Estimation of Surface Temperature and Heat Flux by Inverse Heat Transfer Methods using Internal Temperatures Measured while Radiantly Heating a Carbon/Carbon Specimen up to 1920°F

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Problem Statement and Background

Computational Code Development

- Algorithm derivation
- Data filtering
- Mesh convergence
- Method validation

Analysis of LaRC Radiant Test Data

Concluding Remarks

Future Work
PROBLEM STATEMENT

- Purpose is to estimate surface temperature and heat flux values from internal temperatures measured while radiantly heating a carbon/carbon specimen up to 1920°F
- Initial and boundary conditions (BC) for $T$ and/or $q''$ must be known

**Ideally**, BC’s given on upper and lower surfaces of the material; **direct** problem needs only be solved

**In some cases**, BC’s given internally to material surface; both **direct** and **inverse** problems must be solved

TC: thermocouple
Developed a program to first solve the direct heat conduction problem

- **Direct Problem**: solving between internal measurements
  - Created a mesh between the upper and lower data measurements
  - Calculated $T$ and $q''$ values at nodal points in the mesh and every time step using a one-dimensional, implicit, centered, finite volume method

\[
C_p(x,T)\rho(x) \frac{dT(x,t)}{dt} = \frac{\partial}{\partial x} \left( k(x,T) \frac{dT(x,t)}{dx} \right) + Q(x,t)
\]

\[
T_n^{(m+1)} - T_n^{(m)} \Delta t = \frac{\alpha^{(m)}_{n-1,n}}{2\Delta x} \left( \frac{T_{n-1}^{(m)} - T_n^{(m)} + T_{n-1}^{(m+1)} - T_n^{(m+1)}}{\Delta x} \right) + \frac{\alpha^{(m)}_{n,n+1}}{2\Delta x} \left( \frac{T_{n+1}^{(m)} - T_n^{(m)} + T_{n+1}^{(m+1)} - T_n^{(m+1)}}{\Delta x} \right)
\]

\[
q''_n^{(m+1)} = k_n^{(m)} \left( \frac{T_n^{(m)} - T_{n+1}^{(m)} + T_n^{(m+1)} - T_{n+1}^{(m+1)}}{2\Delta x} \right) + \rho c_{p_n}^{(m)} \Delta x \left( \frac{T_n^{(m+1)} - T_n^{(m)}}{\Delta t} \right)
\]
COMPUTATIONAL METHOD

INVERSE SOLUTION

- Developed a program to then solve the inverse heat conduction problem
  - **Inverse Problem**: using direct problem solution to march to the surface of the material
  - Used work presented by A.S. Carasso, 1992 to select a space marching scheme
  - Space marching allows for the estimation of surface $T$ and $q''$

THE FUTURE 0 SPACE MARCHING TECHNIQUE WAS SELECTED FOR THE INVERSE ANALYSIS.

\[
\begin{align*}
T^{(m+1)}_n &= T^{(m)}_n + \Delta x \frac{q''^{(m)}_n}{k^{(m)}_n} \\
q''^{(m+1)}_n &= q''^{(m)}_n + \Delta x \rho c_p^{(m)} \left( \frac{4T^{(m+1)}_n - T^{(m+2)}_n - 3T^{(m)}_n}{2\Delta t} \right)
\end{align*}
\]

\[
\begin{align*}
T^{(t_f)}_{n+1} &= 2T^{(t_f-1)}_{n+1} - T^{(t_f-2)}_{n+1} \\
q''^{(t_f)}_{n+1} &= 2q''^{(t_f-1)}_{n+1} - q''^{(t_f-2)}_{n+1}
\end{align*}
\]

In this study, local Blackman and Hamming windowed sinc filters were used to reduce noise in the data.

- Noise is reduced at each data point using information from a certain number of data points depending on the selected bandwidth
  - Bandwidth of 0.1 uses 41 data points, 20 on either side of each data point
  - Bandwidth of 0.2 uses 21 data points, 10 on either side of each data point
- Saves on computational time when there is an excessive amount of data points compared to using a global filter

\[
\frac{\partial T(x, t)}{\partial t} = \frac{\partial}{\partial x} \left( k(x, T) \frac{\partial T(x, t)}{\partial x} \right) + Q(x, t)
\]
• Solutions were calculated using multiple iterations of increasing nodal counts (3, 5, 9, 17, 33, etc.)

