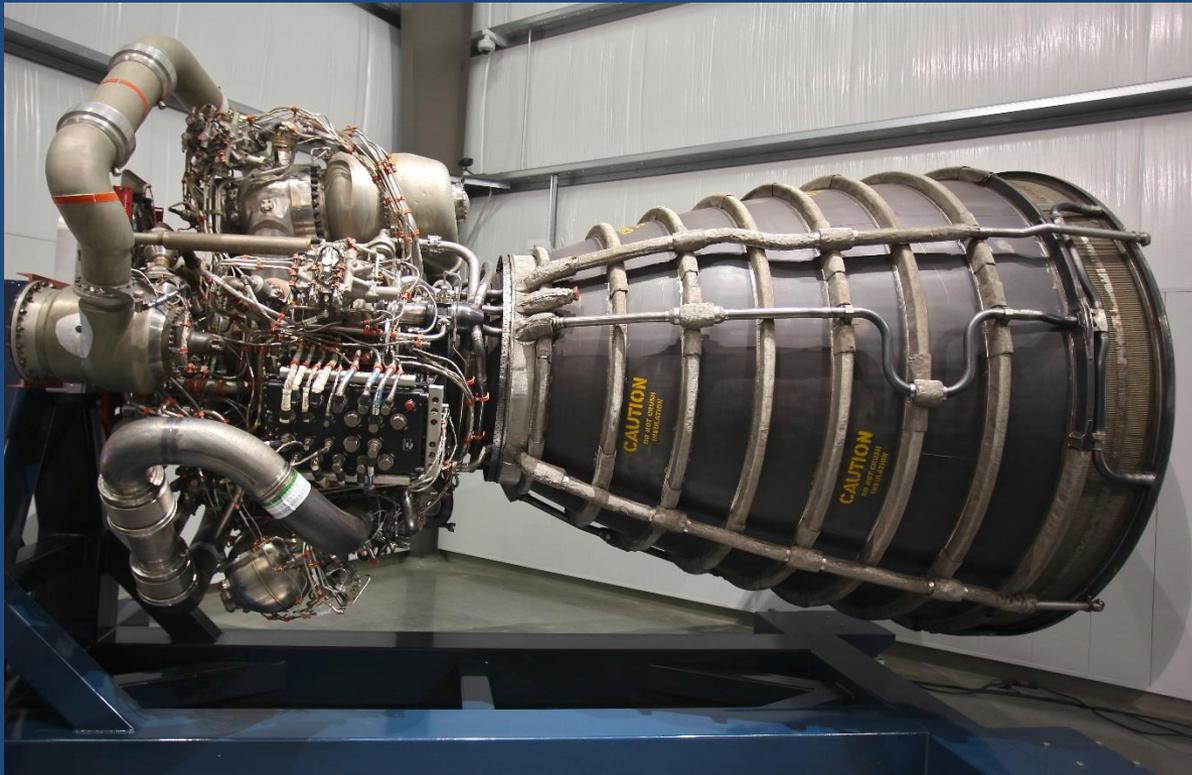




# Structural Dynamics of Rocket Engines



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San Diego, California  
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# Why do I have a job? Why are we here?

2

- From “Characteristics of Space Shuttle Main Engine Failures”, H. Cikanek, AIAA Joint Propulsion Conference, 1987, San Diego, CA
  - “During development and operation of the SSME, 27 ground test failures of sufficient severity to be termed “major incident” have occurred.”
  - “Most SSME failures were a result of design deficiencies stemming from inadequate definition of dynamic loads. High cycle fatigue was the most frequent mechanism leading to failure.”



*Failure of Lox Inlet Splitter to Nozzle Blows Engine Out of Santa Susanna Test Stand*



# Agenda

3

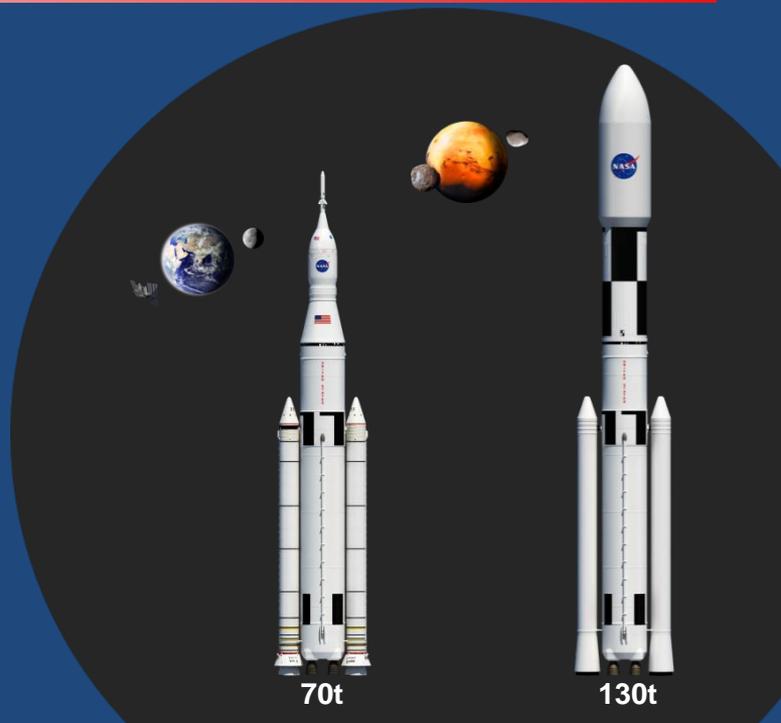
- Introduction to NASA's new SLS
- Short Review of Basics of Structural Dynamics
- The Critical Role of Structural Dynamics in the Design, Analysis, and Testing of Rocket Engines:
  - How Rocket Engines Work
  - Turbomachinery
  - Rocket Nozzles
  - Rocket Engine Loads
  - System Hardware and Propellant Feedlines
- Will need to introduce various Structural Dynamics Analysis Methods throughout presentation – “Two Minute Tutorials”.



# Travelling To and Through Space

## Space Launch System (SLS) – America’s Heavy-lift Rocket

- Provides initial lift capacity of 70 metric tons (t), evolving to 130 t
- Carries the Orion Multi-Purpose Crew Vehicle (MPCV) and significant science payloads
- Supports national and international missions beyond Earth’s orbit, such as near-Earth asteroids and Mars



Solid Rocket  
Booster Test



Friction Stir  
Welding for Core  
Stage



Shell Buckling  
Structural Test



MPCV Stage Adapter  
Assembly



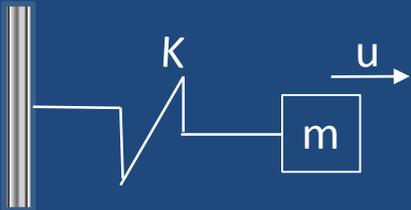
Selective Laser  
Melting Engine  
Parts



RS-25 (SSME) Core  
Stage Engines in  
Inventory

## Test of RS25 Core Stage Engine for Space Launch System

- Free Vibration, Undamped Single Degree of Freedom System



$$\sum F_x = m\ddot{u}$$

$$m\ddot{u} + Ku = 0$$

1) Steady State, simplest, worth remembering:

Assume solution  $u=u(t)$  is of form

$$u(t) = A \cos(\omega t)$$

$$\dot{u}(t) = -A\omega \sin(\omega t)$$

$$\ddot{u}(t) = -A\omega^2 \cos(\omega t)$$

Now plug these equalities into eq of motion:

$$m(-A\omega^2 \cos \omega t) + k(A \cos \omega t) = 0$$

$$A \cos \omega t (k - \omega^2 m) = 0$$

For  $A \cos \omega t = 0$ ,  $A$  has to be  $0$ , i.e., no response ("trivial solution")

Therefore,  $k - \omega^2 m = 0$

$$\omega^2 = \frac{k}{m} \Rightarrow \omega = \sqrt{\frac{k}{m}} \text{ Rad/sec}$$

Define  $\lambda \equiv$  Eigenvalue  $= \omega^2 \equiv$  Natural Frequency<sup>2</sup>

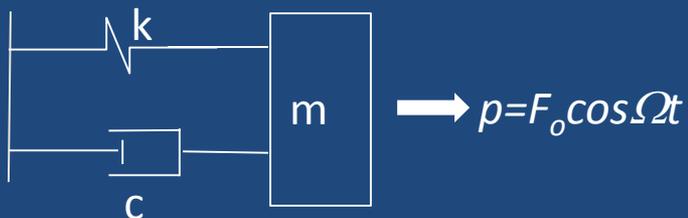
So, solution for  $u=u(t)$  is

$$u(t) = A \cos\left(\sqrt{\frac{k}{m}} t\right)$$

where  $A$  depends on the initial conditions



# Response to Harmonic Excitation



- $\Omega$  = Excitation Frequency
- $p$  = Harmonic Excitation Force
- $\omega$  = System Natural Frequency =  $\sqrt{\frac{k}{m}}$
- $\zeta$  = critical damping ratio =  $\frac{c}{c_{critical}} = \frac{c}{2\sqrt{km}}$

eq. of motion:  $m\ddot{u} + c\dot{u} + ku = F_o \cos \Omega t$

Now, define static response  $U_{st}$  to force  $F_o$  using

$$F_o = kU_{st} \rightarrow U_{st} = \frac{F_o}{k}$$

then we can define the "Complex Frequency Response"

$$\bar{H}(\Omega) = \frac{\text{Dynamic Response } \bar{U}}{\text{Static Response } \bar{U}_{st}} \Rightarrow \bar{U}(\Omega) = \bar{H}(\Omega)\bar{U}_{st}$$

$$|\bar{H}(\Omega)| = \frac{1}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} \quad \text{where we define the Frequency Ratio } r = \frac{\Omega}{\omega}$$

Resonance is defined when  $\Omega = \omega$ , ie,  $r=1$ .

$$\text{At } r=1, |\bar{H}(\Omega)| = \frac{1}{2\zeta} \equiv \text{Quality Factor } Q$$



# Frequency Response Example

Example:

$$F=2; \quad c=0.6; \quad m=1; \quad k=9$$

$$\omega = \sqrt{\frac{k}{m}} = 3$$

$$\zeta = \frac{c}{2\sqrt{km}} = 0.1$$

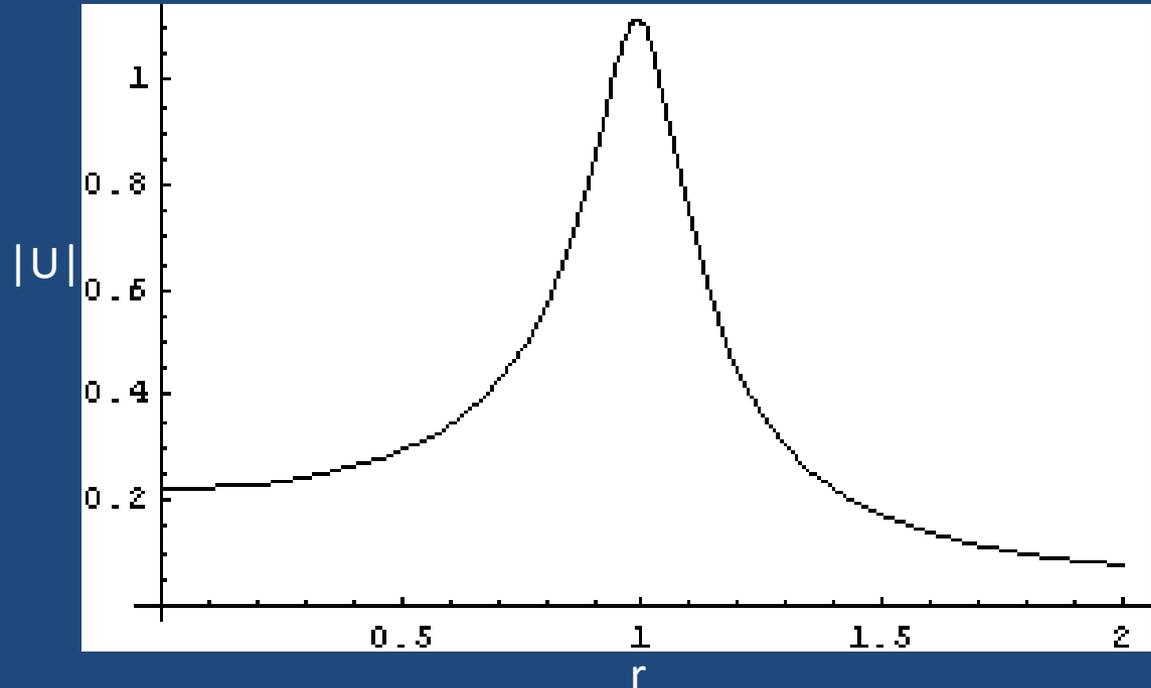
$$U_{static} = \frac{F_0}{k} = 0.222$$

At resonance,  $|U| = Q U_{static} =$

$$|U| = \frac{1}{2\zeta} (.2222) = 1.111$$

For  $\Omega = 2.8$ ,  $r = \frac{\Omega}{\omega} = \frac{2.8}{3} = .9333$ , so

$$|\bar{H}(\Omega)| = \sqrt{\frac{1}{(1-r^2)^2 + (2\zeta r)^2}} = \sqrt{\frac{1}{(1-0.93333^2)^2 + (2*0.1*0.93333)^2}} = 4.408$$



*Demo: Joe- Bob the  
Bungee Jumper*



# Modal Analysis of Multiple DOF Systems

Solutions for Undamped, Free Vibration of MDOF Systems with N dof's.

$$[M]\{\ddot{u}\} + [K]\{u\} = \{0\}$$

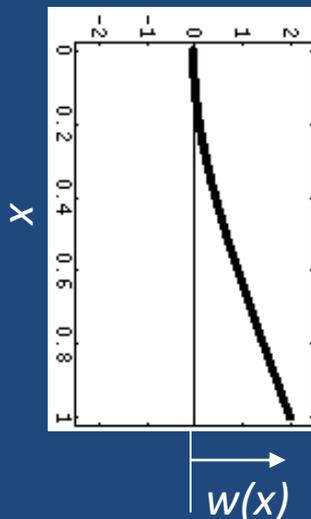
Assume solution of form (m spatial solutions  $\equiv$  eigenvectors = modes)

$$\{u\}_m = \{\phi\}_m e^{i(\omega_m t + \alpha_m)}$$

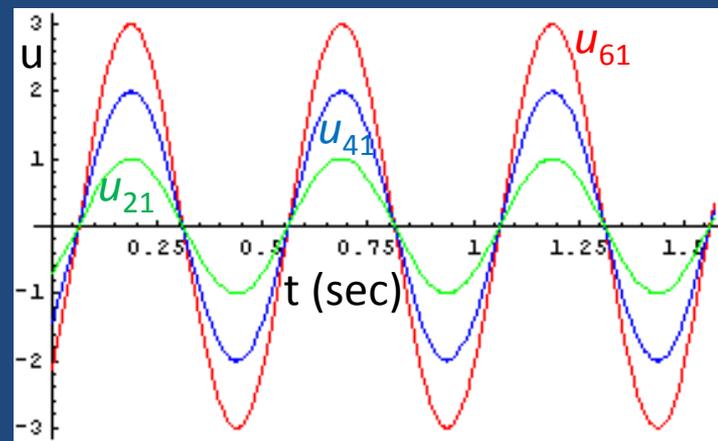
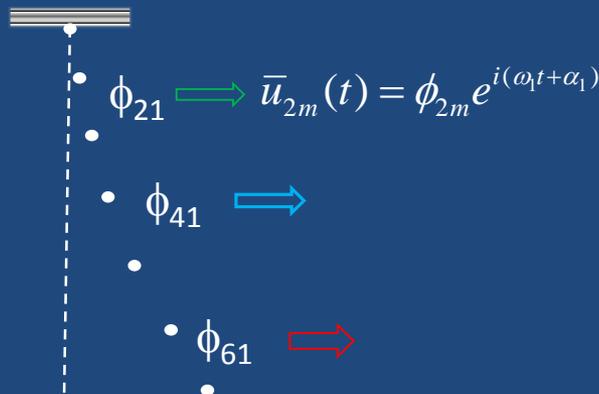
m=1, M, where  $M \leq N$

Continuous

$$w(x) = U_m(x) \Leftrightarrow \{\phi\}_m$$



Discrete MDOF

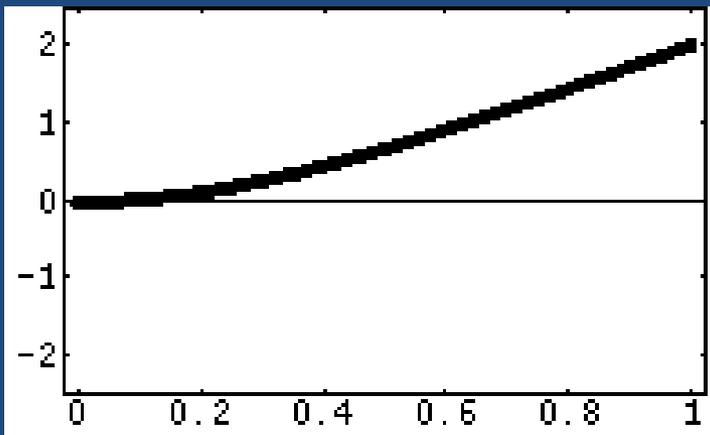




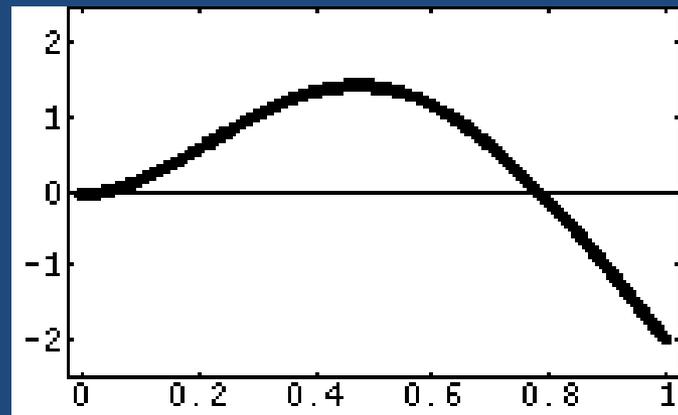
# Other Spatial Solutions are other Mode Shapes

## Clamped-Free Boundary Conditions

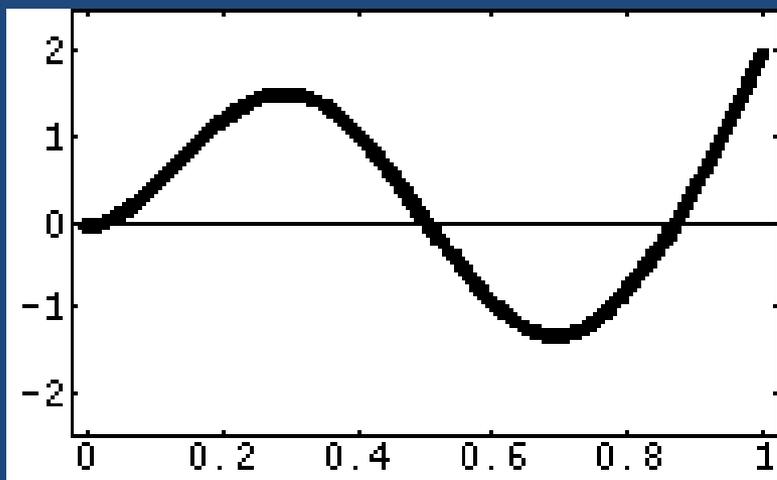
Mode 1  
at  $f_1$  hz



Mode 2  
at  $f_2$  hz



Mode 3  
at  $f_3$  hz





# Generalized Coordinates/Modal Superposition

- Frequency and Transient Response Analysis uses Concept of Modal Superposition using Generalized (or Principal Coordinates).

