Nuclear Cross Sections for Space Radiation Applications

C. M. Werneth\textsuperscript{1}, K. M. Maung\textsuperscript{2}, W. P. Ford\textsuperscript{3}, J. W. Norbury\textsuperscript{1}, and M. D. Vera\textsuperscript{2}

\textsuperscript{1}NASA Langley Research Center
\textsuperscript{2}The University of Southern Mississippi
\textsuperscript{3}The University of Tennessee

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Outline

- Space Radiation Environment
- Lippmann-Schwinger Equation
- Interaction and Parameterizations
- Models
- Results
- Conclusions
INTRODUCTION

- Space Radiation Environment

  Galactic Cosmic Rays
  Solar Particle Events

  Hubble Space Telescope
  Solar Dynamics Observatory

- Accurate cross sections are needed for radiation transport
Lippmann-Schwinger Equation

- Lippmann-Schwinger Equation: \( T = V + VG_0^+ T \)

- For elastic scattering, use equivalent set of coupled equations
  - Elastic Scattering Equation:
    \[
    T = U + UPG_0^+ PT
    \]
  - Optical Potential:
    \[
    U = V + VQG_0^+ QU
    \]
  - Elastic scattering equation in momentum-space
    \[
    T(k', k) = U(k', k) + \int U(k', k'')G_0^+(k'', k)T(k'', k)dk''
    \]

where

\[
G_0^+(k'', k) = \frac{1}{E(k) - E(k'') + i\eta}
\]
**INTERACTION**

- Potential is sum of nucleon-nucleon (NN) interactions

\[
V = \sum_{i=1}^{A_P} \sum_{j=1}^{A_T} v_{ij}
\]

- Write series in terms of pseudo two-body operators

\[
U_{ij} = \tau_{ij} + \tau_{ij} Q G_0^+ Q \sum_{lm} U_{lm}
\]

where

\[
\tau_{ij} = v_{ij} + v_{ij} Q G_0^+ Q \tau_{ij}
\]

- Express \( \tau_{ij} \) in terms of \( t_{ij} \)

\[
\tau_{ij} = t_{ij} + t_{ij} (Q G_0^+ Q - g) \tau_{ij}
\]
### Impulse approximation: $\tau_{ij} \rightarrow t_{ij}$

### Single Scattering

### Optimum Factorization

### Transition amplitude evaluated at beam energy for central potentials

$$U(k', k) = A_PA_T\eta t_{NN}(q, \epsilon)\rho_T(q)\rho_P(q)$$
PARAMETERIZATIONS

- **Nuclear Matter Density**
  - For $A \leq 16$, Harmonic-Well Model\(^1\)
  - For $A > 16$, Two parameter Fermi Model\(^1\)
  - If no data for $A \leq 16$, isotopic average of parameters is used
  - If no data for $A > 16$, Nuclear Droplet Model\(^2\) is used

- **NN transition amplitude\(^3\)**
  - Cross sections
  - Real to imaginary ratio
  - Slope parameters

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\(^2\) Ann. Phys. 84, 186 (1974)

\(^3\) Werneth et al. NASA Technical Publication 2014-218529
MODELS

- **Eikonal (Eik)**
  - High Energy, Forward Scattering
  - Non-relativistic

- **Partial Wave (PW)**
  - Relativistic kinematics easily incorporated
  - Partial wave decomposition is an approximation
  - Numerically unstable for large number of partial wave
  - Finite summation formulas were implemented\(^4\)

- **Three-Dimensional Lippmann-Schwinger\(^5\) (LS3D)**
  - Relativistic kinematics easily incorporated
  - Not an approximation
  - Extended to reactions relevant for space radiation applications\(^6\)

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Unequal Mass Comparisons

$T_{\text{Lab}} = 1$ GeV/n ($p + ^{56}\text{Fe}$)

$T_{\text{Lab}} = 20$ GeV/n ($p + ^{16}\text{O}$)
**Equal Mass Reactions**

COMPARISON TO EXPERIMENT

$T_{\text{Lab}} = 1 \text{ GeV/nucl. (p + } ^{32}\text{S)}$

$T_{\text{Lab}} = 500 \text{ MeV/nucl. (p + } ^{40}\text{Ca)}$


\[ T_{\text{Lab}} = 1047 \text{ MeV/nucl. } (p + ^{58}\text{Ni}) \]

\[ T_{\text{Lab}} = 342 \text{ MeV/nucl. } (^{4}\text{He} + ^{40}\text{Ca}) \]

CONCLUSIONS

- REL kinematic effects depend on mass difference and lab energy
- REL results agree better with experimental data than NR results
- No REL effect observed for equal mass systems
- Equal mass results can be explained with rapidly decaying potential