Multi-Model Ensemble Wake Vortex Prediction

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Multi-Model Ensemble Wake Vortex Prediction

vortex position
decay
circulation
ground effect

NASA
TDP2.1
APA3.4

DLR

probabilistic forecast
Bayesian Model Averaging

deterministic forecast
en-route

Reliability
Ensemble Averaging

wake vortex behaviour
airports

outperform individual models
Revision of P2P - Motivation

wake vortex descent

\[ \Gamma^* = \Gamma / \Gamma_0, \quad z^* = z / b_0, \quad y^* = y / b_0, \quad t^* = t / t_0 \]

- Initial vortex spacing: \( b_0 \)
- Initial vortex descent speed: \( w_0 \)

88 landings

70 landings
Revision of P2P

wake vortex descent

- secondary vortices weakened by 30% after first round
- tertiary vortices weakened by 30% from the beginning \(0.7 \times \Gamma_{sec}\)
- vortex-ground interaction above 0.6 \(b_0\): not yet further investigated
- vortex ground interaction not only distance but also time dependent?
Revision of P2P

\[
\text{bias} = \text{model} - \text{observation}
\]
Revision of P2P
Multi Model Ensemble

Sugar
How to mix several good ingredients?

Water

Lemon Juice

Lemonade
Why not use the best ensemble member exclusively?

- which is the best member?
- in average best performing member can sometimes be the worst one

Can an ensemble outperform its best member?
- success of ensemble appr.: any model can be the best sometimes
- consistently low performing models → no increase of skill

Yes!

Hagedorn et al., 2005
Ensemble Members

NASA-DLR cooperation

**D2P**
- deterministic output of P2P
- based on decaying potential vortex, adapted to LES results (DLR)

**TDP 2.1**
- considers effect of crosswind shear on vortex descent (NASA)

**APA 3.2**
- decay and transport model according to Sarpkaya (NASA)

**APA 3.4**
- reduced effect of stratification (NASA)

Probability that one of the models delivers the best forecast
(in ground-effect, on the basis of rmse for 99 example cases)
Multi-Model Ensemble

Reliability Ensemble Averaging (REA)

\[ \tilde{f} = \tilde{A}(f) = \frac{\sum_i R_i f_i}{\sum_i R_i} \]

\[ R_i = \left[ (R_{B,i})^m \cdot (R_{D,i})^n \right]^{1/(m \cdot n)} = \left\{ \left[ \frac{nv}{\text{abs}(B_i)} \right]^m \left[ \frac{nv}{\text{abs}(D_i)} \right]^n \right\}^{1/(m \cdot n)} \]

- \( f_i \) = forecast of model i
- \( \tilde{A}(f) \) = REA-forecast
- \( R_i \) = reliability factor of model i
- nv = natural variability
- B_i = absolute bias of model i
- D_i = absolute difference between forecast of model i and ensemble mean

Giorgi and Mearns, 2002
Multi-Model Ensemble

Reliability Ensemble Averaging

\[ R_{D,i} \text{ depends on distance to ensemble mean:} \]

\[ \text{ensemble mean} \]

\[ \text{natural variability} \]

if bias or distance to ensemble mean < \( n_v \) → model reliable (\( R_{B,i} \) or \( R_{D,i} = 1 \))

\[ n_v = \text{model resolution limit} \]
Multi-Model Ensemble

Reliability Ensemble Averaging uncertainty bounds:

\[ \tilde{\delta}_f = [\tilde{A}(f_i - \tilde{f})^2]^{1/2} = \left[ \frac{\sum_i R_i (f_i - \tilde{f})^2}{\sum_i R_i} \right]^{1/2} \]

uncertainty bounds depend on ensemble spread

\[ f_+ = \tilde{f} + \tilde{\delta}_f \]

\[ f_- = \tilde{f} - \tilde{\delta}_f \]

according to Giorgi and Mearns, 2002
Application to Wake Vortex Models

Reliability Ensemble Averaging

Training
- mixture of landings from WakeFRA, WakeMUC and WakeOP
- 95 selected cases

\[ R_{B,i} \text{ and } R_{D,i} \]
- \( R_{B,z,i}(t), R_{B,y,i}(t), R_{B,\Gamma,i}(t), R_{D,z,i}(t), R_{D,y,i}(t), R_{D,\Gamma,i}(t) \)
- \( \Delta t^* = 2 \ t_0 \)
- separately for luff and lee vortices
- weights for reliability factors: \( R_{B,z,i} : m=1.0, R_{D,z,i} : n=0.3 \)