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = \{P(t)\}$$

- Mode Superposition Method – transforms to set of uncoupled, SDOF equations that we can solve using SDOF methods.
- First obtain  $[\Phi]_{mass}$ . Then introduce coordinate transformation:

$$\{u\} = \sum_N^M [\Phi] \{\eta\}_M$$

$$[M][\Phi]\{\ddot{\eta}\} + [C][\Phi]\{\dot{\eta}\} + [K][\Phi]\{\eta\} = \{P(t)\}$$

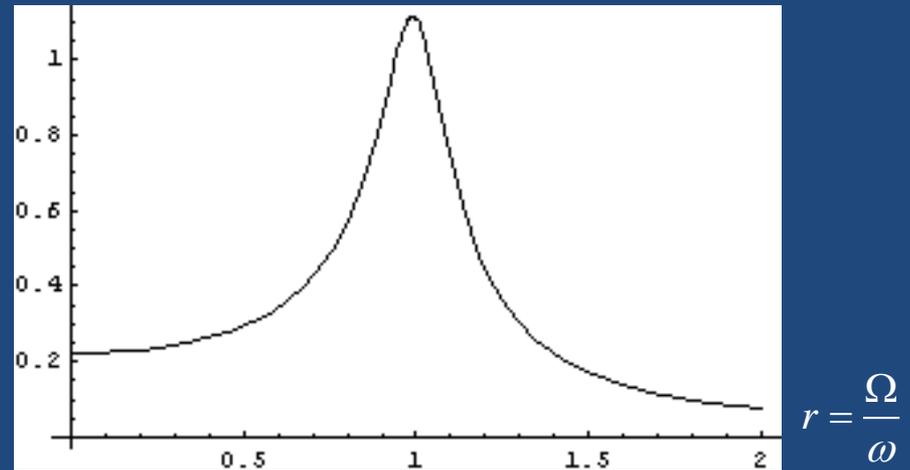


$$[I]\{\ddot{\eta}\} + [C]\{\dot{\eta}\} + [K]\{\eta\} = [\Phi]^T \{P(t)\}$$

Generalized (or Modal) Force - dot product of each mode with excitation force vector  
 - means response directly proportional to similarity of spatial shape of each mode with spatial shape of the force (*Orthogonality*).

for the SDOF equation of motion,  
 $m\ddot{u} + c\dot{u} + ku = p \rightarrow \ddot{u} + 2\zeta\omega\dot{u} + \omega^2u = F_0e^{i\Omega t}$

$$|U(\Omega)| = F_0/k \sqrt{\frac{1}{\left(1 - \left(\frac{\Omega}{\omega}\right)^2\right)^2 + \left(2\zeta\left(\frac{\Omega}{\omega}\right)\right)^2}}$$



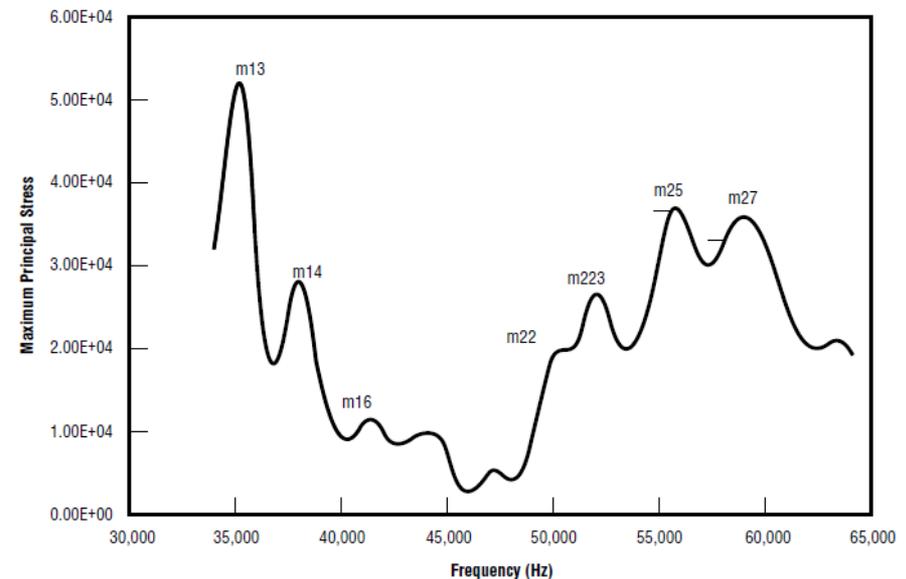
So we get the same equations in  $\eta$ :

$$\ddot{\eta}_m + 2\zeta_m\omega_m\dot{\eta}_m + \lambda_m\eta_m = \{\phi\}_m^T \{P(t)\}$$

$$|\eta_m(t)| = \frac{\{\phi\}_m^T \{F\}}{\lambda_m} \frac{1}{\sqrt{\left(1 - \left(\frac{\Omega}{\omega_m}\right)^2\right)^2 + \left(2\zeta_m\frac{\Omega}{\omega_m}\right)^2}}$$

- For “Frequency Response” Analysis, apply Fourier coefficients coming from CFD such that excitation frequencies match Campbell crossovers.

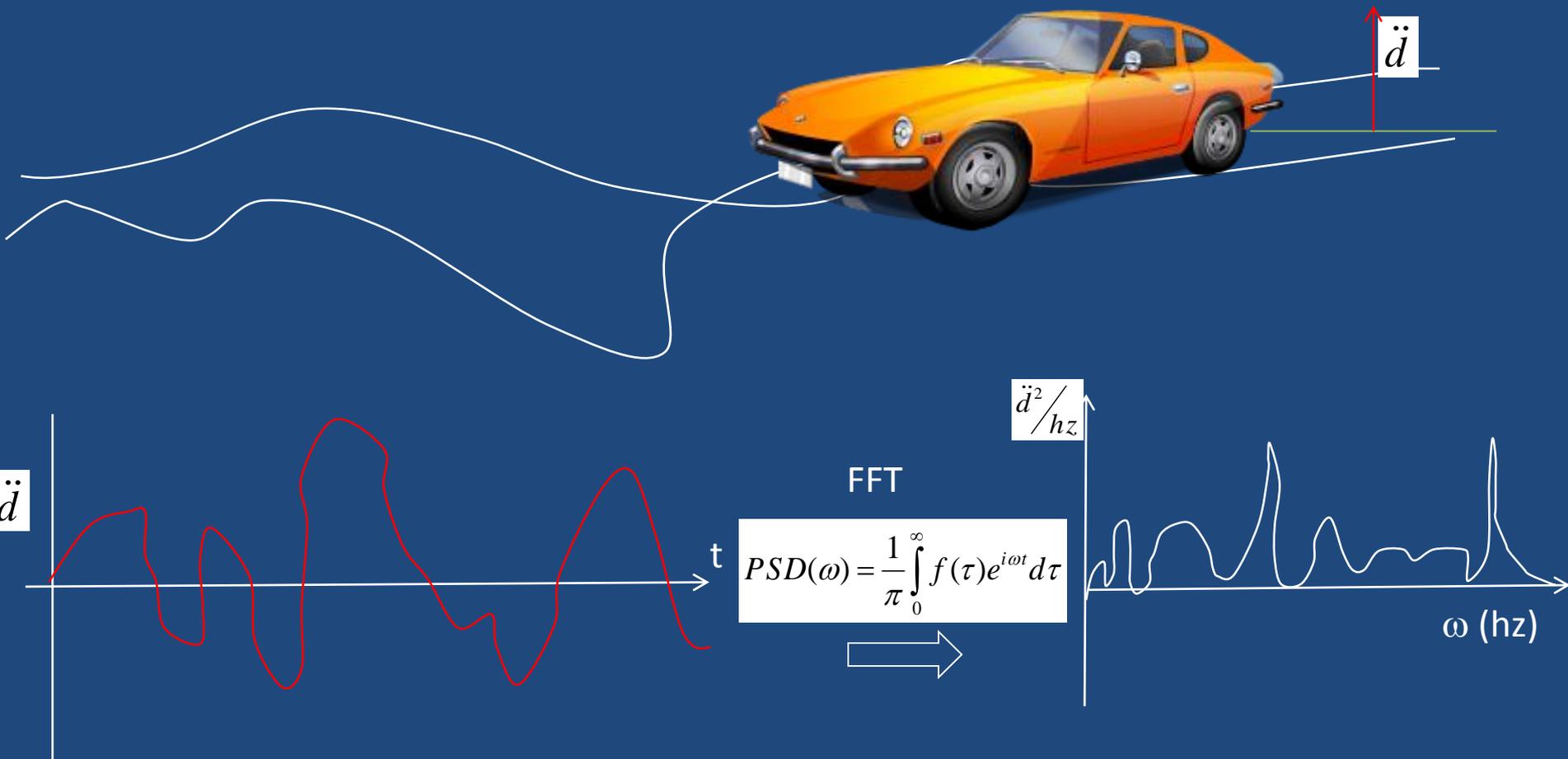
## SSME HPFTP 1<sup>st</sup> Blade Frequency Response





# We Also need to know something about Random Vibrations

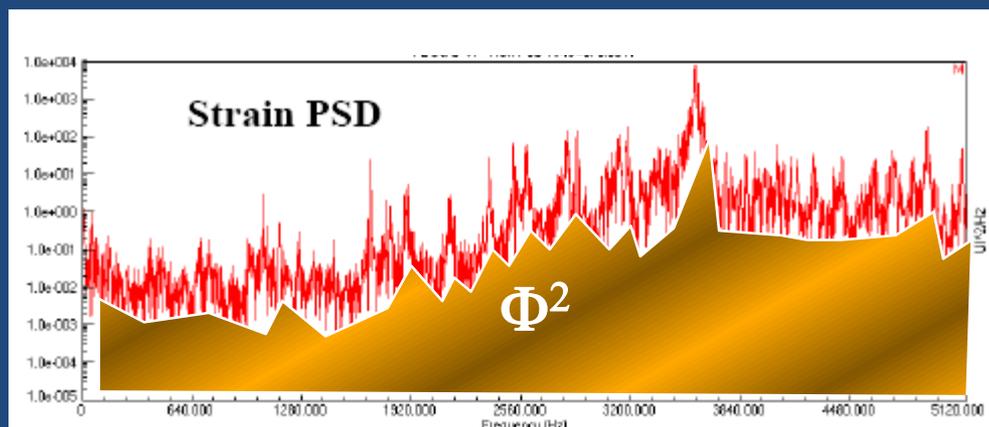
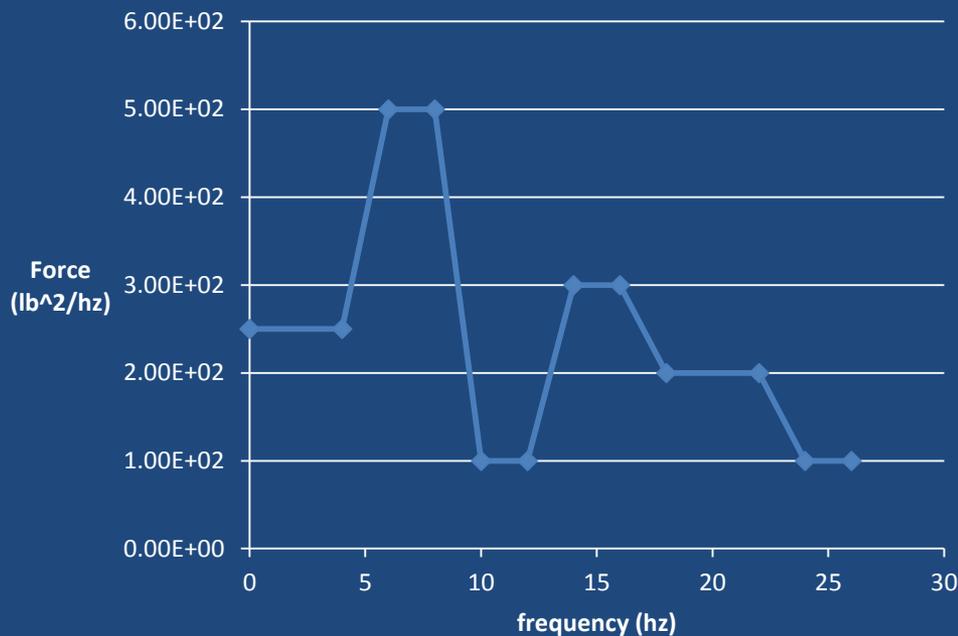
- For structures undergoing random vibration (vibration whose magnitude can only be characterized statistically), random vibration analysis gives the statistical characterization of the response.





# Power Spectral Density and RMS

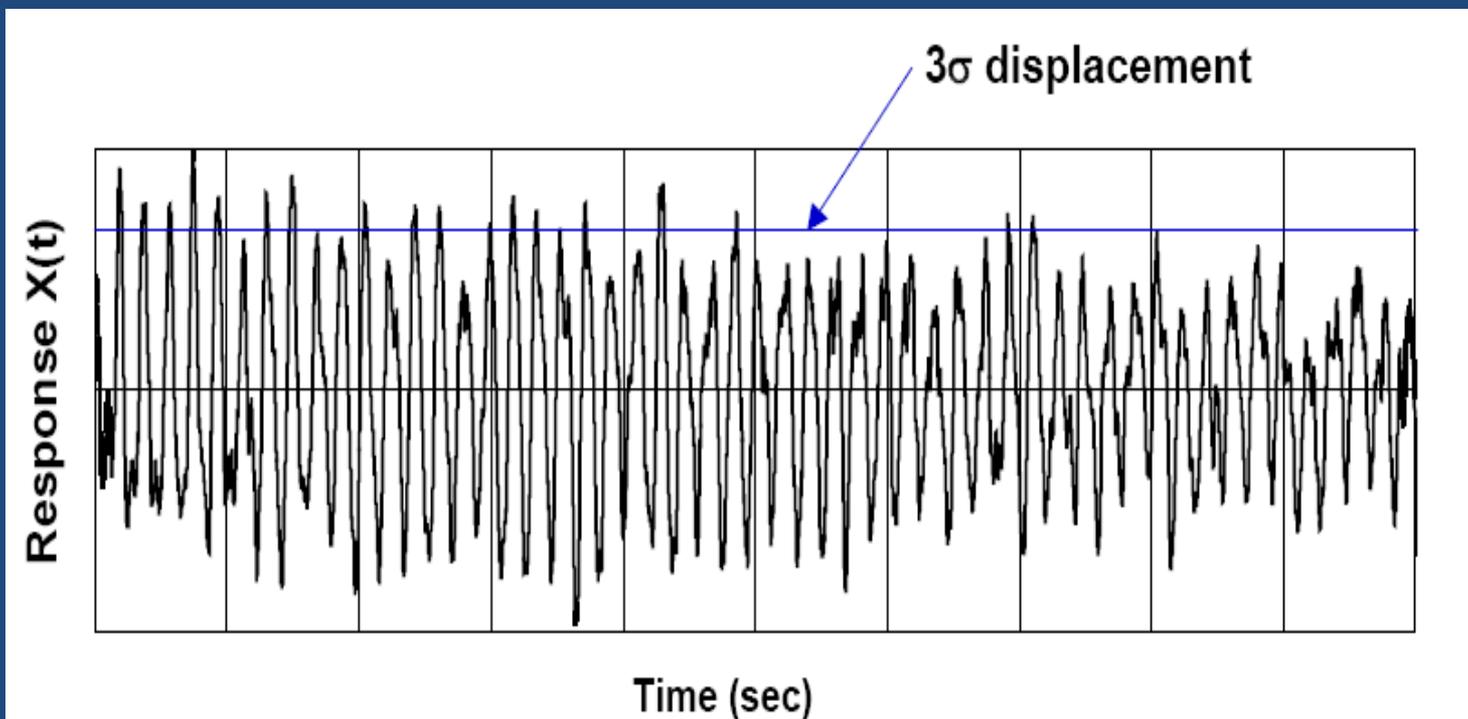
- Perform a Fourier Transform of the excitation to generate a Power Spectral Density (PSD).
- Apply Excitation as series of frequency response analyses, generates response.
- Response will also be in frequency domain, and can be converted to a PSD.
- The area under the PSD curve is defined as the “mean square ( $\Phi^2$ )”.
  - area under each discrete point is “mean square” of sin wave,
  - square root (“root of mean square) of sin =  $.707 \times$  amplitude.
- The RMS of the entire response PSD equals 1 standard deviation of the response for a Gaussian distribution.





# Design for Random Vibration

- By general agreement, the design value for a random response is generally a value that exceeds the response 99.865% of the time.
- This value is 3 sigma for a normal distribution. So we simply multiply the RMS by 3 and use that as our design value.

