International Earth Science Constellation Mission Operations Working Group
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Earth Observation System Flight Dynamics System
Covariance Realism
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• At its User’s Forum on 14 Apr 2015, CARA recommended its users to begin delivering realistic covariances.

• This presentation is a response to that recommendation.

• Aqua and Aura’s covariances have been tuned during times without maneuvers.

• The impact on the probability of collision (on select conjunctions) using a tuned covariance was examined.

• A method to tune covariances through maneuvers is being adopted and will be ready for presentation by the next MOWG.
• **Covariance Realism:**
  – Study the evolution of a set of covariances (propagated into the future beginning with a pre-determined definitive state estimate error) by examining its behavior at equally spaced propagation points.
  – Uses *inferential statistics* in which behavioral conclusions for a large population are drawn using sample data.
  – The Mahalanobis distance of a covariance at a particular propagation point represents the ratio of the predicted minus definitive position difference to the covariance’s prediction.
  – A group of the squares of such calculations should conform to a chi-squared distribution with 3 Degrees-of-Freedom (DoF)

• **Involves the following 3 phases:**
  – Collection/calculation of definitive state estimates through orbit determination.
  – Calculation of covariance realism test statistics at each propagation point.
  – Proper assessment of those test statistics using a hypothesized distribution.
Purpose

- $P_c$ sensitive to Scaling of Primary Covariance:
  - Graph below was presented at the 14 Apr 2015 CARA User's Forum.
  - Depicts $P_c$ differences between nominal value and recalculation with primary covariance rescaled (Scale Factors 0.5 to 2).

- $\sim 2$–$5\%$ of cases show Scaled $P_c$ is greater than the Nominal $P_c$.
  - Impacts operational conclusions.

- A realistic covariance is important.
Definitive State Estimate:
- Best known position and velocity at an epoch time; obtained by passing observations through a Filter or Batch estimator.

Definitive State Estimate Error:
- Uncertainty in the definitive state estimate produced by a Filter or Batch estimator.
- Contained in a Definitive Covariance Matrix.

Predicted State Estimate:
- Position and velocity are propagated to a time \( t \) using a state transition matrix and definitive state estimate at an epoch time \( t_0 \).

Predicted State Estimate Error:
- Uncertainty in the Predicted State Estimate propagated using a force model.
- Contained in a Predictive Covariance Matrix.

Epoch Covariance:
- State Estimate Error at a specific epoch.

Predicted – Definitive State Estimate:
- The difference between the predicted state estimate (propagated from epoch time \( t_0 \)) and the definitive state estimate (obtained through orbit determination) at any time \( t \).
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Gaussian versus 3 DOF Chi-Square Distribution

- A Normal or Gaussian Distribution has a Mean of 0 and Standard Deviation of 1:
  - 50% of values are distributed above and below a Mean of 0

- A Chi-Square Distribution is a multi-variate distribution of the sum of the squares of \( k \) independent standard normal random variables.

- A \( k \) degree-of-freedom (DOF) Chi-Square Distribution has a mean value of \( k \).

- A Chi-Square Distribution with 3 DOF has a Mean of 3 and a Standard Deviation of \( \frac{8}{3} \):
  - 61% of values are distributed below a Mean of 3
The Chi-Square statistic is computed using the vector of predicted – definitive state estimates, $\varepsilon$, and the predicted state estimate error or covariance matrix, $C$:

$$
\chi^2_{3\text{dof}} = \varepsilon C^{-1} \varepsilon^T \approx \left( \frac{\varepsilon_R}{\sigma_R} \right)^2 + \left( \frac{\varepsilon_I}{\sigma_I} \right)^2 + \left( \frac{\varepsilon_C}{\sigma_C} \right)^2
$$

- Inverse of the covariance matrix obtained directly from propagation, thus containing correlation terms
- Predicted State Estimate Error assuming no correlation between variables
- Difference between predicted and definitive state estimates

A perfectly sized covariance should have a Chi-Square equal to 3.

In fact, this first moment of distribution test provides an initial idea of a covariance's departure from reality.

However, a more rigorous Empirical Cumulative Distribution Function (ECDF) Method is adopted for this covariance realism analysis.
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Empirical Cumulative Distribution Function
Method

Quadratic Statistics\(^1\):

- An ECDF method that evaluates how well an empirical distribution corresponds to a parent distribution by examining the summation of a function of the squares of the deviations between the empirical and parent distributions.

- Examples are the Cramér – von Mises, Watson, and Anderson-Darling statistics.

- This analysis uses the more permissive Cramér – von Mises statistic due to the likelihood of outliers.

**P-value and Confidence Interval:**

- \( P \)-value: The likelihood an empirical distribution is drawn from a parent distribution.

- Confidence Interval: A \( p \)-value threshold that indicates a “pass” or “fail”. Normally 2% or higher are accepted.

\[ Q = n \int_{-\infty}^{\infty} \left[ F_n(x) - F(x) \right]^2 \psi(x) dx \]

**Note in this example the Parent Distribution is a 3 DOF Chi-Square Distribution**
• Collect bins of Chi-Square Statistics at each propagation point and examine their Chi-Square distribution.

