OD Parameters Generated by ASW Solutions

- **Solved for: State parameters**
  - Six parameters needed to determine 3-d state fully
  - Cartesian: three position and three velocity parameters in orthogonal system
  - Element: six orbital elements that describe the geometry of the orbit

- **Solved for: Non-conservative force parameters**
  - Ballistic coefficient ($C_D A/m$); describes vulnerability of spacecraft state to atmospheric drag
  - Solar radiation pressure (SRP) coefficient ($C_R A/m$); describes vulnerability of spacecraft state to visible light momentum from sun

- **Considered: ballistic coefficient and SRP consider parameter**
  - Not solved for but “considered” as part of the solution
  - Derived from information outside of the OD itself

- **Covariance matrix includes variances/covariances for all solved-for parameters, with potential alteration by consider parameters**
### Covariance Matrix Construction: Symbolic Example

- Three estimated parameters (a, b, and c)
- Variances of each along diagonal
- Off-diagonal terms the product of two standard deviations and the correlation coefficient ($\rho$); matrix is symmetric

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>…</th>
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</thead>
<tbody>
<tr>
<td>a</td>
<td>$\sigma_a^2$</td>
<td>$\rho_{ab}\sigma_a\sigma_b$</td>
<td>$\rho_{ac}\sigma_a\sigma_c$</td>
<td>…</td>
</tr>
<tr>
<td>b</td>
<td>$\rho_{ab}\sigma_a\sigma_b$</td>
<td>$\sigma_b^2$</td>
<td>$\rho_{bc}\sigma_a\sigma_c$</td>
<td>…</td>
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<tr>
<td>c</td>
<td>$\rho_{ac}\sigma_a\sigma_c$</td>
<td>$\rho_{bc}\sigma_a\sigma_c$</td>
<td>$\sigma_c^2$</td>
<td>…</td>
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</tbody>
</table>
Example Covariance from CDM

- 8 x 8 matrix typical of most ASW updates
  - Some orbit regimes not suited to solution for both drag and SRP; these covariances 7 x 7

- Mix of different units often creates poorly conditioned matrices
  - Condition number of matrix at right is 9.8E+11—terrible!

- Often better numerically (and more intuitive) to separate matrix into sections

- First 3 x 3 portion (amber) is position covariance—often considered separately

\[
\begin{array}{cccccccc}
U & V & W & Udot & Vdot & Wdot & B & AGOM \\
(m) & (m) & (m) & (m/s) & (m/s) & (m/s) & (m^2/kg) & (m^2/kg) \\
6.84E+01 & -2.73E+02 & 6.38E+00 & 2.76E-01 & -7.14E-02 & 8.75E-03 & -3.83E-02 & -3.83E-02 \\
-2.73E+02 & 1.10E+05 & 3.23E+01 & -1.17E+02 & -8.99E-02 & 2.51E-02 & -1.28E-01 & -1.28E-01 \\
6.38E+00 & 3.23E+01 & 4.47E+00 & -3.26E-02 & -6.83E-03 & 1.81E-03 & -3.73E-03 & -3.73E-03 \\
Udot & -1.17E+02 & -3.26E-02 & 1.24E-01 & -2.47E-05 & 1.46E-04 & -1.46E-04 \\
Wdot & 8.75E-03 & 2.51E-02 & 1.81E-03 & -2.47E-05 & -9.39E-06 & 2.06E-05 & -4.39E-06 \\
B & -5.07E-03 & 1.30E+00 & 4.34E-05 & -1.38E-03 & 7.97E-07 & 7.26E-07 & 1.64E-05 & -6.28E-07 \\
AGOM & -3.83E-02 & -1.28E-01 & -3.73E-03 & 1.46E-04 & 4.10E-05 & -4.39E-06 & -6.28E-07 & 2.31E-05 \\
\end{array}
\]
Position Covariance Ellipse

- **Position covariance** defines an “error ellipsoid”
  - Placed at predicted satellite position
  - Square root of variance in each direction defines each semi-major axis (UVW system used here)
  - Off-diagonal terms rotate the ellipse from the nominal position shown

- **Ellipse of a certain “sigma” value contains a given percentage of the expected data points**
  - 1-σ: 19.9%
  - 2-σ: 73.9%
  - 3-σ: 97.1%
  - Note how much lower these are than the univariate normal percentage points
Batch Epoch Covariance Generation (1 of 2)

