Mesh Convergence Requirements for Composite Damage Models

Carlos G. Dávila
Structural Mechanics and Concepts Branch
NASA Langley Research Center
Hampton, VA

Mechanical Engineering Dept.,
TU Eindhoven, NL
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Outline

- Strength or Toughness?
- Cohesive Laws
- Process Zones and Load Redistribution
- Continuum Damage Models
- Transverse Matrix Cracks
- Summary
“Physics” of failure:

- At each scale, damage is described by different physical observations
Effect of Material on Structural Response

- Quasi-isotropic laminates, three-dimensional (3-D) scaling
  - \([45_m/90_m/-45_m/0_m]_{ns}, m, n = 1, 2, 4, 8\)
  - \(d = 1/8\) in., \(1/4\) in., \(1/2\) in.

*Green and Wisnom, 2007

Identical material, different modes of failure!
Scaling: the Effect of Structure Size on Strength

Strength of Materials
(da Vinci, 1505) (Galileo, 1638)

Fracture Mechanics
“weakness is due to the presence of flaws” (Griffith, 1921)

\[ \sigma_u = \sqrt{\frac{E G_c}{\pi a}} \]

Statistical Theory of Size Effect
(Mariotte, 1686)
(Weibull, 1939)
Strength versus toughness:
- Strength and toughness is the result of the interplay between a number of individual mechanisms originating at different length scales.
- Some damage mechanisms inhibit crack propagation.
Size Effect - the Issue of Scale

Scaling from test coupon to structure

Scaling Laws (Z. Bažant)

\[
\log \sigma_n = \frac{E G_c}{\pi a}
\]

Yield or Strength Criteria

Linear Elastic Fracture Mechanics

Normal testing

Range of main practical interest for size effect

Extrapolation that must be anchored in a theory

Number of Tests

Structural Size, in.

86% of data
Cohesive Laws

Two material properties:
- $G_c$ Fracture toughness
- $\sigma_c$ Strength

Bilinear Traction-Displacement Law

$$\int_0^{\delta_c} \sigma(\delta) \, d\delta = G_c$$

Characteristic Length:

$$l_c = \gamma \frac{EG_c}{\sigma_c^2}$$
Mesh Convergence Studies

$$l_c = \gamma \frac{E G_c}{\sigma_c^2}$$

![Diagram showing mesh convergence studies](image)

**Applied Force, N**

**Applied displacement $\Delta$, mm**

- **Analytical solution**
- **$l_{elem} = 2 \ l_c$**
- **$l_{elem} = l_c$**
- **$l_{elem} = l_c/3$**
- **$l_{elem} = l_c/5$**

- Converged meshes
- Non-converged meshes
Crack Length and Process Zone

Force, $F$

LEFM analytical solution

$G_c = \text{constant}$

LEFM error

Quasi-brittle

Steady-state

Initiation

Applied displacement, $\Delta$

\[ l_p = \gamma \frac{EG_c}{\sigma_c^2} \]
Crack Length and Process Zone

Brittle: 
\[ a_0 > 100 \ l_p \]

Quasi-brittle: 
\[ 100 \ l_p \geq a_0 \geq 5l_p \]

Ductile: 
\[ 5 \ l_p > a_0 \]
Fracture-Dominated Failure

\[ G = \frac{\pi \sigma^2 a}{E} \]

Crack propagates unstably once driving force \( G(\sigma, a_0) \) reaches \( G_{lc} \)
Fracture-Dominated Failure: R-Curve

\[ G = \frac{\pi \sigma^2 a}{E} \]

Crack propagates stably when driving force \( G(\sigma, a_0) > G_{\text{Init}} \)

Unstable propagation initiates at \( G_{\text{Init}} < G \leq G_c \)
Length of the Process Zone

Short Tensile Test
Lexan Polycarbonate

Cohesive elements

\[ l_c \approx 0.6 \frac{E G_c}{\sigma_c^2} = 3.4 \text{ mm} \]

A

Symmetry

B

Maximum Load

C

D

h/a_o = 1

\[ l_{pz} = 4.7 \text{ mm} \]

E

F

Symmetry
Cohesive Laws - Prediction of Scale Effects

- The use of cohesive laws to predict the fracture in complex stress fields is explored.
- The bulk material is modeled as either elastic or elastic-plastic.

**Observations:**
- LEFM overpredicts tests for $h/a < 1$

**Graph:**
- Fmax, N vs. $h/a$
- LEFM
- Test (CT Sun)
- Cohesive

Lexan Plexiglass tensile specimens (CT Sun)

$h/a = 1$ (short process zone)

$h/a = 0.25$ (long process zone)

$h/a = 2$

$h/a = 0.5$

$h/a = 0.25$

\[ l_{cz} = 4.7 \text{ mm.} \]

Width
Quasi-Brittle Response and Fracture

Before damage

\[ F = A\sigma = EA\frac{\Delta}{L + \frac{E}{K}} \]

After damage

\[ F = A\sigma = EA\frac{\Delta - \frac{2G_c}{\sigma_c^2}}{L - \frac{2EG_c}{\sigma_c^2} + \frac{E}{K}} \]

For stable fracture under \( \Delta \) control:

\[ \frac{\partial F}{\partial \Delta} \leq 0 \quad \Rightarrow \quad L \leq \frac{2EG_c}{\sigma_c^2} \]

