# Challenges in Adjoint-Based Aerodynamic Design for Unsteady Flows


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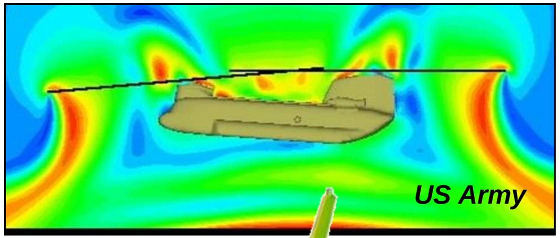
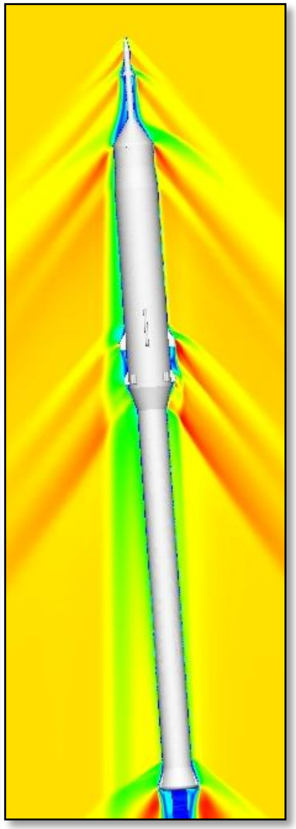
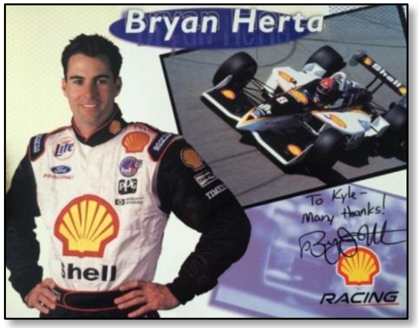
# FUN3D Core Capabilities

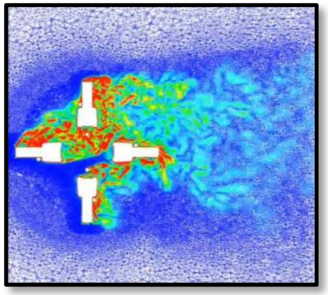
***http://fun3d.larc.nasa.gov***

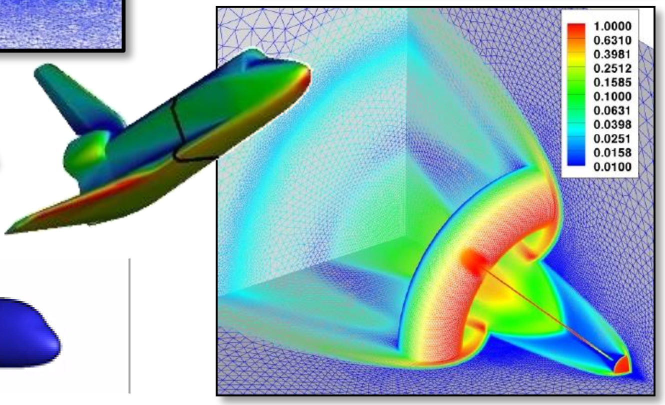
- Established as a research code in late 1980's; now supports numerous internal and external efforts across the speed range
- Solves 2D/3D steady and unsteady Euler and RANS equations on node-based mixed element grids for compressible and incompressible flows
- General dynamic mesh capability: any combination of rigid / overset / morphing grids, including 6-DOF effects
- Aeroelastic modeling using mode shapes, full FEM, CC, etc.
- Constrained / multipoint adjoint-based design and mesh adaptation
- Distributed development team using agile/extreme software practices including 24/7 regression, performance testing
- Capabilities fully integrated, online documentation, training videos, tutorials

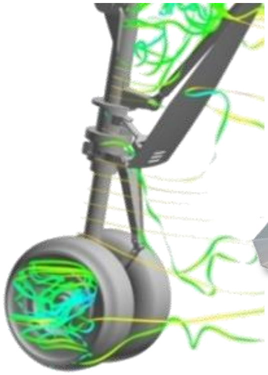


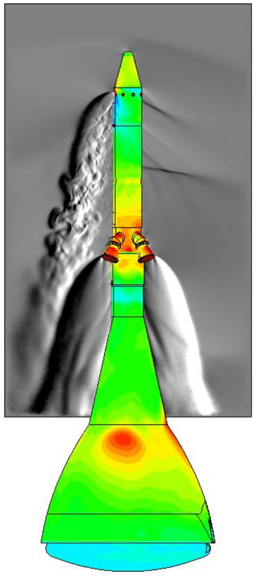



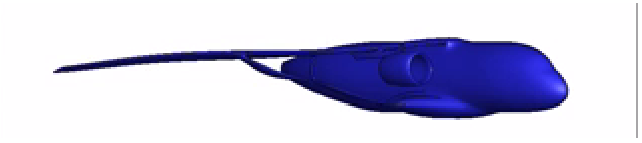








# Conventional Adjoint-Based Design

$$
\begin{aligned}
L(\mathbf{D}, \mathbf{Q}, \mathbf{X}, \mathbf{\Lambda}, \mathbf{\Lambda}_g) &= f\Delta t + \sum_{n=1}^{N} [\mathbf{\Lambda}_g^n]^T \mathbf{G}^n \Delta t \\
&+ \sum_{n=1}^{N} \Bigg\{ [\mathbf{C}_s^n \circ \mathbf{\Lambda}_s^n]^T \Bigg[ a \frac{\mathbf{Q}_s^n - \mathbf{I}_s^n \mathbf{Q}^{n-1}}{\Delta t} \circ \mathbf{V}_s^n \\
&+ c \frac{\mathbf{I}_s^n \mathbf{Q}^{n-2} - \mathbf{I}_s^n \mathbf{Q}^{n-1}}{\Delta t} \circ (\mathbf{I}_s^n \mathbf{V}^{n-2}) \\
&+ d \frac{\mathbf{I}_s^n \mathbf{Q}^{n-3} - \mathbf{I}_s^n \mathbf{Q}^{n-1}}{\Delta t} \circ (\mathbf{I}_s^n \mathbf{V}^{n-3}) \Bigg] \\
&+ [\mathbf{\Lambda}_s^n]^T [\mathbf{R}^n + ((\mathbf{I}_s^n \mathbf{Q}^{n-1}) \circ \mathbf{C}_s^n + \beta \bar{\mathbf{C}}_s^n) \circ \mathbf{R}_{\mathrm{GCL}}^n] \\
&+ [\mathbf{\Lambda}_f^n]^T [\mathbf{A}^n \mathbf{Q}^n] + [\mathbf{\Lambda}_h^n]^T [\mathbf{P}^n \mathbf{Q}^n] \Bigg\} \Delta t \\
&+ (f^0 + [\mathbf{\Lambda}_g^0]^T \mathbf{G}^0 + [\mathbf{\Lambda}^0]^T \mathbf{R}^{\mathrm{in}}) \Delta t
\end{aligned}
$$