• The number of Chi-Square Statistics in each bin should be equal to the number of total propagations.

Each red rectangle or “bin” here contains information of a set of covariances at a unique propagation point.
• State Noise Compensation (SNC) - process noise is added to the propagation of the definitive state estimate in order to account for uncertainty in the force model.

• The predicted state estimate error, $P(t)$, is propagated using linear mapping as follows:

$$ P(t) = \varphi P(t_0) \varphi^T + \Gamma Q \Gamma^T $$

$\varphi, \Gamma = \text{state transition matrices}$

$P(t_0) = \text{definitive state estimate error}$

• The process noise matrix $Q$ is built using variances in acceleration as follows:

$$ Q_{RIC} = \begin{pmatrix} \frac{\Delta T^4}{3} I \cdot \bar{q}_{acc} & \frac{\Delta T^3}{2} I \cdot \bar{q}_{acc} \\ \frac{\Delta T^3}{2} I \cdot \bar{q}_{acc} & \Delta T^2 I \cdot \bar{q}_{acc} \end{pmatrix} \begin{pmatrix} \sigma_R^2 \\ \sigma_I^2 \\ \sigma_C^2 \end{pmatrix} = \begin{pmatrix} \sigma_R \\ \sigma_I \\ \sigma_C \end{pmatrix} $$

$I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$\Delta T = \text{propagation step size}$

$\sigma_{R,I,C} = \text{acceleration variances}$
The following assumptions are made in the study:

- Propagation Date Span: 2 Aug 2014 to 6 Nov 2014.
- Propagation Time Span: 3 Days.
- Maneuvers occur on 27 Aug, 17 Sep, 8 Oct, and 21 Oct 2014 – propagation over these dates are avoided.
- Process noise is kept constant for all propagations.

The study is conducted as follows:

- Select an arbitrary set of acceleration variances and propagate all definitive state estimates using the corresponding process noise.
- Examine the deviation between the ECDF and CDF of the 3 DOF Chi-Square Distribution without outlier identification.
- Perform an outlier identification test and eliminate propagations that contain outliers.
- Resize (by adjusting the process noise) the predicted state estimate error using the total mean RMS error of all remaining propagations (after outlier identification).
- Examine the deviation between the ECDF and CDF of the 3 DOF Chi-Square Distribution (post outlier identification and resized predicted state estimate error).
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RMS Error and Uncertainty
(Without Outlier Identification)

- **Propagation Time Span** – 2 Aug 2014 to 6 Nov 2014
- **Maneuver Dates** – 27 Aug 2014
- **Mean RMS**
  - Predicted State Estimate Error

3/1/2016
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Goodness-Of-Fit Results
(Without Outlier Identification)

- 80 Bins containing Chi-Square statistics equal to the number of propagations (29) are tested by computing their CDF across the 3-day propagation time span.
- A \( p \)-value threshold of 0.02 or 2% is set to determine a statistical pass.
- 54 out of 80 Chi-Square Bins (63.75%) produce \( p \)-value larger than 0.02.
- Statistical failures occur between 2.2 and 3 days in the propagation time span.
- Heavy upper-tail distribution implies covariance is undersized.

3/1/2016
Identify the following potential outliers based on the Normalized In-Track Error at the end (largest disparity in error) of each 3 day propagation:
- 13 Sep 2014
- 28 Sep 2014
- 22 Oct 2014
- 25 Oct 2014

Perform the Rosner Outlier Identification test using the preceding normalized in-track error values.

For a 2% significance level, the outlier test results indicate all 4 propagations are outliers and therefore can be eliminated.

OutlierPositions = [24 25 14 18];
SigOut = [0.0353 0 0.0775 0];

<table>
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<tr>
<th>Position</th>
<th>Start Date</th>
<th>(εᵣ-μᵣ)/σᵣ</th>
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<td>5-Aug-14</td>
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<td>0.250</td>
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<td>4</td>
<td>11-Aug-14</td>
<td>0.396</td>
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<tr>
<td>5</td>
<td>14-Aug-14</td>
<td>0.164</td>
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<tr>
<td>6</td>
<td>17-Aug-14</td>
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<td>7</td>
<td>20-Aug-14</td>
<td>0.456</td>
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<tr>
<td>8</td>
<td>23-Aug-14</td>
<td>0.497</td>
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<tr>
<td>9</td>
<td>29-Aug-14</td>
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<td>1-Sep-14</td>
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<td>27</td>
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<td>0.594</td>
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<tr>
<td>29</td>
<td>6-Nov-14</td>
<td>0.347</td>
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</tbody>
</table>
The Outliers identified by the Rosner test show a direct correlation to solar activity on those dates.