Uncertainty envelope
- initial condition uncertainty added (not considered in original approach):

<table>
<thead>
<tr>
<th>variable</th>
<th>unit</th>
<th>( \sigma ) (standard deviation)</th>
</tr>
</thead>
<tbody>
<tr>
<td>true airspeed</td>
<td>[m/s]</td>
<td>4</td>
</tr>
<tr>
<td>air density</td>
<td>[kg/m³]</td>
<td>0.0048</td>
</tr>
<tr>
<td>weight</td>
<td>[kg]</td>
<td>1300</td>
</tr>
<tr>
<td>z0</td>
<td>[m]</td>
<td>7</td>
</tr>
<tr>
<td>y0</td>
<td>[m]</td>
<td>25</td>
</tr>
</tbody>
</table>

<table>
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<tr>
<th>variable</th>
<th>unit</th>
<th>( \sigma ) (standard deviation)</th>
</tr>
</thead>
<tbody>
<tr>
<td>if initial conditions derived from lidar:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>z0</td>
<td>[m]</td>
<td>9</td>
</tr>
<tr>
<td>y0</td>
<td>[m]</td>
<td>13</td>
</tr>
<tr>
<td>( \Gamma_0 )</td>
<td>[m²/s]</td>
<td>13</td>
</tr>
</tbody>
</table>
Application to Wake Vortex Models

REA natural variability, $\Gamma^*$

\[
\begin{align*}
N^* &= N^*t_0 \\
\epsilon^* &= (\epsilon \cdot b_0)^{1/3}/w_0 \\
v^* &= v/w_0
\end{align*}
\]

<table>
<thead>
<tr>
<th>$N^* &lt; 0.3, \epsilon^* &gt; 0.25$</th>
<th>$N^* &lt; 0.3, \epsilon^* &lt; 0.25$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{obs, \Gamma^*_{luff}}$</td>
<td>0.097 (1073)</td>
</tr>
<tr>
<td>$\sigma_{obs, \Gamma^*_{lee}}$</td>
<td>0.081</td>
</tr>
<tr>
<td></td>
<td>0.094 (956)</td>
</tr>
<tr>
<td></td>
<td>0.096</td>
</tr>
</tbody>
</table>

$\epsilon^* > 0.2, N^* < 0.3$

$\epsilon^* < 0.2, N^* < 0.3$
**Application to Wake Vortex Models**

**REA natural variability, \( z^* \)**

\[
\sigma_{\text{obs}} = \sqrt{\sigma_{e^{Err}}^2 + \sigma_{n^u}^2}
\]

| \( N^* < 0.3, \left| v^* \right| > 0.5 \) | \( N^* < 0.3, \left| v^* \right| < 0.5 \) |
|---------------------------------|---------------------------------|
| \( \sigma_{\text{obs}, z_{\text{iu}}^*} \) | \( \sigma_{\text{obs}, z_{\text{ic}}^*} \) |
| 0.35 (1224) | 0.200 (805) |
| 0.35 | 0.262 |

| \( |v^*| > 1.0, N^* < 0.3 \) | \( |v^*| < 1.0, N^* < 0.3 \) |
|---------------------------------|---------------------------------|
| \( z^* - z_0 \) | \( z^* - z_0 \) |
| ![Graph 1] | ![Graph 2] |
Results

REA forecast
(one single landing)

**enhancement:**

<table>
<thead>
<tr>
<th>rmse $z^<em>_{</em>,TDP}$</th>
<th>$0.158$</th>
</tr>
</thead>
<tbody>
<tr>
<td>rmse $z^<em>_{</em>,REA}$</td>
<td>$0.148$</td>
</tr>
<tr>
<td>rmse $\Gamma^<em>_{</em>,D2P}$</td>
<td>$0.085$</td>
</tr>
<tr>
<td>rmse $\Gamma^<em>_{</em>,REA}$</td>
<td>$0.072$</td>
</tr>
</tbody>
</table>