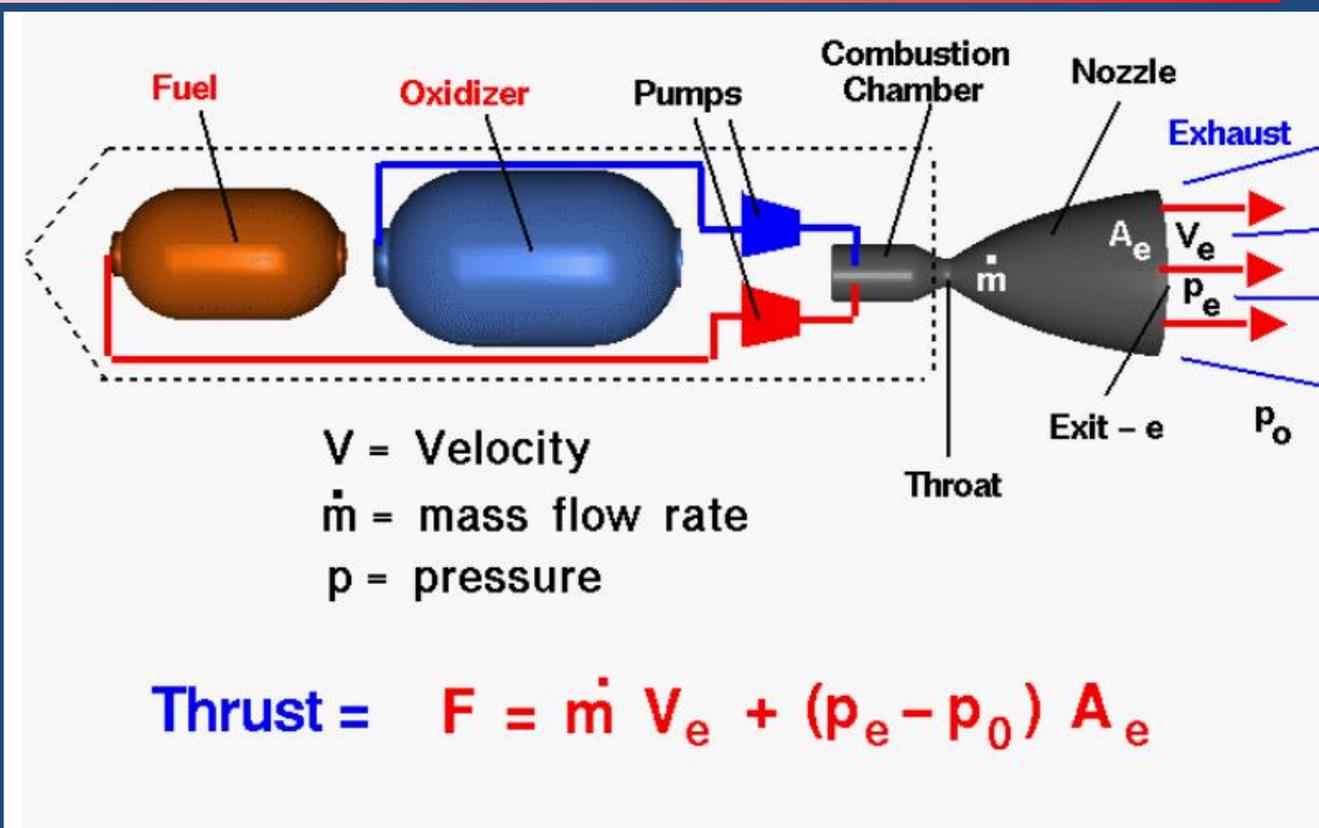


*Probability Density  
Function (like a  
continuous  
histogram) of  
Response*



# How a Rocket Engine Works, and why it needs Structural Dynamic Analysis

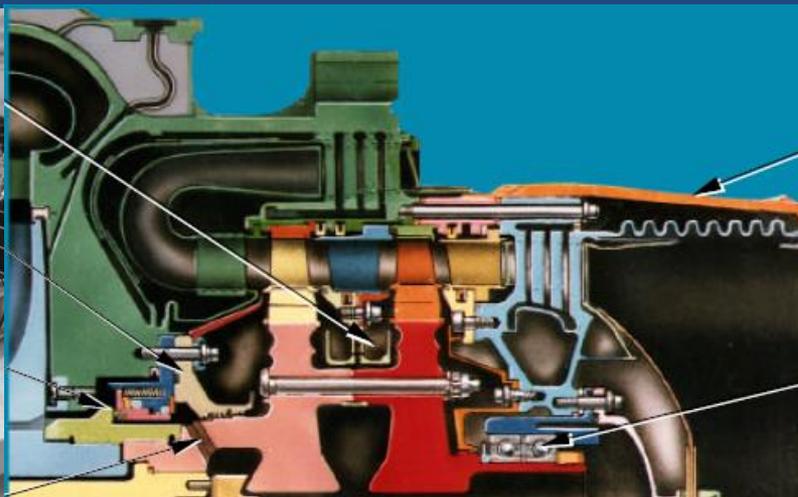
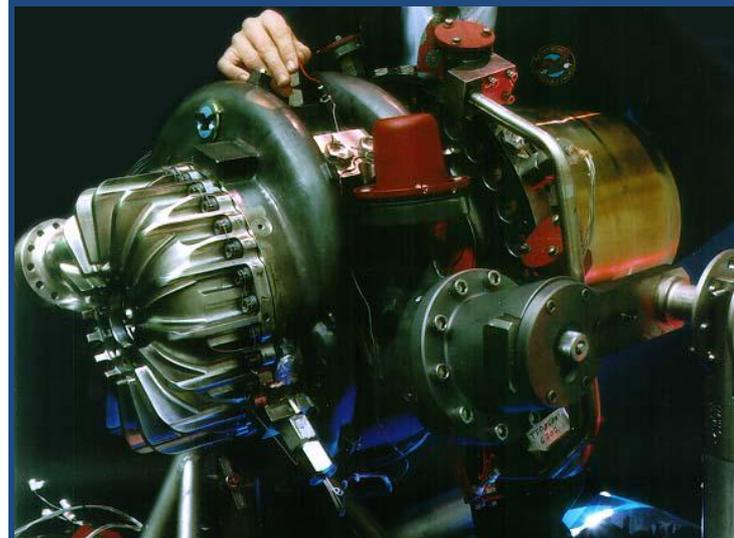
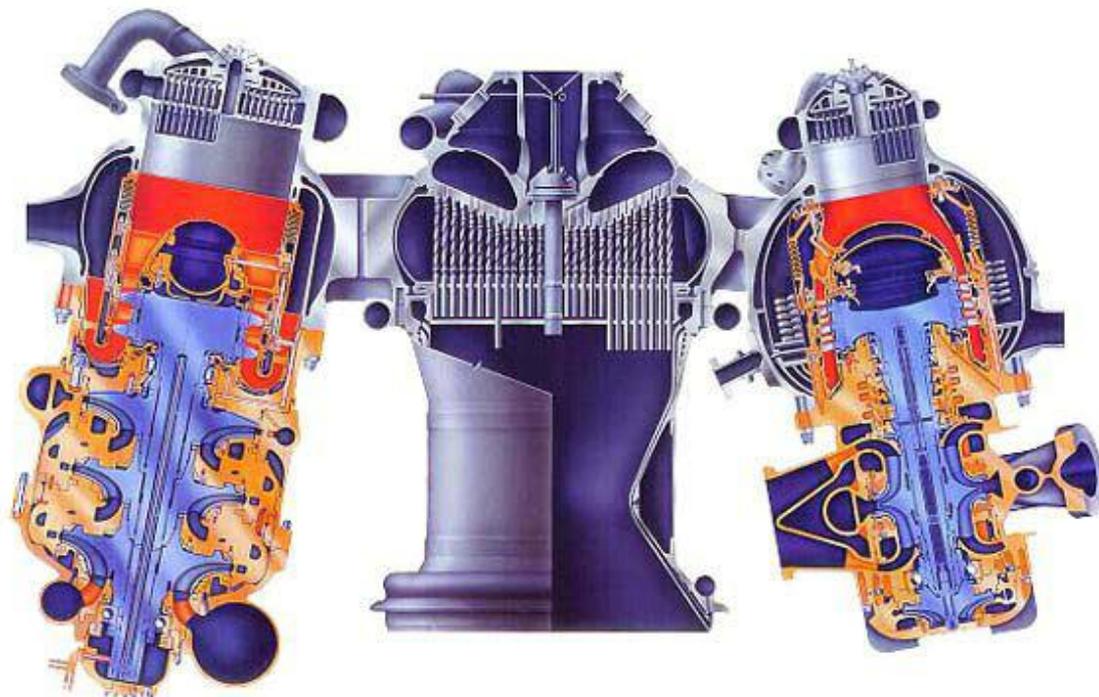
- Liquid Fuel (LH2, Kerosene) and Oxidizer (LO2) are stored in fuel tanks at a few atmospheres.
- Turbines, driven by hot gas created by mini-combustors, tied with shaft to pump, sucks in propellants and increases their pressures to several thousand psi, producing substantial harmonic forces at specific frequencies.
- High pressure propellants sent to Combustion Chamber, which ignites mixture with injectors, produces large forces in a wide band of frequencies, most of which are random.



- Hot gas directed to converging/diverging nozzle to give flow very high velocity for thrust.
- Both the random and the harmonic loads propagate through every component on the engine and last throughout engine operation.



# Structural Dynamics of Turbine Components in Turbopumps



Turbine Components (vanes, stators, blades) experience large harmonic excitations from up & downstream components, and multiples of these counts.



## Motivation is to Avoid High Cycle Fatigue Cracking

- Crack found during ground-test program can stop engine development
- If crack propagates, it could liberate a piece
  - At very high rotational speeds could be catastrophic (i.e., engine will explode)
  - Can cause large unbalance in rotor shaft, driving it unstable, causing engine failure.



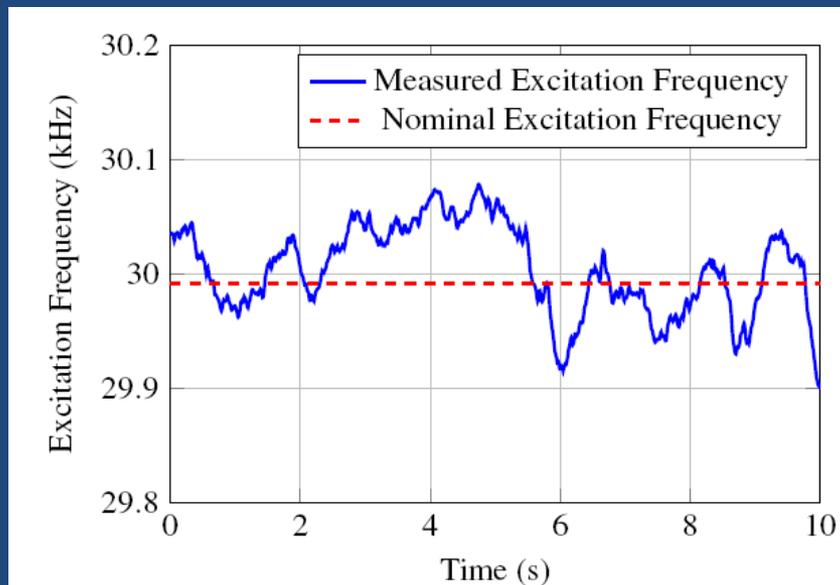


# Characterization of Excitations – Speed Range

- First obtain speed range of operation from performance group.
  - For Rocket Engines, there are generally several “nominal” operating speeds dependent upon phase of mission (e.g., reduce thrust during “Max Q”).
  - However, since flow is the controlling parameter, actual rotational speeds are uncertain (especially during design phase)
  - For new LPS engine being built at MSFC, assuming possible variation +/-5% about each of two operating speeds.

Rated Power Level	70%	100%
Low Range	20759.4	26125
Nominal	21852	27500
High Range	22944.6	28875

In addition, speed generally isn't constant, but instead “dithers”.<sup>†</sup>



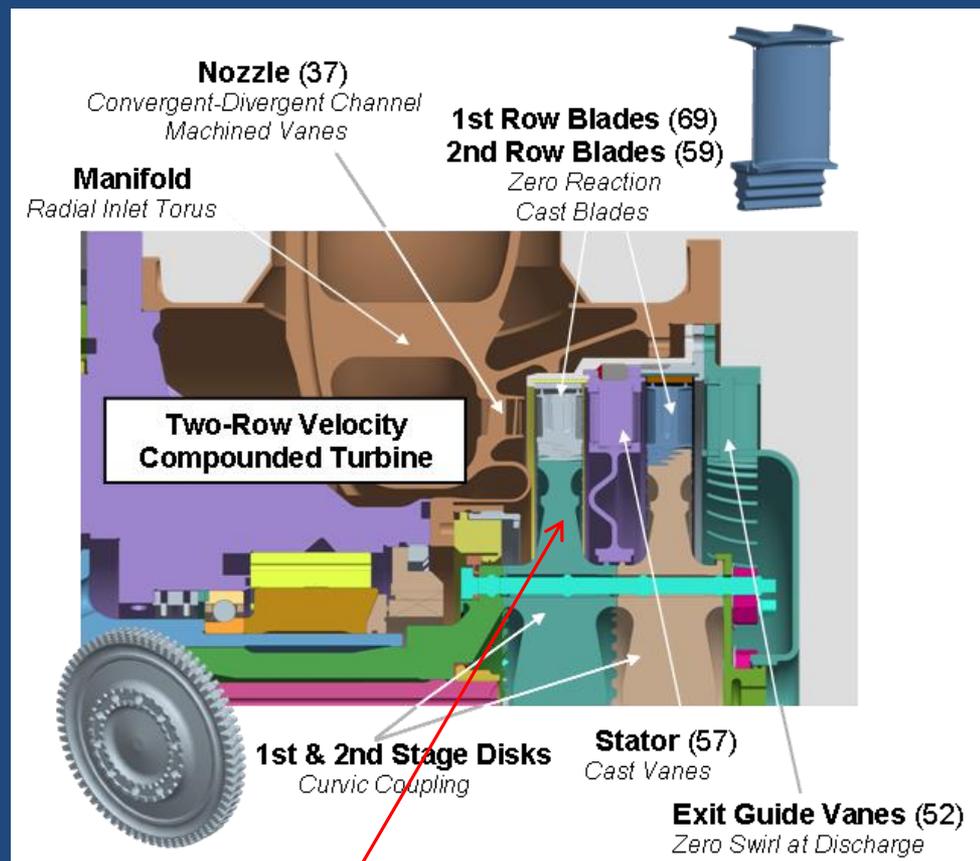
<sup>†</sup>Implementation of Speed Variation in the Structural Dynamic Assessment of Turbomachinery Flow Path Components

Andrew M. Brown, R. Benjamin Davis and Michael K. DeHaye  
*J. Eng. Gas Turbines Power* 135(10), 102503 (Aug 30, 2013)



# Characterization of Fluid Excitation

Sinusoidal excitations =  $(\text{speed } N) * j * d$   
- where  $d$  = Number of flow distortions arising from adjacent upstream and downstream blade and vane counts and “harmonics”  $j=1,2,3$



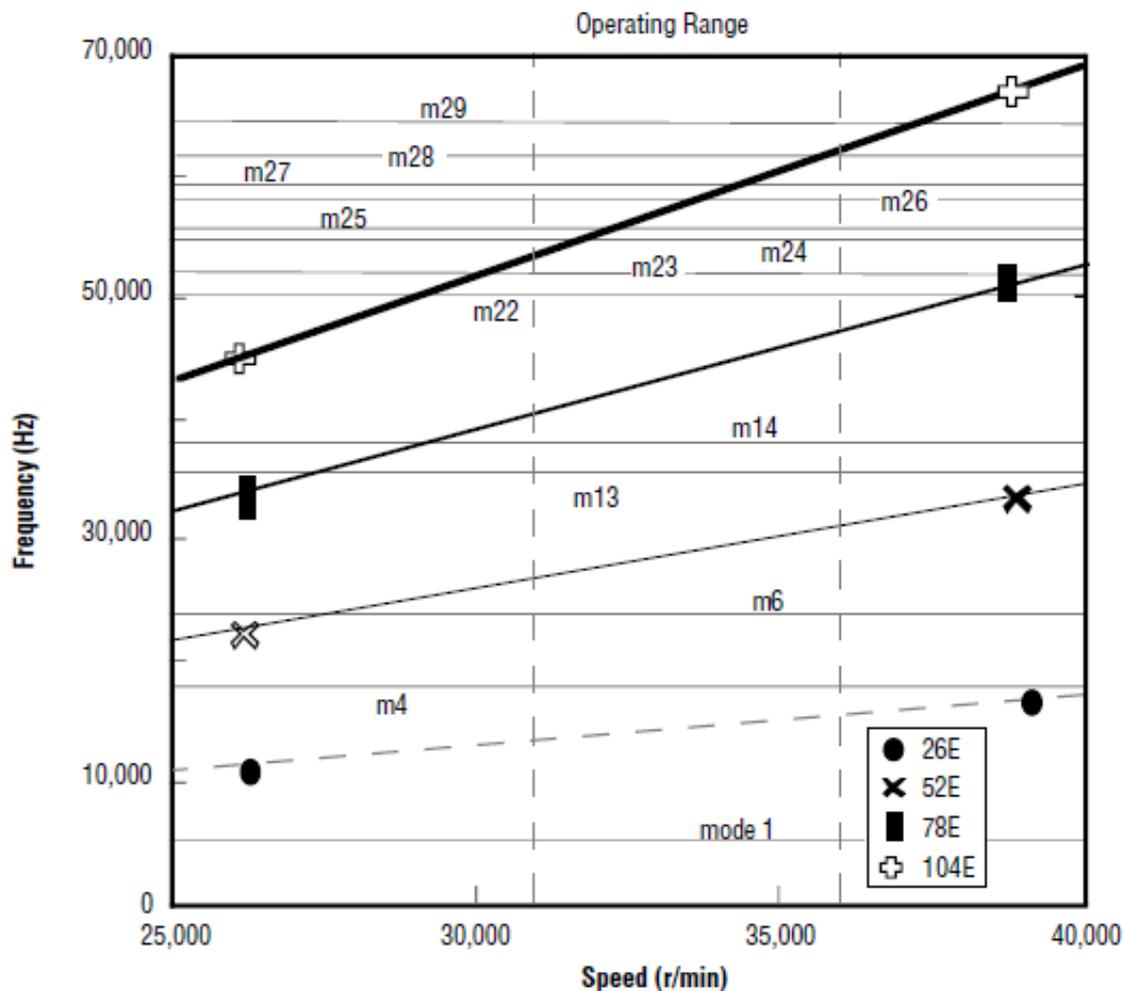
Bladed disk



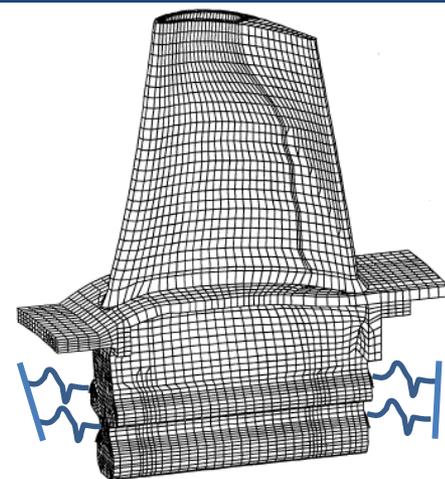


# Create "Campbell Diagram"

- Simplest Version of Campbell Diagram is just a glorified Resonance Chart.



SSME 1<sup>st</sup> Stage Turbine Blade



Here, disk not modelled, spring to ground boundary conditions applied.



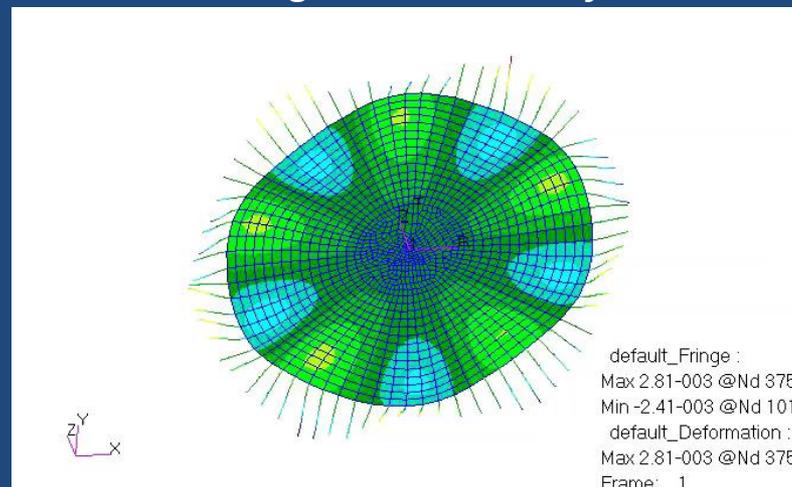
# Cyclic Symmetry and Matching of Nodal Diameter of Modes with Excitation Necessary Condition for Resonance

- For structures with repeating sectors, “Cyclic Symmetry” mathematical transformations enable generation of mode shape of entire structure at huge computational savings.
- These structures exhibit “Nodal Diameter” type modes.
- For disks and disk dominated modes, 5ND Traveling Wave will only excite a 5ND mode

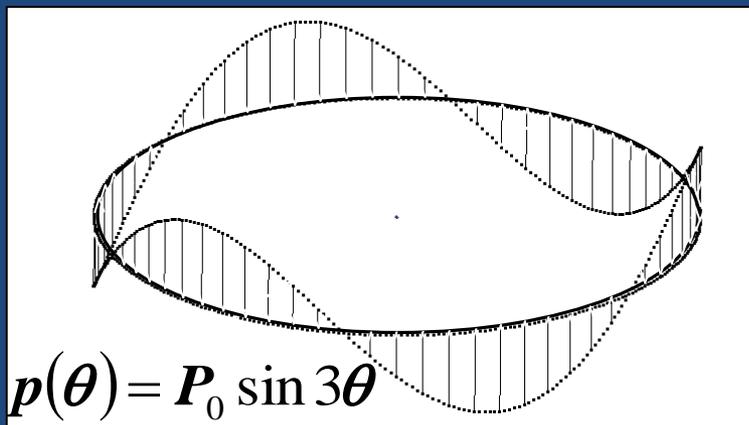


*5ND standing wave mode of Impeller (modal test using holography)*

*5ND travelling wave Mode of Bladed-Disc*



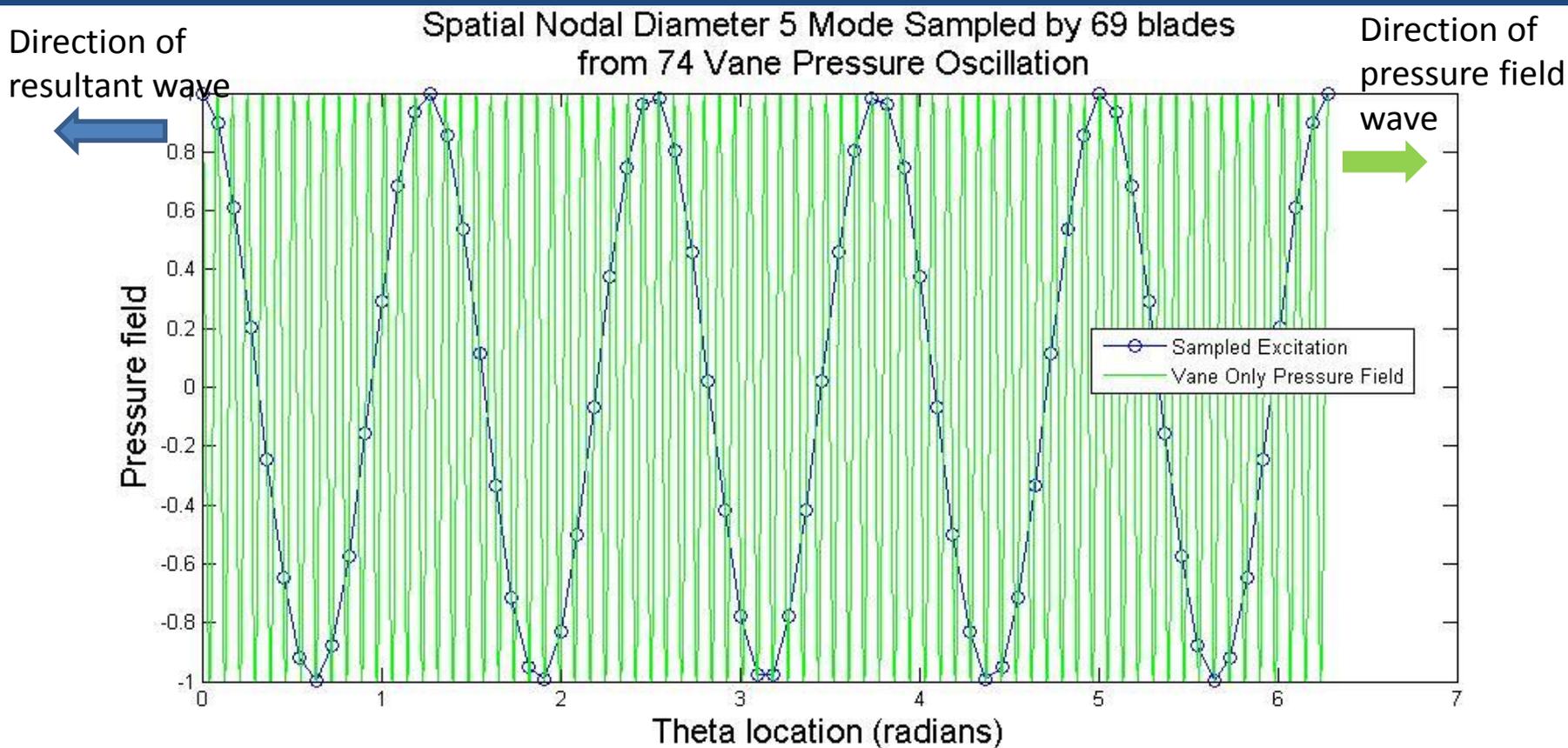
- On the other hand, 3ND excitation (perhaps from pump diffusers) will not excite a 5ND structural mode.