For “long” beams, the response is unstable, dynamic, and independent of \( G_c \)
Outline

• Strength or Toughness?
• Cohesive Laws
• Process Zones and Load Redistribution
• **Continuum Damage Models (CDM)**
• Transverse Matrix Cracks
• Summary
Failure Criteria and Material Degradation

Progressive Failure Analysis

Benefits
- Simplicity (no programming needed)
- Convergence of equilibrium iterations

Drawbacks
- Mesh dependence
- Dependence on load increment
- Ad-hoc property degradation
- Large strains can cause reloading
- Errors due to improper load redistributions
Progressive Failure Analysis

Failure criterion

Elastic property

Residual = E/100

Progressive Damage Analysis – Regularized Softening Laws

Elastic property

Increasing $l_{elem}$
Critical Element size (snap-back):

\[ l_{\text{elem}} \leq \frac{2EG_c}{\sigma_c^2} \]
Advanced Failure Criteria for Laminated Composites

**Failure Criteria:** equations based on stresses and strengths that represent the initiation of damage mechanisms

**LaRC04 Criteria**
- In-situ matrix strength prediction
- Advanced fiber kinking criterion
- Prediction of angle of fracture

-LaRC02 - Dávila, Rose et al., 2003
-LaRC04 - Pinho, et al., 2005
Size Effect and Material Softening Laws

Damage Evolution Laws:
Each damage mode has its own softening response

Two material properties:
- $\sigma_c$ Strength
- $G_c$ Fracture toughness

Material length scale

$$l_c \approx 0.6 \frac{E G_c}{\sigma_c^2}$$
Continuum Damage Model (CDM)  (Maimi/Camanho 2007)

Damage Modes:
- Tension \( F^+ \)
- Compression \( F^- \)

Damage Evolution:
Thermodynamically-consistent material degradation takes into account energy release rate and element size for each mode.

LaRC04 Criteria
- In-situ matrix strength prediction
- Advanced fiber kinking criterion
- Prediction of angle of fracture (compression)
- Criteria used as activation functions within framework of damage mechanics

\[ d_i = 1 - \frac{1}{f_i} \exp \left( A_i \left( 1 - f_i \right) \right) \]

\( f_i \): LaRC04 failure criteria as activation functions
\( i = F^+; F^-; M^{y+}; M^{y-}; M^s \)

Bazant CBT:
\[ A_i = \frac{2l^* X_i^2}{2E_i G_i - l^* X_i^2} \]

Critical (maximum) finite element size:
\[ l^* \leq \frac{2E_i G_i}{X_i^2} \]
Scale Effect in Open Hole Tension (OHT)

Prediction of size effects in notched composites

- Stress-based criteria predict no size effect.
- CDM damage model predicts scale effects w/out calibration.
  
  (P. Camanho, 2007)
Process Zone and Scale Effect in OHT

Both energy and stress criteria are necessary conditions for fracture but neither one nor the other are sufficient. 

(Leguillon, E J of Mech and Solids, 2001)

Scale effect is due to relative size of process zone

\[ l_p = \gamma \frac{EG_c}{\sigma_c^2} \]
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Transverse matrix cracks propagate in thickness direction and in the fiber direction.

Cracks shield the adjacent material and prevent new cracks.

Shielded area
Matrix Cracking – In Situ Effect

Potential crack plane, with crack nucleus

Transverse Strength of 90° Ply, GPa

- Thin ply model
- Thick ply model
- Unidirectional

T300/944
- [±25/90°]s
- [25/−25/90°]s
- [90°]s
- Onset of delamination
- [0/90°/0]


Inner 90° Ply Thickness 2a, mm

Thickness propagation
Longitudinal propagation

28/34
Material Inhomogeneity

Initial crack density in a uniformly stressed laminate is strictly a function of material inhomogeneity:

- Strength scaled by $f$, Fracture toughness scaled by $f^2$
- Constant $f$ along each crack path

$f(x)$

Inhomogeneity applied to 3 levels of mesh refinement:

- 10 elts.
- 2 elts.
- 1 elt.

F Leone, 2015
Analyses with 3 levels of mesh refinement
**Transverse Matrix Cracks w/ One Element Per Ply**

Multi-element model: correct crack evolution

Conventional single-element: no opening w/out delam.

Modified single-element: correct Energy Release Rate

**Equation:**

\[ K \approx \frac{4E_2}{\pi^2 t} \]

Van der Meer, F.P. & Dávila, C., JCM, 2013
Crack Initiation, Densification, and Saturation

\( \sigma = 182 \text{ MPa} \)

\( \sigma = 273 \text{ MPa} \)

\( \sigma = 372 \text{ MPa} \)

\( \sigma = 679 \text{ MPa} \)

Van der Meer, F.P. & Dávila, C., JCM, 2013
Comparison of X-FEM Single-Ply Model and Full 3-D

$\sigma = 372$ MPa

Van der Meer, F.P. & Dávila, C., JCM, 2013
Summary

- **Strength and fracture issues are:**
  - Interrelated
  - Subject to size effects
- **Cohesive laws:**
  - Account for strength and fracture toughness
  - More than 3 elements are required in the process zone
- **Continuum damage models**
  - Snap-back imposes maximum element size
- **Transverse matrix cracks:**
  - More than one element per ply is required
- **Other mesh issues?**