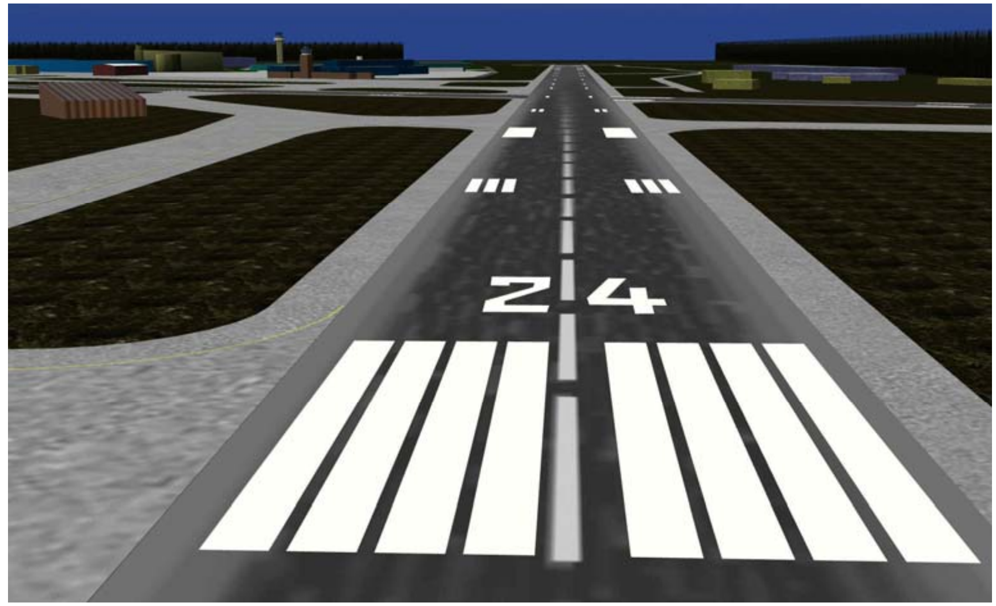
Standard Check-cases for Six-Degree-of-Freedom Flight Vehicle Simulations

Presented by
Michael Madden
NASA Langley Research Center
and
Bob Shelton
NASA Johnson Space Flight Center

Outline



- Problem Description
- Approach
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- Scenarios
  - Atmospheric
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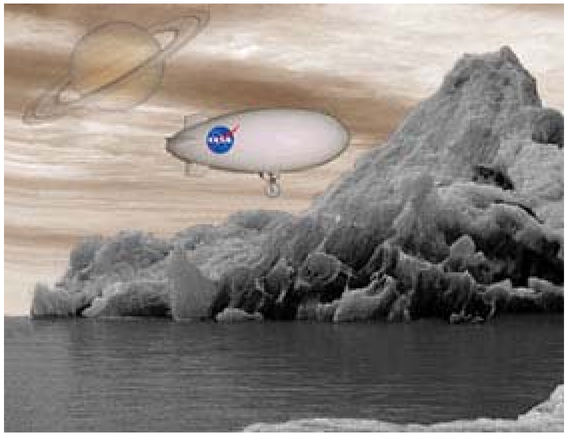
- Comparison Data
- Participating Simulations
- Selected Results
- Conclusions
- Data Repository on NESC Academy Web-Site

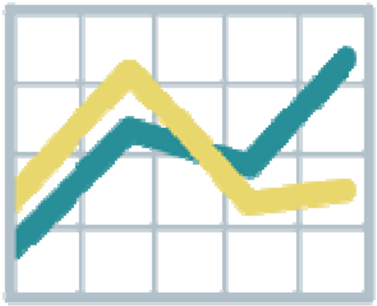
Problem Description

- The different NASA, industry and commercial flight simulation tools sometimes provide substantially different results when applied to the same flight problem.
- Some differences can be traced to errors in the implementation of equations of motion (kinematics and dynamics) and of the geodetic, gravitational, and atmosphere models.
- Currently, there are no standard benchmark check-cases for verification of flight simulation tools. This increases risk in relying on these tools for flight prediction and design in support of NASA's flight projects.



Approach

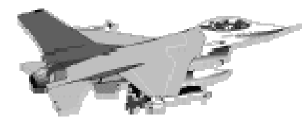
- Agree on set of atmospheric and exo-atmospheric scenarios
  - A scenario defines the vehicle model, vehicle initial conditions, vehicle maneuvers, the environment models (including geodesy, atmosphere and gravity), and duration.
- Agree on output variables to compare and the time history recording exchange format (CSV)

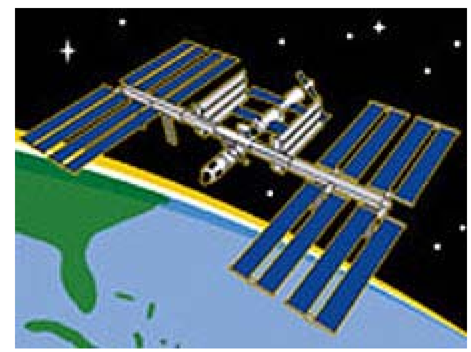


- Develop unambiguous reference models
  - Reference models encoded in ANSI/AIAA S-119 format
- Generate resulting time-histories
- Compare resulting time-histories
- Refine results
  - Identify and eliminate disagreements on scenario configuration
  - Attempt to identify remaining differences in results
- Publish reference trajectory information
  - May be several 'in-family'
  - Anonymize results so that a trajectory cannot be traced to a given simulation tool
  - Publically accessible at http://nescacademy.nasa.gov/flightsim

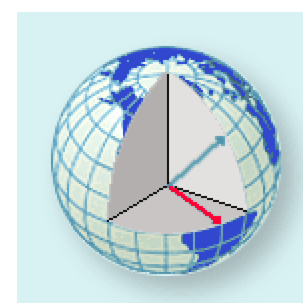
Reference models

- Vehicle models
  - Two Sphere models
    - One sphere in English units for atmospheric cases
    - One sphere in SI units for exo-atmospheric cases
  - Tumbling brick
  - Cylinder
  - Single-engine fighter w/ simple control system
  - Representative International Space Station mass
  - Two-stage rocket w/ constant thrust





- Environment models
  - 2 Geodesy (planetary) models (round and WGS-84)
  - 4 Gravity models (constant, $1/R^2$, J2, and GEM-T1)
  - 3 Atmospheric models (constant, US 1976 and MET)



- Two Inertial Frames (True-of-Date and J2000 – IERS FK5)

Scenarios – Atmospheric

Scenario Matrix for EOM Verification Assessment project (Atmospheric)