# “Blade/Vane” Interaction causes different ND excitation

- Sampling by discrete number of points on structure of pressure oscillation results in spatial Nodal Diameter excitation at the difference of the two counts.
- E.g., a 74 wave number pressure field (coming from 2x37 vanes), exciting 69 blades results in a Nodal Diameter mode of  $69-74=-5$ , where sign indicates direction of traveling 5ND wave (*plot courtesy Anton Gagne*).



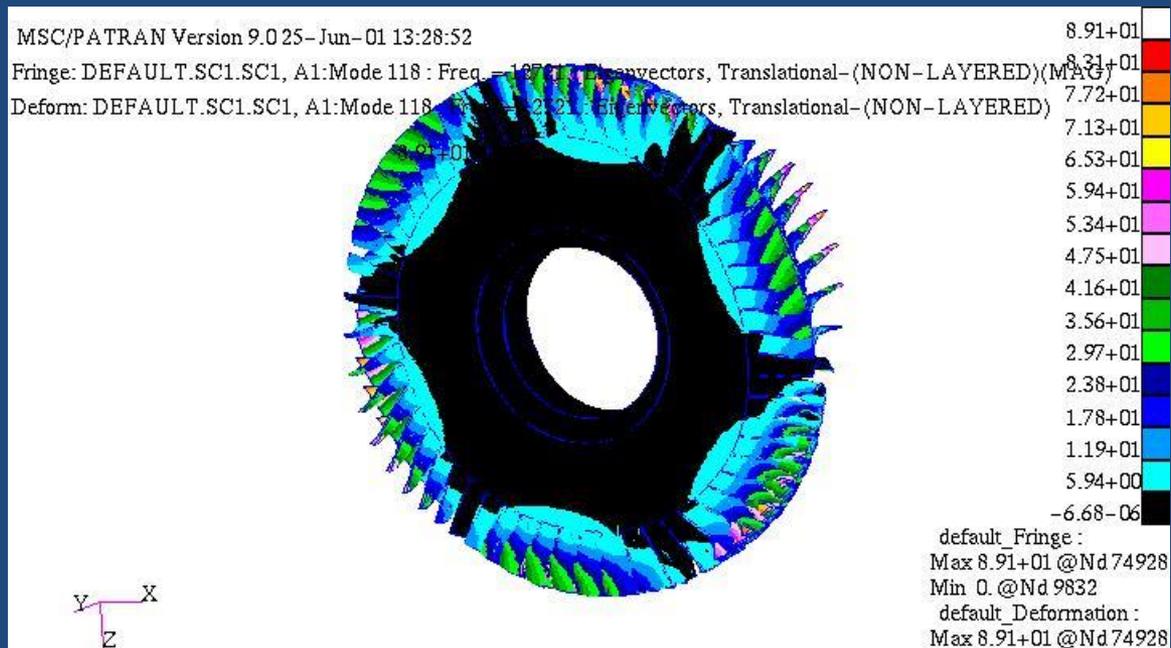


# For Cyclically Symmetric Structures with Coupling, Identification of Nodal Diameters in Modes Required

## Tyler-Sofrin Blade-Vane Interaction Charts

Upstream Nozzle						Downstream					
Multiples	37	74	111	148		Stator Multiples	57	114	171	228	
Blade multiples						Blade multiples					
	69	32	-5	N/A	N/A		69	12	N/A	N/A	N/A
	138	N/A	N/A	27	-10		138	N/A	24	-33	N/A
	207	N/A	N/A	N/A	N/A		207	N/A	N/A	N/A	-21

- 74N excites 5ND mode at 40,167 hz
- 4 revolution CFD analysis such that primary temporal Fourier Component  $F_0 e^{i\Omega t}$  has that frequency.





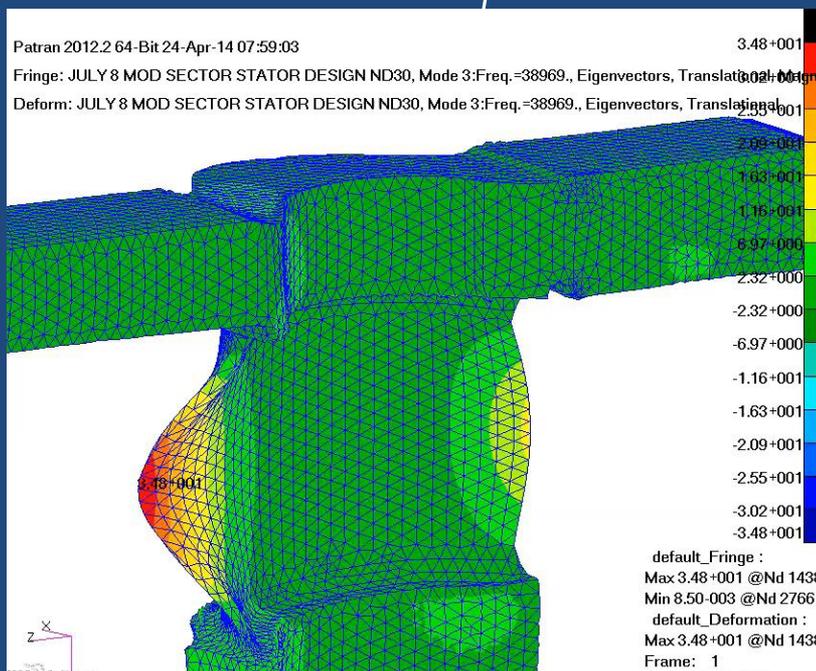
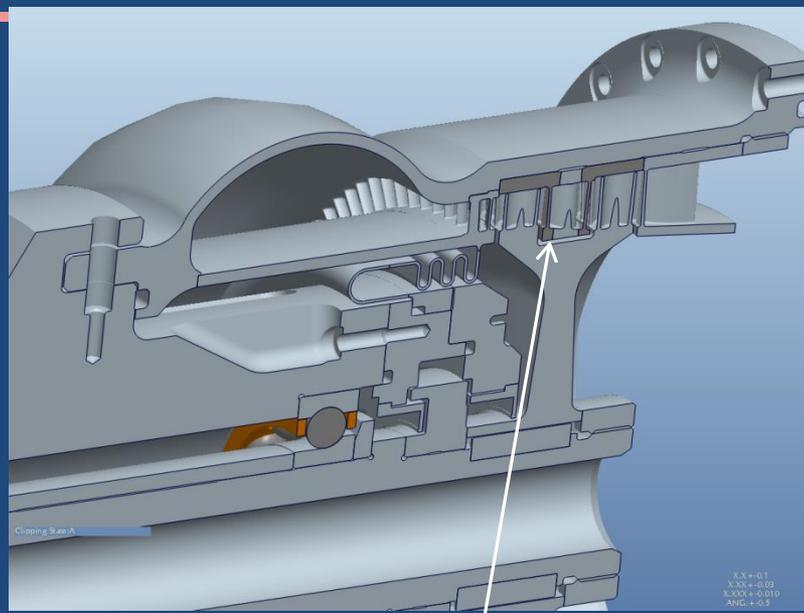
# Modal Analysis has Multiple Uses

- Redesign Configuration to move excitations ranges away from natural frequencies
- Redesign component to move resonances out of operating range.
- Put in enough damping to significantly reduce response
- Use as first step in “Forced Response Analysis” (applying forces and calculating structural response).



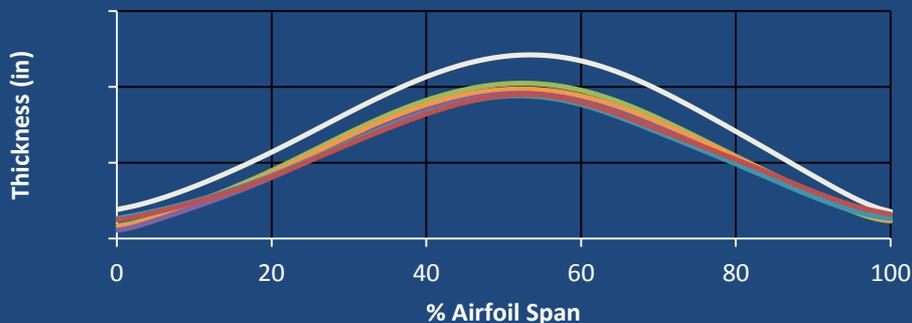
# LPSP Turbine Stator Redesign to Avoid Resonance

- Modal analysis of original design indicated resonance with primary mode by primary forcing function.
  - Since excitation simultaneously from upstream and downstream blades, critical to change design to avoid resonance.
  - Extensive optimization effort performed to either move natural frequency out of range and/or change count of turbine blades to move excitation.



## Stator Airfoil Thickness Changes

- Initial (R02)
- R03b2
- R03b2\_t2
- R03b2\_t4
- R03b2\_t5
- R03b2\_t6

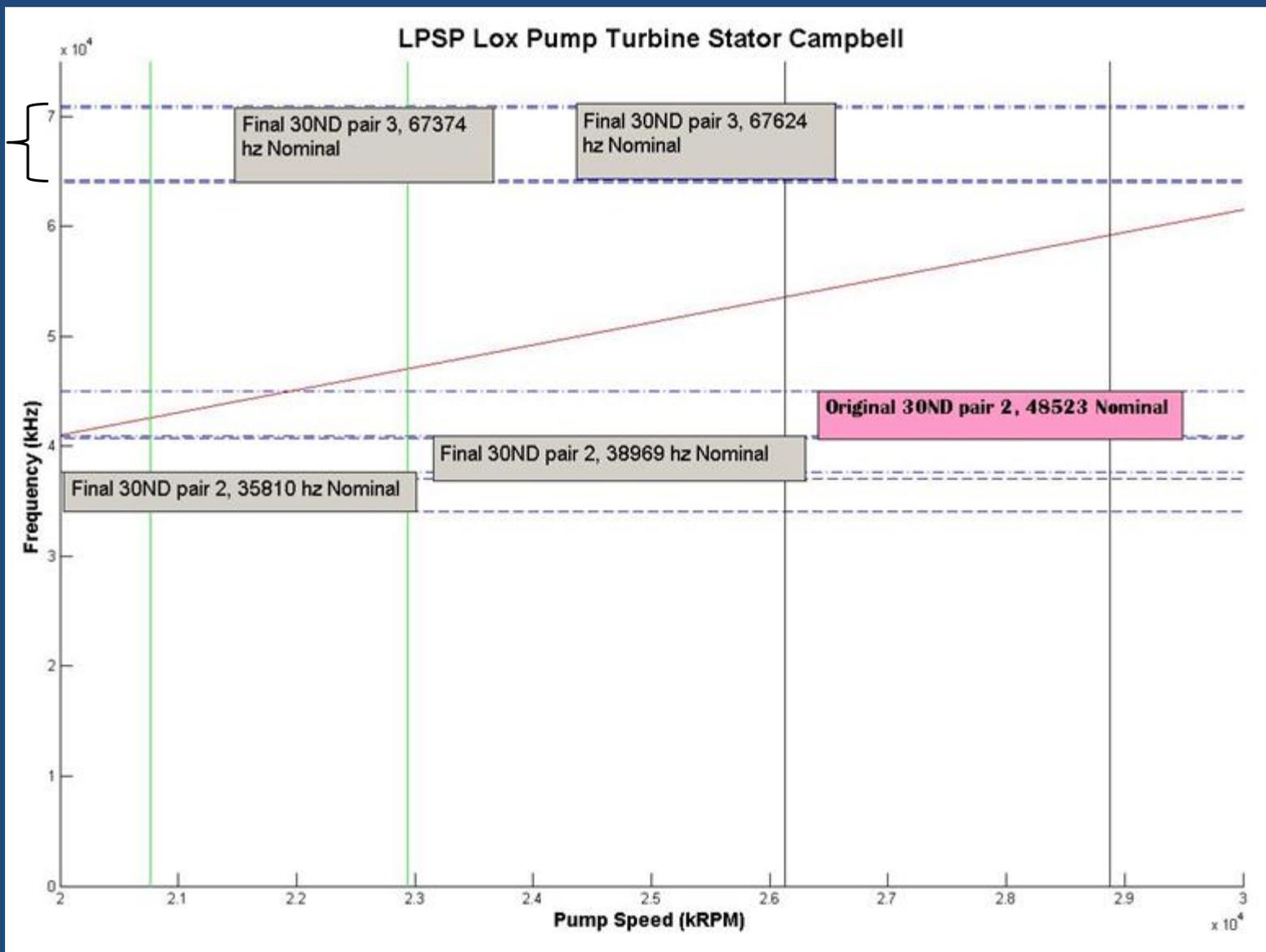


Courtesy D. O'Neal



# Final and Original Campbell of Modes for Stator Vane 30ND Family

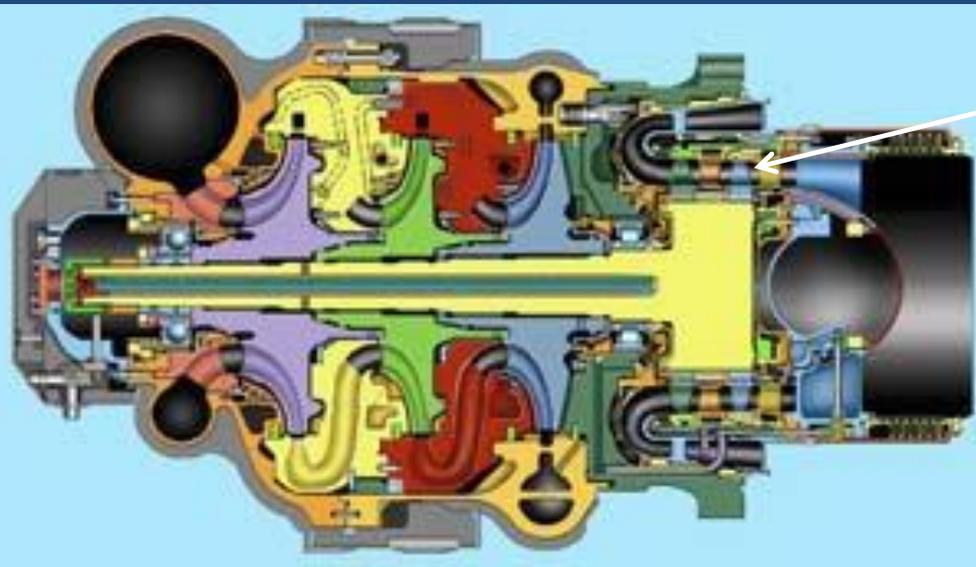
Range of +/- 5% on natural frequencies to account for modeling uncertainty



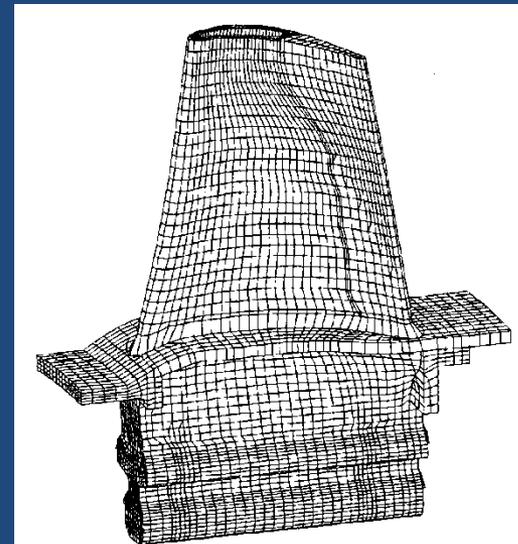


# Can Also Use Modal Analysis in Failure Investigations

- Examination of **Modal Stress** Plots provides link to location of observed cracking.



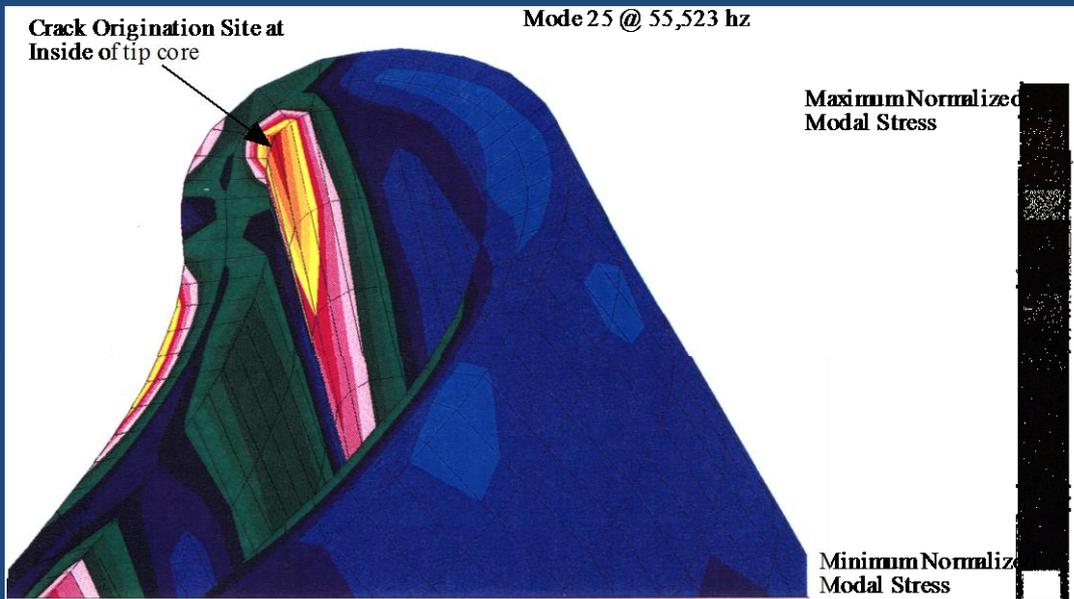
SSME  
HPFTP  
1<sup>st</sup> Stage  
Turbine  
Blade



Modal displacement

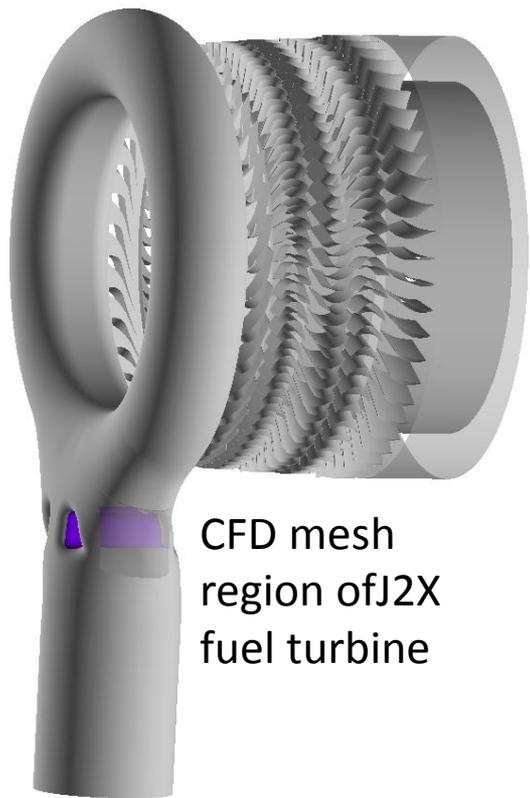
Modal stress

$$\phi^m = \begin{Bmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_N \end{Bmatrix}^m \rightarrow \phi_\sigma^m = \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \vdots \\ \sigma_N \end{Bmatrix}^m$$