- Flow field and grid adjoint equations derived for the time-dependent Navier-Stokes equations on arbitrary combinations of static/rigidly moving/deforming overset grids undergoing parent-child motion
- The following terms are included in the Lagrangian
  - Objective function
  - Grid terms
  - Higher-order temporal terms
  - Fluxes
  - Geometric Conservation Law term
  - Overset interpolation terms
  - Initial conditions
- Implemented by hand and verified using complex variables

*$\circ$ is the Hadamard vector multiplication operator; see*
*Nielsen, E.J. and Diskin, B., "Discrete Adjoint-Based Design for Unsteady Turbulent Flows on Dynamic Overset Unstructured Grids," AIAA Journal, Vol. 51, No. 6, June 2013.*

# Conventional Adjoint-Based Design

- After linearizing the Lagrangian and solving the flow and grid adjoint equations, the desired sensitivities are computed as follows

$$\frac{\partial L}{\partial \mathbf{D}} = \frac{\partial f}{\partial \mathbf{D}} \Delta t + \sum_{n=1}^{N} [\mathbf{\Lambda}_g^n]^T \frac{\partial \mathbf{G}^n}{\partial \mathbf{D}} \Delta t$$

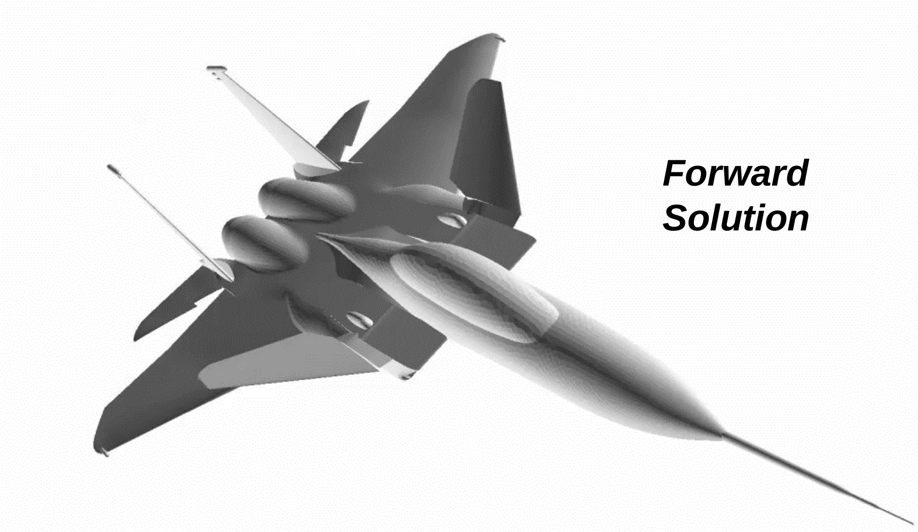
$$+ \sum_{n=1}^{N} [\mathbf{\Lambda}_s^n]^T \left[ \frac{\partial \mathbf{R}^n}{\partial \mathbf{D}} + ((\mathbf{I}_s^n \mathbf{Q}^{n-1}) \circ \mathbf{C}_s^n + \beta \bar{\mathbf{C}}_s^n) \odot \frac{\partial \mathbf{R}_{\mathrm{GCL}}^n}{\partial \mathbf{D}} \right] \Delta t$$

$$+ \left( [\mathbf{\Lambda}_g^0]^T \frac{\partial \mathbf{G}^0}{\partial \mathbf{D}} + [\mathbf{\Lambda}^0]^T \left[ \frac{\partial \mathbf{R}^{\mathrm{in}}}{\partial \mathbf{D}} \right] \right) \Delta t$$

*$\odot$ is the extension of the Hadamard operator to vector-matrix multiplication where the vector on the left multiplies each column in the matrix on the right.*

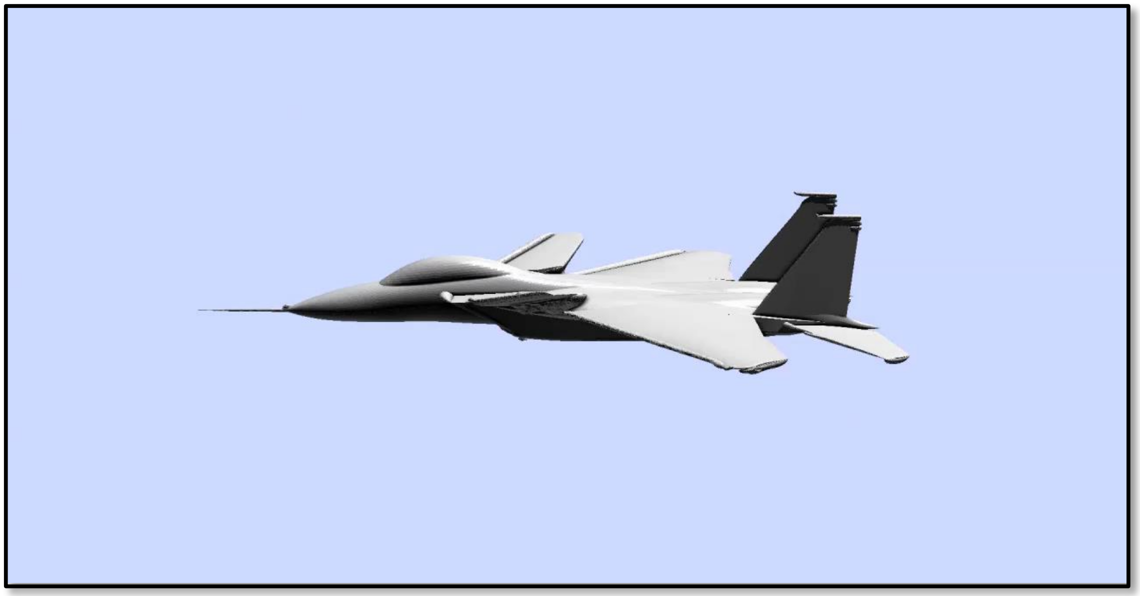
# Examples

***Forward / Reverse Solutions for F-15***



***Forward Solution***

- Transonic turbulent flow over modified F-15 configuration
- Propulsion effects included as well as simulated aeroelastic deformations of canard/wing/h-tail
- Objective is lift-to-drag ratio



