Multi-Model Ensemble Wake Vortex Prediction

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Multi-Model Ensemble Wake Vortex Prediction

- Vortex position
- Decay
- Circulation
- Ground effect
- APA3.2
- D2P
- NASA
- TDP2.1
- APA3.4
- DLR

Probabilistic forecast

- Wake vortex behaviour
- Reliability
- Ensemble
- Averaging

Deterministic forecast

- En-route
- Airports
- Outperform
- Individual models

Bayesian Model Averaging
Revision of P2P - Motivation

wake vortex descent

\[ \Gamma^* = \Gamma/\Gamma_0, \quad z^* = z/b_0, \quad y^* = y/b_0, \quad t^* = t/t_0 \]

- \( b_0 \): initial vortex spacing
- \( w_0 \): initial vortex descent speed

88 landings

70 landings
Revision of P2P

wake vortex descent

- secondary vortices weakened by 30 % after first round
- tertiary vortices weakened by 30 % from the beginning \((0.7 \times \Gamma_{sec})\)
- vortex-ground interaction above 0.6 \(b_0\): not yet further investigated
- vortex ground interaction not only distance but also time dependent?
Revision of P2P

![Graphs showing bias in model predictions compared to observations for different locations and configurations.](image)
Revision of P2P
Sugar

How to mix several good ingredients?

Water

Lemon Juice

Lemonade
Why not use the best ensemble member exclusively?

- which is the best member?
- in average best performing member can sometimes be the worst one

Can an ensemble outperform its best member?

- success of ensemble appr.: any model can be the best sometimes
- consistently low performing models → no increase of skill

Yes!

Hagedorn et al., 2005
Ensemble Members

**NASA-DLR cooperation**

**D2P**
- deterministic output of P2P
- based on decaying potential vortex, adapted to LES results (DLR)

**TDP 2.1**
- considers effect of crosswind shear on vortex descent (NASA)

**APA 3.2**
- decay and transport model according to Sarpkaya (NASA)

**APA 3.4**
- reduced effect of stratification (NASA)

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**Probability that one of the models delivers the best forecast**
(in ground-effect, on the basis of rmse for 99 example cases)
Multi-Model Ensemble

Reliability Ensemble Averaging (REA)

\[ \tilde{f} = \tilde{A}(f) = \frac{\sum_i R_i f_i}{\sum_i R_i} \]

- \( R_i \) = reliability factor of model i
- \( f_i \) = forecast of model i
- \( \tilde{A}(f) \) = REA-forecast
- \( n \) = natural variability
- \( B_i \) = absolute bias of model i
- \( D_i \) = absolute difference between forecast of model i and ensemble mean

Giorgi and Mearns, 2002
Multi-Model Ensemble

Reliability Ensemble Averaging

\[ R_{D,i} \text{ depends on distance to ensemble mean:} \]

If bias or distance to ensemble mean < \( nv \) → model reliable (\( R_{B,i} \) or \( R_{D,i} = 1 \))

\( \rightarrow \text{nv = model resolution limit} \)
Multi-Model Ensemble

Reliability Ensemble Averaging uncertainty bounds:

\[ \tilde{\delta}_f = \left[ \tilde{A}(f_i - \tilde{f})^2 \right]^{1/2} = \left[ \frac{\sum R_i (f_i - \tilde{f})^2}{\sum R_i} \right]^{1/2} \]

uncertainty bounds depend on ensemble spread

\[ f_+ = \tilde{f} + \tilde{\delta}_f \]

\[ f_- = \tilde{f} - \tilde{\delta}_f \]

according to Giorgi and Mearns, 2002
Application to Wake Vortex Models

Reliability Ensemble Averaging

Training
- mixture of landings from WakeFRA, WakeMUC and WakeOP
- 95 selected cases