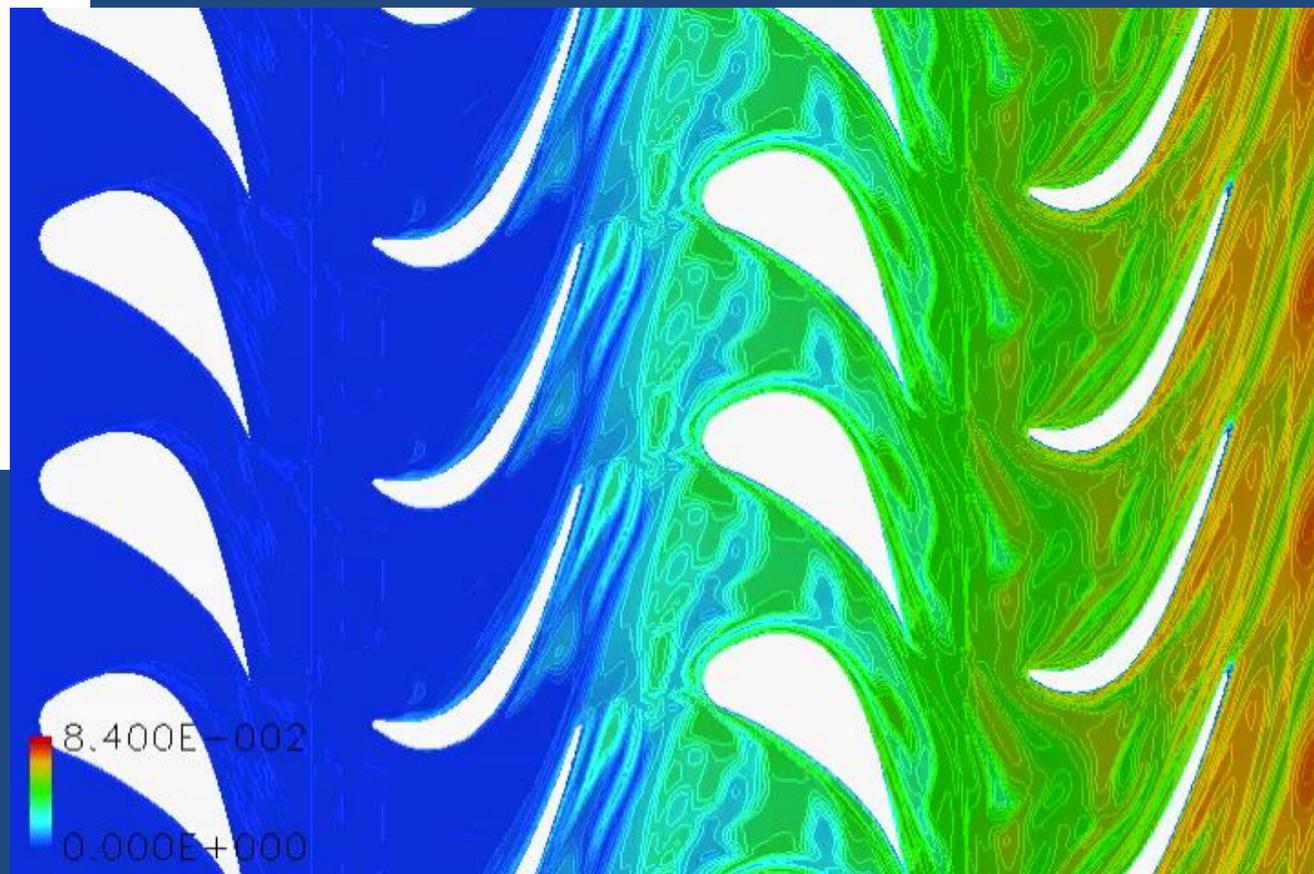


# If Forced Response Analysis Required, need Forcing Function on Turbine Components from CFD



CFD mesh region of J2X fuel turbine

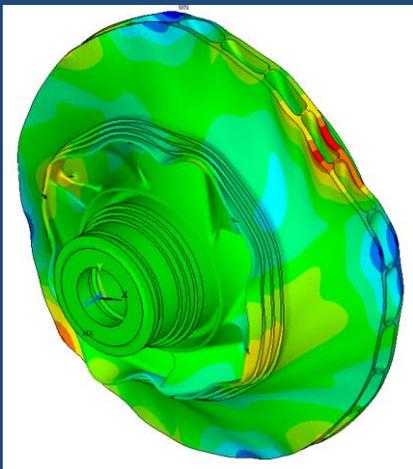
Ref: Wikipedia



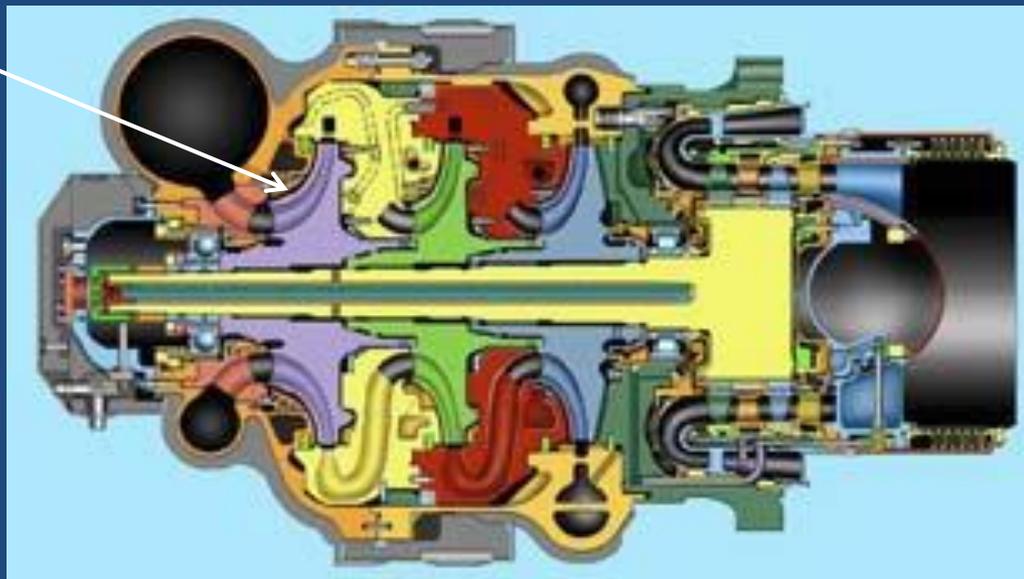


# Forced Response Analysis in Failure Investigations

- SSME HPFTP 1<sup>st</sup> Stage Impeller.

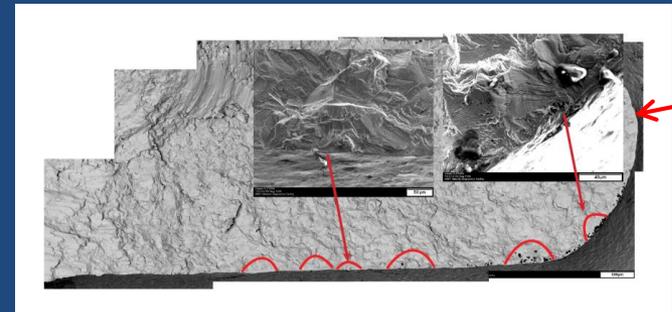
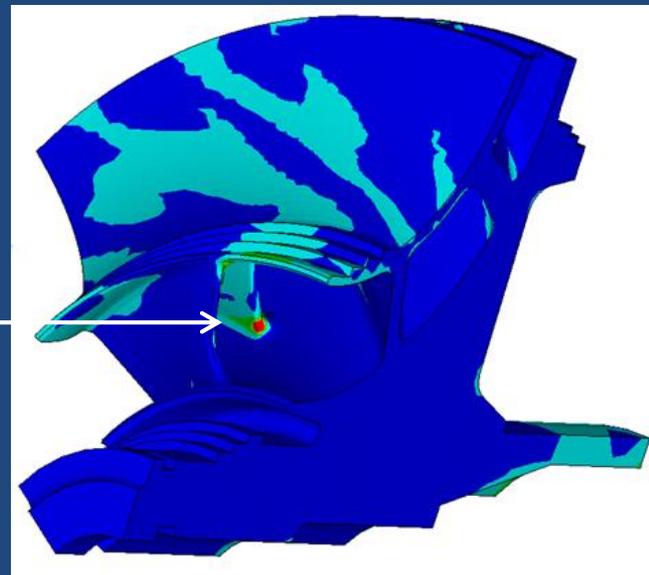
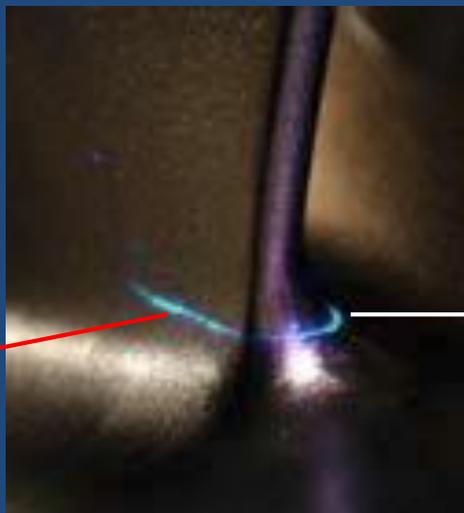


Mode shape



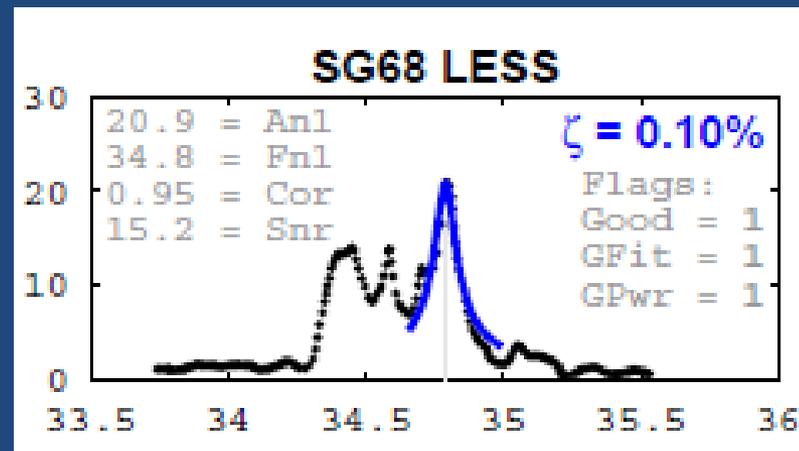
Crack location 1<sup>st</sup> splitter

Frequency Response Analysis



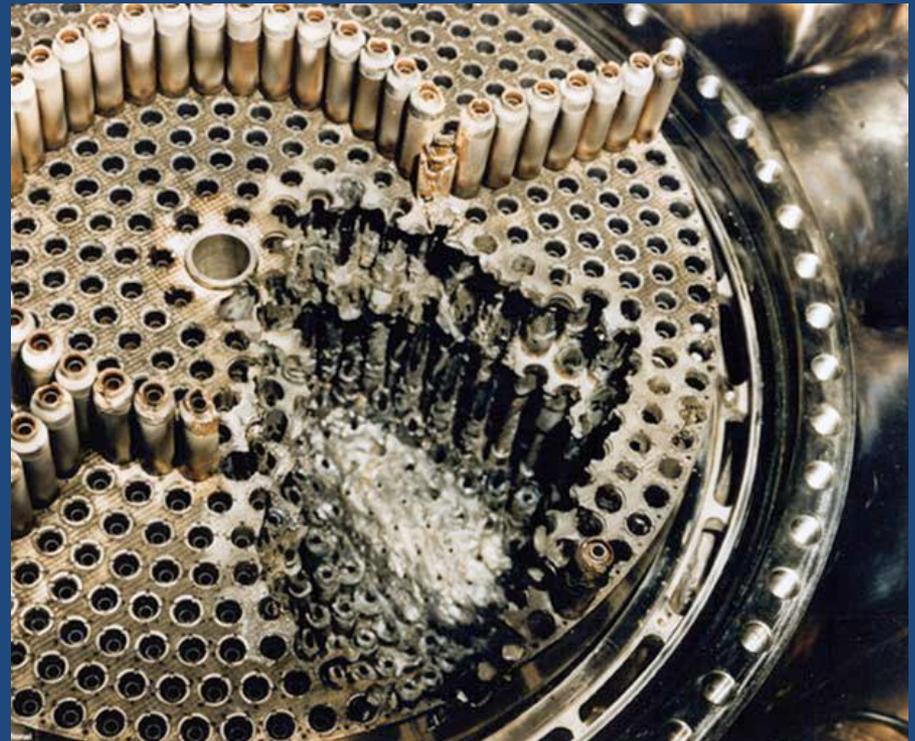
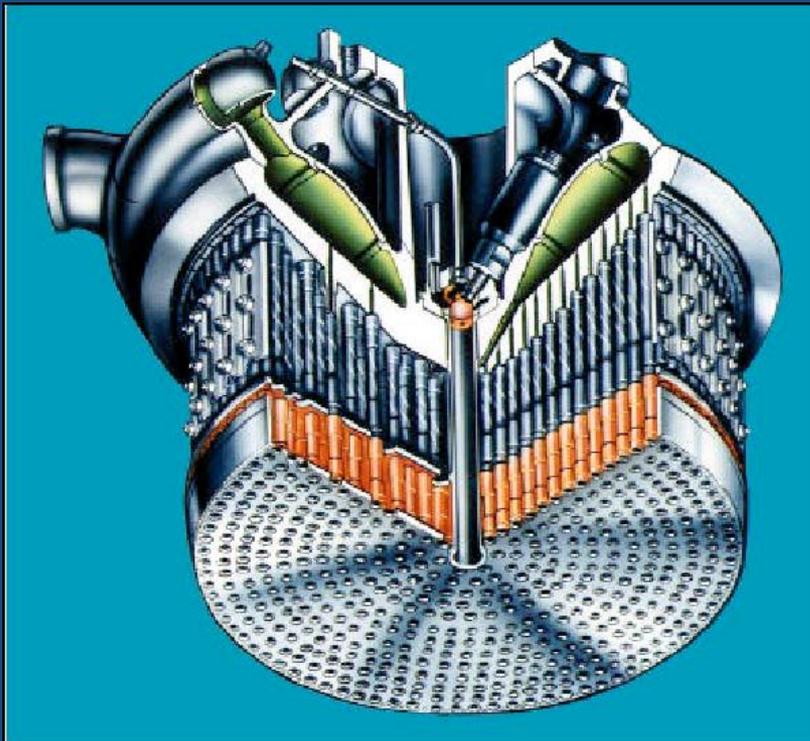
- Damping is critical parameter for forced response prediction, so “whirligig” test program used to obtain data.
- Whirligig is mechanically-driven rotor with bladed-disk excited by pressurized orifice plate simulate blade excitation.
- Key assumption is that this reflects true configuration.

- SDOF Curve fit technique applied to selected top-responding blades to derive damping from response.

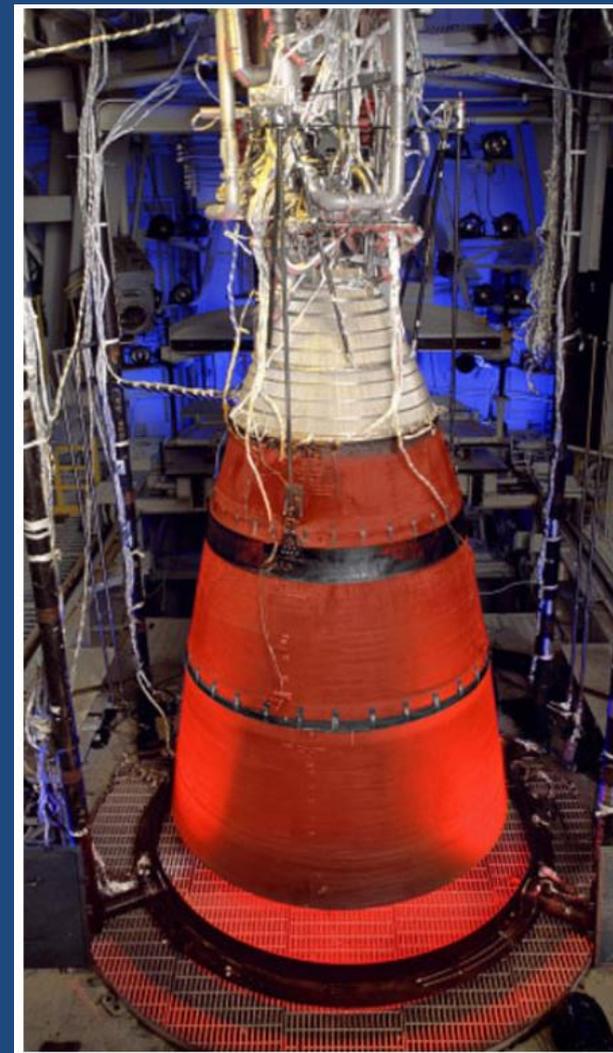


## Injectors in Main Combustion Chamber

- Test of SSME in 1981 failed due to burn through of 149 liquid oxidizer Injectors , caused by high-cycle fatigue cracking.
- Failure investigation showed design insufficient to withstand huge random load caused by combustion and flow induced vibration from hot gas (flow by cylinders causing vortex shedding).



- Nozzle is a major portion of the overall dynamics of the engine, frequently the structural backbone if components mounted onto it.
- Accurate assessment of Nozzle response critical for evaluating both HCF and Ultimate.
- Nozzle material complex
  - Tube-wall construction filled with liquid hydrogen
  - Graphite phenolic composite, Young's Modulus can be highly temperature dependent
  - Exotic high temperature metals, still close to melting.
- Upper Stage Engine Designs can be unusual to allow for optimization during ascent
  - RL10B has extension that stows until deployed; undeployed configuration has very active modes that were challenge to prove ok during ascent.