***Reverse Solution***

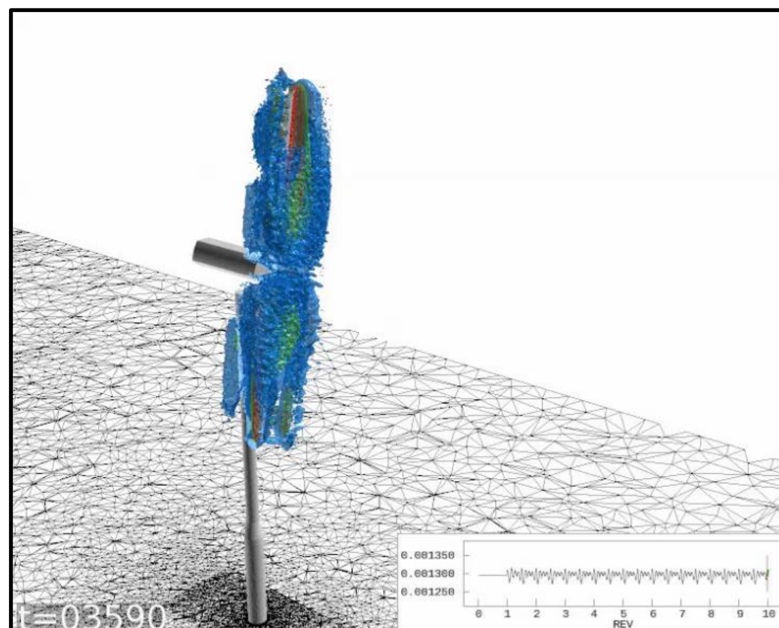
# Examples

***Forward / Reverse Solutions for Wind Turbine***

**_Forward Solution_**

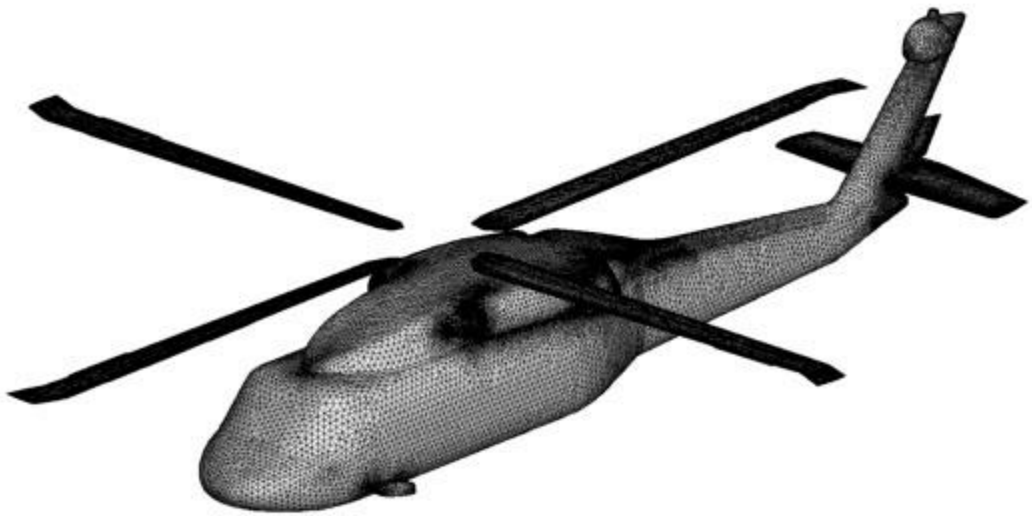
- Incompressible turbulent flow over NREL Phase VI wind turbine
- Overset grids used to model rotating blade system
- Objective function is based on the torque

**_Reverse Solution_**



# UH-60A Blackhawk Helicopter

***Overview***



- Composite grid consists of 9,262,941 nodes / 54,642,499 tetrahedra
- Compressible RANS: $M_{tip}$=0.64, $Re_{tip}$=7.3M, $\mu$=0.37, $\alpha$=0.0º
- Blade pitch has child motion governed by collective and cyclic control inputs:

$$\underbrace{\theta}_{\text{Blade pitch}} = \underbrace{\theta_c}_{\text{Collective}} + \underbrace{\theta_{1c}}_{\text{Lateral cyclic}} \cos\psi + \underbrace{\theta_{1s}}_{\text{Longitudinal cyclic}} \sin\psi$$

- Baseline value of all control inputs is zero

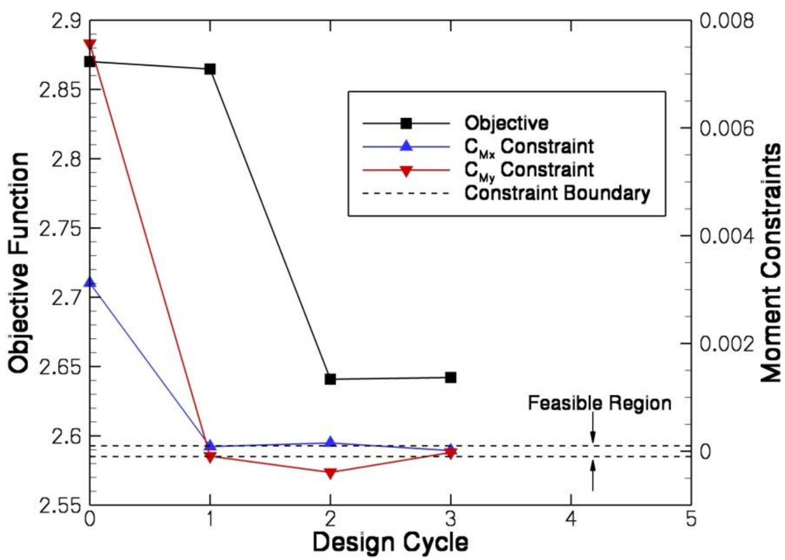
# UH-60A Blackhawk Helicopter

***Problem Definition and Results***

- Objective is to maximize $\bar{C}_L$ while satisfying trim constraints over second rev:

$$\min f = \left[ \left( \frac{1}{360} \sum_{n=361}^{720} C_L^n \right) - 2.0 \right]^2 \Delta t \quad \text{such that} \quad \begin{aligned} g_1 &= \frac{1}{360} \sum_{n=361}^{720} C_{M_x}^n \Delta t = 0 \\ g_2 &= \frac{1}{360} \sum_{n=361}^{720} C_{M_y}^n \Delta t = 0 \end{aligned}$$

- Separate adjoint solutions required for all three functions
- 67 design variables include 64 thickness and camber variables across the blade planform, plus collective and cyclic control inputs up to $\pm 7^o$