**probability levels according to**

- 99 testcases
- WakeFRA & WakeOP
Results

REA reliability factors (one single landing)

- No correlation between $R_D$ and $R_B$ can be found!
Results

REA forecast reliability (one single landing)

\[ \tilde{\rho} = \frac{\sum R_i^2}{\sum R_i} \]

- low reliability for \( y \)-forecast
- high reliability for \( z \)-forecast
- medium reliability for \( \Gamma \)-forecast
## Results

### REA scoring

- 99 randomly chosen cases
- skill factor $s_i$:
  \[
  s_i = \frac{\sum_{p=1}^{n} rmse_{e,p} / rmse_{i,p}}{n} - 1
  \]

<table>
<thead>
<tr>
<th>median</th>
<th>$rms \Gamma_{luff}^*$</th>
<th>$rms \Gamma_{lee}^*$</th>
<th>$rms y_{luff}$</th>
<th>$rms y_{lee}^*$</th>
<th>$rms \dot{z}_{luff}^*$</th>
<th>$rms \dot{z}_{lee}^*$</th>
<th>$s$</th>
</tr>
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<tbody>
<tr>
<td>REA</td>
<td>0.116</td>
<td>0.099</td>
<td>0.433</td>
<td>0.409</td>
<td>0.163</td>
<td>0.149</td>
<td>0.000</td>
</tr>
<tr>
<td>TDP 2.1</td>
<td>0.127</td>
<td>0.106</td>
<td>0.590</td>
<td>0.415</td>
<td>0.237</td>
<td>0.147</td>
<td>-0.122</td>
</tr>
<tr>
<td>APA 3.4</td>
<td>0.160</td>
<td>0.107</td>
<td>0.602</td>
<td>0.413</td>
<td>0.179</td>
<td>0.154</td>
<td>-0.127</td>
</tr>
<tr>
<td>APA 3.2</td>
<td>0.204</td>
<td>0.140</td>
<td>0.612</td>
<td>0.410</td>
<td>0.179</td>
<td>0.154</td>
<td>-0.190</td>
</tr>
<tr>
<td>D2P</td>
<td>0.122</td>
<td>0.120</td>
<td>0.406</td>
<td>0.408</td>
<td>0.140</td>
<td>0.166</td>
<td>-0.016</td>
</tr>
</tbody>
</table>
Results

REA scoring

- 99 randomly chosen cases
- skill factor $s_i$:

$$ s_i = \frac{\sum_{p=1}^{n} \frac{rms_{e,p}}{rms_{i,p}}}{n} - 1 $$

2nd best
best

<table>
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<tr>
<th>median</th>
<th>$\text{rms } \Gamma^*_{lu ff}$</th>
<th>$\text{rms } \Gamma^*_{lee}$</th>
<th>$\text{rms } y^*_{lu ff}$</th>
<th>$\text{rms } y^*_{lee}$</th>
<th>$\text{rms } z^*_{lu ff}$</th>
<th>$\text{rms } z^*_{lee}$</th>
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<td>0.149</td>
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</tr>
<tr>
<td>DEA</td>
<td>0.131</td>
<td>0.093</td>
<td>0.525</td>
<td>0.411</td>
<td>0.173</td>
<td>0.149</td>
<td>-0.048</td>
</tr>
</tbody>
</table>

advanced MME approach outperforms Direct Ensemble Average (DEA)
**PDD of models and ensemble**

**overconfident ensemble**: too narrow ensemble spread

**well-dispersed ensemble**: coverage of full spectrum of possible solutions

Weigel et al., 2008, Hagedorn et al., 2004

well-dispersed model forecasts → rmse improvement

overconfident ensemble → small or no rmse improvement
Conclusion

- ensemble **can improve quality** of wake vortex forecasts in average
- however **only 1.6 %** improvement compared to best model
  
  **reason:** ensemble is **overconfident** for $z^*$ and $y^*$

- **but:** models might behave differently in particular ambient weather conditions and out-of-ground → investigation with pdds
Further Development

- How does a good training data set look like?
- Can the results be further improved by distinguishing various ambient weather conditions?
- How does the Bayesian Model Averaging (BMA) perform?