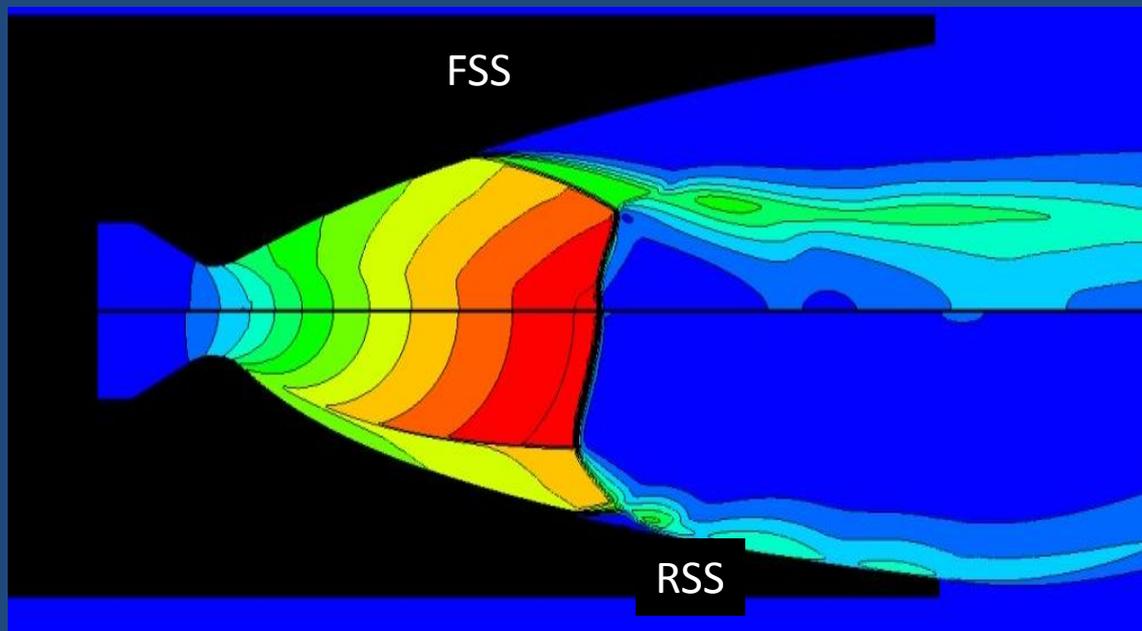
*Ref: Impact of Dynamics on the Design of the RL-10B-2 Extendible Carbon-carbon Exit Cone, Mary Baker et.al, 1998*



# “Side Loads” in Rocket Nozzles is Major Fluid/Structural Dynamic Interaction Issue

- Start-up, shut-down, or sea-level testing of high-altitude engines, ambient pressure higher than internal nozzle wall pressures.
- During transient, pressure differential moves axially down nozzle.
- At critical  $p_{\text{wall}}/p_{\text{ambient}}$ , flow separates from wall - Free Shock Separation (FSS), induces “Side Load”.
- Flow can reattach to wall - Restricted Shock Separation (RSS).
- RSS generally larger than FSS.
- Primary Nozzle Failure Mode for most Rocket Engines is Buckling due to Side Loads during Start-Up and Shut-Down

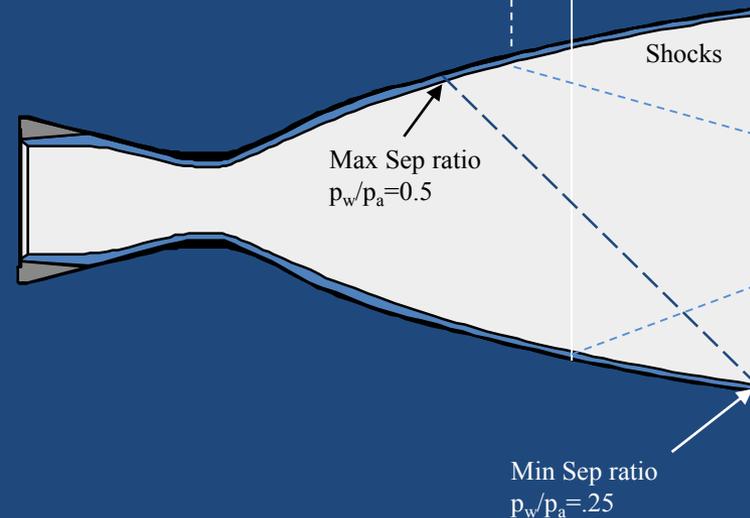
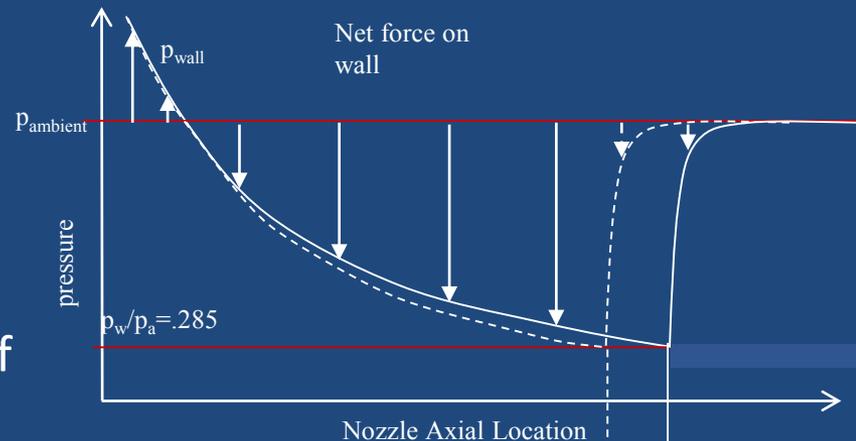
Mach number simulation from CFD





# Calculation of Dynamic Forcing Function

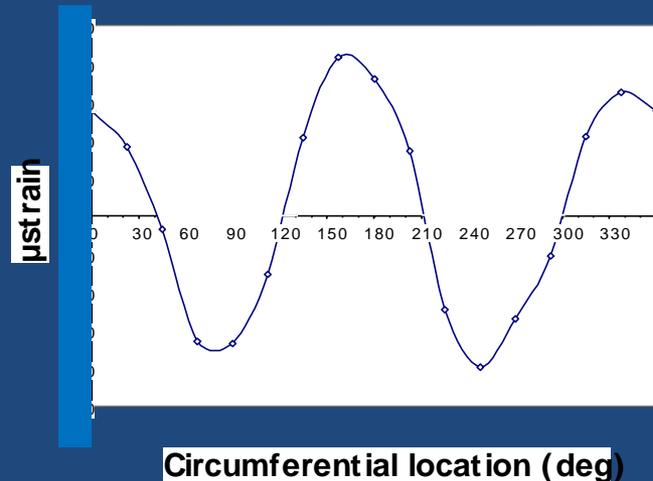
- Boundary layer separation of low-pressure internal fluid flow from inner wall of nozzle
- In-rushing ambient pressure at uneven axial locations causes large transverse shock load
- Caused failures of both nozzle actuating systems (Japanese H4 engine) and sections of the nozzle itself (SSME).
- Existing Side Load calculation method
  - Assumes separation at two different axial stations, integrates the resultant  $\Delta P \cdot dA$  loads.
- Method calibrated to maximum and minimum possible separation locations to be intentionally conservative.



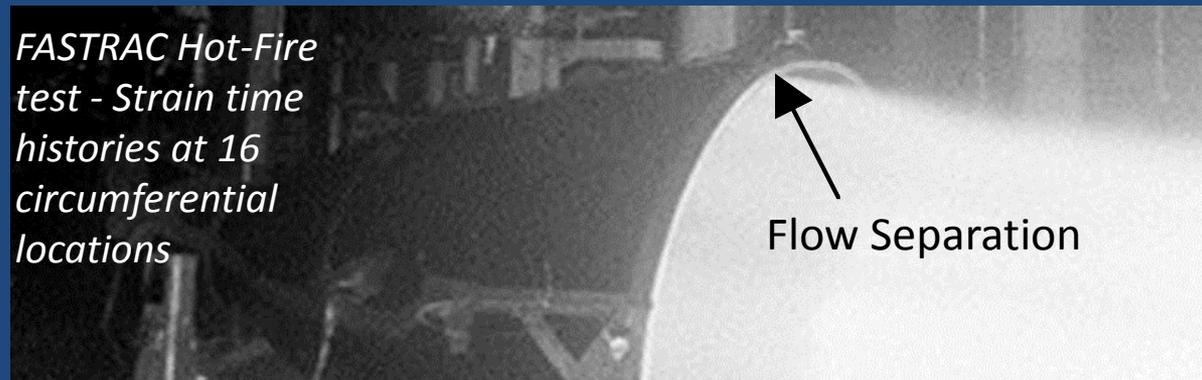


# MSFC Side Loads Research Program - 1998

- FASTRAC engine designed to operate in overexpanded condition during ground test.
- Didn't have funding to pay for vacuum clamshell.
- Test/analysis program initiated with goal of obtaining physics-based, predictable value.
- Strain-gauge measurements taken on nozzle during hot-fire test
- Flow separation clearly identified at Steady-State Operation.

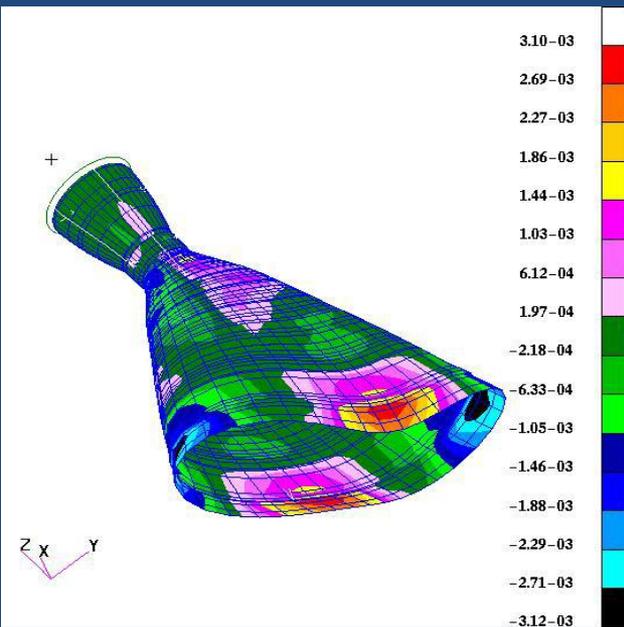
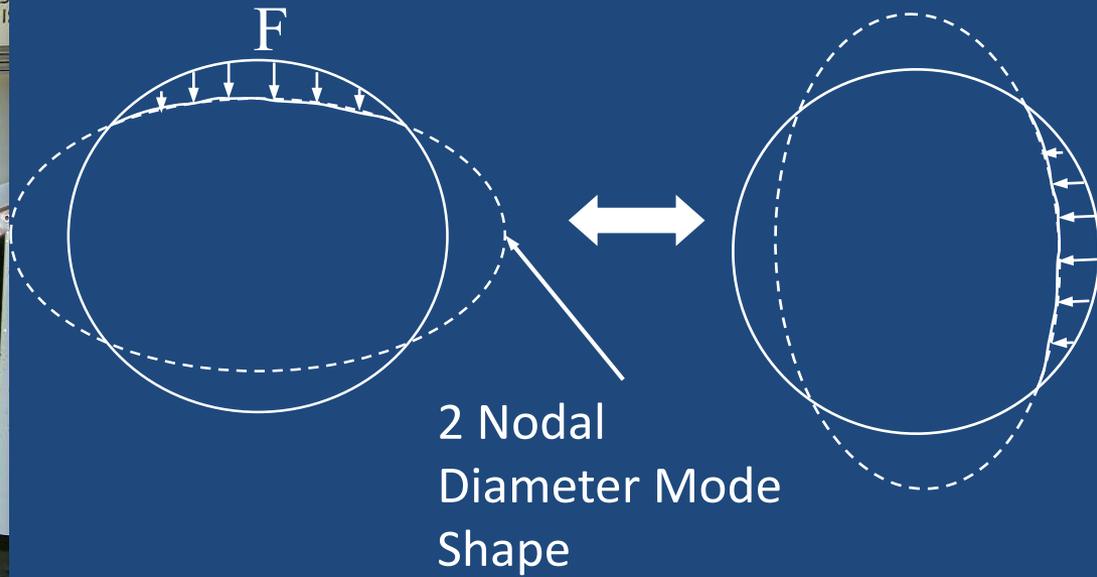
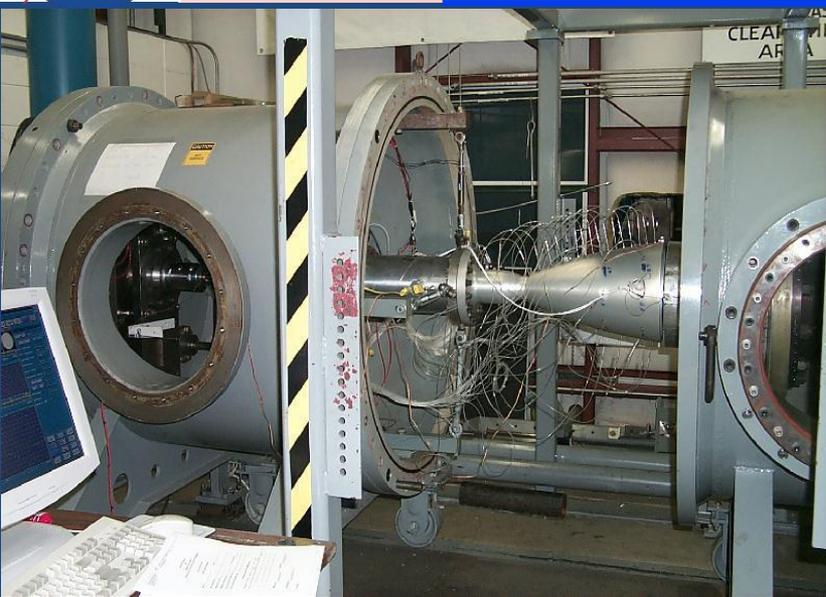


*FASTRAC Hot-Fire test - Strain time histories at 16 circumferential locations*

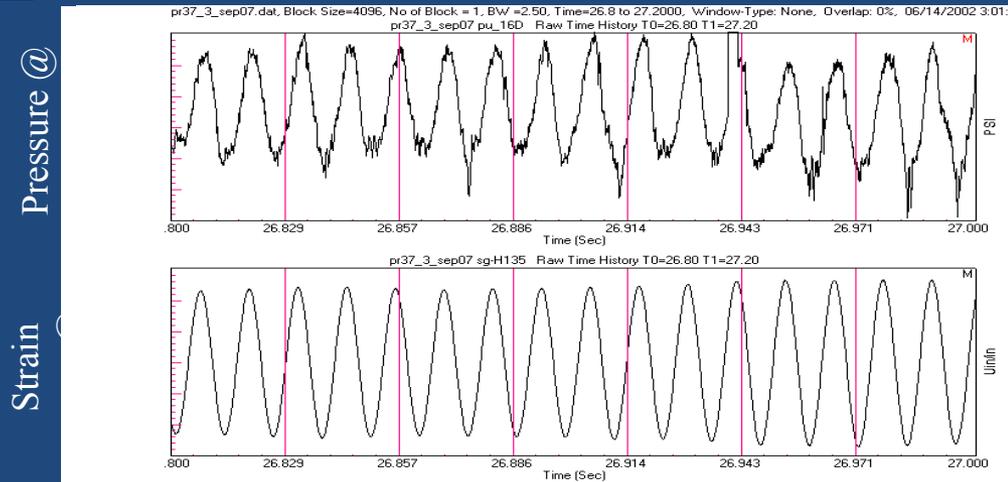




# Designed Cold-Flow Sub Scale Tests to investigate Fluid/Structure Interaction & Feedback during Steady-State Separation.



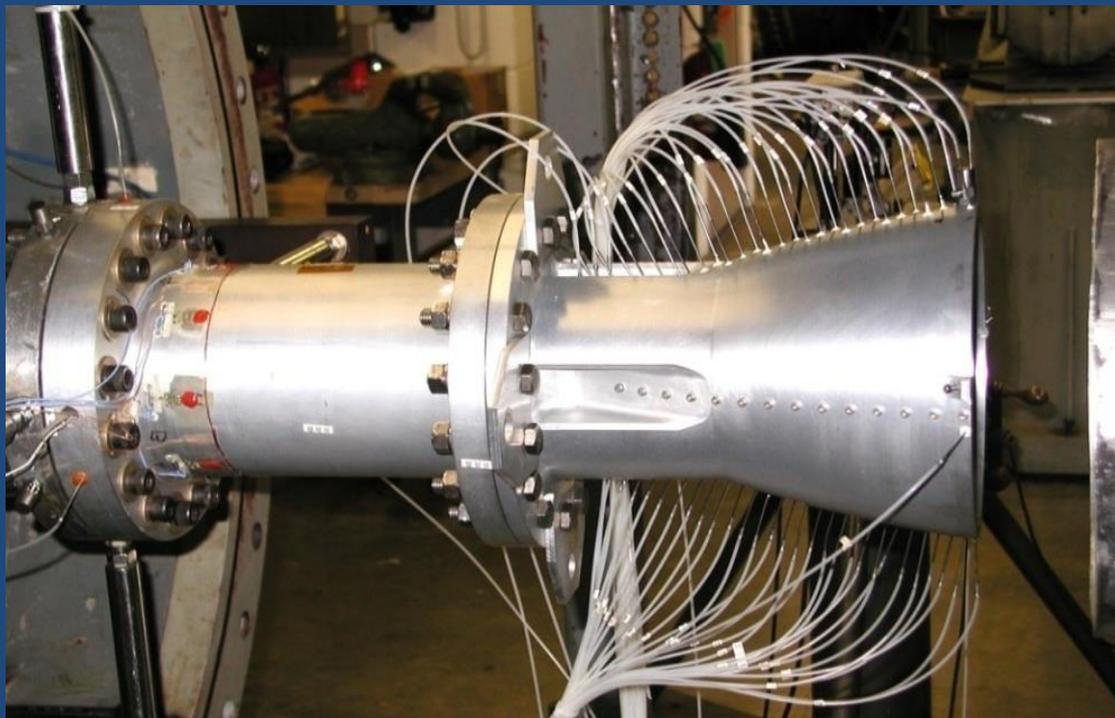
- Video, Pressure and strain-gage data from thin-wall nozzle show self-excited vibration loop tying structural 2ND mode and flow separation.





# Follow-on Testing to Measure Magnitude of Side Loads

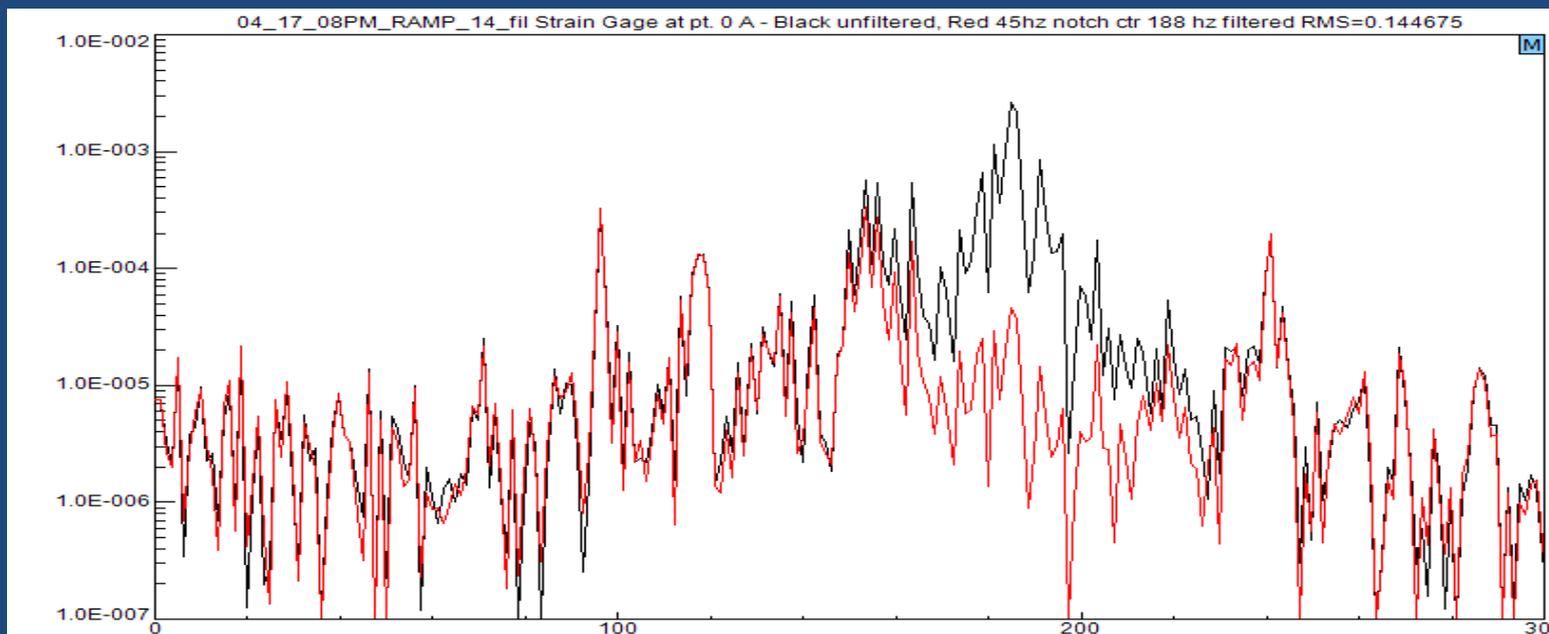
- Cold-flow air through sub-scale nozzles replicates non-dimensional fluid parameters; Expansion Ratio,  $NPR = p_{\text{chamber}} / p_{\text{ambient}}$ .
- Simply measuring forces using pressure transducers impractical.
- “Side Load” measurement setup based on Frey, et.al., 2000, consisting of very stiff nozzle (“lumped mass”) attached to flexible “strain tube”.
- Accelerometers and pressures measured in nozzle, strains measured on strain tube.
- Hypothesis: system is SDOF, measure response, back-calculate forcing function FRF. (*“Easier said than done”*)





# Generation of Reduced Side Loads Estimate for TIC Nozzle

- Modal test performed on system to generate Transfer Function  $H(f)$ . Acceleration PSD's on nozzle tip measured during test.
- For SDOF,  $[S_{\ddot{x}\ddot{x}}(f)] = [H^*(f)][S_{F_{gen}F_{gen}}(f)][H(f)]^T$
- Filter out resonantly amplified spectra for each nozzle contour, SL based on remaining signal. I.E., just use the loading at non-resonant frequencies .