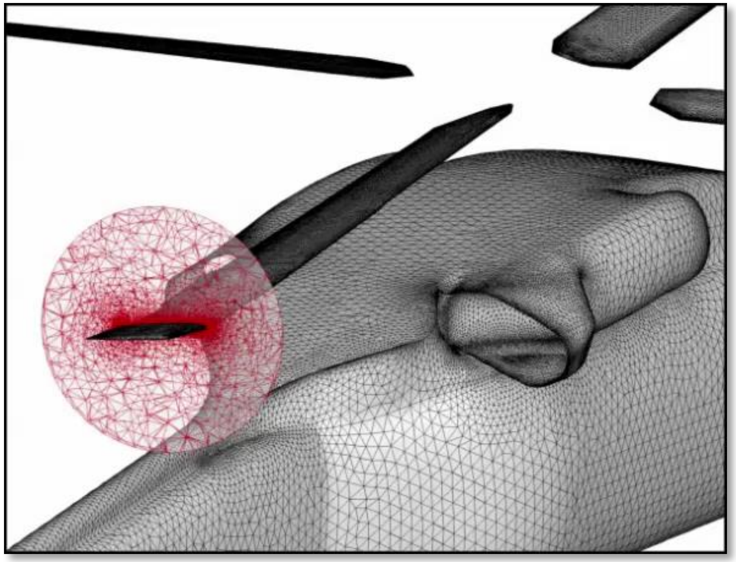


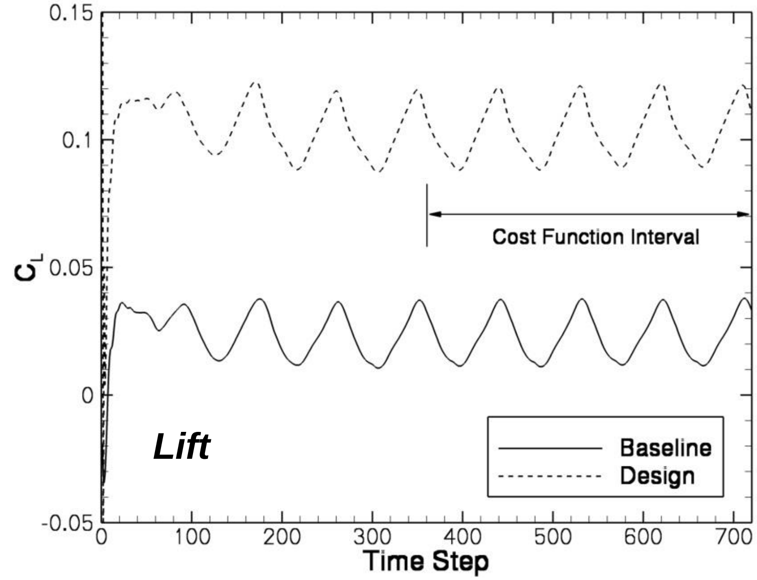

| | $\bar{C}_L$ | Flow Solves (2 hrs) | Adjoint Solves (3 hrs) | Total Time |
|---|---|---|---|---|
| Baseline | 0.023 | - | - | - |
| Design | 0.103 | 4 | 4 | 0.8 days (38,400 CPU hrs) |

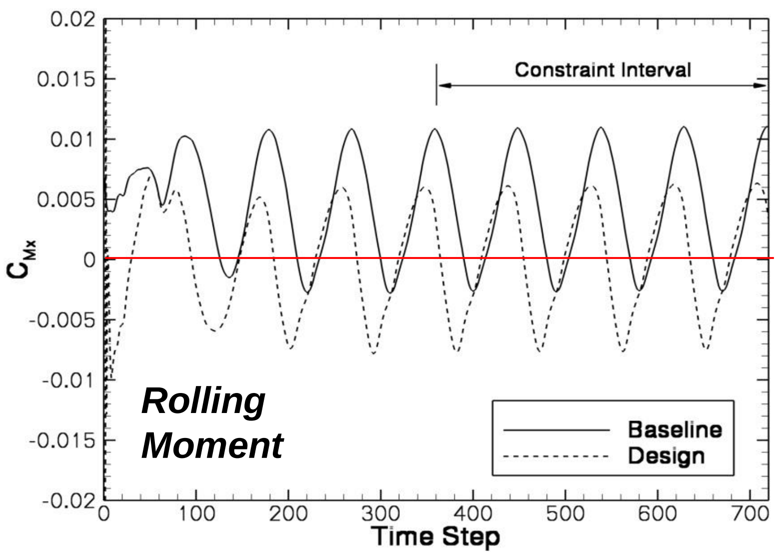
- Feasible region is quickly located
- Both moment constraints are satisfied within tolerance at the optimal solution
- Final controls: $\theta_c$=6.71º, $\theta_{1c}$=2.58º, $\theta_{1s}$=-7.00º

