Elastic and Piezoelectric Properties of Boron Nitride Nanotube Composites
Part II: Finite Element Model

H. Alicia Kim\textsuperscript{1}, Robert Hardie\textsuperscript{2}, Vesselin Yamakov\textsuperscript{3}, Cheol Park\textsuperscript{4}

\textsuperscript{1}Associate Professor, UC San Diego, CA, USA
\textsuperscript{2}University of Bath, UK
\textsuperscript{3}National Institute of Aerospace, VA, USA
\textsuperscript{4}NASA Langley Research Center, VA, USA
Background: Boron Nitride Nanotube (BNNT)

• Our interest is in piezoelectric properties.
• Nitrogen atoms are more electronegative than boron atoms.
• Polarisation is cancelled out due to chiral symmetry.
• Strain induces polarisation field.
• Polarisation creates electric charge across a nanotube.
• Inherently multiscale
Research Aim

To investigate a suitable fidelity of a Representative Volume Element (RVE) Finite Element Model (FEM) of multiple Boron Nitride NanoTubes (BNNTs) in a matrix.
2D FE Model

- Uniform distribution
- Random distribution
- Volume fraction

Amount of stiff material (BNNT)

Unit cell

- 2D area, 3D solid cylinder, 3D hollow tubes
- Reference – Analytical solution for finite length cylindrical inclusions at many orientations by Tandon and Weng (1976)
## Material Properties

<table>
<thead>
<tr>
<th>Property</th>
<th>BNNT</th>
<th>Matrix Polymer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus, $E$ (GPa)</td>
<td>900</td>
<td>1.8</td>
</tr>
<tr>
<td>Poison’s ratio</td>
<td>0.3</td>
<td>0.39</td>
</tr>
<tr>
<td>Axial piezoelectric constant, $e$ (C/m$^2$)</td>
<td>0.2</td>
<td>-</td>
</tr>
<tr>
<td>Dielectric constant, $b$ (pF/m)</td>
<td>159.3</td>
<td>79.6</td>
</tr>
</tbody>
</table>

Elasticity Constant for 2D Models

Young's modulus (GPa)

Volume Fraction

Area ratio
Hollow tube
Solid cylinder
Tandon-Weng
Single Nanotube
Elasticity Constant for 2D Models

![Graph showing the relationship between Poisson's ratio and Volume Fraction for different models: Area ratio, Hollow tube, Solid cylinder, Tandon-Weng, and Single Nanotube.](image)
3D FE Model

- Coupled field tetrahedral elements
- BNNTs modelled as:
  1) Solid cylinders
  2) Hollow tubes

\[
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\sigma_{12} \\
D_1 \\
D_2
\end{bmatrix}
=
\begin{bmatrix}
C_{11}^* & C_{12}^* & 0 & -e_{11}^* & -e_{11}^* \\
C_{21}^* & C_{22}^* & 0 & -e_{11}^* & -e_{11}^* \\
0 & 0 & C_{66}^* & -e_{11}^* & -e_{11}^* \\
0 & 0 & b_{11}^* & 0 & 0 \\
0 & 0 & 0 & b_{22}^* & 0
\end{bmatrix}
\begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\varepsilon_{12} \\
E_1 \\
E_2
\end{bmatrix}
\]
Young’s Modulus
Elasticity Constant, $C_{11}$

![Graph showing the relationship between $C_{11}$ and Volume Fraction for different distributions and geometries.](image-url)
Elasticity Constant, $C_{12}$
Elasticity Constant, $C_{22}$
Elasticity Constant, $C_{66}$
Piezoelectric Constant

![Graph showing the relationship between piezoelectric constant ($e_{11}$) and volume fraction.]
Conclusion

• 2D uniform distribution model can offer a first order understanding of the effective elastic and piezoelectric properties
• Volume fraction based on filled solids was most appropriate for 2D model
• Differences between 3D models with solid cylinders and with hollow tubes insignificant
• $C_{11}$ and $e_{11}$ most sensitive to the volume fraction