- The “Ruf-Brown Knockdown Factor” ☺

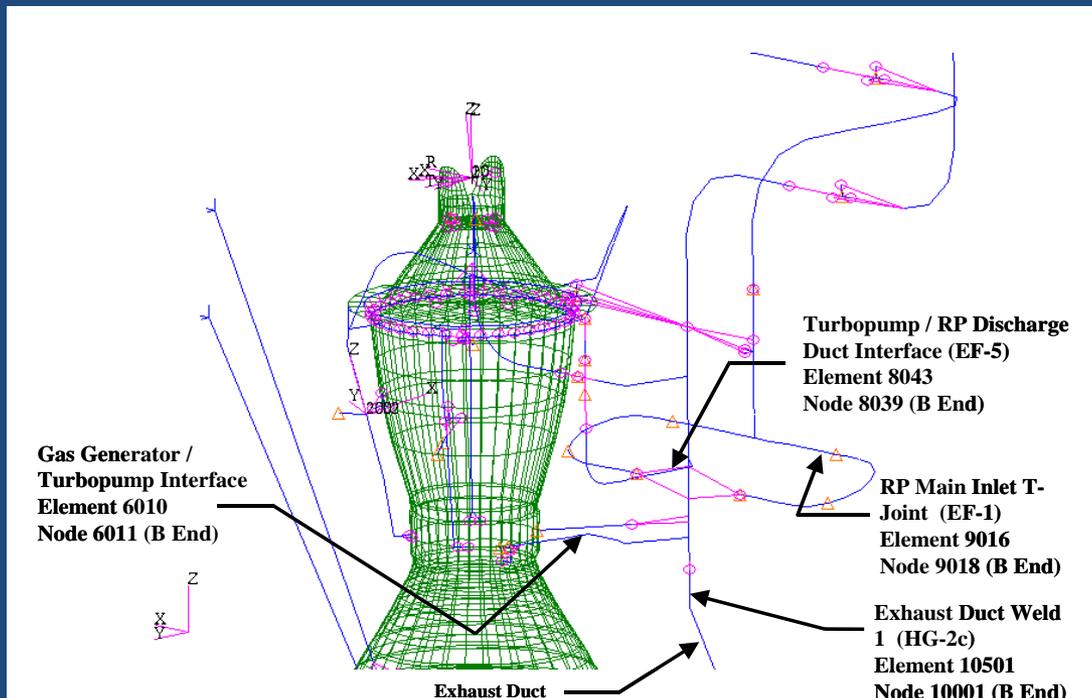
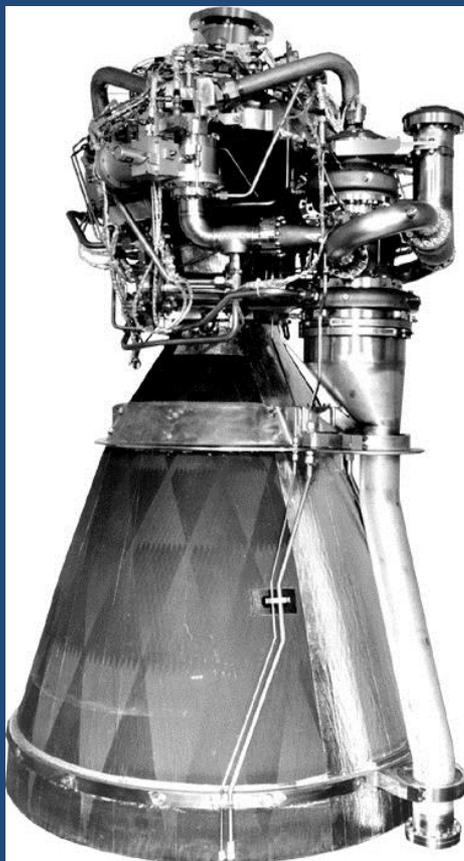
$$SL(tic) = .8 * SL(toc)$$

Calculate factor based on measured “static side loads” that have dynamically amplified portion filtered out.



# Engine System Structural Dynamic Loads

Random and the harmonic loads propagate through every component on the engine and last throughout engine operation, so Engine System Model required.

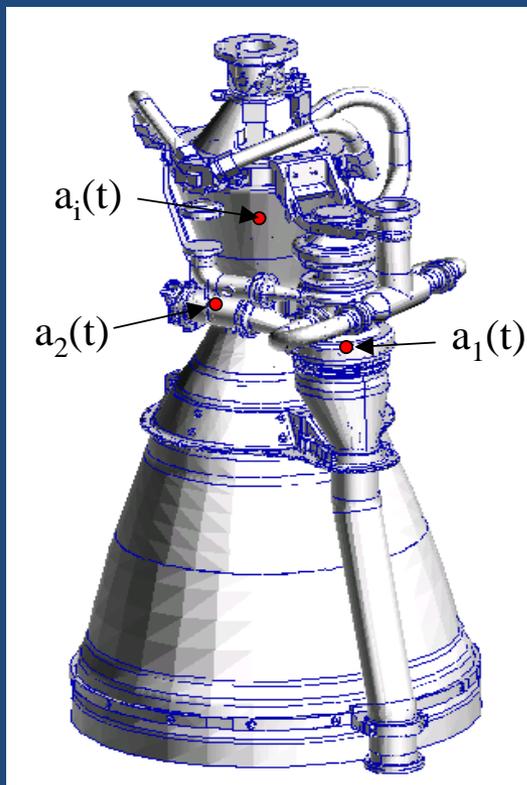




# Engine Dynamic Mechanical Loads

## Engine Self-Induced Loads

- Forces acting on engine result from extremely complex processes: combustion pressures, fluid flow, rotating turbomachinery
- For steady-state operation there are two types of dynamic environments: sinusoidal (resulting from turbomachinery) and random, which typically dominate.
- With current level technology, impracticable to quantify these forces with enough precision to conduct a true transient dynamic analysis.



- However, we can measure the engine dynamic environment (i.e., accelerations) at key locations in the engine near primary vibration sources
- For a new engine, data from “similar” previous engine designs is scaled to define an engine vibration environment.

*Some content of this section courtesy Dr. Eric Christensen, DCI Inc.*



# Definition of Dynamic Environment

- Frequency Content and Characteristics, Amplitude, and Location of Dynamic Environment needed for initial design and as revised during life of program.
- For Random Component, structure-borne vibration from combustion devices is mechanically induced, so Initial Environment based on similar previous engines using “Barrett Criteria”

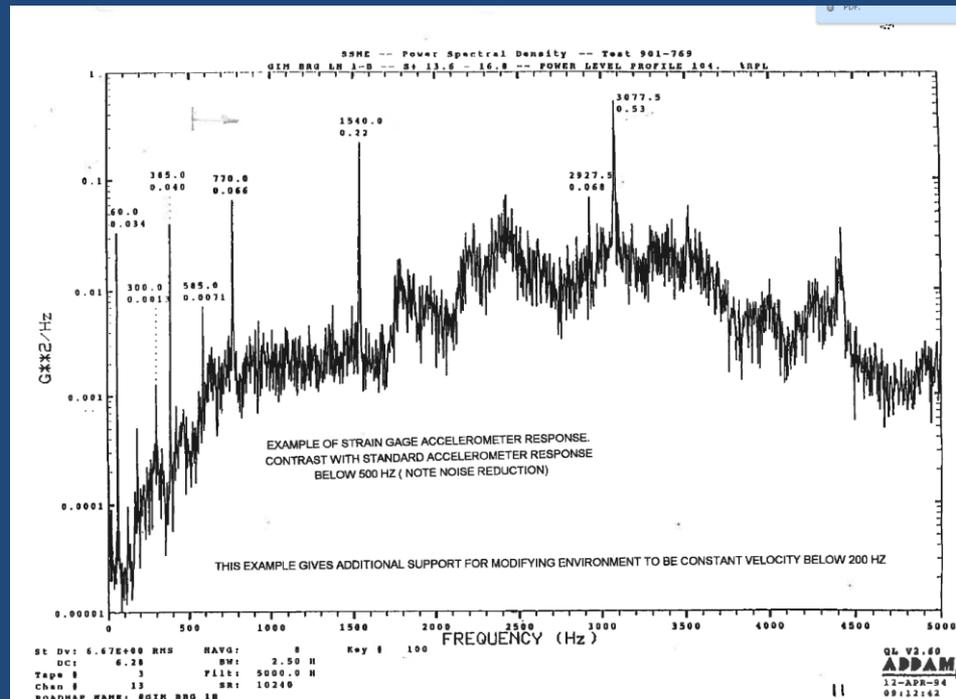
$$G_n(f) = G_r(f) \frac{N_n T_n V_n W_r}{N_r T_r V_r W_n} F$$

- $W_n$  = weight of new structure
- $W_r$  = weight of reference structure
- $N_n$  = number of engines on stage of interest for new structure
- $N_r$  = number of engines on stage of interest for reference structure
- $T_n$  = thrust of engines on stage of interest for new structure
- $T_r$  = thrust of engines on stage of interest for reference structure
- $V_n$  = exhaust velocity of engines on stage of interest for new structure
- $V_r$  = exhaust velocity of engines on stage of interest for reference structure



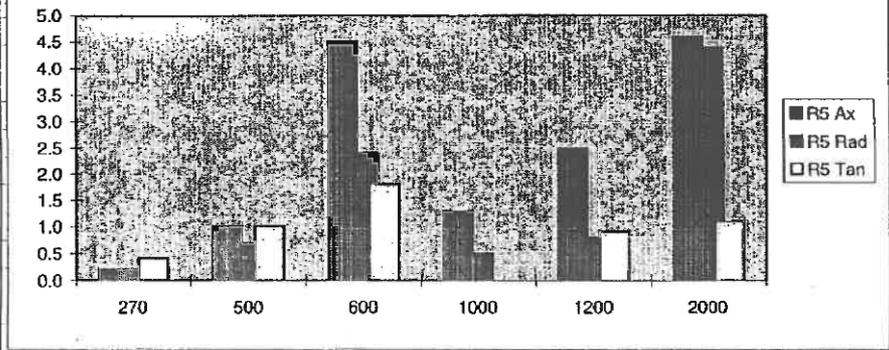
# Data Used to Revise Environment

- Accelerometer measurements taken during hot-fire testing of engine used to update or create environments.
- Specification created by “enveloping” this data.
- For engines with multiple sources of excitation (thrust chamber, turbomachinery), different excitation criteria used for each “zone”.



	Frequency	R5 Ax	R5 Rad	R5 Tan
	270	0.2	0.2	0.4
	500	1.0	0.7	1.0
	600	4.5	2.4	1.8
	1000	1.3	0.5	
	1200	2.5	0.8	0.9
	2000	4.6	4.4	1.1

Sinusoidal Dwells (R5 Zone B)

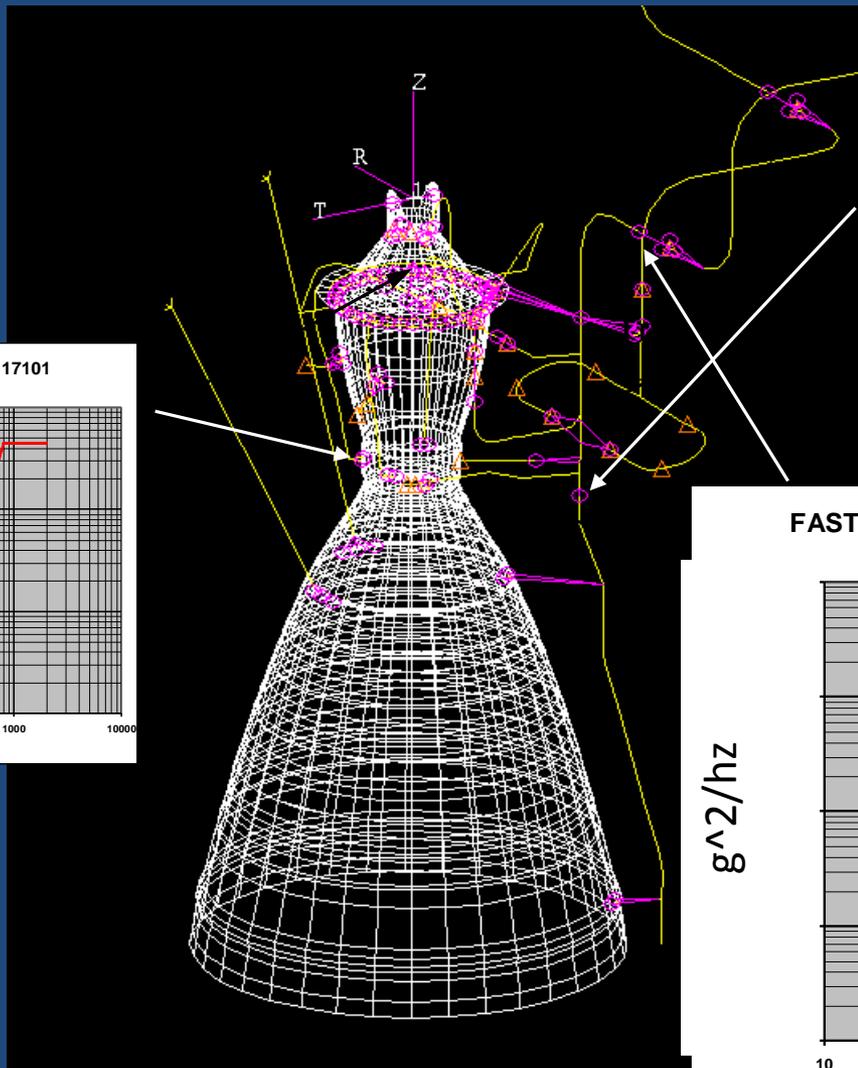


- Harmonic excitation obtained by taking peaks from overall data signal, then calculating the RMS of the sine using the PSD magnitudes of the peak & adjacent bins.

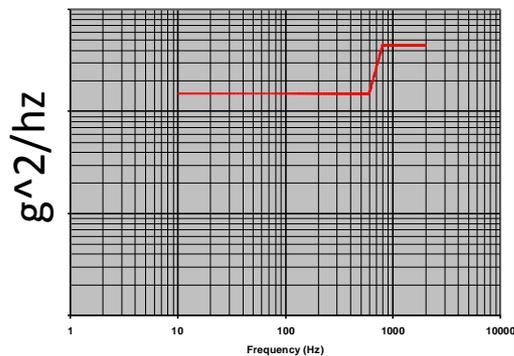
$$g_{rms} = \sqrt{bw(A_1 + A_2 + A_3)}$$

# Specified Environment at Different Zones

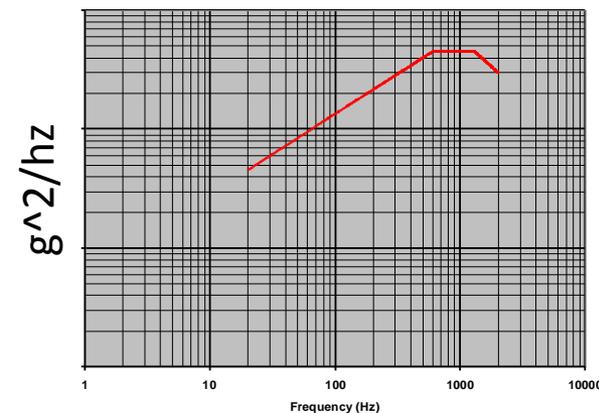
- Acceleration data is enveloped to capture uncertainties thus defining a vibration environment, standard has been between 20-2000hz.



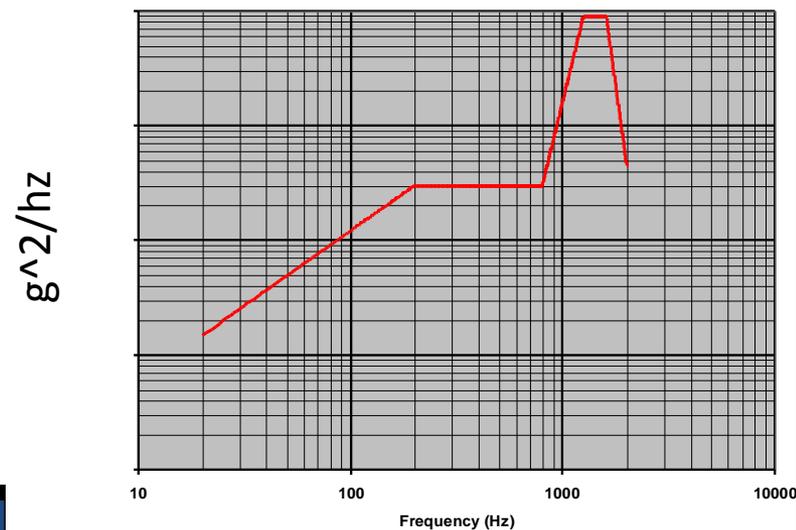
FASTRAC ENGINE - Manifold Node 17101



FASTRAC ENGINE - Turbopump Node 75008



FASTRAC ENGINE - Gas Generator Node 6006





# Typical MC-1 Engine Load Set

<b>Glue Bracket 3</b>	Shear 1	Shear 2	Axial	Bending 1	Bending 2	Torque
<b>GB-3</b>	(lbs)	(lbs)	(lbs)	(in-lbs)	(in-lbs)	(in-lbs)
Sine X	97	7	0	3	78	72
Sine Y	91	7	0	3	98	70
Sine Z	119	5	0	2	78	52
<b>Sine Peak (RSS)</b>	<b>178</b>	<b>11</b>	<b>0</b>	<b>5</b>	<b>148</b>	<b>113</b>
3 sig Random X	450	113	0	16	25	1475
3 sig Random Y	781	66	0	9	41	828
3 sig Random Z	155	1	0	4	1101	6
<b>Random Peak (RSS)</b>	<b>915</b>	<b>130</b>	<b>0</b>	<b>19</b>	<b>1102</b>	<b>1692</b>
<b>Stringer Bracket 3 (Lower Support)</b>						
<b>SB-6</b>						
Sine X	18	8	11	8	17	2
Sine Y	12	4	10	7	11	1
Sine Z	11	12	8	3	28	3
<b>Sine Peak (RSS)</b>	<b>24</b>	<b>15</b>	<b>17</b>	<b>11</b>	<b>34</b>	<b>4</b>
3 sig Random X	35	333	6	85	1349	52
3 sig Random Y	60	192	10	145	775	29
3 sig Random Z	12	1	11	83	6	0
<b>Random Peak (RSS)</b>	<b>70</b>	<b>384</b>	<b>16</b>	<b>187</b>	<b>1556</b>	<b>59</b>
<b>Stringer Bracket 3 (Upper Support)</b>						
<b>SB-5</b>						
Sine X	59	7	21	81	9	21
Sine Y	58	5	21	80	6	26
Sine Z	43	4	16	59	5	25
<b>Sine Peak (RSS)</b>	<b>93</b>	<b>9</b>	<b>34</b>	<b>129</b>	<b>12</b>	<b>42</b>
3 sig Random X	44	447	117	93	1557	69
3 sig Random Y	76	256	202	160	893	38
3 sig Random Z	139	2	1002	322	4	0
<b>Random Peak (RSS)</b>	<b>165</b>	<b>515</b>	<b>1029</b>	<b>371</b>	<b>1795</b>	<b>79</b>



# Calculating System Dynamic Loads

- Try to reproduce the engine environment by forcing engine response to match the measured (enveloped) accelerations
- Several ways this can be done
  - System Direct Approach
    - Directly apply an enforced acceleration at the points where environments are defined (Fastrac, RS-68). If modeling of component too difficult, use Shock Spectra to obtain maximum response.
  - System Equivalent Applied Force Methods
    - Determine a set of applied forces that will reproduce the measured environment.
  - Component Approach
    - Calculate loads on a component basis by fixing both ends and exciting entire structure with random load
- Most methods used to date result in loads which are almost always over-conservative.



