Estimation of Surface Temperature and Heat Flux by Inverse Heat Transfer Methods using Internal Temperatures Measured while Radiantly Heating a Carbon/Carbon Specimen up to 1920°F

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OUTLINE

Problem Statement and Background

Computational Code Development
  • Algorithm derivation
  • Data filtering
  • Mesh convergence
  • Method validation

Analysis of LaRC Radiant Test Data

Concluding Remarks

Future Work
PROBLEM STATEMENT

- Purpose is to estimate surface temperature and heat flux values from internal temperatures measured while radiantly heating a carbon/carbon specimen up to 1920°F
- Initial and boundary conditions (BC) for $T$ and/or $q''$ must be known

**Ideally**, BC’s given on upper and lower surfaces of the material; **direct** problem needs only be solved

**In some cases**, BC’s given internally to material surface; both **direct** and **inverse** problems must be solved

TC: thermocouple
Developed a program to first solve the direct heat conduction problem

- **Direct Problem**: solving between internal measurements
  - Created a mesh between the upper and lower data measurements
  - Calculated $T$ and $q''$ values at nodal points in the mesh and every time step using a one-dimensional, implicit, centered, finite volume method

\[
C_p(x,T)\rho(x) \frac{\partial T(x,t)}{\partial t} = \frac{\partial}{\partial x} \left( k(x,T) \frac{\partial T(x,t)}{\partial x} \right) + Q(x,t)
\]

\[
\frac{T_n^{(m+1)} - T_n^{(m)}}{\Delta t} = \frac{\alpha_{n-1,n}^{(m)}}{2\Delta x} \left( \frac{T_{n-1}^{(m)} - T_n^{(m)} + T_{n-1}^{(m+1)} - T_n^{(m+1)}}{\Delta x} \right) + \frac{\alpha_{n,n+1}^{(m)}}{2\Delta x} \left( \frac{T_{n+1}^{(m)} - T_n^{(m)} + T_{n+1}^{(m+1)} - T_n^{(m+1)}}{\Delta x} \right)
\]

\[
q''_{n}^{(m+1)} = k_n^{(m)} \left( \frac{T_n^{(m)} - T_{n+1}^{(m)} + T_{n+1}^{(m+1)} - T_n^{(m+1)}}{2\Delta x} \right) + \rho c_p n^{(m)} \Delta x \left( \frac{T_n^{(m+1)} - T_n^{(m)}}{\Delta t} \right)
\]
COMPUTATIONAL METHOD

INVERSE SOLUTION

- Developed a program to then solve the inverse heat conduction problem
  - Inverse Problem: using direct problem solution to march to the surface of the material
    - Used work presented by A.S. Carasso, 1992 to select a space marching scheme
    - Space marching allows for the estimation of surface $T$ and $q''$

THE FUTURE 0 SPACE MARCHING TECHNIQUE WAS SELECTED FOR THE INVERSE ANALYSIS.

\[
\begin{align*}
T_{\tilde{n}+1}^{(m)} &= T_{\tilde{n}}^{(m)} + \Delta x \frac{q_{\tilde{n}}''(m)}{k_{\tilde{n}}(m)} \\
q_{\tilde{n}+1}'' &= q_{\tilde{n}}'' + \Delta x \rho c_p(n) \left( \frac{4T_{\tilde{n}}^{(m+1)} - T_{\tilde{n}}^{(m+2)} - 3T_{\tilde{n}}^{(m)}}{2\Delta t} \right) \\
T_{\tilde{n}+1}^{(t_f)} &= 2T_{\tilde{n}+1}^{(t_f-1)} - T_{\tilde{n}+1}^{(t_f-2)} \\
q_{\tilde{n}+1}'' &= 2q_{\tilde{n}+1}'' - q_{\tilde{n}+1}''
\end{align*}
\]

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In this study, local Blackman and Hamming windowed sinc filters were used to reduce noise in the data. Noise is reduced at each data point using information from a certain number of data points depending on the selected bandwidth:

- Bandwidth of 0.1 uses 41 data points, 20 on either side of each data point
- Bandwidth of 0.2 uses 21 data points, 10 on either side of each data point

Saves on computational time when there is an excessive amount of data points compared to using a global filter.
Solutions were calculated using multiple iterations of increasing nodal counts (3, 5, 9, 17, 33, etc.).

Variance between $T$ values at the current and previous nodal count was calculated over the entire range of time at the midpoint between the upper and lower boundary.