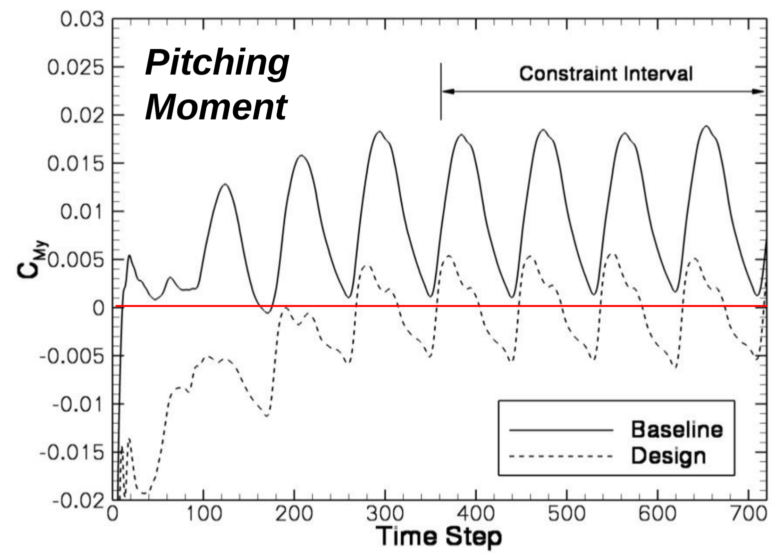
# UH-60A Blackhawk Helicopter

***Results***





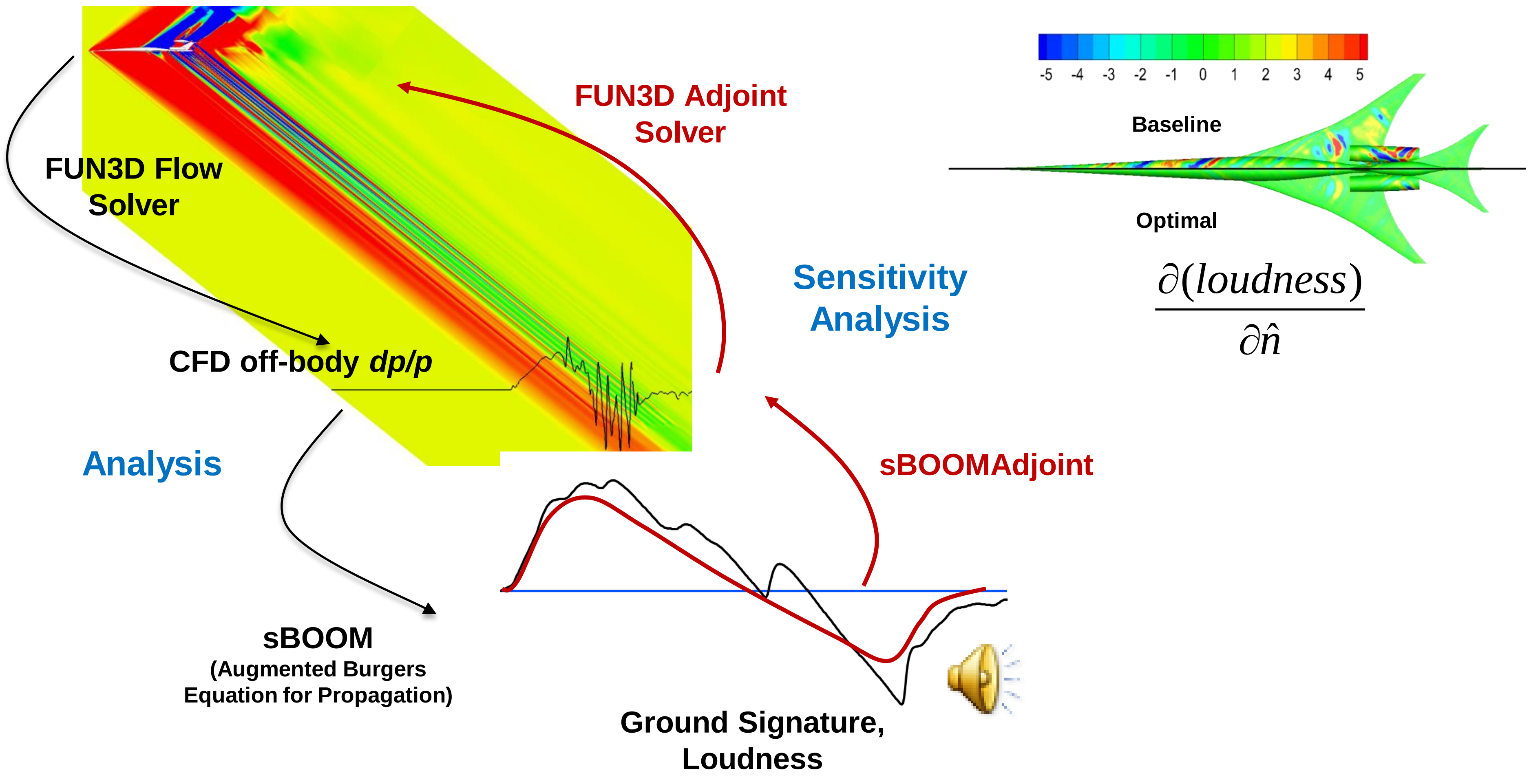



# Multidisciplinary Design

## *Sonic Boom Mitigation*

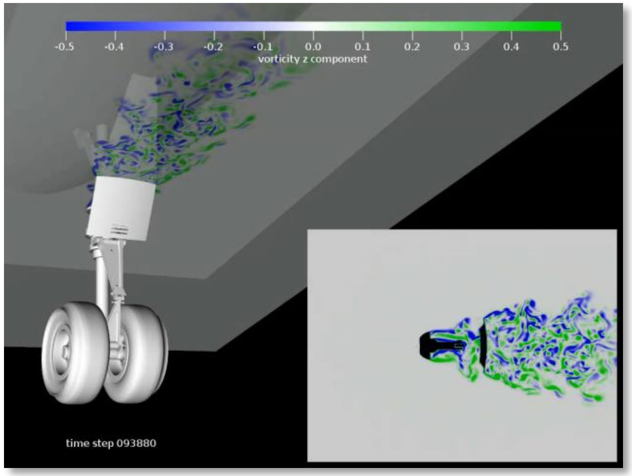
- Multidisciplinary discrete adjoint has been very successful for sonic boom mitigation - discrete derivatives of ground-based metrics with respect to OML
- Many other disciplines being considered / pursued



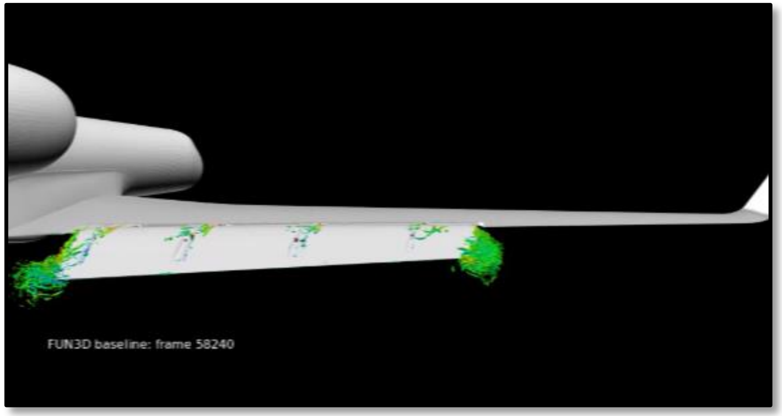
# Challenges for Unsteady Problems

- Extensive linearization and infrastructure effort, particularly for dynamic and overset grids

- Sheer cost – every simulation is now a time-dependent run
  - For steady flows, terms could be computed once and stored for efficiency
  - Unsteady flows require these linearizations to be recomputed at every time step




- Need for entire forward solution
  - Brute force it: Store to disk (big data)
  - Recompute it: Store periodically, recompute intermediate steps as needed (checkpointing)
  - Approximate it: Store periodically, interpolate intermediate steps as needed




- Chaotic flows

# Goal of Current Work

***Compute sensitivities of infinite time averages for chaotic flows***

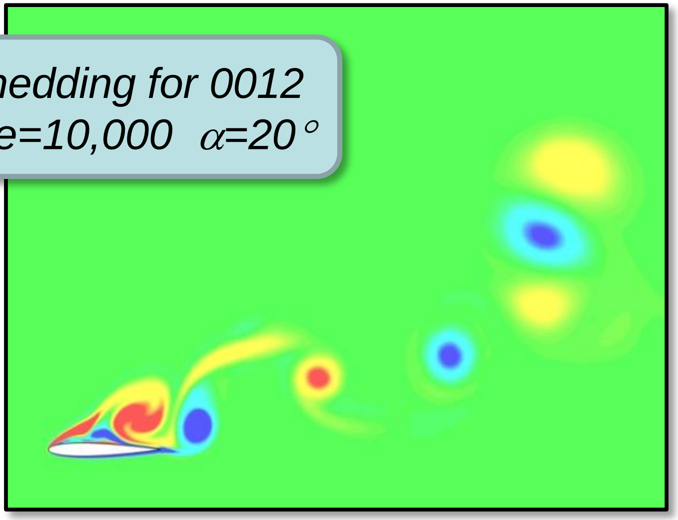
- Theory exists that states these sensitivities are well-defined and bounded

