Mission Operations Working Group

International Earth Science Constellation Mission Operations Working Group
April 13-15, 2016

Earth Observation System Flight Dynamics System
Covariance Realism

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https://ntrs.nasa.gov/search.jsp?R=20160005170 2019-06-03T03:19:22+00:00Z
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Motivation

• At its User’s Forum on 14 Apr 2015, CARA recommended its users to begin delivering realistic covariances.

• This presentation is a response to that recommendation.

• Aqua and Aura’s covariances have been tuned during times without maneuvers.

• The impact on the probability of collision (on select conjunctions) using a tuned covariance was examined.

• A method to tune covariances through maneuvers is being adopted and will be ready for presentation by the next MOWG.
• **Covariance Realism:**
  – Study the evolution of a set of covariances (propagated into the future beginning with a pre-determined definitive state estimate error) by examining its behavior at equally spaced propagation points.
  – Uses *inferential statistics* in which behavioral conclusions for a large population are drawn using sample data.
  – The Mahalanobis distance of a covariance at a particular propagation point represents the ratio of the predicted minus definitive position difference to the covariance’s prediction.
  – A group of the squares of such calculations should conform to a chi-squared distribution with 3 Degrees-of-Freedom (DoF)

• **Involves the following 3 phases:**
  – Collection/calculation of definitive state estimates through orbit determination.
  – Calculation of covariance realism test statistics at each propagation point.
  – Proper assessment of those test statistics using a hypothesized distribution.
• **$P_c$ sensitive to Scaling of Primary Covariance:**
  – Graph below was presented at the 14 Apr 2015 CARA User’s Forum.
  – Depicts $P_c$ differences between nominal value and recalculation with primary covariance rescaled (Scale Factors 0.5 to 2).

![Pc Sensitivity to Scaling of Primary Covariance (2013 Data)](image)

• ~2–5% of cases show Scaled $P_c$ is greater than the Nominal $P_c$.
  – Impacts operational conclusions.

• A realistic covariance is important.
Definitive State Estimate:
- Best known position and velocity at an epoch time; obtained by passing observations through a Filter or Batch estimator.

Definitive State Estimate Error:
- Uncertainty in the definitive state estimate produced by a Filter or Batch estimator.
- Contained in a Definitive Covariance Matrix.

Predicted State Estimate:
- Position and velocity are propagated to a time \( t \) using a state transition matrix and definitive state estimate at an epoch time \( t_0 \).

Predicted State Estimate Error:
- Uncertainty in the Predicted State Estimate propagated using a force model.
- Contained in a Predictive Covariance Matrix.

Epoch Covariance:
- State Estimate Error at a specific epoch.

Predicted – Definitive State Estimate:
- The difference between the predicted state estimate (propagated from epoch time \( t_0 \)) and the definitive state estimate (obtained through orbit determination) at any time \( t \).
A Normal or Gaussian Distribution has a Mean of 0 and Standard Deviation of 1:
- 50% of values are distributed above and below a Mean of 0

A Chi-Square Distribution is a multi-variate distribution of the sum of the squares of $k$ independent standard normal random variables.
- A $k$ degree-of-freedom (DOF) Chi-Square Distribution has a mean value of $k$.
- A Chi-Square Distribution with 3 DOF has a Mean of 3 and a Standard Deviation of $8/3$:
  - 61% of values are distributed below a Mean of 3
The Chi-Square statistic is computed using the vector of predicted \( \varepsilon \) – definitive state estimates, and the predicted state estimate error or covariance matrix, \( C \):

\[
\chi^2_{3 \text{dof}} = \varepsilon C^{-1} \varepsilon^T \approx \left( \frac{\varepsilon_R}{\sigma_R} \right)^2 + \left( \frac{\varepsilon_I}{\sigma_I} \right)^2 + \left( \frac{\varepsilon_C}{\sigma_C} \right)^2
\]

- A perfectly sized covariance should have a Chi-Square equal to 3.
- In fact, this first moment of distribution test provides an initial idea of a covariance's departure from reality.
- However, a more rigorous Empirical Cumulative Distribution Function (ECDF) Method is adopted for this covariance realism analysis.
Quadratic Statistics\(^1\):

- An ECDF method that evaluates how well an empirical distribution corresponds to a parent distribution by examining the summation of a function of the squares of the deviations between the empirical and parent distributions.
- Examples are the Cramér – von Mises, Watson, and Anderson-Darling statistics.
- This analysis uses the more permissive Cramér – von Mises statistic due to the likelihood of outliers.

**P-value and Confidence Interval:**

- **P-value:** The likelihood an empirical distribution is drawn from a parent distribution.
- **Confidence Interval:** A \( p \)-value threshold that indicates a “pass” or “fail”. Normally 2% or higher are accepted.

\[ Q = n \int_{-\infty}^{\infty} [F_n(x) - F(x)]^2 \psi(x) dx \]

**Note in this example the Parent Distribution is a 3 DOF Chi-Square Distribution**
• Collect bins of Chi-Square Statistics at each propagation point and examine their Chi-Square distribution.
• The number of Chi-Square Statistics in each bin should be equal to the number of total propagations.