# Direct Approach

- Apply engine acceleration environments directly to the model.
- Constrain nodes to have a given random acceleration PSD

$X_f$  = Free DOF

$X_s$  = Support DOF where accelerations are applied

$$\begin{bmatrix} M_{ff} & M_{fs} \\ M_{sf} & M_{ss} \end{bmatrix} \begin{Bmatrix} \ddot{X}_f \\ \ddot{X}_s \end{Bmatrix} + \begin{bmatrix} C_{ff} & C_{fs} \\ C_{sf} & C_{ss} \end{bmatrix} \begin{Bmatrix} \dot{X}_f \\ \dot{X}_s \end{Bmatrix} + \begin{bmatrix} K_{ff} & K_{fs} \\ K_{sf} & K_{ss} \end{bmatrix} \begin{Bmatrix} X_f \\ X_s \end{Bmatrix} = \begin{Bmatrix} 0 \\ F_s \end{Bmatrix}$$

$$M_{ff} \ddot{X}_f + C_{ff} \dot{X}_f + K_{ff} X_f = -M_{fs} \ddot{X}_s(t) - C_{fs} \dot{X}_s(t) - K_{fs} X_s(t)$$

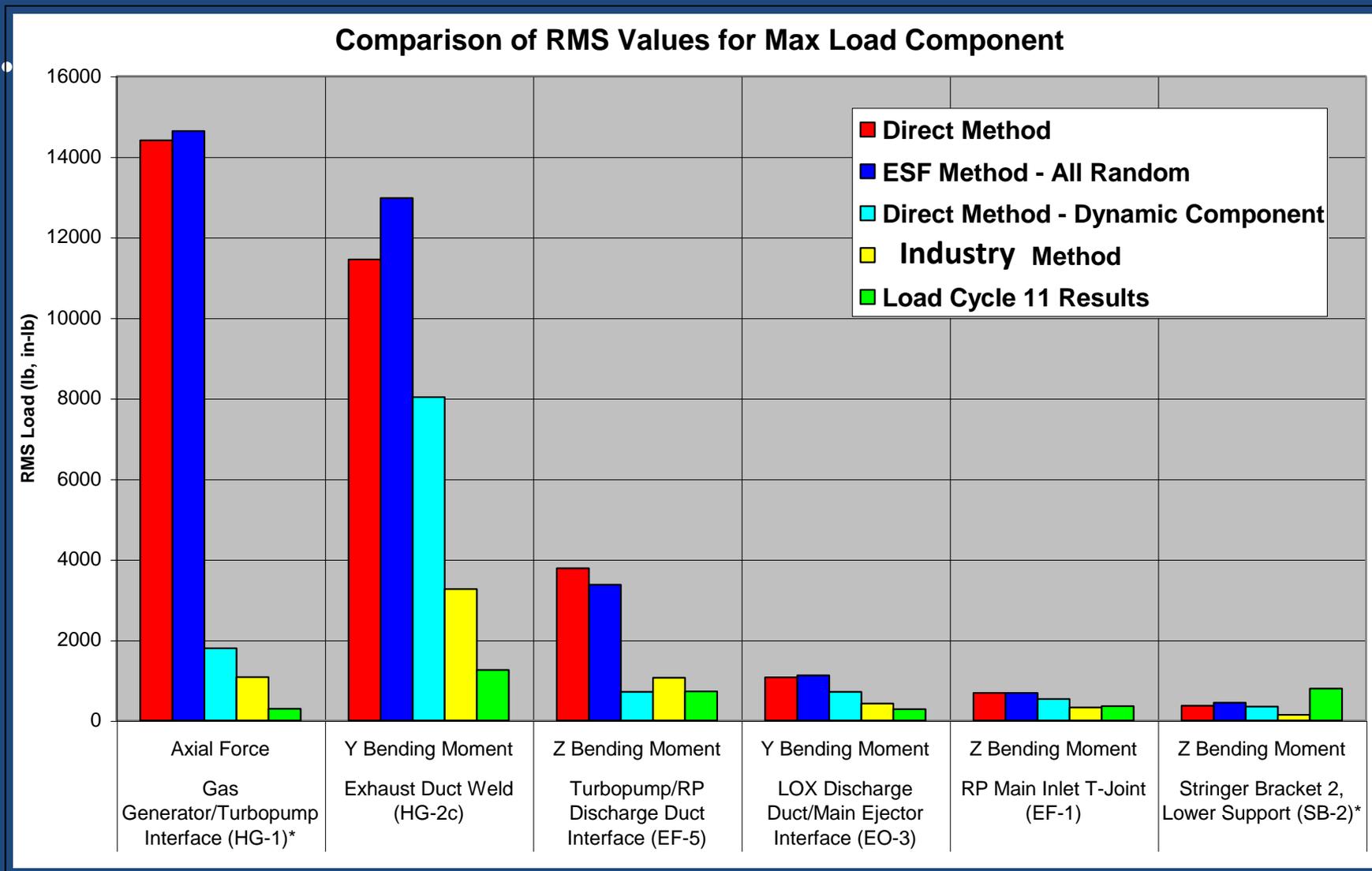
- Solve first equation using the NASTRAN (Finite Element Code) random analysis methods to give desired response.

$$F_s(t) = M_{sf} \ddot{X}_f + M_{ss} \ddot{X}_s + C_{sf} \dot{X}_f + C_{ss} \dot{X}_s + K_{sf} X_f + K_{ss} X_s$$

- Second equation is “pseudo-static portion” that isn’t real in a rocket engine (only in an earthquake!), so remove it from total response.



# New Methodology Significantly Reduces Loads



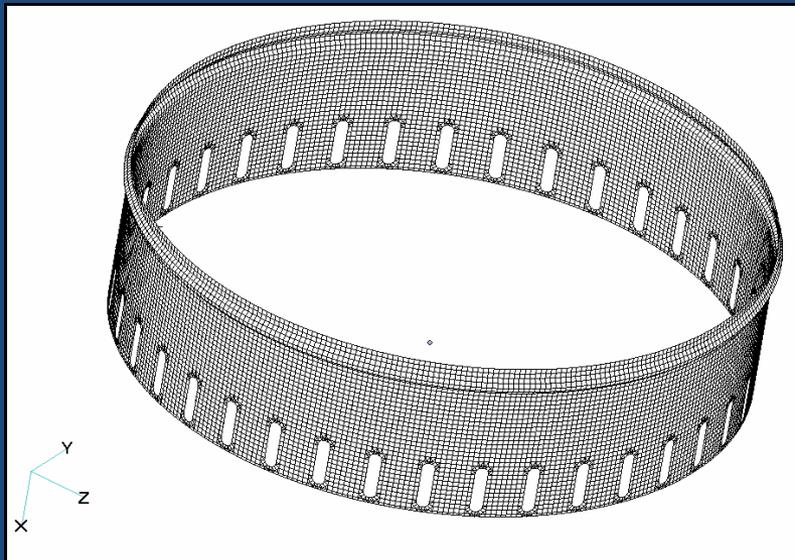
Method tested by performing multi-axis shaker test of Fastrac, enabling measurement of response and excitation





# St. Dynamics Tasks in Failure Investigation of Cracked Flowliners

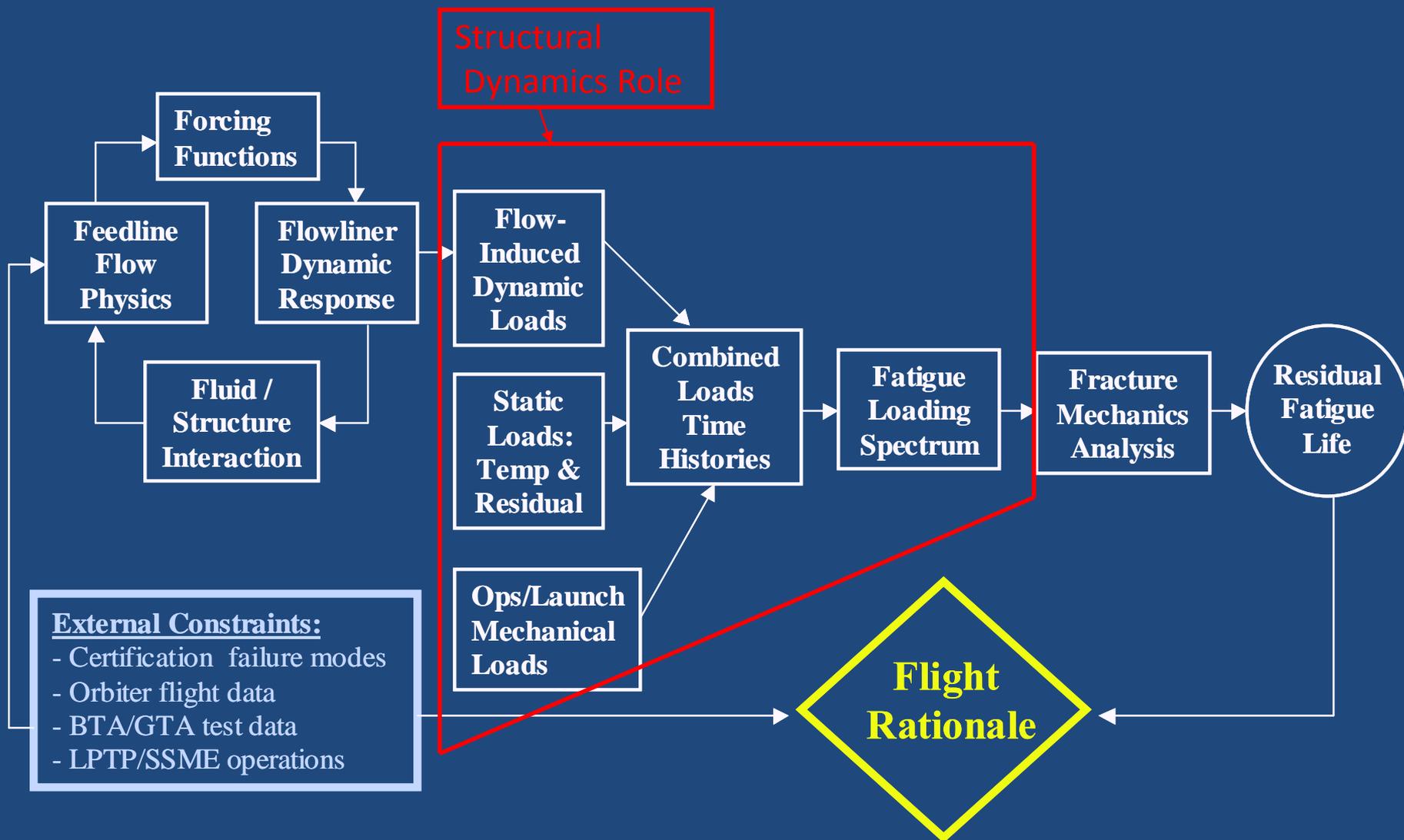
- Assess loads and environments on flowliner
  - Difficult to characterize highly dynamic, cavitating, cryogenic flow environment
  - Analyze hot fire tests data (flow induced environments)
  - Develop loading spectra (X lbs at Y hz for Z sec) for fracture analysis.
- Assess Dynamic Response
  - Finite Element Models created, modal analysis
  - Identify relevant modes for each flight condition
  - Assess strain transfer factors (test measured locations at mid ligament to crack initiation / field stress)





# Flowchart of Analyses

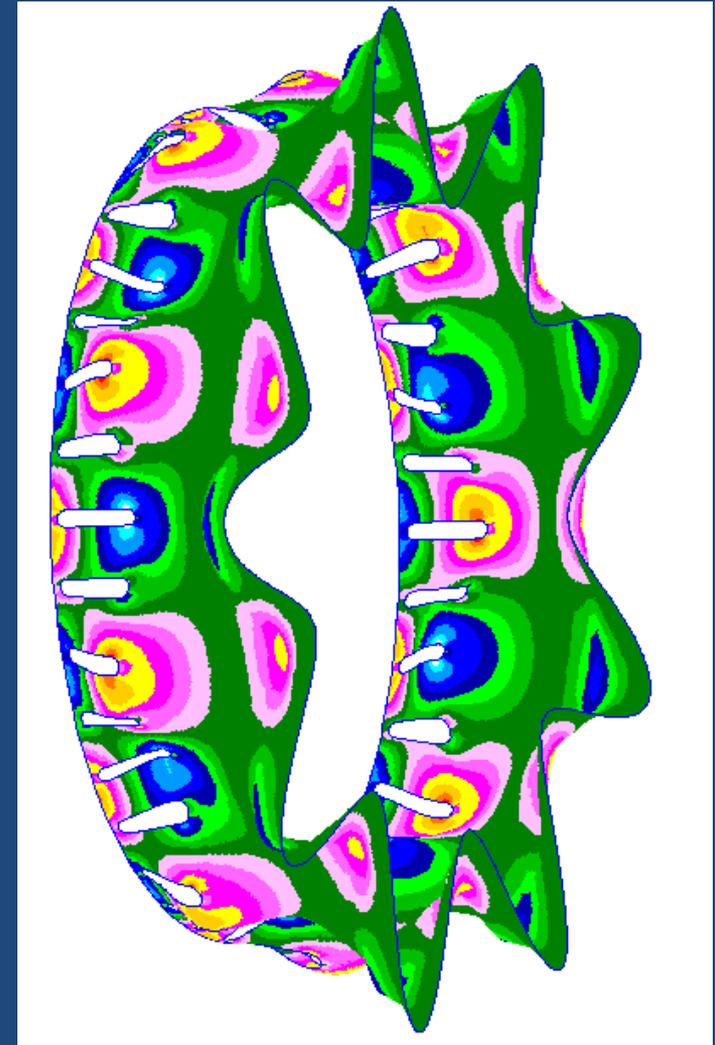
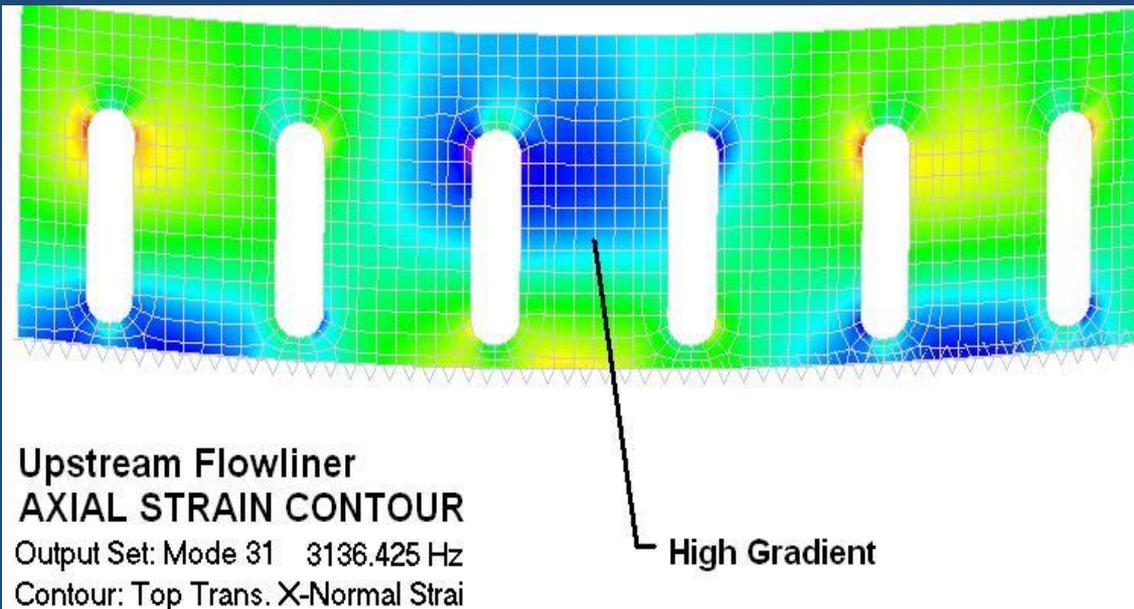
Huge NASA-wide team assembled. Structural Dynamic team played key role:





# Flowliner Dynamic Analysis Results

- Dynamic analysis determined source of cracking was several modes excited by downstream inducer blade count and cavitation.
- Tested flowliner dynamic response to validate models.
- Performed fracture analysis and computed expected service life based upon observed crack sizes. Solution was improved and more frequent inspections.



- Use Instrumented Hammer to do quick impact onto structure, which contains broadband frequency content.
- Response measured using an accelerometer or laser vibrometer.
- Fourier Transform of response/excitation (FRF) generated.
- Imaginary part of FRF at each location gives magnitude of mode shape.

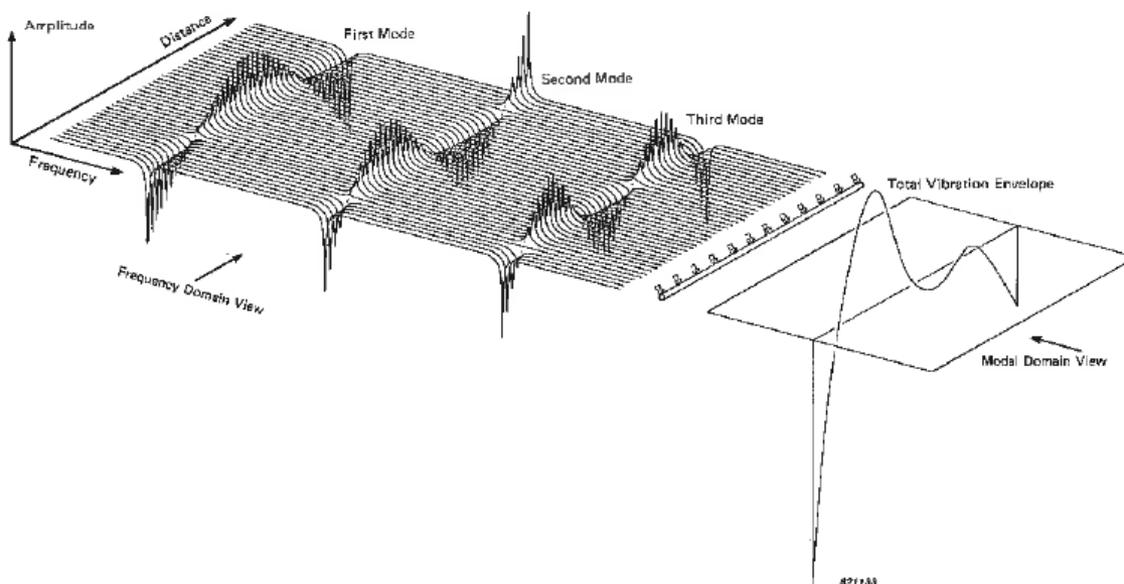
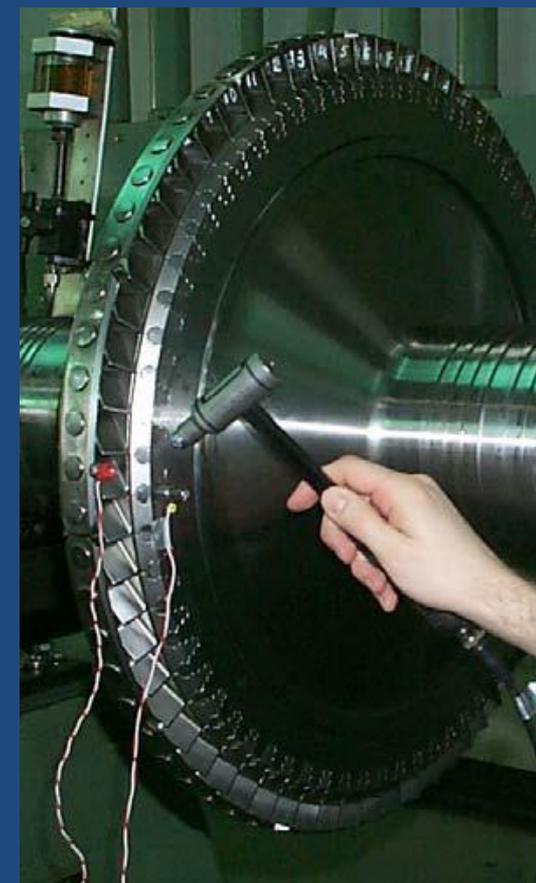


Fig.21. Three dimensional view illustrating the frequency and the modal domain

- Compare test & analytical mode shapes, update if necessary.



# Conclusion

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- Structural Dynamics is one of the Critical Disciplines for the successful Design, Development, Testing of Rocket Engines.
- It is applied from the smallest component (turbine blades), all the way to the entire engine and propellant feedlines.
- Successful application of Structural Dynamics requires extensive knowledge of Fourier Techniques, Linear Algebra, Random Variables, Finite Element Modeling, and essentials of SDOF and MDOF vibration theory.
- Working knowledge of Fluid Dynamics and Data Analysis also extremely useful.
- It all pays off when you get to see a successful engine firing!