
<td>10.60-11.19</td>
<td>100 (30)</td>
</tr>
<tr>
<td>12***</td>
<td>9.42-9.8</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>10.1-10.6</td>
<td>2</td>
<td>10 – TIR</td>
<td>10.60-11.19</td>
<td>100 (30)</td>
</tr>
<tr>
<td>14</td>
<td>10.8-11.6</td>
<td>2</td>
<td>10 - TIR</td>
<td>10.60-11.19</td>
<td>100 (30)</td>
</tr>
<tr>
<td>15</td>
<td>11.8-12.8</td>
<td>2</td>
<td>11 - TIR</td>
<td>11.50-12.51</td>
<td>100 (30)</td>
</tr>
<tr>
<td>16</td>
<td>13.0-13.6</td>
<td>2</td>
<td>11 – TIR</td>
<td>11.50-12.51</td>
<td>100 (30)</td>
</tr>
</tbody>
</table>

***Atmospheric band that does not see to the ground
Multispectral Landsat Chip Library

ABI Point Database

- 530 GCP features

Source of base map: ArcGIS online map service: http://www.arcgis.com/home/item.html?id=10df2279f9684e4a9f6a7f08febac2a9

Latitude, Longitude, Landsat Path/Row, GCP ID and Landmark
Specified in CSV file
Multispectral Landsat Chip Library

Landsat Scene Database

- 456 Landsat-8 scenes

Multi-seasonal scenes per ground control point (GCP), projected to fixed grid for each satellite location, and stored in directories for IPATS ingest

Source of base map: ArcGIS online map service: http://www.arcgis.com/home/item.html?id=10df2279f9684e4a9f6a7f08febac2a9
Multispectral Landsat Chip Library

Examples of FG Calculations for Satellite Positions

Satellite Position 137° W

Satellite Position 89.5° W

Satellite Position 75° W

Landsat data available from U.S. Geological Survey.

ABI pre-launch rehearsal proxy images provided by GOES-R ground project using AIPS test data
Multispectral Landsat Chip Library

*Chip Output*

- Save each chip as a flat binary image and ENVI header

Data available from U.S. Geological Survey
Preliminary IPATS Test Results

Misregistrations

- Focus of initial IPATS science testing has been demonstrating expected behavior for NAV tests – comparing an ABI chip to the Landsat database
- The factor by which the ABI chip is upsampled is called the sub pixel factor (SPF)
- Raw misregistrations will change as a function of SPF due to resolution changes, but parabolic refined misregistrations should stay relatively constant

Median misregistration computed using parabolic refinement of CC peak location in correlation plane is largely independent of SPF, as expected
Preliminary IPATS Test Results

Dispersion of Misregistrations

• Misregistration dispersion is measured as the median absolute deviation (MAD) of misregistrations and compared to MU
• Since MU is far lower than parabolically refined MAD misregistrations, MAD is likely measuring true INR dispersion
• Large Landsat chips and image evaluation regions contribute to small MU

Misregistration dispersion as measured by MAD is largely independent of SPF, as expected
Summary

- IPATS nearing end of development phase
- Extensive test and optimization is in progress
- Initial performance results are as expected
- IPATS has been designed and implemented to meet its objectives:
  - *Capability to generate all INR metrics of interest, including individual measurements and statistics*
  - *User selectable algorithmic processing path and alternative component algorithms for maximum flexibility within unified architecture*
  - *Capability to estimate measurement uncertainty and filter out INR errors most contaminated by measurement error*
  - *Data manipulation, filtering and visualization capabilities*
  - *Output database capture of all results, including information necessary to reproduce results*

*IPATS shows great promise as a highly effective tool for GOES-R ABI and GLM INR performance assessment*
Backup Slides
IPSE Backup
Image pair identification (1 of 2)

Automatic selection of image pairs for each evaluation type

- **NAV**
  - *All input images paired against one or more images from Multispectral Landsat Chip Library*

- **FFR**
  - *Images sorted by nominal start of collection time (from filename)*
  - *For each input, find next image of same type, satellite position, band, …*
  - *For Mesoscale images, ensure minimum amount of geographic overlap*

- **CCR**
  - *Images grouped by nominal start of collection time*
  - *IPSE identifies image pairs from each group matching user specified band pairs and desired edge enhancement for that band pair*
    - *e.g., Compare Band 1 to Band 2 using Sobel edge enhancement*
Image pair identification (2 of 2)

Automatic selection of image pairs for each evaluation type

- **SSR**
  - *For each Mesoscale image, find*
    - another like Mesoscale image (e.g., same band, Mesoscale type)
    - collected 30 seconds ± user specified delta after initial image
    - with minimum geographic overlap
  - *Define evaluation locations contained within overlap region*
  - *Filter locations such that location is in different swath in each image*

![Diagram showing image pair identification process](image)

- Exclude regions not entirely in a single swath
- Exclude regions in the same swath in both images
IPSE common evaluator (1 of 2)