Each red rectangle or “bin” here contains information of a set of covariances at a unique propagation point.
• State Noise Compensation (SNC) - process noise is added to the propagation of the definitive state estimate in order to account for uncertainty in the force model.

• The predicted state estimate error, $P(t)$, is propagated using linear mapping as follows:

$$P(t) = \varphi P(t_0) \varphi^T + \Gamma Q \Gamma^T$$

$\varphi, \Gamma =$ state transition matrices

$P(t_0) =$ definitive state estimate error

• The process noise matrix $Q$ is built using variances in acceleration as follows:

$$Q_{RIC} = \begin{pmatrix}
\frac{\Delta T^4}{3} I \cdot \bar{q}_{acc} & \frac{\Delta T^3}{2} I \cdot \bar{q}_{acc} & \frac{\Delta T^3}{2} I \cdot \bar{q}_{acc} \\
\frac{\Delta T^3}{2} I \cdot \bar{q}_{acc} & \Delta T^2 I \cdot \bar{q}_{acc} & \Delta T^2 I \cdot \bar{q}_{acc} \\
\end{pmatrix}$$

$$\bar{q}_{acc} = \begin{pmatrix}
\sigma \dot{\mathbf{r}} \\
\sigma \dot{\mathbf{I}} \\
\sigma \dot{\mathbf{C}} \\
\end{pmatrix}$$

$I = \begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
\end{pmatrix}$

$\Delta T =$ propagation step size

$\sigma_{R,I,C} =$ acceleration variances
• **The following assumptions are made in the study:**
  – Propagation Time Span: 3 Days.
  – Maneuvers occur on 27 Aug, 17 Sep, 8 Oct, and 21 Oct 2014 – propagation over these dates are avoided.
  – Process noise is kept constant for all propagations.

• **The study is conducted as follows:**
  – Select an arbitrary set of acceleration variances and propagate all definitive state estimates using the corresponding process noise.
  – Examine the deviation between the ECDF and CDF of the 3 DOF Chi-Square Distribution without outlier identification.
  – Perform an outlier identification test and eliminate propagations that contain outliers.
  – Resize (by adjusting the process noise) the predicted state estimate error using the total mean RMS error of all remaining propagations (after outlier identification).
  – Examine the deviation between the ECDF and CDF of the 3 DOF Chi-Square Distribution (post outlier identification and resized predicted state estimate error).
• **Propagation Time Span** – 2 Aug 2014 to 6 Nov 2014

• **Maneuver Dates** – 27 Aug 2014
   17 Sep 2014
   08 Oct 2014
   21 Oct 2014

• **Mean RMS Predicted State Estimate Error**
80 Bins containing Chi-Square statistics equal to the number of propagations (29) are tested by computing their CDF across the 3-day propagation time span.

- A p-value threshold of 0.02 or 2% is set to determine a statistical pass.
- 54 out of 80 Chi-Square Bins (63.75%) produce p-value larger than 0.02.
- Statistical failures occur between 2.2 and 3 days in the propagation time span.
- Heavy upper-tail distribution implies covariance is undersized.
• Identify the following potential outliers based on the Normalized In-Track Error at the end (largest disparity in error) of each 3 day propagation:
  – 13 Sep 2014
  – 28 Sep 2014
  – 22 Oct 2014

• Perform the Rosner Outlier Identification test using the preceding normalized in-track error values.

• For a 2% significance level, the outlier test results indicate all 4 propagations are outliers and therefore can be eliminated.

OutlierPositions = [24 25 14 18];
SigOut = [0.0353 0 0.0775 0];
The Outliers identified by the Rosner test show a direct correlation to solar activity on those dates.

At this time, it appears FDS Propagation is not equipped to predict persistently high solar activity or a dramatic drop in the geomagnetic index.
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RMS Error and Uncertainty
(With Outlier Identification)

- **Propagation Time Span** – 02 Aug 2014 to 06 Nov 2014

Mean RMS Predicted State Estimate Error
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Goodness-Of-Fit Results
(With Outlier Identification)

- 80 Bins containing Chi-Square statistics equal to the number of propagations (25) are tested by computing their CDF across the 3-day propagation time span.
- A $p$-value threshold of 0.02 or 2% is set to determine a statistical pass.
- 70 out of 80 Chi-Square Bins (87.50%) produce $p$-value larger than 0.02.
- Statistical failures occur between 0.6 and 0.75 days in the propagation time span.
In-Track Acc. Variance required for realistic covariances appears to follow the seasonal change in solar activity.