2012-07-27

Scenario		Purpose	Gravitation	Geodesy	Atmosphere	Winds
1	Dropped sphere with no drag	Gravitation model, translational EOM	J2	WGS-84 rotating	US Std 1976	still air
2	Tumbling brick with no damping in vacuum	checks rotational EOM	J2	WGS-84 rotating	US Std 1976	still air
3	Tumbling brick with dynamic damping, no drag	checks inertial coupling effects	J2	WGS-84 rotating	US Std 1976	still air
4	Dropped sphere with constant CD, no wind	Simplest model	$1/R^2$	Round non-rotating	US Std 1976	still air
5	Dropped sphere with constant CD, no wind	Adds rotation	$1/R^2$	Round rotating	US Std 1976	still air
6	Dropped sphere with constant CD, no wind	Adds ellipsoid	J2	WGS-84 rotating	US Std 1976	still air
7	Dropped sphere with constant CD + steady wind	Adds wind effects	J2	WGS-84 rotating	US Std 1976	steady wind
8	Dropped sphere with constant CD + 2D wind shear	Adds 2D winds	J2	WGS-84 rotating	US Std 1976	f(alt)
9	Ballistically launched sphere eastward along equator	checks translational EOM	J2	WGS-84 rotating	US Std 1976	still air
10	Ballistically launched sphere northward along prime meridian from equator	checks Coriolis	J2	WGS-84 rotating	US Std 1976	still air
11	Simple linear aero model in trimmed flight across planet (subsonic)	checks aero-related equations, e.g. Mach, calibrated airspeed	J2	WGS-84 rotating	US Std 1976	still air
12	Simple linear aero model in trimmed flight across planet (supersonic)	checks aero-related equations, e.g. Mach, calibrated airspeed	J2	WGS-84 rotating	US Std 1976	still air
13	Maneuvering flight of 6DOF rigid aircraft with non-linear aerodynamics (subsonic)	checks multidimensional table lookups, alpha-dot, beta-dot, Mach etc.	J2	WGS-84 rotating	US Std 1976	still air
14	Maneuvering flight of 6DOF rigid aircraft with non-linear aerodynamics (supersonic)	checks multidimensional table lookups, alpha-dot, beta-dot, Mach etc.	J2	WGS-84 rotating	US Std 1976	still air
15	Circular flight around North pole	checks propagation near singularity, crossing dateline	J2	WGS-84 rotating	US Std 1976	still air
16	Circular flight around equator/dateline intersection	checks for proper sign and wind-up of heading, etc.	J2	WGS-84 rotating	US Std 1976	still air
17	Two-stage rocket to orbit	checks staging, high-altitude atmosphere table	J2	WGS-84 rotating	1976/MET	f(alt)

Scenarios – Exo-atmospheric

Scenario Matrix for EOM Verification Assessment project (Exo-atmospheric)

2012-07-27

	Scenario	Purpose	Gravitation	Atmosphere	3rd Body	Model
1	Earth Modeling Parameters	Environmental constants	N/A	N/A	N/A	N/A
2	Keplerian Propagation	Integration, RNP, orientation	$1/R^2$	N/A	N/A	ISS
3A	Spherical Harmonic Gravity: 4x4	4x4 Harmonic gravity model	4x4	N/A	N/A	ISS
3B	Spherical Harmonic Gravity: 8x8	8x8 Harmonic gravity model	8x8	N/A	N/A	ISS
4	Planetary Ephemeris	Third body gravitation	$1/R^2$	N/A	sun, moon	ISS
5A	Atmosphere: Min. Solar Activity	Free molecular flow	$1/R^2$	MET	N/A	ISS
5B	Atmosphere: Mean. Solar Activity	Free molecular flow	$1/R^2$	MET	N/A	ISS
5C	Atmosphere: Max. Solar Activity	Free molecular flow	$1/R^2$	MET	N/A	ISS
6A	Const Density Drag	Response to constant force	$1/R^2$	const	N/A	sphere
6B	Aero Drag with Dyn. Atmos.	Response to dynamic drag	$1/R^2$	MET	N/A	sphere
6C	Plane Change Maneuver	Response to propulsion firing	$1/R^2$	N/A	N/A	cylinder
6D	Earth Departure Maneuver	Response to propulsion firing	$1/R^2$	N/A	N/A	cylinder
7A	Combined Translational Test: 4x4 Gravity	Translation response	4x4	N/A	sun, moon	sphere
7B	Combined Translational Test: 8x8 Gravity	Translation response	8x8	N/A	sun, moon	sphere
7C	Combined Translational Test: 4x4 Gravity w/ Drag	Translation response	4x4	MET	sun, moon	sphere
7D	Combined Translational Test: 8x8 Gravity w/ Drag	Translation response	8x8	MET	sun, moon	sphere
8A	Rotation Test: No rotation rate	Integration methods for rotation	$1/R^2$	N/A	N/A	ISS
8B	Rotation Test: Initial rotation rate	Integration methods for rotation	$1/R^2$	N/A	N/A	ISS
9A	Torque w/ no initial rotation	Rotational response	$1/R^2$	N/A	N/A	ISS
9B	Torque w/ initial rotation	Rotational Response	$1/R^2$	N/A	N/A	ISS
9C	Torque + Force w/ no initial rotation rate	Rotational Response	$1/R^2$	N/A	N/A	ISS
9D	Torque + Force w/ initial rotation rate	Rotational Response	$1/R^2$	N/A	N/A	ISS
10A	Gravity Gradient: circular orbit, no initial rotation rate	Gravity gradient modeling	$1/R^2$	N/A	N/A	cylinder
10B	Gravity Gradient: circular orbit, initial rotation rate	Gravity gradient modeling	$1/R^2$	N/A	N/A	cylinder
10C	Gravity Gradient: elliptical orbit, no initial rotation rate	Gravity gradient modeling	$1/R^2$	N/A	N/A	cylinder
10D	Gravity Gradient: elliptical orbit, initial rotation rate	Gravity gradient modeling	$1/R^2$	N/A	N/A	cylinder
FULL	Integrated 6-DOF Orbital Motion	Combined effects response	8x8	MET	sun, moon	ISS

Example - Atmospheric Scenario 1

Case	Dropped sphere with no drag				**A01**
Geodesy	WGS-84 rotating		**Duration:**	30 s	
Gravitation	J2				
Atmosphere	US 1976 STD; no wind				
Vehicle	Dragless sphere				
Initial States	**Position**	**Velocity**	**Attitude**	**Rate**	
Inertial[1][2]	(R, 0, 0)	(0, ω*R, 0)	(0, $-\pi/2$, 0)	(0, 0, 0)	
Geocentric	(0, 0, R)	(0, 0, 0)	(0, 0, 0)	($-\omega$, 0, 0)	
Geodetic	(0, 0, 30,000)	(0, 0, 0)	(0, 0, 0)	($-\omega$, 0, 0)	
Ground ref pt	(0, 0, 0)	--	--	--	
Ground relative	(0, 0, 30,000)	(0, 0, 0)	(0, 0, 0)	($-\omega$, 0, 0)	
Notes	[1] normative (primary) reference frame; others for convenience [2] R = ER + 30,000 ft				

Example – Exo-atmospheric Scenario

- Simulation
  - Simulation Duration: 28,800 seconds
  - Data Collection Rate: 60 seconds
- Vehicle
  - Orbital State: Vector A: Typical Orbit (D.1.1)
  - Mass Properties: Mass Properties C (D.2.3)
- Dynamics
  - Rotational Propagation: No
    - Initial Rotation Rate: LVLH
  - External Torques: No
  - External Forces: No
- Environmental Models
  - Gravity Model: On
    - Order: Spherical
    - Planetary Ephemeris: Off
    - Sun/Moon Perturbations: Off
  - Gravity Gradient Torque: Off
  - Atmospheric Model: On
    - F10.7: 128.8
    - Geomagnetic Index: 15.7
  - Aerodynamic Drag Model: Off
    - Coefficient of Drag: N/A
    - Cross-sectional Area: N/A

Comparison Data

- Results stored in comma-separated value (CSV)
  - Emerged as the common format that all teams could supply
- States/values to be stored are:
  - Elapsed simulation time
  - Vehicle rigid-body states (angular & linear velocities and positions)
    - Relative to inertial frame
    - Relative to geodetic frame (atmospheric)
    - Relative to orbit (exo-atmospheric)
  - Output variables – gravitation, aerodynamic forces and moments, atmospheric properties (density, temperature, pressure)
  - Storage frequency is scenario dependent
  - Precision to 10 significant digits, minimum

Participating Simulations

- Core - Dryden Flight Research Center
- JEOD - Johnson Space Center
- LaSRS++ - Langley Research Center
- MAVERIC - Marshall Space Flight Center
- POST-II - Langley Research Center
- VMSRTE - Ames Research Center
- JRBSim – Open-Source