$R_{B,i}$ and $R_{D,i}$
- $R_{B,z,i}(t)$, $R_{B,y,i}(t)$, $R_{B,\Gamma,i}(t)$, $R_{D,z,i}(t)$, $R_{D,y,i}(t)$, $R_{D,\Gamma,i}(t)$
- $\Delta t^* = 2 \, t_0$
- separately for luff and lee vortices
- weights for reliability factors: $R_{B,z,i} : m=1.0$, $R_{D,z,i} : n=0.3$

Uncertainty envelope
- initial condition uncertainty added (not considered in original approach):

<table>
<thead>
<tr>
<th>variable</th>
<th>unit</th>
<th>$\sigma$ (standard deviation)</th>
</tr>
</thead>
<tbody>
<tr>
<td>true airspeed</td>
<td>[m/s]</td>
<td>4</td>
</tr>
<tr>
<td>air density</td>
<td>[kg/m³]</td>
<td>0.0048</td>
</tr>
<tr>
<td>weight</td>
<td>[kg]</td>
<td>1300</td>
</tr>
<tr>
<td>$z_0$</td>
<td>[m]</td>
<td>7</td>
</tr>
<tr>
<td>$y_0$</td>
<td>[m]</td>
<td>25</td>
</tr>
</tbody>
</table>

<table>
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<tr>
<th>variable</th>
<th>unit</th>
<th>$\sigma$ (standard deviation)</th>
</tr>
</thead>
<tbody>
<tr>
<td>if initial conditions derived from lidar:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$z_0$</td>
<td>[m]</td>
<td>9</td>
</tr>
<tr>
<td>$y_0$</td>
<td>[m]</td>
<td>13</td>
</tr>
<tr>
<td>$\Gamma_0$</td>
<td>[m²/s]</td>
<td>13</td>
</tr>
</tbody>
</table>
Application to Wake Vortex Models

REA natural variability, $\Gamma^*$

\[ N^* = N^* t_0 \]
\[ \varepsilon^* = (\varepsilon \cdot b_0)^{1/3}/w_0 \]
\[ \nu^* = \nu/w_0 \]

<table>
<thead>
<tr>
<th>$N^* &lt; 0.3, \varepsilon^* &gt; 0.25$</th>
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</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{\text{obs}, \Gamma^*_{\text{luff}}}$</td>
<td>0.097 (1073)</td>
</tr>
<tr>
<td>$\sigma_{\text{obs}, \Gamma^*_{\text{lee}}}$</td>
<td>0.081</td>
</tr>
</tbody>
</table>

\( \varepsilon^* > 0.2, N^* < 0.3 \)

\( \varepsilon^* < 0.2, N^* < 0.3 \)
Application to Wake Vortex Models

REA natural variability, $z^*$

$$\sigma_{obs} = \sqrt{\sigma_{err}^2 + \sigma_{nu}^2}$$

| $N^* < 0.3$, $|v^*| > 0.5$ | $N^* < 0.3$, $|v^*| < 0.5$ |
|-----------------|-----------------|
| $\sigma_{obs,z_{luff}}$ | 0.35 (1224) | 0.200 (805) |
| $\sigma_{obs,z_{lee}}$ | 0.35 | 0.262 |


| $|v^*| > 1.0$, $N^* < 0.3$ | $|v^*| < 1.0$, $N^* < 0.3$ |
|-----------------|-----------------|
| $z^* - z_0$ | $z^* - z_0$ |

<table>
<thead>
<tr>
<th>$t^*$</th>
<th>$t^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>7</td>
<td>7</td>
</tr>
</tbody>
</table>
Results

REA forecast
(one single landing)

Enhancement:

- $\text{rmse}_{z^*,TDP} = 0.158$
- $\text{rmse}_{z^*,REA} = 0.148$
- $\text{rmse}_{\Gamma^*,D2P} = 0.085$
- $\text{rmse}_{\Gamma^*,REA} = 0.072$