**Percentage of Chi-Square Statistics above 2%**

**Magnitude of the In-Track Acceleration Variance**

- PM – 73.75%
- CH – 77.5%
- PM – 75%
- CH – 67.5%
- PM – 93.75%
- CH – 85%
- PM – 70%
- CH – 77.5%
- PM – 81.25%
- CH – 82.5%
- PM – 81.5%
- CH – 70%
- PM – 86.25%
- CH – 86.25%
- PM – 86.25%
- CH – 86.25%
- PM – 70%
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Impact of a Realistic Covariance on the Probability of Collision ($P_C$)

- Selected several conjunctions with similar primary and secondary object uncertainties (a rare occurrence) 2.5 – 3 days to TCA.
- Replaced the OCM ASSET covariance with a Tuned O/O covariance.
- Kept miss distance equal to the OCM ASSET solution.
- Examined the changes in uncertainties and their impact on the $P_C$.

| Time of Closest Approach (UTC) | OCM Creation Time (UTC) | OCM Age at TCA (Days) | O/O Cov. Start Date (UTC) | O/O Cov. Age at TCA (Days) | Radial Miss (m) | In-Track Miss (m) | Cross-Track Miss (m) | O/O Radial Cov (m) | O/O In-Track Cov (m) | O/O Cross-Track Cov (m) | OCM ASSET Radial Cov (m) | OCM ASSET In-Track Cov (m) | OCM ASSET Cross-Track Cov (m) | Sec, Object Radial Cov (m) | Sec, Object In-Track Cov (m) | Sec, Object Cross-Track Cov (m) | $P_c$ w/ O/O Cov. | $P_c$ w/ OCM ASSET Cov. |
|-------------------------------|-------------------------|-----------------------|---------------------------|---------------------------|----------------|-----------------|-----------------|----------------|-----------------|-----------------|----------------|-----------------|----------------|----------------|---------------------------|----------------|----------------|----------------|----------------|----------------|----------------|------------------|
| 03/26/14 03:45:13            | 03/23/14 03:02:57       | 3.03                  | 03/23/14 12:00:00         | 2.66                      | 94.1           | 2807.6          | -1771           | 5.0            | 518.2           | 8.6             | 13.2            | 1303.5          | 3.4             | 5.3             | 836             | 6.8            | 1.0E-19          | 1.0E-14          |
| 04/07/14 00:22:40            | 04/04/14 03:27:10       | 2.87                  | 04/04/14 12:00:00         | 2.52                      | -299.3         | 9766.4          | 2619.4          | 4.5            | 546.7           | 7.3             | 8.8             | 942.3           | 6.9             | 10.9            | 292             | 16.7           | 1.3E-71          | 1.1E-87          |
| 12/22/14 18:32:31            | 12/20/14 03:10:30       | 2.64                  | 12/20/14 12:00:00         | 2.27                      | 380.6          | -2325.2         | -1989.9         | 4.9            | 513.6           | 6.9             | 17.4            | 755.4           | 4.4             | 13.4            | 526             | 7.4            | 4.9E-67          | 2.2E-63          |
| 03/22/15 02:51:29            | 03/20/15 02:51:29       | 2.51                  | 03/20/15 12:00:00         | 2.13                      | 416.6          | -3288.1         | 976.9           | 4.9            | 434             | 7.9             | 39.0            | 647.2           | 21.0            | 17.7            | 380             | 13.3           | 5.9E-119         | 9.0E-42          |
| 05/11/15 14:08:45            | 05/08/15 14:28:10       | 2.99                  | 05/08/15 12:00:00         | 3.09                      | -33.9           | 984             | 994.6           | 4.2            | 246.4           | 7.9             | 10.0            | 427.4           | 2.8             | 8.3             | 153             | 6.5            | 4.7E-12          | 1.4E-06          |
| 05/11/15 14:08:45            | 05/08/15 17:37:35       | 2.85                  | 05/08/15 12:00:00         | 3.09                      | -29.4           | 1020.9          | 1029.4          | 4.2            | 246.4           | 7.9             | 9.6             | 394.7           | 2.8             | 8.3             | 153             | 6.5            | 6.4E-13          | 2.4E-07          |
| 05/11/15 14:08:45            | 05/09/15 01:00:08       | 2.55                  | 05/08/15 12:00:00         | 3.09                      | -26.3           | 995.9           | 1002.8          | 4.2            | 246.4           | 7.9             | 9.8             | 321.6           | 2.8             | 7.2             | 116             | 6.2            | 1.1E-13          | 1.6E-09          |
Conclusion:

- Aqua and Aura are ready to start delivering tuned covariances.
- Covariance realism tuning is sensitive to outliers but can be tuned for up to 3 months at a time.
- Provided the primary and secondary object uncertainties are similar, an impact on the Pc is evident – similar uncertainties involve well-tracked secondary objects.

Future Work:

- Interpolate definitive state estimates and add them to prediction – definitive state estimate.
- Resampling Investigation – Take 1,000 random subsets of a passed Chi-Square Bin and determine the p-values of each test.
- Complete analysis for covariance propagation through maneuvers
- Complete analysis for Terra and GPM.
- Test tuned results with 7-day propagations.
- Group together outliers during high solar events and determine if they conform to a Gaussian distribution – Look at the possibility of increasing process noise during high solar events to more accurately model the predicted state estimate error.