SELECT RESULTS

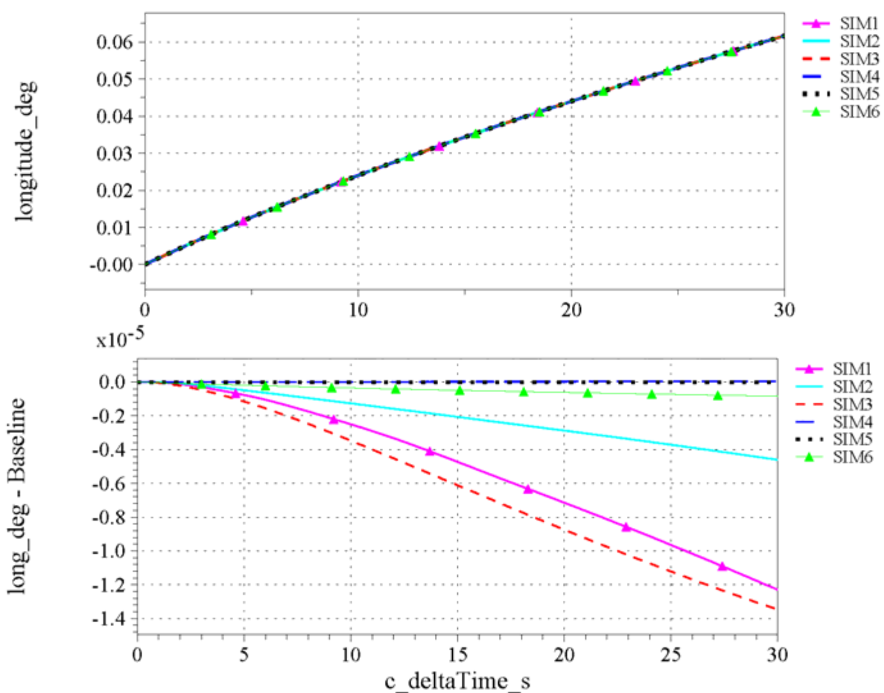
Results are anonymized using designations SIM 1 through SIM 6 for the atmospheric cases and SIM A through SIM D for the orbital cases. Designation order does not match order of tools on slide 8.

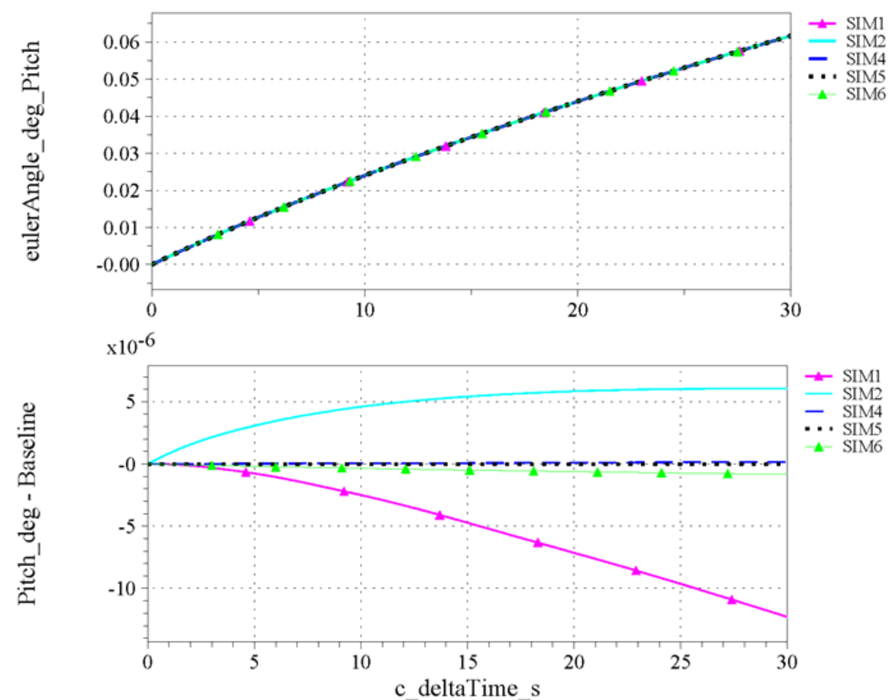
Atmospheric Scenario 9 (1/3)

Case	Sphere launched ballistically eastward along equator				**A09**
Geodesy	WGS-84 rotating		**Duration**:	30 s	
Gravitation	J2				
Atmosphere	US 1976 STD; no wind				
Vehicle	Sphere with constant C_D				
Initial States	**Position**	**Velocity**[1][2]	**Attitude**	**Rate**	
Inertial[2]	(R, 0, 0)	$(V_o, \omega^*R+V_o, 0)$	$(-\pi/2, 0, \pi/2)$	$(-\omega, 0, 0)$	
Geocentric	(0, 0, R)	$(0, V_o, -V_o)$	(0, 0, 90)	(0, 0, 0)	
Geodetic	(0, 0, 0)	$(0, V_o, -V_o)$	(0, 0, 90)	(0, 0, 0)	
Ground ref pt	(0, 0, 0)	--	--	--	
Ground relative	(0, 0, 0)	$(0, V_o, -V_o)$	(0, 0, 90)	(0, 0, 0)	
Notes	[1] R = Earth equatorial radius [2] V_o = 1,000 ft/s (45 degree initial vertical trajectory)				

Atmospheric Scenario 9 (2/3)

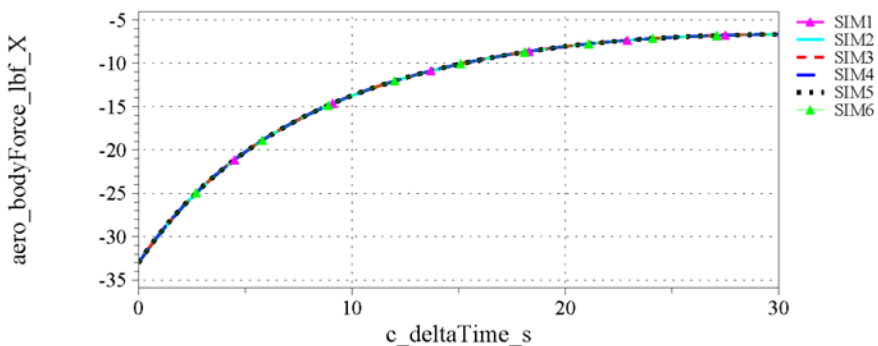
- Due to East trajectory with initial zero rotation rate relative to Earth, pitch angle grows with longitude
- Longitude difference represents separation of up to 5 ft
  - See next slide for explanation
- Pitch angle difference equal to longitude difference except SIM 2
  - SIM 2 has additional difference in integration error for Euler angles