One evaluation module performs all evaluation types

- Single image cropping, resampling, comparison implementation minimizes inadvertent differences in evaluation
- Determine portion of image necessary to evaluate specified region and pad region based on evaluation parameters
  - *For cross correlator based on maximum anticipated registration error*
  - *For peak refinement based on refinement kernel size*
  - *To avoid edge artifacts in resampling or local averaging*
  - *To avoid edge artifacts in edge enhancement*
IPSE common evaluator (2 of 2)

One evaluation module performs all evaluation types

Load needed pixels from input files

Resample to target resolution

Apply edge enhancement & crop (if specified)

Calculate IPC and MU on common area

Calculate cross-correlation
Putting NAV into the common evaluator

Adjust evaluation region down so padding can bring it back up

• Common evaluator assumes unlimited image data surrounding evaluation region
• Except that bounds of Landsat chip determines bounds of evaluation region
• Inset region based on necessary padding and calculate usable interior evaluation size
• e.g., 28 pixel wide 4× resolution Landsat chip results in a maximum 3 pixel wide evaluation region in the ABI image

• Common evaluator builds 18 pixel region from Landsat chip, 3 pixels from ABI, resamples and processes images to target space, performs correlation as in all other evaluation types
Image Pair Comparison module

*IPSE image quality, collection quality based filtering*

- Before images are processed through cross-correlator (computationally expensive), IPSE performs a quality check on the pair
- If images are sufficiently different (high measurement uncertainty) or if solar zenith angle not suitable for high quality collection, IPSE will skip evaluation
- Fact of image pair and location selection logged to IPRR
ODAT Backup
IPATS Output Data Analysis Tool (ODAT)

Data Analysis

- Data is internally stored in Python Pandas DataFrame objects
- Pandas group by functionality is used to group data by combinations of the input parameters such as image date and band
- Stock statistics provided by Pandas are used such as the min, max, mean, median, and standard deviation of the registration error
- Custom statistics are provided by extending the Pandas DataFrame object with custom code
- Several outlier rejection methods are implemented by extending the DataFrame objects
MLCL Backup
Multispectral Landsat Chip Library

• Input CSV File
  – Contains Feature Lon/Lat (GCP), Landsat Scene Path/Row And Unique ID

• Cycles Through Each GCP
  1. Subset the corresponding full Landsat scene into a 4200x4200 image centered on the GCP
  2. Compute each image’s longitude, latitude and height and convert to ortho-rectified ABI fixed grid (FG) coordinates for the three GOES-R satellite positions (137°W, 89.5°W and 75°W)
  3. Subset the result into a rectangle with no “background” pixel values
    • Convert digital numbers (DN) into at-satellite radiances
    • Average into a pixel ground sample distance (GSD) 12 times finer than the corresponding ABI spectral band
  4. Subset the output further to make certain that the chip is perfectly nested within the native ABI pixel grid

Data available from U.S. Geological Survey
Computation of fixed grid angles

1. Convert Landsat UTM to geographic (lat/long) coordinates

2. Extract height-above-ellipsoid (HAE) topography from SRTM 90m dataset

3. Convert geographic to Earth-Centered, Earth-Fixed (ECEF) coordinates

4. Convert ECEF to fixed grid coordinates (angles)

GOES-R Ground Segment Product Users’ Guide, DCN 7035538 R C.1, Figure 5.1.2.1
http://www.goes-r.gov/resources/docs.html
Multispectral Landsat Chip Library
Calculating FG Angles with Terrain Correction

Non-Terrain Calculation

Aerospace Calculation Using SRTM Geoid Heights

Data available from U.S. Geological Survey
Multispectral Landsat Chip Library

**CSV Output**

- Populate an output CSV Database with Metadata for IPATS
  - *Separate record for every spectral chip*

