Nuclear Cross Sections for Space Radiation Applications

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Outline

- Space Radiation Environment
- Lippmann-Schwinger Equation
- Interaction and Parameterizations
- Models
- Results
- Conclusions
**INTRODUCTION**

- **Space Radiation Environment**
  - Galactic Cosmic Rays
  - Solar Particle Events

- **Accurate cross sections are needed for radiation transport**
**Lippmann-Schwinger Equation**

- Lippmann-Schwinger Equation: \( T = V + VG_0^+ T \)
- For elastic scattering, use equivalent set of coupled equations
  - Elastic Scattering Equation:
    \[
    T = U + UPG_0^+ PT
    \]
  - Optical Potential:
    \[
    U = V + VQG_0^+ QU
    \]
- Elastic scattering equation in momentum-space
  \[
  T(k', k) = U(k', k) + \int U(k', k'') G_0^+(k'', k) T(k'', k) dk''
  \]
  where
  \[
  G_0^+(k'', k) = \frac{1}{E(k) - E(k'') + i\eta}
  \]


**Interaction**

- Potential is sum of nucleon-nucleon (NN) interactions

\[ V = \sum_{i=1}^{A_P} \sum_{j=1}^{A_T} v_{ij} \]

- Write series in terms of pseudo two-body operators

\[ U_{ij} = \tau_{ij} + \tau_{ij} QG_0^+ Q \sum_{lm} U_{lm} \]

where

\[ \tau_{ij} = v_{ij} + v_{ij} QG_0^+ Q \tau_{ij} \]

- Express \( \tau_{ij} \) in terms of \( t_{ij} \)

\[ \tau_{ij} = t_{ij} + t_{ij} (QG_0^+ Q - g) \tau_{ij} \]
Impulse approximation: $\tau_{ij} \rightarrow t_{ij}$

Single Scattering

Optimum Factorization

Transition amplitude evaluated at beam energy for central potentials

$$U(k', k) = A_P A_T \eta t_{NN}(q, \epsilon) \rho_T(q) \rho_P(q)$$
PARAMETERIZATIONS

- Nuclear Matter Density
  - For $A \leq 16$, Harmonic-Well Model\(^1\)
  - For $A > 16$, Two parameter Fermi Model\(^1\)
  - If no data for $A \leq 16$, isotopic average of parameters is used
  - If no data for $A > 16$, Nuclear Droplet Model\(^2\) is used

- NN transition amplitude\(^3\)
  - Cross sections
  - Real to imaginary ratio
  - Slope parameters

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\(^2\) Ann. Phys. 84, 186 (1974)
\(^3\) Werneth et al. NASA Technical Publication 2014-218529
**MODELS**

- **Eikonal (Eik)**
  - High Energy, Forward Scattering
  - Non-relativistic

- **Partial Wave (PW)**
  - Relativistic kinematics easily incorporated
  - Partial wave decomposition is an approximation
  - Numerically unstable for large number of partial wave
  - Finite summation formulas were implemented\(^4\)

- **Three-Dimensional Lippmann-Schwinger\(^5\) (LS3D)**
  - Relativistic kinematics easily incorporated
  - Not an approximation
  - Extended to reactions relevant for space radiation applications\(^6\)

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**Unequal Mass Comparisons**

\[ T_{\text{Lab}} = 1 \text{ GeV/n (} p + ^{56}\text{Fe)} \]

\[ T_{\text{Lab}} = 20 \text{ GeV/n (} p + ^{16}\text{O)} \]

Graphs showing differential cross sections \( \frac{d\sigma}{d\Omega} \) in \( \text{mb/sr} \) as a function of \( \theta_{\text{CM}} \) in degrees.
**EQUAL MASS REACTIONS**

Comparison to Experiment

\[ T_{\text{Lab}} = 1 \text{ GeV/nucl.} \ (p + ^{32}\text{S}) \]

\[ T_{\text{Lab}} = 500 \text{ MeV/nucl.} \ (p + ^{40}\text{Ca}) \]

**COMPARISON TO EXPERIMENT**

\[ T_{\text{Lab}} = 1047 \text{ MeV/nucl.} \ (p + {}^{58}\text{Ni}) \]

\[ T_{\text{Lab}} = 342 \text{ MeV/nucl.} \ ({}^{4}\text{He} + {}^{40}\text{Ca}) \]

CONCLUSIONS

- REL kinematic effects depend on mass difference and lab energy
- REL results agree better with experimental data than NR results
- No REL effect observed for equal mass systems
- Equal mass results can be explained with rapidly decaying potential