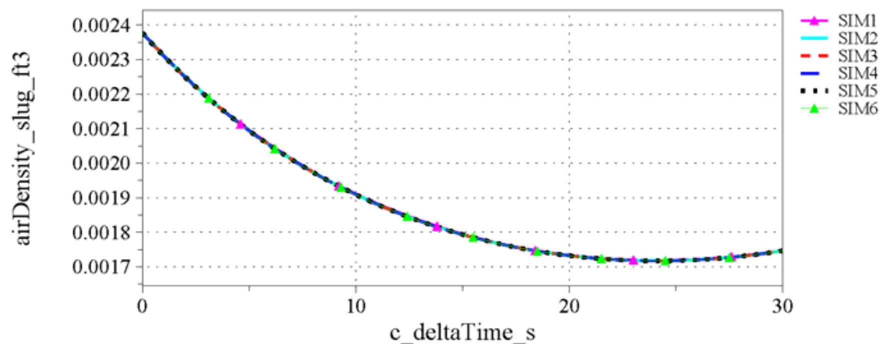


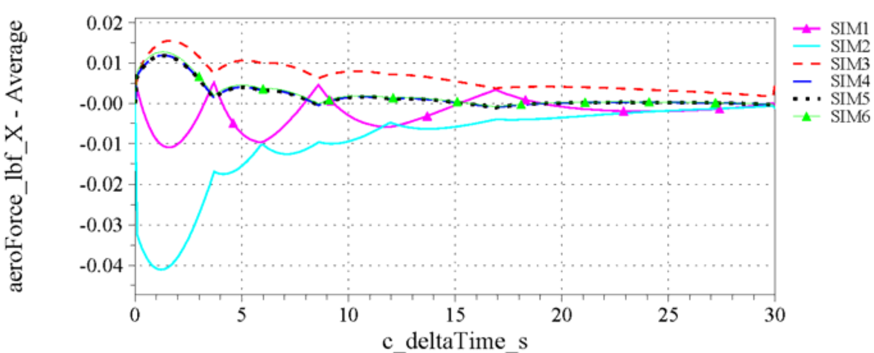


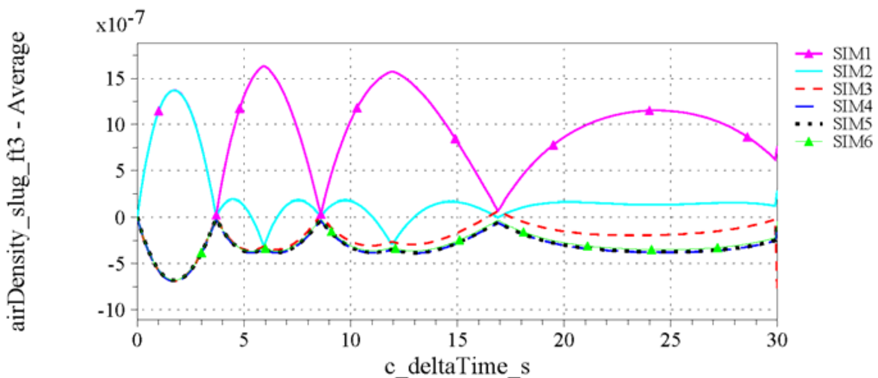
Atmospheric Scenario 9 (3/3)

- Aerodynamic drag differences are driver of translational difference in SIM1 and SIM2
- Differences in atmospheric density contribute to drag differences
  - SIM 1 and SIM 2 both use a table lookup for 1976 Atmosphere Model
  - Density is a non-linear function in the 1976 Atmosphere Model
- SIM 2 exhibits recording lag of one frame for drag
- SIM 3 drag difference due to velocity and altitude difference
- Drag model feedback assumed to amplify trajectory integration error or other unidentified EOM difference for SIM 3 and, to a lesser extent, SIM 1









Atmospheric Scenario 11 (1/3)

Case	Subsonic winged flight – trimmed straight & level			**A11**
Geodesy	WGS-84 rotating		**Duration**:	30 s
Gravitation	J2			
Atmosphere	US 1976 STD; no wind			
Vehicle	Simplified F-16 model; stability augmentation off			
Initial States	**Position**	**Velocity**[2]	**Attitude**	**Rate**
Inertial	[4195599.3, -16425651.4, 12242837.8] ft	[1527.1, 632.8, 323.5] ft/s	[3]	[3]
ECEF	[4195599.3, -16425651.4, 12242837.8] ft	[329.3, 326.9, 323.5] ft/s	[3]	[3]
Geodetic	10,000 ft over FFA (10,013 MSL)	[V_o, V_o, 0]	[3]	[3]
Ground ref pt	FFA[1]	--	--	--
Ground relative	[0, 0, 10000] ft	[393.0, 406.9, 0] ft/s	[3]	[3]
Notes	[1] FFA is [36°01.09' N, 75°40.28' W, 13 ft] with 1° W variation [2] V_o = 400 ft/s with a track angle of 45° true or 46° magnetic; V_{total} = 565.7 ft/s. [3] Attitude and angular rate will depend on the trim solution for the simulation. The geodetic angular rate should be at or near zero.			

Atmospheric Scenario 11 (2/3)

- Trim algorithm determines initial attitude and rotational rate
- Simulations had different targets for rotational rate
  - SIM 2 targeted zero inertial rotational rate
  - SIM 4 targeted zero roll and yaw rate relative to the Earth with pitch rate set to maintain pitch angle as vehicle flew over surface of the Earth
  - SIM 5 targeted a rotational rate in all three axes intended to maintain all three Euler angles as vehicle flew over curved surface of the Earth
- SIM 2 has initial roll angle because J2 gravitation vector is aligned to geodetic normal rather than geocentric radial
  - Roll angle is nearly equal to difference between geodetic and geocentric latitude
  - Trim is aligning vehicle lift vector with resultant direction of gravity

	Roll Rate	Pitch Rate	Yaw Rate
SIM 2	0.00000	0.00000	0.00000
SIM 4	0.00250	-0.00395	-0.00234
SIM 5	0.00253	-0.00394	-0.00314

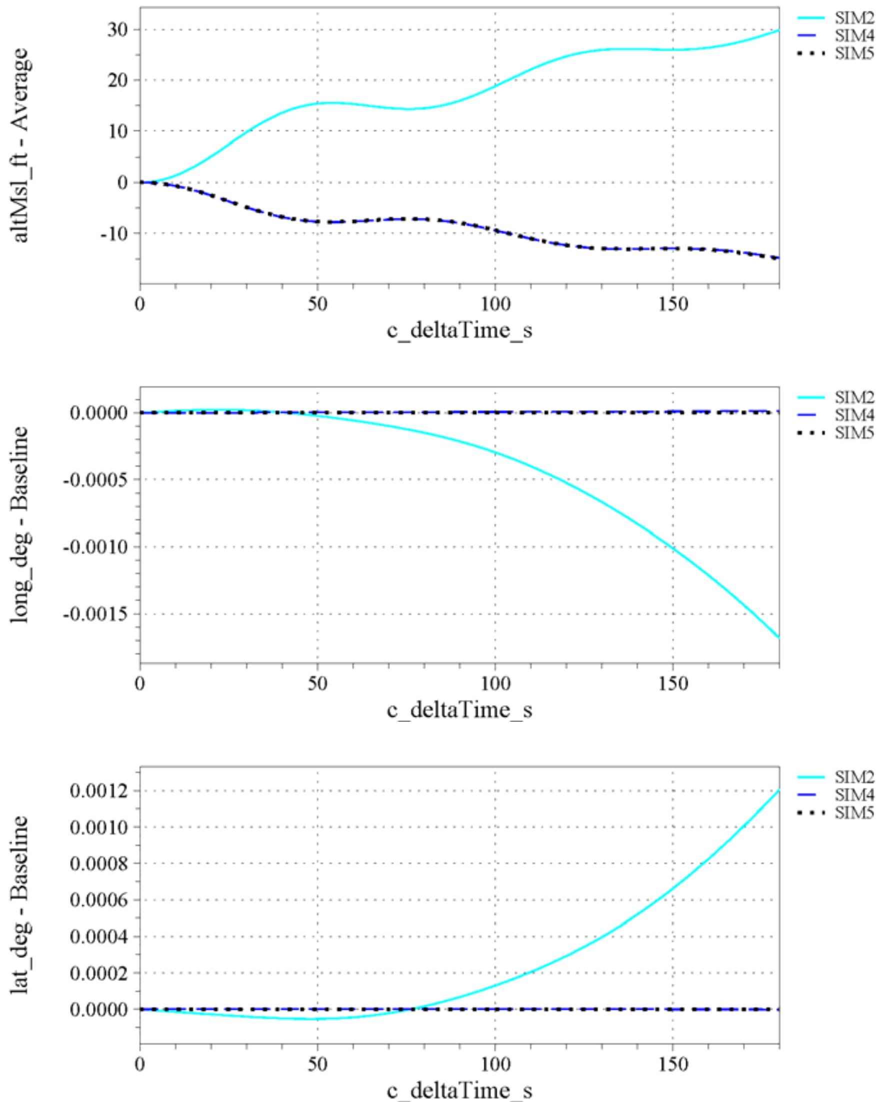
Inertial Rotational Rate at T = 0 (deg/s)