• Variance between $T$ values at the current and previous nodal count was calculated over the entire range of time at the midpoint between the upper and lower boundary

• Solution was considered to be converged once the variance was less than 0.01 K
METHOD VALIDATION
COMPARISON TO EXACT ANALYTICAL SOLUTION

• To solve for an exact, analytical solution the following assumptions were made:
  – Constant thermal properties \( \frac{\partial T(x, t)}{\partial t} = \alpha \frac{\partial^2 T(x, t)}{\partial x^2}, \alpha = \frac{k}{C_p \rho} \)
  – No internal sources of heat

• Spatial range: \( 0 \leq x \leq L \)

• Constant initial condition: \( T(x, 0) = T_0 \)

• Constant boundary conditions: \( T(0, t) = T_1 \) and \( T(L, t) = T_2 \)

• Solved for the unique analytical solution over a range of \( t = 0, 60 \) seconds at varying spatial locations of \( x = \frac{L}{4}, \frac{L}{2} \), and \( \frac{3L}{4} \)

**TEMPERATURE**
\[
T(x, t) = T_1 + (T_2 - T_1)\frac{x}{L} + \frac{2}{\pi} \sum_{j=1}^{\infty} \frac{1}{j} \left[ (-1)^j (T_2 - T_0) - (T_1 - T_0) \right] \sin \left( \frac{j\pi x}{L} \right) e^{-k\left(\frac{j\pi}{L}\right)^2 t}
\]

**HEAT FLUX**
\[
q''(x, t) = -k \left[ (T_2 - T_1)\frac{1}{L} + \frac{2}{\pi} \sum_{j=1}^{\infty} \frac{1}{j} \left[ (-1)^j (T_2 - T_0) - (T_1 - T_0) \right] \right] \left( \frac{j\pi x}{L} \right) \cos \left( \frac{j\pi x}{L} \right) e^{-k\left(\frac{j\pi}{L}\right)^2 t}
\]

<table>
<thead>
<tr>
<th>Mock Temperature and Thermal Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_1 (K) )</td>
</tr>
<tr>
<td>Test I</td>
</tr>
<tr>
<td>Test II</td>
</tr>
</tbody>
</table>
METHOD VALIDATION
COMPARISON RESULTS

Temperature vs. Time
Test II, $\Delta t = 0.05$ seconds

This temperature difference plot depicts the temperature difference between the analytical and computational solutions over the entire time domain.
METHOD VALIDATION
COMPARISON RESULTS

Heat Flux vs. Time
Test II, $\Delta t = 0.05$ seconds

- Converged: $x = L/4$
- Analytical: $x = L/4$
- Converged: $x = L/2$
- Analytical: $x = L/2$
- Converged: $x = 3L/4$
- Analytical: $x = 3L/4$
TC PLUG DESCRIPTIONS

• Carbon/carbon (C/C) material, slightly different for each plug
• For accuracy of solution
  – Material properties must be accurate
  – TC depths must be accurate
  – **Analysis must represent the physics** (1D analysis, 1D physics)

Horizontal-ply plug

SAME MATERIAL and
SAME FIBER ORIENTATION

Vertical-ply plug

DIFFERENT MATERIAL and/or
DIFFERENT FIBER ORIENTATION
LaRC RADIANT TEST SETUP

- 6 in. × 6 in. × ½ in. carbon/carbon test sample placed directly on ½ in. alumina insulation on top of water-cooled plate; heated with quartz lamps
- Measured internal $T$ using horizontal-ply and vertical-ply TC plugs, each with 4 embedded TC’s; data sampled at 10 Hz
- 12 different tests conducted with target $T$ of 500°F to 1920°F, Test 9 results discussed
LaRC RADIANT TEST 9
HORIZONTAL-PLY RESULTS

• Front surface $q''$ estimated by direct problem with front/back surface $T$ as BC’s
• Front surface $T$ data, estimated $q''$ assumed to be the correct surface values
• TC plug data analyzed with combinations of TC1, TC2, TC3, and TC4 as BC’s
   – Surface estimations compared to the assumed correct values
   – Results indicated ACCEPTABLE inverse analysis $T$ and $q''$ surface estimations
LaRC RADIANT TEST 9
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LaRC RADIANT TEST 9
VERTICAL-PLY RESULTS

- Data analyzed using thermal properties of the plug and of the surrounding material with TC2/TC4 as BC’s

Analysis demonstrated a 1D modeling technique can only consistently predict accurate surface $T$ and $q''$ when the TC plug properties and fiber orientation are the same as the surrounding material, hence approximating 1D physics.
LaRC RADIANT TESTS

CONCLUDING REMARKS

- Developed computational code to solve 1D direct and inverse heat conduction problems
  - Used one-dimensional, implicit, centered, finite volume method for direct problem
  - Used space marching techniques for inverse problem
  - Validated direct problem accuracy by comparing to an exact analytical solution
  - Accounted for data filtering and mesh convergence
- Applied 1D analysis to real experimental data
- Demonstrated solution has better accuracy when using TC plugs manufactured with same material and fiber orientation (approximately 1D physics) vs. when manufactured with different material and/or fiber orientation (non-1D physics)
  - Analysis yielded small errors for $T$, $q''$ when using horizontal-ply plug
  - Analysis yielded large errors for $T$, $q''$ when using vertical-ply plug
FUTURE WORK

- Assume a BC on the upper surface of the material, \( x = x_0 \)
- Using the direct problem computational formula (one-dimensional, implicit, centered, finite volume method), solve for thermal solution between the material surface and lower BC using \( x_0 \) and \( x_2 \) as BC’s
- Adjust \( x_0 \) estimation until thermal solution at \( x = x_1 \) matches TC measurement