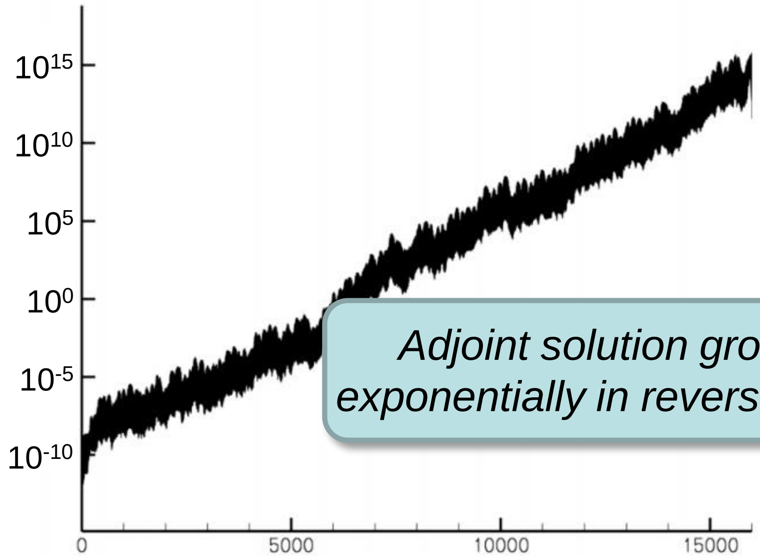
**Why does conventional approach not work?**

For chaotic flows:

- The finite time average approaches the infinite time average
- The sensitivity for a finite time average does not approach the sensitivity for the infinite time average

*Chaotic shedding for 0012*
*$M_\infty$=0.1 Re=10,000 $\alpha$=20$^o$*






*Adjoint solution grows exponentially in reverse time*

# Approach

- Least-Squares Shadowing (LSS) method proposed by Wang and Blonigan
  - Key assumption is ergodicity of the simulation: long time averages are essentially independent of the initial conditions
  - Also assumes existence of a shadowing trajectory
- The LSS formulation involves a linearly-constrained least squares optimization problem which results in a set of KKT equations
- Preliminary LSS exploration for fluids applications

***Define the following quantities:***

$\boldsymbol{Q_i} \equiv$ Vector of conserved variables at time level i

$\boldsymbol{R_i} \equiv$ Vector of spatial residuals at time level i

$\mathbf{V} \equiv$ Matrix of cell volumes

$t \equiv$ Time

$f_i \equiv$ Objective function at time level i

# LSS System

$$
\begin{pmatrix}
\begin{pmatrix} \mathbf{V} & & & & \\ & \mathbf{V} & & & \\ & & \ddots & & \\ & & & \mathbf{V} & \\ & & & & \mathbf{V} \end{pmatrix} & & \begin{pmatrix} -\mathbf{V}/\Delta t & & & \\ \mathbf{G}_1^T & -\mathbf{V}/\Delta t & & \\ & \mathbf{G}_2^T & \ddots & \\ & & \ddots & -\mathbf{V}/\Delta t \\ & & & \mathbf{G}_m^T \end{pmatrix} \\
& \begin{pmatrix} \alpha^2 & & \\ & \alpha^2 & \\ & & \ddots \end{pmatrix} & \begin{pmatrix} \boldsymbol{R}_1^T & & & \\ & \boldsymbol{R}_2^T & & \\ & & \ddots & \\ & & & \boldsymbol{R}_m^T \end{pmatrix} \\
\begin{pmatrix} -\mathbf{V}/\Delta t & \mathbf{G}_1 & & & \\ & -\mathbf{V}/\Delta t & \mathbf{G}_2 & & \\ & & \ddots & \ddots & \\ & & & -\mathbf{V}/\Delta t & \mathbf{G}_m \end{pmatrix} & \begin{pmatrix} \boldsymbol{R}_1 & & \\ & \boldsymbol{R}_2 & \\ & & \ddots \end{pmatrix} &
\end{pmatrix}
\begin{pmatrix}
\begin{pmatrix} \boldsymbol{\Phi}_0 \\ \boldsymbol{\Phi}_1 \\ \vdots \\ \vdots \\ \boldsymbol{\Phi}_m \end{pmatrix} \\
\begin{pmatrix} \Psi_1 \\ \Psi_2 \\ \vdots \\ \Psi_m \end{pmatrix} \\
\begin{pmatrix} \Lambda_1 \\ \Lambda_2 \\ \vdots \\ \Lambda_m \end{pmatrix}
\end{pmatrix}
= -
\begin{pmatrix}
\begin{pmatrix} \boldsymbol{g}_0 \\ \boldsymbol{g}_1 \\ \vdots \\ \vdots \\ \boldsymbol{g}_m \end{pmatrix} \\
\begin{pmatrix} h_1 \\ h_2 \\ \vdots \\ h_m \end{pmatrix} \\
\begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}
\end{pmatrix}
$$

***Tangent solution***

***Parameters from shadowing lemma***

***Adjoint solution***

$$\mathbf{G} = \mathbf{V}/\Delta t + \partial \boldsymbol{R}/\partial \mathbf{Q}$$

$$\boldsymbol{g} = \partial f/\partial \mathbf{Q}$$

$\boldsymbol{h}$ is related to time dilation

$\alpha$ is a regularization parameter

***This is a globally coupled space-time problem, where each sub-row represents a time level***

# Reduced LSS System

- To determine sensitivities, we need the LSS adjoint solution
- Use a Schur complement approach to arrive at a reduced system for the LSS adjoint variables:

Writing the previous system as

$$\begin{pmatrix} \mathbf{V} & 0 & \mathbf{B}^T \\ 0 & \alpha^2\mathbf{I} & \mathbf{C}^T \\ \mathbf{B} & \mathbf{C} & 0 \end{pmatrix} \begin{pmatrix} \mathbf{\Phi} \\ \mathbf{\Psi} \\ \mathbf{\Lambda} \end{pmatrix} = - \begin{pmatrix} \boldsymbol{g} \\ \boldsymbol{h} \\ \mathbf{0} \end{pmatrix}$$

The LSS adjoint solution can be determined from

$$\left[ \mathbf{B}\mathbf{V}^{-1}\mathbf{B}^T + \frac{1}{\alpha^2}\mathbf{C}\mathbf{C}^T \right] \mathbf{\Lambda} = -\mathbf{B}\mathbf{V}^{-1}\boldsymbol{g} - \frac{1}{\alpha^2}\mathbf{C}\boldsymbol{h}$$

- This remains a globally coupled space-time problem
- $\mathbf{B}\mathbf{B}^T$ increases the fill of the matrix
- Furthermore, the system is dense due to $\mathbf{C}\mathbf{C}^T$ term

# Sensitivity Evaluation

- To determine sensitivities, we evaluate the conventional sensitivity expression using the LSS adjoint solution
- Conventional terms related to initial conditions drop out

$$\frac{\partial L}{\partial \mathbf{D}} = \frac{\partial f}{\partial \mathbf{D}} \Delta t + \sum_{n=1}^{N} [\mathbf{\Lambda}_g^n]^T \frac{\partial \mathbf{G}^n}{\partial \mathbf{D}} \Delta t$$