	Phi	Theta	Psi
SIM 2	-0.17183	2.643331	45
SIM 4	0.00000	2.63873	45
SIM 5	0.00000	2.63893	45

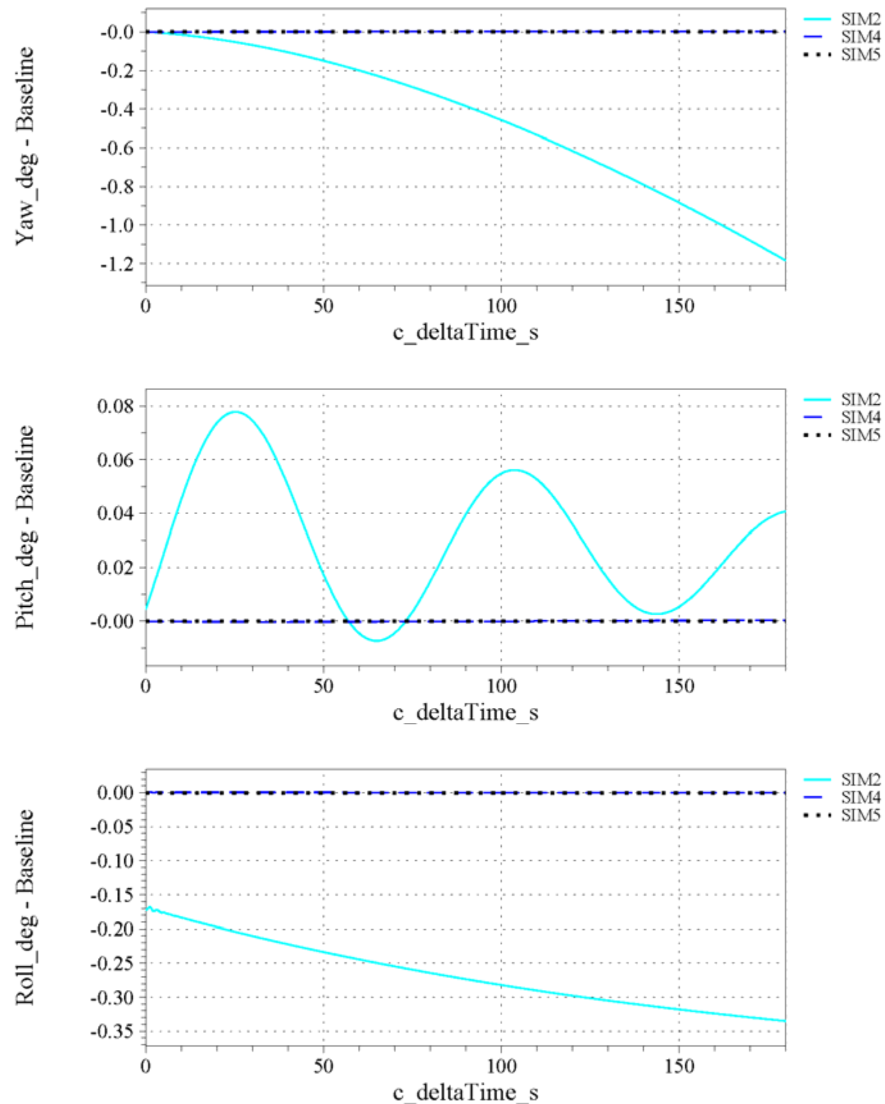
Euler Angles at T = 0 (deg)

Atmospheric Scenario 11 (3/3)

Geodetic Altitude, Longitude, Latitude Difference Plots



Yaw Angle (psi), Pitch Angle (theta), Roll Angle (phi) Difference Plots

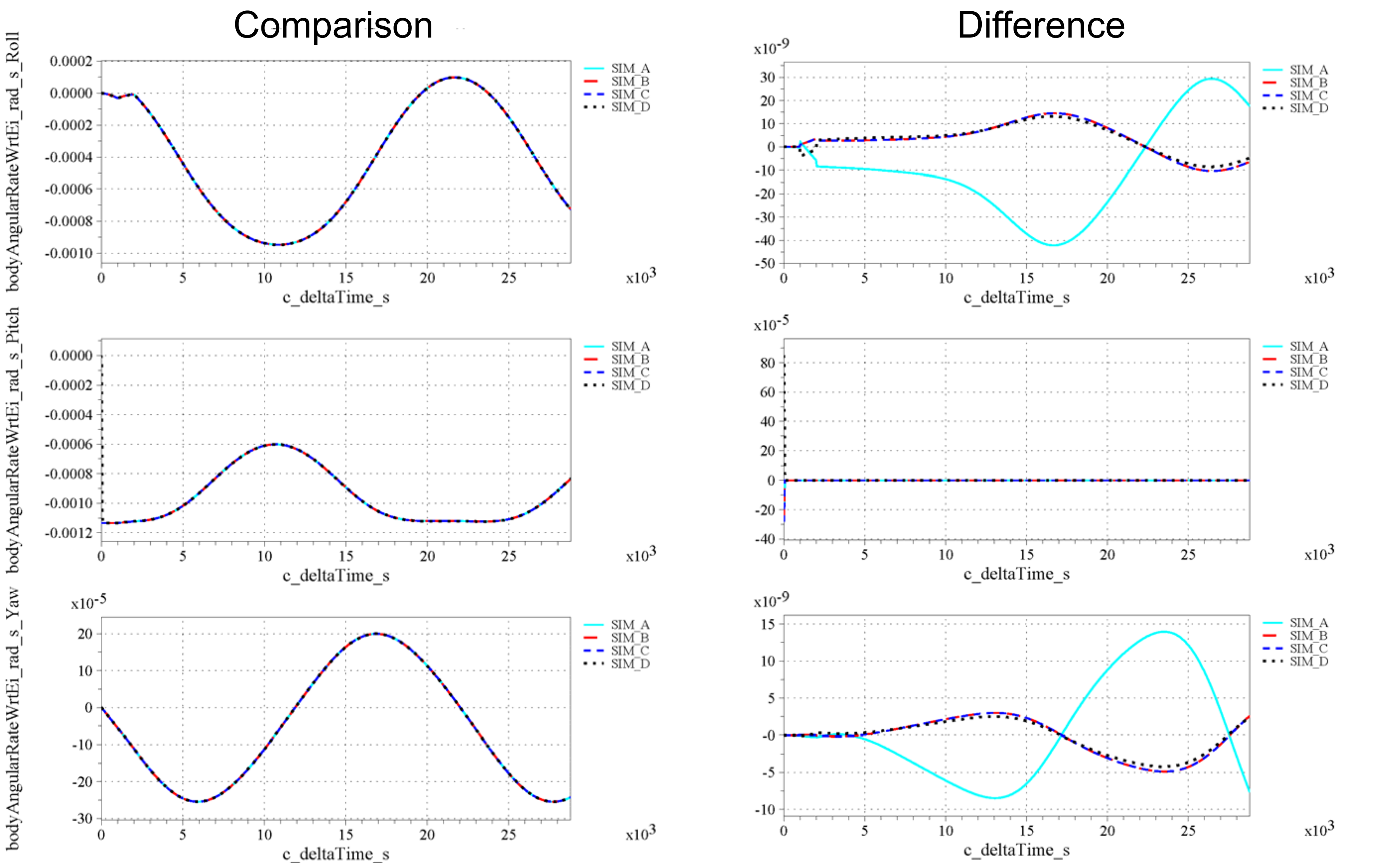


Orbital Scenario 9D (1/5)