• **Batch least-squares update (ASW method) uses the following minimization equation**
  
  \[ \text{dx} = (A^TWA)^{-1}A^TWb \]

  • dx is the vector of corrections to the state estimate
  • A is the time-enabled partial derivative matrix, used to map the residuals into state-space
  • W is the “weighting” matrix that provides relative weights of observation quality (usually \(1/\sigma\), where \(\sigma\) is the standard deviation generated by the sensor calibration process)
  • b is the vector of residuals (observations – predictions from existing state estimate)

• **Covariance is the collected term (A^TWA)^{-1}**
  
  – A the product of two partial derivative matrices:
  
  \[ A = \frac{\partial (\text{obs})}{\partial X_0} = \frac{\partial (\text{obs})}{\partial X} \frac{\partial X}{\partial X_0} \]

  • First term: partial derivatives of observations with respect to state at obs time
  • Second term: partial derivatives of state at obs time with respect to epoch state
Batch Epoch Covariance Generation (2 of 2)

• Formulated this way, this covariance matrix is called an *a priori* covariance
  – A does not contain actual residuals, only transformational partial derivatives
  – So \((A^TWA)^{-1}\) is a function only of the amount of tracking, times of tracks, and sensor calibration relative weights among those tracks
    • Not a function of the actual residuals from the correction

• Limitations of *a priori* covariance
  – Does not account well for unmodeled errors, such as transient atmospheric density prediction errors
    • Because not examining actual fit residuals
  – W-matrix only as good as sensor calibration process
    • Principal weakness of present process, but expected to be improved eventually with JSpOC Mission System (JMS) upgrades
ASW Covariance Propagation

- Covariance in VCM is virginal (unaltered) covariance
- When propagating VCM covariance, the propagator
  - Scales the covariance by the weighted RMS if it is greater than unity
    - \( C^* = C \times \text{WRMS}^2 \)
    - Was an early attempt to improve covariance realism; not clear this is still a good idea
  - Applies the consider parameter to the ballistic coefficient variance
    - \( C^*(7,7) = C^*(7,7) + Cpd^2 \)
    - More later on how this value is determined
  - May apply a consider parameter to the solar radiation pressure variance
    - \( C^*(9,9) = C^*(9,9) + Cps^2 \)
    - Presently not used (Cps set to 0)
  - Propagates the altered covariance using linearized dynamics
    - \( \Phi \times C^{**} \times \Phi^T \)
    - Converts propagated matrix from equinoctial to Cartesian coordinates
Altered Covariance Positions

- Ballistic coefficient consider parameter (DCP) applied to ballistic coefficient variance (orange)

- If used, solar radiation pressure consider parameter applied to solar radiation pressure variance (purple)

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<th>W</th>
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<th>Wdot</th>
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<th>AGOM</th>
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<td></td>
<td>(m)</td>
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<td>(m)</td>
<td>(m/s)</td>
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Dynamic Consider Parameter (DCP)

• **Specifies global error in the atmospheric density forecast**
  – Parameterizes percent RMS error in terms of
    • Satellite height (perigee altitude)
    • Geomagnetic activity ($a_p$ and Dst)
  – Density forecast error combination of solar/geomagnetic indices and DCA
    • Directly compared numerous forecast densities to actual density
    • Discretized heights from 200 km to 1000 km averaged over lat / lon
    • Found most variation parameterizable via $a_p$ conditions (versus $F_{10}$)
    • Functions optimized for 3-day predictions—this is the tuning point!

• **Determines satellite-specific frontal area variation in prediction**
  – Quantifies ballistic coefficient RMS error through satellite histories
    • Looks back in time up to a year in most cases (upfront preprocessing)
    • Ascertains error / target for 3-day predictions (accounting for time-lags)

• **Combine the two uncertainty components to obtain DCP value**
  – Additive in variance sense as the root sum of squares
B Consider Parameter Values

![Graph showing dRho STD and Dst with different parameter values.

- Dst (Storms) <= -75
- 25 < ap < 50
- 10 < ap <= 25
- 0 < ap <= 10

Current 12% for Altitude (km) range 200 to 1000.]
DCP Components RMS Uncertainty

DCP for Day 14203

- Density Forecast Error
- Frontal Area Variation
- DCP

Percentile of Drag Satellites

0 10 20 30 40 50 60 70 80 90 100