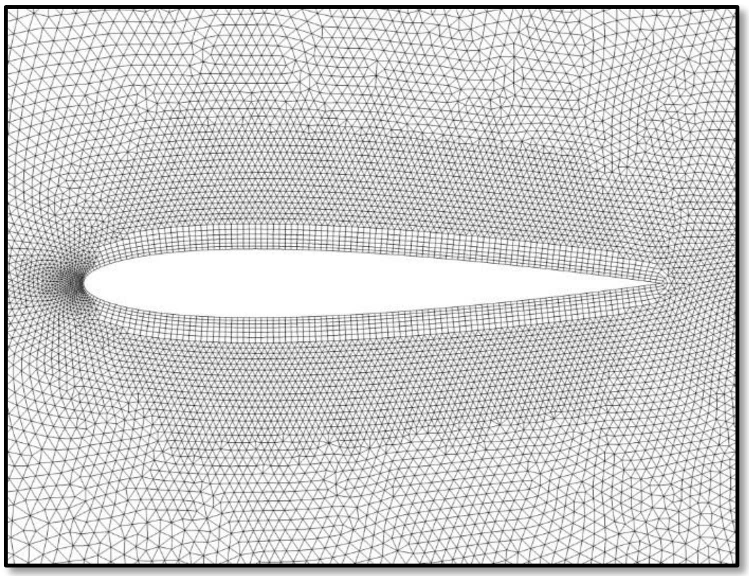
$$+ \sum_{n=1}^{N} [\mathbf{\Lambda}_s^n]^T \left[ \frac{\partial \mathbf{R}^n}{\partial \mathbf{D}} + ((\mathbf{I}_s^n \mathbf{Q}^{n-1}) \circ \mathbf{C}_s^n + \beta \bar{\mathbf{C}}_s^n) \odot \frac{\partial \mathbf{R}_{\mathrm{GCL}}^n}{\partial \mathbf{D}} \right] \Delta t$$

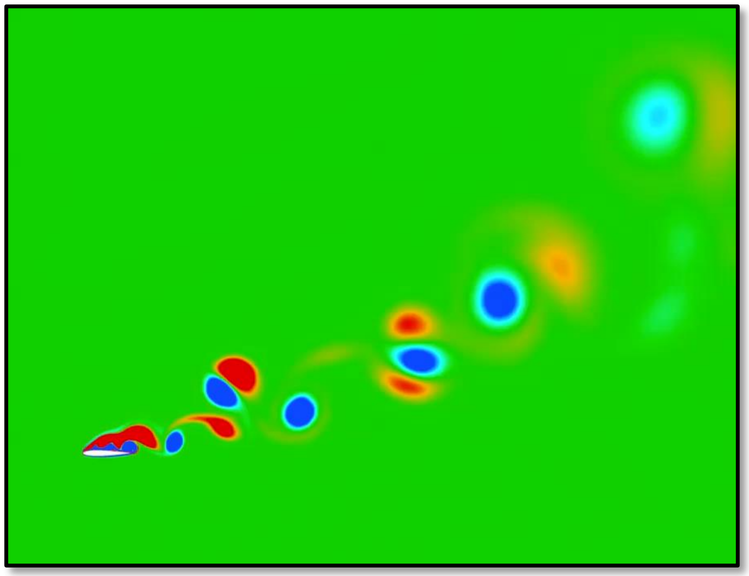
$$\cancel{+ \left( [\mathbf{\Lambda}_g^0]^T \frac{\partial \mathbf{G}^0}{\partial \mathbf{D}} + [\mathbf{\Lambda}^0]^T \left[ \frac{\partial \mathbf{R}^{\mathrm{in}}}{\partial \mathbf{D}} \right] \right) \Delta t}$$