Solution was considered to be converged once the variance was less than 0.01 K.
METHOD VALIDATION
COMPARISON TO EXACT ANALYTICAL SOLUTION

- To solve for an exact, analytical solution the following assumptions were made:
  - Constant thermal properties \( \frac{\partial T(x, t)}{\partial t} = \alpha \frac{\partial^2 T(x, t)}{\partial x^2}, \alpha = \frac{k}{C_p \rho} \)
  - No internal sources of heat
- Spatial range: \( 0 \leq x \leq L \)
- Constant initial condition: \( T(x, 0) = T_0 \)
- Constant boundary conditions: \( T(0, t) = T_1 \) and \( T(L, t) = T_2 \)
- Solved for the unique analytical solution over a range of \( t = [0, 60] \) seconds at varying spatial locations of \( x = \frac{L}{4}, \frac{L}{2}, \) and \( \frac{3L}{4} \)

**TEMPERATURE** \( T(x, t) = T_1 + (T_2 - T_1) \frac{x}{L} + \frac{2}{\pi} \sum_{j=1}^{\infty} \frac{1}{j} [(-1)^j (T_2 - T_0) - (T_1 - T_0)] \sin \left( \frac{j \pi x}{L} \right) e^{-k \left( \frac{j \pi}{L} \right)^2 t} \)

**HEAT FLUX** \( q''(x, t) = -k \left[ (T_2 - T_1) \frac{1}{L} + \frac{2}{\pi} \sum_{j=1}^{\infty} \frac{1}{j} [(-1)^j (T_2 - T_0) - (T_1 - T_0)] \right] \left( \frac{j \pi x}{L} \right) \cos \left( \frac{j \pi x}{L} \right) e^{-k \left( \frac{j \pi}{L} \right)^2 t} \)

<table>
<thead>
<tr>
<th>Mock Temperature and Thermal Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_1 ) (K)</td>
</tr>
<tr>
<td>Test I</td>
</tr>
<tr>
<td>200</td>
</tr>
<tr>
<td>Test II</td>
</tr>
<tr>
<td>200</td>
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</tbody>
</table>
This temperature difference plot depicts the temperature difference between the analytical and computational solutions over the entire time domain.
METHOD VALIDATION
COMPARISON RESULTS

Heat Flux vs. Time
Test II, $\Delta t = 0.05$ seconds

- Converged: $x = L/4$
- Analytical: $x = L/4$
- Converged: $x = L/2$
- Analytical: $x = L/2$
- Converged: $x = 3L/4$
- Analytical: $x = 3L/4$
TC PLUG DESCRIPTIONS

- Carbon/carbon (C/C) material, slightly different for each plug
- For accuracy of solution
  - Material properties must be accurate
  - TC depths must be accurate
  - Analysis must represent the physics (1D analysis, 1D physics)

**Horizontal-ply plug**

SAME MATERIAL and
SAME FIBER ORIENTATION

**Vertical-ply plug**

DIFFERENT MATERIAL and/or
DIFFERENT FIBER ORIENTATION
LaRC RADIANT TEST SETUP

• 6 in. × 6 in. × ½ in. carbon/carbon test sample placed directly on ½ in. alumina insulation on top of water-cooled plate; heated with quartz lamps

• Measured internal $T$ using horizontal-ply and vertical-ply TC plugs, each with 4 embedded TC’s; data sampled at 10 Hz

• 12 different tests conducted with target $T$ of 500°F to 1920°F, Test 9 results discussed
LaRC RADIANT TEST 9
HORIZONTAL-PLY RESULTS

- Front surface $q''$ estimated by direct problem with front/back surface $T$ as BC's
- Front surface $T$ data, estimated $q''$ assumed to be the correct surface values
- TC plug data analyzed with combinations of TC1, TC2, TC3, and TC4 as BC's
  - Surface estimations compared to the assumed correct values
  - Results indicated ACCEPTABLE inverse analysis $T$ and $q''$ surface estimations

Temperature vs. Time
Front Surface Comparison, LaRC Test 9

Heat Flux vs. Time
Front Surface Comparison, LaRC Test 9
LaRC RADIANT TEST 9
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LaRC RADIANT TEST 9
VERTICAL-PLY RESULTS

• Data analyzed using thermal properties of the plug and of the surrounding material with TC2/TC4 as BC’s

Analysis demonstrated a **1D modeling technique** can only **consistently predict accurate surface** $T$ and $q''$ when the **TC plug properties and fiber orientation are the SAME as the surrounding material**, hence approximating 1D physics.
LaRC RADIANT TESTS
CONCLUDING REMARKS

• Developed computational code to solve 1D direct and inverse heat conduction problems
  – Used one-dimensional, implicit, centered, finite volume method for direct problem
  – Used space marching techniques for inverse problem
  – Validated direct problem accuracy by comparing to an exact analytical solution
  – Accounted for data filtering and mesh convergence
• Applied 1D analysis to real experimental data
• Demonstrated solution has better accuracy when using TC plugs manufactured with same material and fiber orientation (approximately 1D physics) vs. when manufactured with different material and/or fiber orientation (non-1D physics)
  – Analysis yielded small errors for $T$, $q''$ when using horizontal-ply plug
  – Analysis yielded large errors for $T$, $q''$ when using vertical-ply plug
FUTURE WORK

• Assume a BC on the upper surface of the material, \( x = x_0 \)
• Using the direct problem computational formula (one-dimensional, implicit, centered, finite volume method), solve for thermal solution between the material surface and lower BC using \( x_0 \) and \( x_2 \) as BC’s
• Adjust \( x_0 \) estimation until thermal solution at \( x = x_1 \) matches TC measurement