<table>
<thead>
<tr>
<th>NAME_S16</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ORIGLAT_R</td>
<td>Input Latitude of GCP</td>
</tr>
<tr>
<td>ORIGLON_R</td>
<td>Input Longitude of GCP</td>
</tr>
<tr>
<td>ID_R</td>
<td>ID in the CSV just made as 1000 + 1 for each record in the input</td>
</tr>
<tr>
<td>NEWLAT_R</td>
<td>Latitude of GCP (this will be updated if the GCP is Moved)</td>
</tr>
<tr>
<td>NEWLON_R</td>
<td>Longitude of GCP (this will be updated if the GCP is Moved)</td>
</tr>
<tr>
<td>FILENAME_S128</td>
<td>The Full output Landsat Bandname i.e LC80250402014136LGN00_B1_&lt;satPos&gt;_.&lt;id&gt;.dat (one for each of the 3 sat positions)</td>
</tr>
<tr>
<td>ROWS_U</td>
<td>Rows in the output</td>
</tr>
<tr>
<td>COLS_U</td>
<td>Cols in the output</td>
</tr>
<tr>
<td>BANDS_U</td>
<td>Bands (this will always be 1 now!)</td>
</tr>
<tr>
<td>PROJLON_R</td>
<td>The satellite longitude that this was mapped for (-137, -89.5 or -75 deg)</td>
</tr>
<tr>
<td>PROJLAT_R</td>
<td>The satellite latitude that this was mapped for (Currently always 0)</td>
</tr>
<tr>
<td>BANDNUM_U</td>
<td>Landsat Band Number</td>
</tr>
<tr>
<td>ANGGSD_R</td>
<td>Angular GSD (radians) ((factors in the RSMULT))</td>
</tr>
<tr>
<td>RSMULT_R</td>
<td>Resolution Multiplication Factor (decided that it's always 12)</td>
</tr>
<tr>
<td>MIN_X_R</td>
<td>Minimum X Fixed Grid Angle Value (radians) - Center of Lower Left Pixel</td>
</tr>
<tr>
<td>MAX_X_R</td>
<td>Maximum X Fixed Grid Angle Value - Center of Upper Right Pixel</td>
</tr>
<tr>
<td>MIN_Y_R</td>
<td>Minimum Y Fixed Grid Angle Value - Center of Lower Left Pixel</td>
</tr>
<tr>
<td>MAX_Y_R</td>
<td>Maximum Y Fixed Grid Angle Value - Center of Upper Right Pixel</td>
</tr>
<tr>
<td>WAVELENGTH_R</td>
<td>Landsat wavelength</td>
</tr>
<tr>
<td>WAVEUNITS_S48</td>
<td>Micrometers</td>
</tr>
<tr>
<td>CIRRUS_R</td>
<td>% of Pixels that are marked as Cirrus</td>
</tr>
<tr>
<td>CLOUD_R</td>
<td>% of Pixels that are marked as Clouds</td>
</tr>
<tr>
<td>SNOW_R</td>
<td>% of Pixels that are marked as Snow</td>
</tr>
<tr>
<td>DATE_D</td>
<td>YYY-MM-DD</td>
</tr>
<tr>
<td>COMMENTS_S1024</td>
<td>Comments including if had to move the Original GCP</td>
</tr>
</tbody>
</table>

| NAME_S16 | ORIGLAT_R | ORIGLON_R | GAP_ID_U | NEWLAT_R | NEWLON_R | FILENAME_S128 | ROWS_U | COLS_U | BANDS_U | PROJLON_R | PROJLAT_R | BANDNUM_U | ANGGSD_R | RSMULT_R | MIN_X_R | MAX_X_R | MIN_Y_R | MAX_Y_R | WAVELENGTH_R | WAVEUNITS_S48 | CIRRUS_R | CLOUD_R | SNOW_R | DATE_D | COMMENTS_S1024 |
|----------|------------|-----------|----------|----------|----------|---------------|--------|--------|---------|------------|-----------|-----------|----------|---------|---------|--------|--------|---------|--------|--------------|-------------|---------|--------|--------|--------|----------------|
| P39837   | 33.207524  | -115.600484 | 1003    | 33.20738143 | -115.6006501 | LC80250402014136LGN00_B1_<satPos>_.<id>.dat | 1164   | 1176   | 1       | -75       | 0         | 4         | 1.16670E-06 | 12     | -0.99286417 | -0.99286417 | 0.00020583 | 0.00020583 | 0.655   | Micrometers | 0.0001   | 0.368   | 0.2024 | 3/15/2014 | 36 |
Computation of fixed grid angles

• Convert geographic to Earth-Centered, Earth-Fixed (ECEF) coordinates
  \[
  X = (N(\phi) + H) \cdot \cos(\phi) \cdot \cos(\lambda)
  \]
  \[
  Y = (N(\phi) + H) \cdot \cos(\phi) \cdot \sin(\lambda)
  \]
  \[
  Z = (N(\phi) \cdot (1 - e^2) + H) \cdot \sin(\phi)
  \]

• Convert ECEF to fixed grid coordinates (angles) for spacecraft position
  \( \mathbf{p} = (p_x, p_y, p_z) \)
  \[
  \mathbf{w} = (p_x, p_y, p_z)/|\mathbf{p}|
  \]
  \[
  \mathbf{v} = (0,0,1)
  \]
  \[
  \mathbf{u} = \mathbf{v} \times \mathbf{w} = (-p_y, p_x, 0)/|\mathbf{p}|
  \]

• Define
  \[
  \mathbf{A} = \begin{bmatrix}
  u_1 & v_1 & w_1 \\
  u_2 & v_2 & w_2 \\
  u_3 & v_3 & w_3
  \end{bmatrix}, \quad \mathbf{B} = \mathbf{A}^T
  \]

• Per pixel,
  \[
  \mathbf{u}_1 = \begin{bmatrix} X - p_1 \\ Y - p_2 \\ Z - p_3 \end{bmatrix}
  \]
  \[
  \mathbf{v}_1 = \begin{bmatrix} \sin \theta \\ \sin \varphi \cos \theta \\ -\cos \varphi \cos \theta \end{bmatrix}
  \]
  \[
  \mathbf{v}_1 = \mathbf{B} \mathbf{u}_1
  \]

• \((\theta, \varphi)\) are the fixed grid coordinate for pixel at \((X, Y)\) in ECEF coordinates