High Resolution Image Reconstruction from Projection of Low Resolution Images Differing in Sub-pixel Shifts

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What is Super Resolution (SR)?

• A High Resolution (HR) image computed from several observed low-resolution (LR) images that differ in sub-pixel shifts and rotation is called Super Resolution (SR).

• SR Increases high frequency components and removing the degradations caused by the imaging process of the low resolution camera.
Why Super Resolution?

• Higher spatial resolution gives more details
• Spatial Resolutions cannot be increased indefinitely by hardware
• The limitations of spatial resolution come from
  o Diffraction of optical system
  o Signal to noise ratio of the sensor system
  o Spacecraft orbits in space-borne imaging
Approaches to Computing SR

• Frequency Domain Approach

• Spatial Domain Approaches
  • Non-Iterative approach: Interpolation and Restoration
  • Statistical Approaches:
    o Maximum A posterior (MAP)
    o Maximum Likelihood (MLE)

• Wavelet-Based construction

• Set Theoretic Methods
Spatial Domain Approaches to SR

- Iterative Back Projection (IBP)
- Interpolations
  - Nearest Neighbor (NN)
  - Inverse Distance Weighted (IDW) Interpolation
- Maximum Likelihood Estimation (MLE)
- Interpolation using Radial Basis Functions (RBF)
Imaging Model

• High Resolution (HR) Image
  • Let $x$ be the HR image sought (in 1-D vector form scanned in lexicographical order)

• Low Resolution (LR) Images
  • Let $y_1, y_2, \ldots, y_k$ be LR images that differ in translation and rotation w.r.t image $y_1$ (reference image)
  • Let $Y$ be concatenation of $y_1, y_2, \ldots, y_k$

• Each of the LR images ($y_k$) is given by
  • $y_k = D_k H_k F_k x + v_k$
    
    $F_k$ : motion matrix
    $H_k$ : Blurring matrix
    $D_k$: down sampling
    $V_k$ : noise term
Imaging Model (2)

• The previous equation can be written as
  \[ y = Mx + V \]

  o \( M = [D_1 H_1 F_1 \ D_2 H_2 F_2 \ ... \ D_k H_k F_k]^t \)
  o \( y \) is concatenation of \( y_1, \ldots, y_k \)
  o \( V \) concatenation of noise components \( V_1, \ldots, V_k \)
  o Given \( y \), to determine \( x \); which is an ill-posed problem.
  o Also in imaging system the matrices used in the above equations are not known.
The Generative Model of LR Images

\[ y_i = T_i BD(x) + n_i, \]

T is sub-pixel translation, B is blurring, and D down sample operators

\( n_i \) is noise
Spatial Domain Approach to SR

Figure 1. Block diagram reconstructing HR image from LR images
Sub-pixel Registration

- Uses Discrete Wavelet Transforms (DWT)
- Decomposes the Images into LR image and High Frequency Components in Horizontal and Vertical Directions (LL,LH,HL,HH) in multi-resolution fashion into a number of levels thus building a hierarchy low resolution images
- The decomposition is done for both (reference and input) images that are to be registered.
- At each level of decomposition features are extracted from low-frequency and high-frequency subbands for reference and input images and transformation function is computed using correlations.
- The transformation function is improved iteratively using images from the hierarchy.
Interpolation onto HR Grid

• Interpolation algorithms used to project the LR images onto HR
  • Nearest Neighbor (NN) Interpolation
  • Inverse Distance Weighted (IDW) Interpolation
  • Radial Basis Functions (RDF) Interpolation
Deblurring and Noise filter

• Simulation of LR images from Original Image includes a know Point Spread Function (PSF) and a noise component as shown in generative model LR images
• The reconstructed image deblurring is obtained by deconvolving using Weiner filter
• A low pass filter is used on the reconstructed SR image to filter out high frequency noise present in LR images
Statistical Methods in Computing SR image

• Maximum Likelihood Estimation (MLE)
  • In this method the SR image is estimated from a given set of LR images differing in sub-pixel shifts
  • \( p(x|y_1, y_2, \ldots y_k) = \frac{p(y_1, y_2, \ldots y_k|x) p(x)}{\int_x p(y_1, y_2, \ldots y_k|x) p(x)} \)
  • \( x \) is SR image, \( y_1, y_2, \ldots y_k \) are LR images
  • Log Likelihood function \( L(x) = \log(p(y_1, y_2, \ldots y_k|x) p(x)) \)
  • MLE assumes flat a prior \( p(x) \)
  • \( x' = \arg\max_x L(x) \)
  • Every pixel on HR grid is the one that maximizes the \( L(x) \)
Statistical Methods in Computing SR image (2)