- Simulation
  - Simulation Duration: 28,800 seconds
  - Data Collection Rate: 60 seconds
- Vehicle
  - Orbital State: Near Circular Orbit
    - Time = 2007/11/20 00:00:00 UTC
    - r_x = −4315967.74 m
    - r_y = 960356.20 m
    - r_z = 5167269.53 m
    - v_x = 129.091037 m/s
    - v_y = −7491.513855 m/s
    - v_z = 1452.515654 m/s
  - Mass Properties: Representative ISS Mass
- Dynamics
  - Rotational Propagation: Yes
    - Initial Rotation Rate: LVLH
  - External Torques: 10Nm for 1000 s at 1000 s about body X axis
  - External Forces: 10 N for 1000s at 1000s along body X axis
- Environmental Models
  - Gravity Model: On
    - Order: Spherical
    - Planetary Ephemeris: Off
    - Sun/Moon Perturbations: Off
  - Gravity Gradient Torque: Off
  - Atmospheric Model: On
    - F10.7: 128.8
    - Geomagnetic Index: 15.7
  - Aerodynamic Drag Model: Off

Orbital Scenario 9D (2/5)

Inertial Rotational Rate



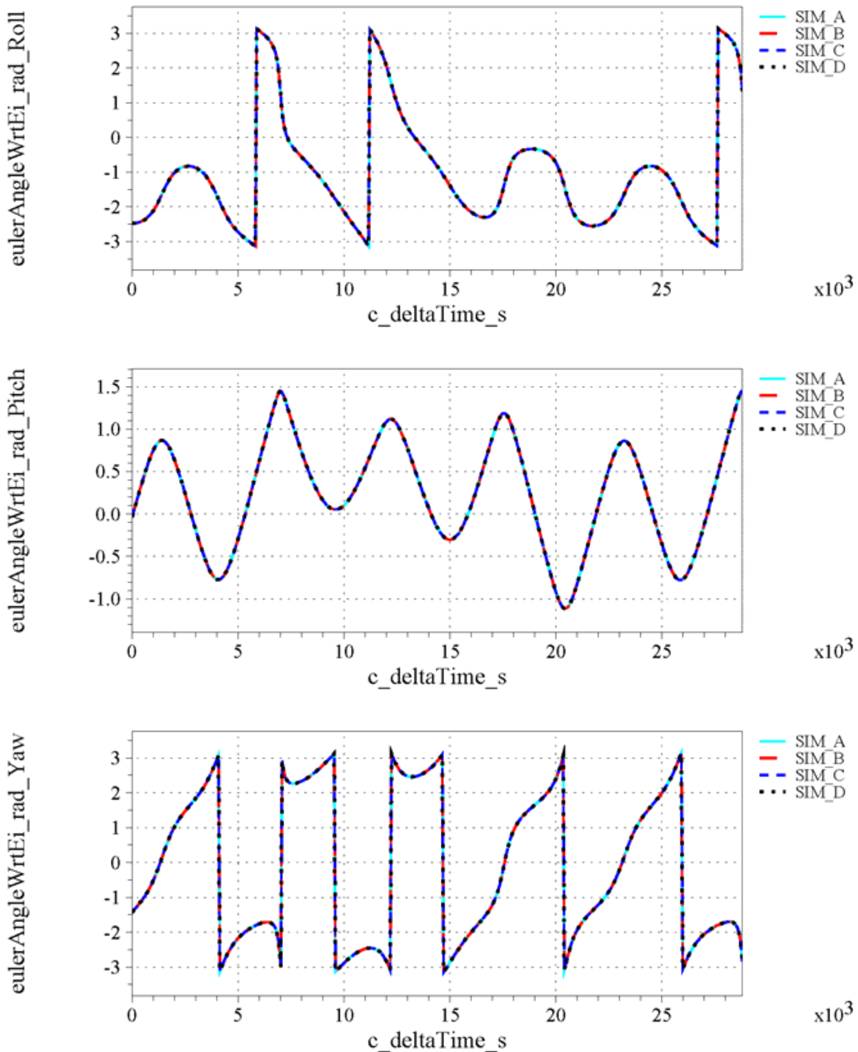
Orbital Scenario 9D (3/5)

- Force and torque are modeled as a square pulse
  - The integration error of a numerical technique can increase at the discontinuous leading and trailing edges of the square pulse.
- The angular rate difference chart shows the sudden change in integration error at the leading and trailing edge of the torque
  - Angular rate differences exhibit growth at leading edge of torque
  - SIM D rotation rate rejoins SIM B and C at trailing edge but interim difference has a lasting affect on attitude
  - SIM A exhibits difference in rotational rate from leading to trailing edge; this drives a permanent difference in rotational rate and attitude
- Translational and rotational dynamics are coupled since thrust is applied along a fixed body axis during torque
  - SIM D orbit after combined thrust-torque pulse differs just enough to separate SIM D from other simulations by 2 meters at end of run
  - SIM A manages to achieve same orbit as SIM B & C despite differences in rotation rate during thrust-torque pulse

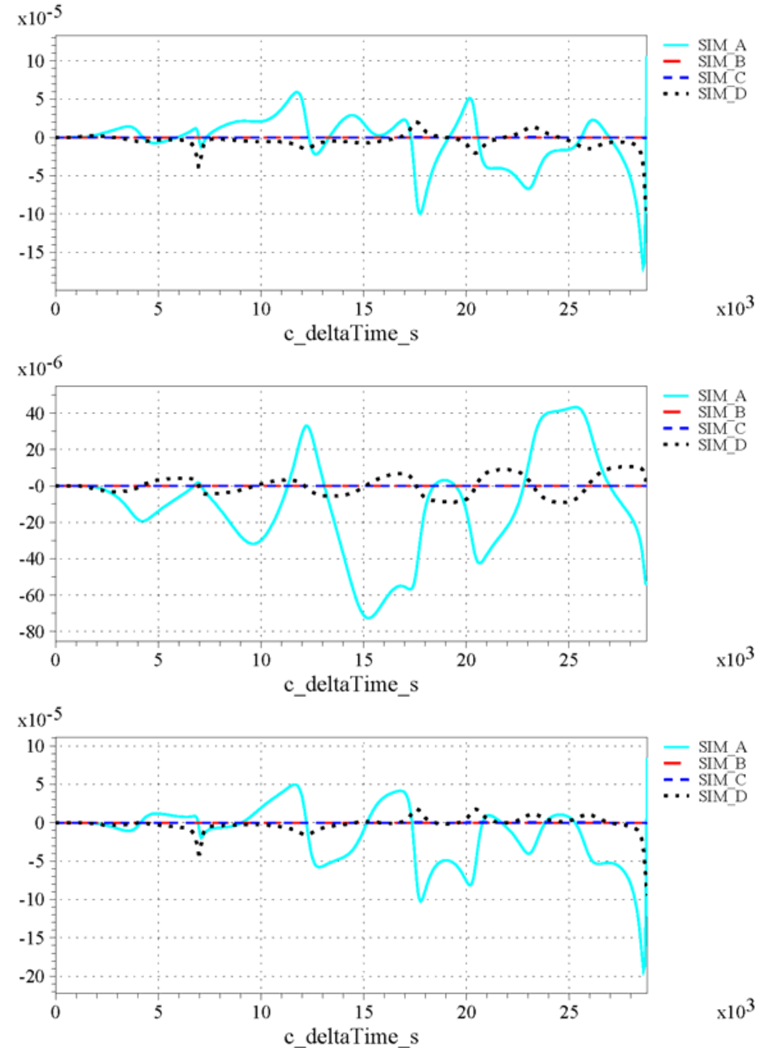
Orbital Scenario 9D (4/5)

