Nuclear Cross Sections for Space Radiation Applications

C. M. Werneth\textsuperscript{1}, K. M. Maung\textsuperscript{2}, W. P. Ford\textsuperscript{3}, J. W. Norbury\textsuperscript{1}, and M. D. Vera\textsuperscript{2}

\textsuperscript{1}NASA Langley Research Center
\textsuperscript{2}The University of Southern Mississippi
\textsuperscript{3}The University of Tennessee

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Outline

- Space Radiation Environment
- Lippmann-Schwinger Equation
- Interaction and Parameterizations
- Models
- Results
- Conclusions
**INTRODUCTION**

- Space Radiation Environment
  - Galactic Cosmic Rays
  - Solar Particle Events

- Accurate cross sections are needed for radiation transport

![Hubble Space Telescope](image1.png)

![Solar Dynamics Observatory](image2.png)
Lippmann-Schwinger Equation

- Lippmann-Schwinger Equation: \( T = V + VG_0^+ T \)
- For elastic scattering, use equivalent set of coupled equations
  - Elastic Scattering Equation:
    \[
    T = U + UPG_0^+ PT
    \]
  - Optical Potential:
    \[
    U = V + VQG_0^+ QU
    \]
- Elastic scattering equation in momentum-space
  \[
  T(k', k) = U(k', k) + \int U(k', k'') G_0^+(k'', k) T(k'', k) dk''
  \]
  where
  \[
  G_0^+(k'', k) = \frac{1}{E(k) - E(k'') + i\eta}
  \]
Potential is sum of nucleon-nucleon (NN) interactions

\[
V = \sum_{i=1}^{A_P} \sum_{j=1}^{A_T} v_{ij}
\]

- Write series in terms of pseudo two-body operators

\[
U_{ij} = \tau_{ij} + \tau_{ij} Q G_0^+ Q \sum_{lm} U_{lm}
\]

where

\[
\tau_{ij} = v_{ij} + v_{ij} Q G_0^+ Q \tau_{ij}
\]

- Express \( \tau_{ij} \) in terms of \( t_{ij} \)

\[
\tau_{ij} = t_{ij} + t_{ij} (Q G_0^+ Q - g) \tau_{ij}
\]
Nucleus-Nucleus Scattering

\[ k \phi_{AP}^0 (AP) \rightarrow P_P (AP - 1) k' \phi_{AP}^0 (AP) \]

\[ -k' \phi_{AT}^0 (AT) \rightarrow P_T (AT - 1) p_2 \]

\[ p_1 = \frac{q}{2} + \frac{q}{2} p_2 - \frac{q}{2} \]

\[ U(k', k) = A_PA_T \eta t_{NN}(q, \epsilon) \rho_T(q) \rho_P(q) \]

- Impulse approximation: \( \tau_{ij} \rightarrow t_{ij} \)
- Single Scattering
- Optimum Factorization
- Transition amplitude evaluated at beam energy for central potentials
PARAMETERIZATIONS

- Nuclear Matter Density
- For $A \leq 16$, Harmonic-Well Model\(^1\)
- For $A > 16$, Two parameter Fermi Model\(^1\)
- If no data for $A \leq 16$, isotopic average of parameters is used
- If no data for $A > 16$, Nuclear Droplet Model\(^2\) is used

- NN transition amplitude\(^3\)
  - Cross sections
  - Real to imaginary ratio
  - Slope parameters

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\(^3\) Werneth et al. NASA Technical Publication 2014-218529
MODELS

- Eikonal (Eik)
  - High Energy, Forward Scattering
  - Non-relativistic
- Partial Wave (PW)
  - Relativistic kinematics easily incorporated
  - Partial wave decomposition is an approximation
  - Numerically unstable for large number of partial wave
  - Finite summation formulas were implemented\(^4\)
- Three-Dimensional Lippmann-Schwinger\(^5\) (LS3D)
  - Relativistic kinematics easily incorporated
  - Not an approximation
  - Extended to reactions relevant for space radiation applications\(^6\)

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Unequal Mass Comparisons

\( T_{\text{Lab}} = 1 \) GeV/n (p + \(^{56}\)Fe)

\( T_{\text{Lab}} = 20 \) GeV/n (p + \(^{16}\)O)
**Comparison to Experiment**

\[ T_{\text{Lab}} = 1 \, \text{GeV/nucl.} \quad (p + ^{32}\text{S}) \]

\[ T_{\text{Lab}} = 500 \, \text{MeV/nucl.} \quad (p + ^{40}\text{Ca}) \]

- Alkhazov et al.
- LS3D (NR)
- LS3D (REL)

Comparison to Experiment

$T_\text{Lab} = 1047$ MeV/nucl. (p $^\text{58}$Ni)

$T_\text{Lab} = 342$ MeV/nucl. ($^\text{4}$He $^\text{40}$Ca)

**Conclusions**

- REL kinematic effects depend on mass difference and lab energy
- REL results agree better with experimental data than NR results
- No REL effect observed for equal mass systems
- Equal mass results can be explained with rapidly decaying potential