At this time, it appears FDS Propagation is not equipped to predict persistently high solar activity or a dramatic drop in the geomagnetic index.
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RMS Error and Uncertainty
(With Outlier Identification)

- **Propagation Time Span** – 02 Aug 2014 to 06 Nov 2014

**Mean RMS**
- Predicted State
- Estimate Error
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Goodness-Of-Fit Results
(With Outlier Identification)

- 80 Bins containing Chi-Square statistics equal to the number of propagations (25) are tested by computing their CDF across the 3-day propagation time span.
- A $p$-value threshold of 0.02 or 2% is set to determine a statistical pass.
- 70 out of 80 Chi-Square Bins (87.50%) produce $p$-value larger than 0.02.
- Statistical failures occur between 0.6 and 0.75 days in the propagation time span.

A 0.26 $p$-value represents an excellent result.
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Aqua and Aura Seasonal Covariance Tuning
Jan 2014 – Oct 2015

Magnitude of the In-Track Acceleration Variance required for realistic covariances appears to follow the seasonal change in solar activity.

**Percentage of Chi-Square Statistics above 2%**

- **PM**
  - Jan 2014
  - Oct 2015

- **CH**
  - Jan 2014
  - Oct 2015

**F10.7 Solar Flux**

- **Geomagnetic Index (Ap)**

**Date**

- 11/22/2013
- 3/2/2014
- 6/10/2014
- 9/18/2014
- 12/27/2014
- 4/6/2015
- 7/15/2015
- 10/23/2015

**Solar Activity**

- 0
- 50
- 100
- 150
- 200
- 250

**In-Track Acc. Variance**

- 0.00E+00
- 1.00E-10
- 2.00E-10
- 3.00E-10
- 4.00E-10
- 5.00E-10
- 6.00E-10
- 7.00E-10

**Percentage of Chi-Square Statistics above 2%**

- PM – 93.75%
  - CH – 85%

- PM – 92.5%
  - CH – 86.25%

- PM – 92.5%
  - CH – 86.25%

- PM – 86.25%
  - CH – 86.25%

- PM – 70%
  - CH – 77.5%

- PM – 77.5%
  - CH – 77.5%

- PM – 75%
  - CH – 67.5%

- PM – 81.25%
  - CH – 82.5%

- PM – 81.25%
  - CH – 82.5%

- PM – 81.5%
  - CH – 70%

- PM – 93.75%
  - CH – 85%

- PM – 75%
  - CH – 67.5%

- PM – 81.25%
  - CH – 82.5%

- PM – 81.5%
  - CH – 70%
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Impact of a Realistic Covariance on the Probability of Collision ($P_c$)

- Selected several conjunctions with similar primary and secondary object uncertainties (a rare occurrence) 2.5 – 3 days to TCA.
- Replaced the OCM ASSET covariance with a Tuned O/O covariance.
- Kept miss distance equal to the OCM ASSET solution.
- Examined the changes in uncertainties and their impact on the $P_c$.

<table>
<thead>
<tr>
<th>Time of Closest Approach (UTC)</th>
<th>OCM Creation Time (UTC)</th>
<th>OCM Age at TCA (Days)</th>
<th>O/O Cov. Start Date (UTC)</th>
<th>O/O Cov. Age at TCA (Days)</th>
<th>O/O Radial Miss (m)</th>
<th>In-Track Miss (m)</th>
<th>Cross-Track Miss (m)</th>
<th>O/O Radial Cov (m)</th>
<th>O/O In-Track Cov (m)</th>
<th>O/O Cross-Track Cov (m)</th>
<th>OCM ASSET Radial Cov (m)</th>
<th>OCM ASSET In-Track Cov (m)</th>
<th>OCM ASSET Cross-Track Cov (m)</th>
<th>Sec, Object Radial Cov (m)</th>
<th>Sec, Object In-Track Cov (m)</th>
<th>Sec, Object Cross-Track Cov (m)</th>
<th>$P_c$ w/ O/O Cov.</th>
<th>$P_c$ w/ OCM ASSET Cov.</th>
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<td>03/23/14 03:02:57</td>
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<td>03/23/14 12:00:00</td>
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<td>1.6E-09</td>
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</tbody>
</table>

3/1/2016
Conclusion:

– Aqua and Aura are ready to start delivering tuned covariances.
– Covariance realism tuning is sensitive to outliers but can be tuned for up to 3 months at a time.
– Provided the primary and secondary object uncertainties are similar, an impact on the Pc is evident – similar uncertainties involve well-tracked secondary objects.

Future Work:

– Interpolate definitive state estimates and add them to prediction – definitive state estimate.
– Resampling Investigation – Take 1,000 random subsets of a passed Chi-Square Bin and determine the p-values of each test.
– Complete analysis for covariance propagation through maneuvers
– Complete analysis for Terra and GPM.
– Test tuned results with 7-day propagations.
– Group together outliers during high solar events and determine if they conform to a Gaussian distribution – Look at the possibility of increasing process noise during high solar events to more accurately model the predicted state estimate error.