Probability levels according to:
- 99 testcases
- WakeFRA & WakeOP

- 89.1 %
- 89.1 %
- 62.9 %
- 62.6 %
- 69.7 %
- 75.6 %
Results

REA reliability factors (one single landing)

no correlation between $R_D$ and $R_B$ can be found!
Results

REA forecast reliability (one single landing)

\[ \tilde{\rho} = \frac{\sum R_i^2}{\sum R_i} \]

- low reliability for y - forecast
- high reliability for z - forecast
- medium reliability for \( \Gamma \) - forecast
Results

REA scoring

- 99 randomly chosen cases
- skill factor $s_i$: 

$$s_i = \frac{\sum_{p=1}^{n} \frac{rmse_{e,p}}{rmse_{i,p}}}{n} - 1$$

<table>
<thead>
<tr>
<th>median</th>
<th>$\text{rms } \Gamma_{luf}^*$</th>
<th>$\text{rms } \Gamma_{lee}^*$</th>
<th>$\text{rms } y_{luf}$</th>
<th>$\text{rms } y_{lee}$</th>
<th>$\text{rms } z_{luf}^*$</th>
<th>$\text{rms } z_{lee}^*$</th>
<th>$s$</th>
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<tr>
<td>REA</td>
<td>0.116</td>
<td>0.099</td>
<td>0.433</td>
<td>0.409</td>
<td>0.163</td>
<td>0.149</td>
<td>0.000</td>
</tr>
<tr>
<td>TDP 2.1</td>
<td>0.127</td>
<td>0.106</td>
<td>0.590</td>
<td>0.415</td>
<td>0.237</td>
<td><strong>0.147</strong></td>
<td>-0.122</td>
</tr>
<tr>
<td>APA 3.4</td>
<td>0.160</td>
<td>0.107</td>
<td>0.602</td>
<td>0.413</td>
<td>0.179</td>
<td>0.154</td>
<td>-0.127</td>
</tr>
<tr>
<td>APA 3.2</td>
<td>0.204</td>
<td>0.140</td>
<td>0.612</td>
<td>0.410</td>
<td>0.179</td>
<td>0.154</td>
<td>-0.190</td>
</tr>
<tr>
<td>D2P</td>
<td><strong>0.122</strong></td>
<td><strong>0.120</strong></td>
<td><strong>0.406</strong></td>
<td><strong>0.408</strong></td>
<td><strong>0.140</strong></td>
<td><strong>0.166</strong></td>
<td>-0.016</td>
</tr>
</tbody>
</table>
Results

REA scoring

- 99 randomly chosen cases
- skill factor $s_i$:

$$ s_i = \frac{1}{n} \sum_{p=1}^{n} \frac{r m s e_{e,p}}{r m s e_{i,p}} - 1 $$

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</tr>
<tr>
<td>DEA</td>
<td>0.131</td>
<td>0.093</td>
<td>0.525</td>
<td>0.411</td>
<td>0.173</td>
<td>0.149</td>
<td>-0.048</td>
</tr>
</tbody>
</table>

advanced MME approach outperforms Direct Ensemble Average (DEA)
PDD of models and ensemble

**overconfident ensemble:**
too narrow ensemble spread

**well-dispersed ensemble:**
coverage of full spectrum of possible solutions

Weigel et al., 2008, Hagedorn et al., 2004

well-dispersed model forecasts → rmse improvement

overconfident ensemble → small or no rmse improvement
Conclusion

- ensemble **can improve quality** of wake vortex forecasts in average
- however **only 1.6 %** improvement compared to best model
  - **reason:** ensemble is **overconfident** for z* and y*
- **but:** models might behave differently in particular ambient weather conditions and out-of-ground → investigation with pdds
Further Development

- How does a good training data set look like?
- Can the results be further improved by distinguishing various ambient weather conditions?
- How does the Bayesian Model Averaging (BMA) perform?