• Maximum A Posterior (MAP) method

\[ p(x|y_1, y_2, \ldots, y_k) = \frac{p(y_1, y_2, \ldots, y_k|x) p(x)}{\int_x p(y_1, y_2, \ldots, y_k|x) p(x)} \]

\[ x' = \arg\max_x p(y_1, y_2, \ldots, y_k|x)p(x) \]

• This method allows inducing priors in estimating \( x' \)

• The results of MLE and MAP are similar
Iterative Back Projection (IBP)

- Initial HR \((x')\)image is reconstructed from the reference LR image \((y_1)\) by replicating pixels

- From \(x'\) the LR images are generated using the regenerative model \((y'_1, y'_2, \ldots, y'_k)\)

- \(x' = x' + \sum_{i=2}^{k} (y_i - y'_i)\)
Projection of LR images on HR Grid

• Assume the HR grid is $2n \times 2n$ where LR images are $n \times n$
• Number of LR images differing in sub-pixel shift are $2^2$
• HR grid $kn \times kn$, requires at $k^2$ LR images for optimal reconstruction
• All LR images are assumed to differ in subpixel shifts that are different from each other.
• The subpixel shifts are global
Projection of LR images on HR Grid: NN Reconstruction

\[ x' = U^2(y1) \]

\[ U^2 \] is upsample operator by 2

\[ x' = U^2(y1) + \sum_{k=2}^{4} U^2(T_k(y_k)) \]

\( T_k \) Translation operator
Projection LR images on HR Grid using IDW

• Unlike in NN, all the LR images contribute to the estimation of every missing pixels of HR grid
• Pixel value depends on distance of the LR image pixels from the missing HR grid point by inverse distance
  \[ x' = U^2(y_1) + \sum_{k=2}^{4} U^2(w_k y_k) \]
  \[ w_k \text{ is inverse distance of } y_k \text{ from missing positions of } x' \]
  \[ \sum_{k=2}^{4} w_k = 1 \]
Interpolation using Radial Basis Functions (RBF)

• A radial basis function (RBF) is a real valued function whose value depends on the distance from a given point $x_i$ ($x$ a variable not to be confused with image variable used for HR image).

• $\phi(x, x_i) = \phi(||x - x_i||)$

• When the distance function is Gaussian, the RBF is called Gaussian RBF

• $\phi(x, x_i) = e^{-(x-x_i)^2}$
Interpolation using Radial Basis Functions (RBF)

• The reference LR image ($nxn$) is used to fill up every other HR grid ($2nx2n$) pixels along the row and column.
• The remaining LR images are used to estimate the 3 missing cells of 2x2 HR grid as follows
• $x(i) = \sum_{k=2}^{4} \phi(i - T(k)) * y_k$
• $x(i) = \sum_{j=2}^{k} e^{-||i-T(j)||} y_j$
• In the above equation vector $i$ is the position of the missing pixel from HR grid (0,1 for horizontal, 1,0 for vertical and 1,1 for diagonal)
• $T(k)$ is the translation of LR image, $y_k$ with respect to $i$. 
Experimental Results

- Simulation of LR images from Chesapeake Bay Landsat Image
Results
**Results (SSE)**

\[ y_2: \ tr_2 = 0.9, \ tc_2 = 0.24 \]
\[ y_3: \ tr_3 = 0.12, \ tc_3 = 0.82 \]
\[ y_4: \ tr_4 = 0.9, \ tc_4 = 0.55 \]

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Results (SSE)

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y2: \( tr_2 = 0.08, \ tc_2 = 0.08 \)

y3: \( tr_3 = 0.17, \ tc_3 = 0.84 \)

y4: \( tr_4 = 0.64, \ tc_4 = 0.56 \)
Concluding Remarks

• Various algorithms are implemented to compare their performance in reconstructing HR image from a set of LR images differing subpixel shifts.

• Interpolation using RBFs performed the best in most cases.

• HR reconstruction accuracy depends on the subpixel shifts and number of LR images.

• We have experimented with four LR images to increase the spatial resolution by a factor of 2.

• Further research is required to improve the spatial resolution by an arbitrary number with a given number of LR images.