# Problem Definition

Shedding NACA 0012

$M_\infty$=0.1  Re=10,000  $\alpha$=20°

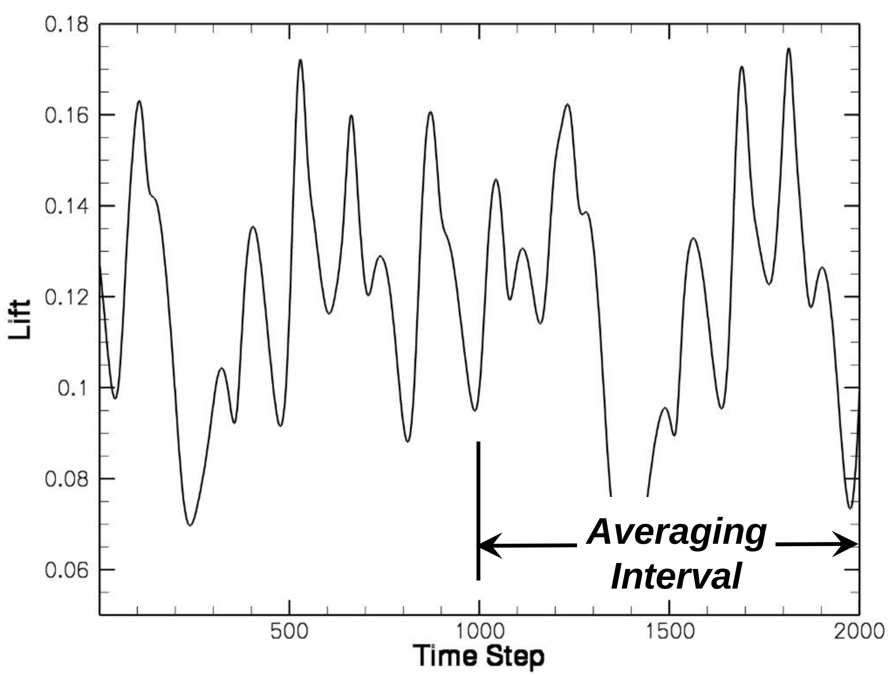




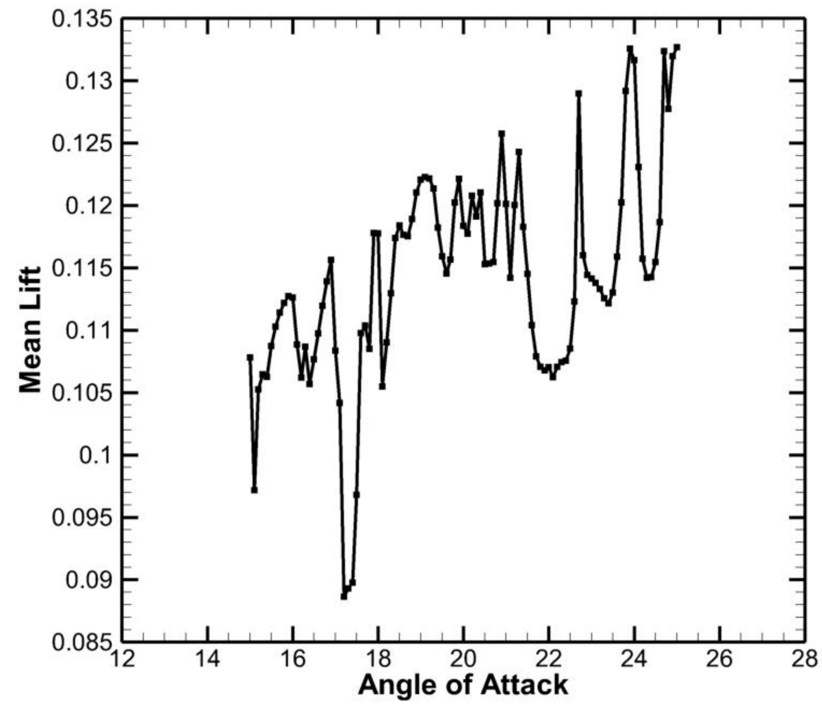
- Unstructured mesh consisting of 102,940 grid points with 100,139 prisms and 1,144 hexes in spanwise direction
- Relatively coarse wall spacing to alleviate stiffness in LSS system
- Laminar Navier-Stokes equations with second-order spatial discretization
- First-order backward differencing in time for LSS simplicity

# Problem Definition

***Lift vs Time Step***




***Lift vs Alpha***




- Simulation started from chaotic initial solution to improve ergodicity
- Objective is to maximize time-averaged lift over final 1,000 time steps

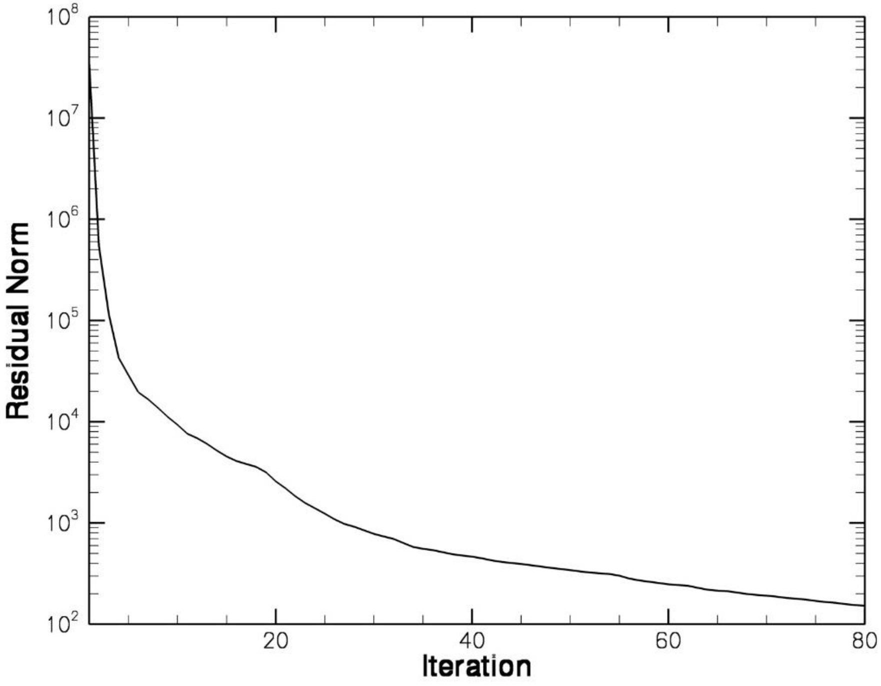
# Approach

- Execute FUN3D flow/adjoint solvers to output data to disk for use in LSS: nonlinear residual vectors and Jacobians of residual and objective function
- For this tiny problem, the raw dataset is ~1.1 TB (in-core requirement much larger)
- Developed standalone LSS solver, where partitioning is performed in time with a single time plane per core
  - Assume the spatial discretization fits on a single core for simplicity
- Global GMRES solver used with a local ILU(0) preconditioner for each time plane, with $\mathbf{CC}^T$ term neglected in preconditioner
- Execution was constrained to a subset of the cores available on each 128 GB Haswell node to provide sufficient memory for solving the LSS adjoint system
- Checked discrete consistency of LSS implementation using complex variables
- This complex variable test does not provide the same rigor for LSS as for conventional adjoint implementations; additional verification approaches needed

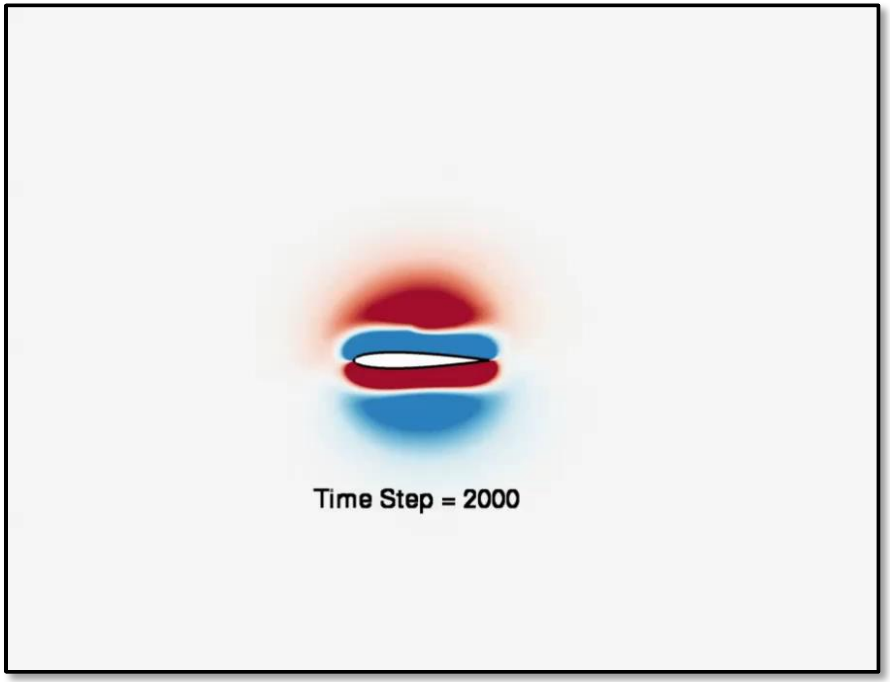
# Solution of LSS Adjoint System

- After ~30 minutes for I/O, solution converges 5 orders of magnitude in ~30 mins on 2,000 cores
- Solution remains bounded

***Convergence of LSS Adjoint System***




***LSS Adjoint Solution for Energy Equation***



# Current Status and Future Outlook

- Assess if (or how well) ergodicity assumption is satisfied for this problem
- Evaluate quality of computed sensitivities
- Attempt design optimization

- How to afford extension of LSS to realistic problems?

***Thank you to the organizers for having us!***