source: Raftery et al., 2005
Backup
Wake Vortex Predictions

Motivation

1. - optimization of tactical separation at airports
   - hazard warning system

2. - Wake Encounter Avoidance & Advisory System (WEAA)
   - “Free Flight”
Ensemble Methoden
Bayesian Model Averaging

\[
P(B) = \text{Wahrscheinlichkeit des Eintretens von } B
\]

\[
P(B|A) = \text{Wahrscheinlichkeit für } B, \text{ unter Vorraussetzung } A
\]

PDF = Probability Density Function (Wahrscheinlichkeitsdichtefunktion)

\[
P(B) = \sum_{n} P(B \cap A_n) = \sum_{n} P(A_n) P(B|A_n)
\]

Beispiel:
Wir befinden uns auf einem Schiff:
- wir wollen die Position B bestimmen
- 3 Crew-Mitglieder (A1,A2,A3) wissen wie es geht, haben aber unterschiedliche Methoden

according to Grimmett and Welsh., 1986
Ensemble Methoden
Bayesian Model Averaging

Law of total probability:

\[ P(B) = \sum_{n} P(B \cap A_n) = \sum_{n} P(A_n) P(B|A_n) \]

<table>
<thead>
<tr>
<th>Methode</th>
<th>A1</th>
<th>A2</th>
<th>A3</th>
</tr>
</thead>
<tbody>
<tr>
<td>individuelle Wahrscheinlichkeit, dass die Methode Erfolg hat: P(B</td>
<td>A_n)</td>
<td>0.6</td>
<td>0.9</td>
</tr>
<tr>
<td>Wahrscheinlichkeit, dass wir A1, A2 or A3 fragen: P(A_n)</td>
<td>0.2</td>
<td>0.5</td>
<td>0.3</td>
</tr>
</tbody>
</table>

\[ s = \sum_{n} v_n * t_n \]

\[ P(B) = 0.78 \]
Ensemble Methoden
Bayesian Model Averaging

Law of total probability:

\[ P(B) = \sum_{n} P(B \cap A_n) = \sum_{n} P(A_n) P(B|A_n) \]

<table>
<thead>
<tr>
<th>Methode</th>
<th>A1</th>
<th>A2</th>
<th>A3</th>
</tr>
</thead>
<tbody>
<tr>
<td>PDF der Methode (Modell- Unsicherheiten): ( P(B</td>
<td>A_n) )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wahrscheinlichkeit, dass wir A1, A2 or A3 fragen: ( P(A_n) )</td>
<td>0.1</td>
<td>0.6</td>
<td>0.3</td>
</tr>
</tbody>
</table>

\[ s = \sum_{n} v_n \cdot t_n \]
Ensemble Methoden
Bayesian Model Averaging

Law of total probability:  
\[ P(B) = \sum_{n} P(B \cap A_n) = \sum_{n} P(A_n) P(B|A_n) \]

Angewandt auf Vorhersage-Modelle:

\[ A_n = \text{Modell } n \]
\[ B = \text{vorherzusagende Größe} \]
\[ B^T = \text{Trainings-Daten} \]
\[ P(A_n) = \text{Wahrscheinlichkeit, dass } A_n \text{ das beste Modell ist} \]

(Gewichtungsfaktor, basierend auf \( B^T \))

\[ P(B|A_n) = \text{PDF of } A_n \text{ alone (Gaussian distribution, given that } A_n \text{ is the best forecast)} \]

\[ \Delta \text{ gewichtete Summe von Wahrscheinlichkeitsdichtefunktionen (PDFs)} \]

Annahme: es gibt immer ein bestes Ensemble-Glied

according to Raftery et al., 2005
Ensemble Methoden
Bayesian Model Averaging

BMA applied on 48-h surface temperature forecast (bias corrected)

- ensemble forecast
- individual model PDF
- individual model forecast
- 90% interval
- verification

source: Raftery et al., 2005
Multi-Model Ensemble

benefit

- increase deterministic skill
- predict forecast skill
- provide probabilistic forecast

individual model forecasts

average

probabilistic envelope
Multi-Model Ensemble