(source: Raftery et al., 2005)
Backup
Wake Vortex Predictions

Motivation

1. - optimization of tactical separation at airports
   - hazard warning system

2. - Wake Encounter Avoidance & Advisory System (WEAA)
   - “Free Flight”
Ensemble Methoden
Bayesian Model Averaging

\[
P(\mathcal{B}) = \text{Wahrscheinlichkeit des Eintretens von } \mathcal{B}
\]

\[
P(\mathcal{B}|\mathcal{A}) = \text{Wahrscheinlichkeit für } \mathcal{B}, \text{ unter Vorraussetzung } \mathcal{A}
\]

\[
\text{PDF} = \text{Probability Density Function (Wahrscheinlichkeitsdichtefunktion)}
\]

Law of total probability:

\[
P(\mathcal{B}) = \sum_{n} P(\mathcal{B} \cap \mathcal{A}_n) = \sum_{n} P(\mathcal{A}_n) P(\mathcal{B}|\mathcal{A}_n)
\]

Beispiel:

Wir befinden uns auf einem Schiff:
- wir wollen die Position B bestimmen
- 3 Crew-Mitglieder (A1,A2,A3) wissen wie es geht, haben aber unterschiedliche Methoden

according to Grimmett and Welsh., 1986
### Ensemble Methoden

**Bayesian Model Averaging**

#### Law of total probability:

\[
P(B) = \sum_{n} P(B \cap A_n) = \sum_{n} P(A_n) P(B|A_n)
\]

<table>
<thead>
<tr>
<th>Methode</th>
<th>A1</th>
<th>A2</th>
<th>A3</th>
</tr>
</thead>
<tbody>
<tr>
<td>individuelle Wahrscheinlichkeit, dass die Methode Erfolg hat: (P(B</td>
<td>A_n))</td>
<td>0.6</td>
<td>0.9</td>
</tr>
<tr>
<td>Wahrscheinlichkeit, dass wir A1, A2 or A3 fragen: (P(A_n))</td>
<td>0.2</td>
<td>0.5</td>
<td>0.3</td>
</tr>
</tbody>
</table>

\[s = \sum_{n} v_n * t_n\]

\[P(B) = 0.78\]
Ensemble Methoden
Bayesian Model Averaging

Law of total probability:

\[ P(B) = \sum_{n} P(B \cap A_n) = \sum_{n} P(A_n) P(B|A_n) \]

<table>
<thead>
<tr>
<th>Methode</th>
<th>A1</th>
<th>A2</th>
<th>A3</th>
</tr>
</thead>
<tbody>
<tr>
<td>PDF der Methode (Modell- Unsicherheiten): ( P(B</td>
<td>A_n) )</td>
<td>![Graph of A1]</td>
<td>![Graph of A2]</td>
</tr>
<tr>
<td>Wahrscheinlichkeit, dass wir A1, A2 or A3 fragen: ( P(A_n) )</td>
<td>0.1</td>
<td>0.6</td>
<td>0.3</td>
</tr>
</tbody>
</table>
Ensemble Methoden
Bayesian Model Averaging

Law of total probability:

\[ P(B) = \sum_{n} P(B \cap A_n) = \sum_{n} P(A_n) P(B|A_n) \]

**Angewandt auf Vorhersage-Modelle:**

- \( A_n \): Modell \( n \)
- \( B \): vorherzusagende Größe
- \( B^T \): Trainings-Daten
- \( P(A_n) \): Wahrscheinlichkeit, dass \( A_n \) das beste Modell ist
  
  \( \text{(Gewichtungsfaktor, basierend auf } B^T) \)
- \( P(B|A_n) \): PDF of \( A_n \) alone (Gaussian distribution, given that \( A_n \) is the best forecast)

\( \triangleq \) gewichtete Summe von Wahrscheinlichkeitsdichtefunktionen (PDFs)

Annahme: es gibt immer ein bestes Ensemble-Glied

according to Raftery et al., 2005
Ensemble Methoden
Bayesian Model Averaging

BMA applied on 48-h surface temperature forecast (bias corrected)

- ensemble forecast
- individual model PDF
- individual model forecast
- 90% interval
- verification

source: Raftery et al., 2005
Multi-Model Ensemble

**benefit**

- increase deterministic skill
- predict forecast skill
- provide probabilistic forecast
Multi-Model Ensemble