Inertial Euler Angles

Comparison



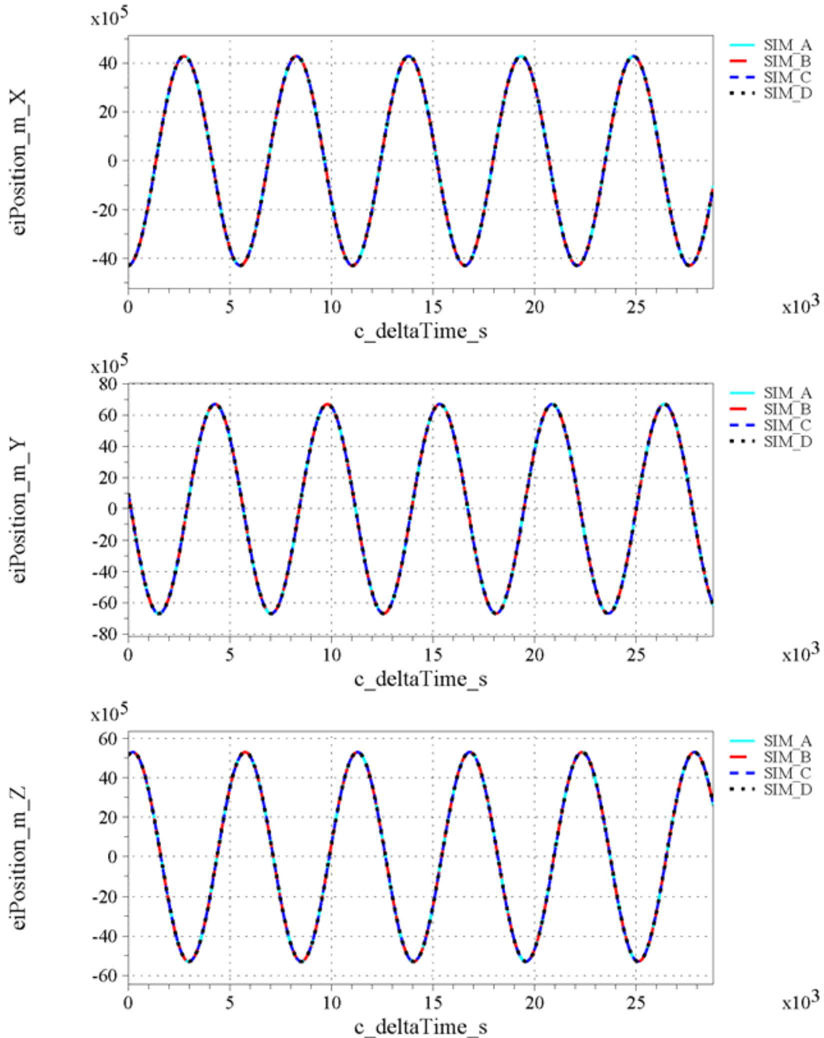
Difference



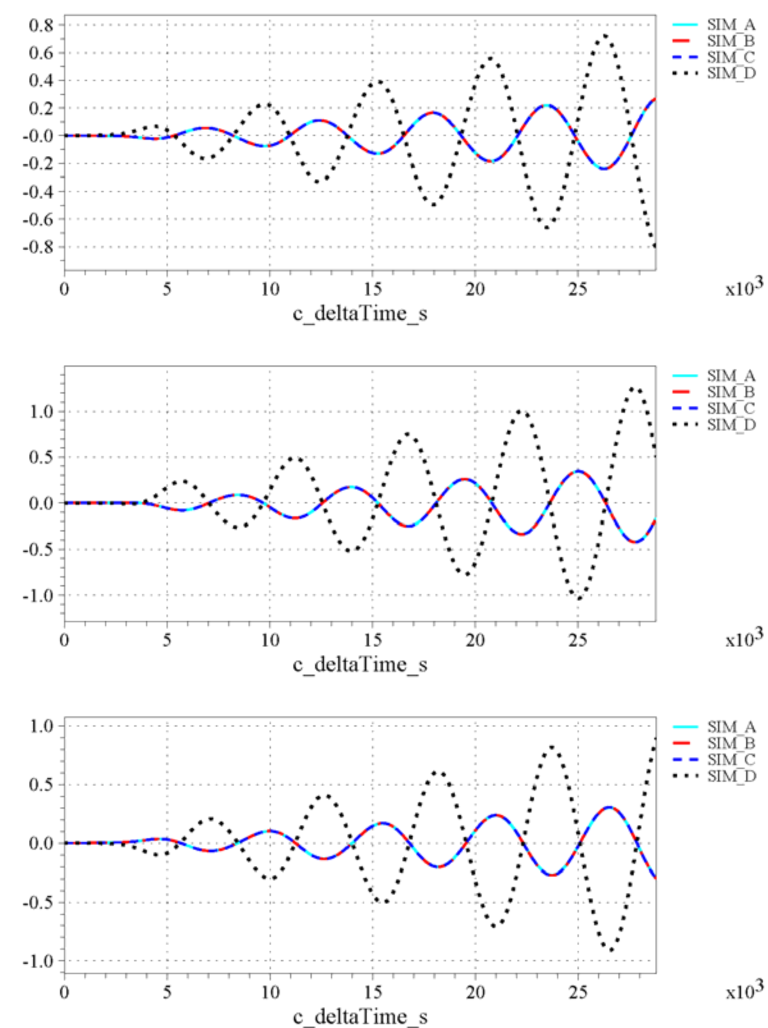
Orbital Scenario 9D (5/5)

Inertial Position

Comparison



Difference



Conclusions (1/2)

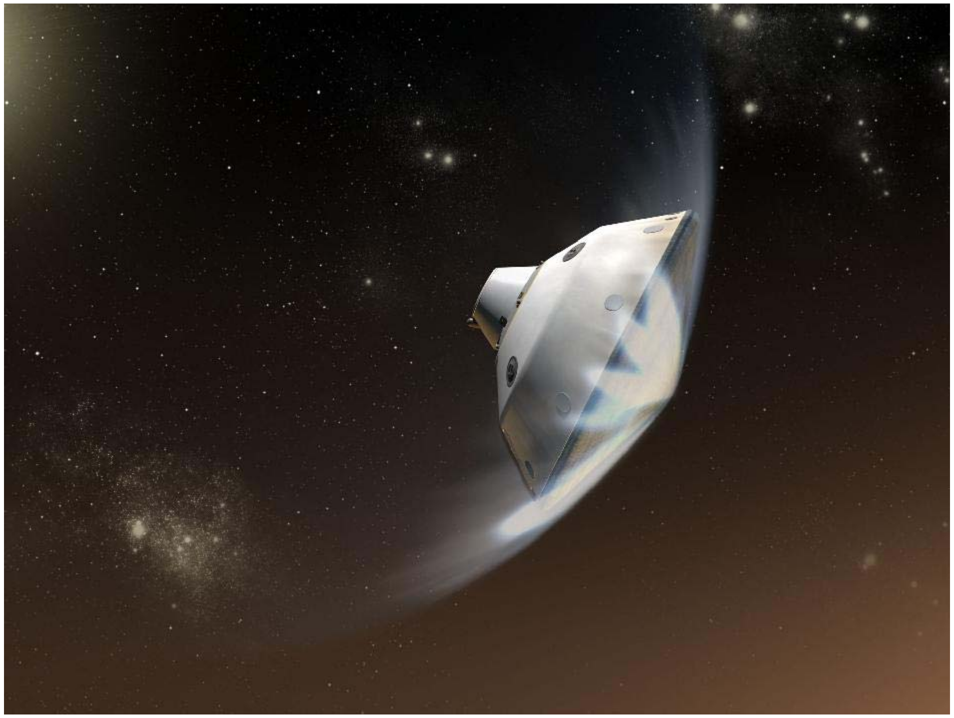
- 25+ rounds of comparison and refinement across the seven simulation tools were necessary to achieve the level of matching presented
- Remaining differences in results have been traced to:
  - Lingering differences in scenario configuration or in simulation parameters (e.g. physical constants, unit conversions)
  - Differences in integration error
  - tabular versus equation-based atmosphere models
  - Differing versions for the MET atmosphere model
  - Heritage simplifications derived from a flat or spherical Earth assumption
  - Differing targets for trim algorithms
  - Differing results from gravitation model including differences in direction of J2 gravitation vector
- Other Lessons Learned
  - Modeling even simple vehicles posed challenges. Teams introduced differences in model implementation, scenario configuration, and modeling parameters.
  - Precise specification of scenarios would be assisted by a standard for specifying the state vector of a 6-DOF flight simulation.

Conclusions (2/2)

- Other Lessons Learned (cont.)
  - In the atmospheric cases, each tool obtained a fair match with at least two other tools. Correlation was closest among tools targeting similar problem domains.
  - Trajectories in the orbital cases matched well, but minor differences remain.
  - Atmospheric trajectories did not match as well as orbital trajectories. Atmospheric flight is highly non-linear, and more tools participated in the atmospheric cases.
  - The effort to develop a validation data set for multiple test cases across numerous simulation tools was estimated at one year and took two and half.
  - Teams employed different definitions for similar sounding variable names in the recorded data. This caused miscommunication in early comparisons. Simulation comparisons would benefit from employing ANSI/AIAA S-119-2011 for unambiguous definition of recorded variables.
  - Every team made at least one improvement to their simulation tool as a result of running the check cases.
  - NASA succeeded in producing a verification data set for 6-DOF flight simulations using the check cases presented. Additional scenarios and results would improve the value of the data set. Future needs include supersonic maneuvering flight and atmospheric re-entry scenarios.

Accessing the Check Cases

- The check case descriptions and data from the initial participating simulations is publically available at URL http://nescacademy.nasa.gov/flightsim/
- Check cases descriptions are detailed in Volume II of NASA/TM-2015-218675
- Trajectory data from the initial participants is provided as zipped CSV files



Questions?

Full Data Set Available at http://nescacademy.nasa.gov/flightsim

BACKUP SLIDES

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Team Members

E. Bruce Jackson
Dr. Robert Shelton
Dr. A. A. Jackson
Manuel P. Castro
Deleena M. Noble
Michael M. Madden
Soumyo Dutta
Daniel K. Litton
Richard W. Powell
Eric M. Queen
Jeremy D. Schidner
William A. Sellers
Scott A. Striepe
William Chung
Joseph P. White
John Aguirre
Curtis J. Zimmerman
Emily K. Lewis
Scott E. Reardon
Nghia Vuong
Michael J. Weinstein
Jon S. Berndt

Reference models – Geodesy

- Round
  - Constant radius sphere with same surface area as WGS-84
    $R_2 = 6.3710071809 \times 10^6$ m (2.0902255199×10^7 ft)
- WGS-84
  - Ellipsoidal Earth with a semi-major (equatorial) radius, flattening parameter, coefficient of eccentricity, and average sidereal rotation
  - WGS-84 defining parameters[3]

equatorial radius	a	6,378,137 m	20,925,600 ft
flattening parameter	$1/f$	298.257223563	
Gravitational constant	GM	$3.986004418 \times 10^{14}$ m^3/s^2	$1.407644311 \times 10^{16}$ ft^3/s^2
Angular rotation rate	ω	7.292115×10^{-5} rad/s	4.178073×10^{-3} deg/s

  - Derived parameters

    $e^2 = 6.69437999014 \times 10^{-3}$ (derived from f) “first eccentricity squared”

Reference models – Coordinate Frames (1/2)

- Earth-Centered Earth-Fixed (ECEF)
  - **X** axis points from the center of the Earth to the intersection of equator and prime meridian.
  - **Z** axis points from the center of the Earth to the geographic North Pole.
  - **Y** = **Z** x **X**
- Two Earth-Centered Inertial (ECI) frames
  - True-of-Date
    - ECI and ECEF axes are aligned at simulation start.
    - Used in atmospheric cases.
  - J2000
    - Modeled on the mean equator and mean equinox of the epoch at noon on 1 Jan 2000 Terrestrial Time.
    - Formally defined with respect to extra-galactic quasar sources (FK5 frame).
    - The IERS publishes code and data to transform J2000 to ECEF
    - Used in exo-atmospheric cases.

Reference models – Coordinate Frames (2/2)

- Ellipsoidal planet adds challenge to calculate position in several coordinate frames: inertial, geocentric, geodetic
  - Conversion between inertial and geocentric is closed form
  - Conversion from geodetic to geocentric coordinates is closed form
  - Conversion from geocentric to geodetic coordinates often uses an iterative solution.

Frame	**Coord. type**	**Coordinates**	**Acronym**	**S-119 ID**
Inertial	Rectangular	X, Y, Z	XYZ	`ei`
Geocentric	Spherical	ψ, λ, r	ULR	`ge`
Geodetic	Spherical	ϕ, λ, h	LLH	`--`
Local	Rectangular	N, E, D	NED	`ll`
Body	Rectangular	x, y, z	xyz	`body`
Ground-relative	Rectangular	x_{fe}, y_{fe}, z_{fe}	--	`fe`
Orbit (LVLH)	Rectangular	x_o, y_o, z_o	--	`vo`

Reference models – Gravitation

1. Constant gravity – A fixed value at all locations.
   - Use unit g (9.80665 m/s^2 or 32.1740 ft/s^2) , which approximates free fall due to gravitation less centrifugal relief due to Earth's rotation.
2. Inverse-square law – Gravitation varies inversely with the square of the radius from the center of the Earth
   - Use WGS-84 μ = 3.986004418 x 10^{14} m^3/s^2 (1.407644311 x 10^{16} ft^3/s^2)
3. J2 – includes the first zonal harmonic fluctuation of gravitation to approximate non-spherical shape of Earth
   - It is a function of geodetic latitude and geocentric radius.
   - Gravitation has two dimensions: radial (inward to Earth center) and latitudinal (Northward)
   - Use J2 = 1.08262982 x 10^{-3}, derived from WGS-84 value of $C_{2,0}$
4. GEM-T1 – Goddard Earth Model T1.
   - Full spherical harmonic expansion of the Earth's gravitation using GEM-T1 published coefficients.
   - Taken to degree and order of 4 x 4 or 8 x 8 in exo-atmospheric cases.

Reference models –Atmosphere

- 1976 U.S. Standard Atmosphere
  - Can be implemented as equations or tables
  - Normally given as a function of geometric height (Z)
  - Translated into S-119 model
- Marshall Engineering Thermosphere Model (MET)
  - Developed by Marshall Space Flight Center for engineering applications
  - modified Jacchia 1970 model that includes some spatial and temporal variation patterns of the Jacchia 1971 model
  - Computes thermospheric densities, temperatures, gravitational accelerations, and specific heats

References

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5. Nguyen, Luat, et. al.: “Simulator Study of a Stall/Post-Stall Characteristics of a Fighter Airplane with Relaxed Static Stability,” NASA TP 1538, 1979
6. Burtch, Robert: "A Comparison of Methods Used in Rectangular to Geodetic Coordinate Transformations," presented at the ACSM Annual Conference and Technology Exhibition, Orlando, FL, April 21-26, 2006. http://www.ferris.edu/faculty/burtchr/papers/cartesian_to_geodetic.pdf
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