Mesh Convergence Requirements for Composite Damage Models

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17 March 2016
Outline

• Strength or Toughness?
• Cohesive Laws
• Process Zones and Load Redistribution
• Continuum Damage Models
• Transverse Matrix Cracks
• Summary
Scales of Damage in Composites

“Physics” of failure:

- At each scale, damage is described by different physical observations
Effect of Material on Structural Response

- Quasi-isotropic laminates, three-dimensional (3-D) scaling
  - $[45_m/90_m/-45_m/0_m]_{ns}$, $m, n = 1, 2, 4, 8$
  - $d = 1/8 \text{ in.}, 1/4 \text{ in.}, 1/2 \text{ in.}$

*Green and Wisnom, 2007

Identical material, different modes of failure!
Scaling: the Effect of Structure Size on Strength

Strength of Materials
(da Vinci, 1505) (Galileo, 1638)

Fracture Mechanics
“weakness is due to the presence of flaws” (Griffith, 1921)

\[ \sigma_u = \sqrt{\frac{E G_c}{\pi a}} \]

Statistical Theory of Size Effect
(Mariotte, 1686) (Weibull, 1939)
Strength versus toughness:
- Strength and toughness is the result of the *interplay* between a number of individual mechanisms originating at different length scales.
- Some damage mechanisms *inhibit* crack propagation.
Size Effect - the Issue of Scale

Scaling from test coupon to structure

Scaling Laws
(Z. Bažant)

\[ \log \sigma_n \]

\[ \log D' \]

\[ \log D \]

Structural Size, in.

Number of Tests

Range of main practical interest for size effect

Extrapolation that must be anchored in a theory

86% of data

Yield or Strength Criteria

Normal testing

Linear Elastic Fracture Mechanics

\[ \sigma_u = \sqrt{\frac{EG_c}{\pi a}} \]
Cohesive Laws

Two material properties:
- $G_c$ Fracture toughness
- $\sigma_c$ Strength

Bilinear Traction-Displacement Law

$$\int_0^{\delta_c} \sigma(\delta) \, d\delta = G_c$$

Characteristic Length:

$$l_c = \frac{E G_c}{\sigma_c^2}$$
Mesh Convergence Studies

\[ l_c = \gamma \frac{E G_c}{\sigma^2_c} \]

- **Converged meshes**
- **Non-converged meshes**

- Applied Force, N
- Applied displacement \( \Delta \), mm

- **Analytical solution**
- \( l_{\text{elem}} = 2 l_c \)
- \( l_{\text{elem}} = l_c \)
- \( l_{\text{elem}} = l_c / 3 \)
- \( l_{\text{elem}} = l_c / 5 \)

Converged meshes
Crack Length and Process Zone

LEFM analytical solution

\[ G_c = \text{constant} \]

\[ 1_p = \gamma \frac{E G_c}{\sigma_c^2} \]

Initiation

Steady-state

Quasi-brittle

LEFM error

Force, F vs. Applied displacement, \( \Delta \)
Crack Length and Process Zone

Brittle:
\[ a_0 > 100 \, l_p \]

Quasi-brittle:
\[ 100 \, l_p \geq a_0 \geq 5l_p \]

Ductile:
\[ 5 \, l_p > a_0 \]
Fracture-Dominated Failure

\[ G = \frac{\pi \sigma^2 a}{E} \]

Crack propagates unstably once driving force \( G(\sigma, a_0) \) reaches \( G_{lc} \).
Fracture-Dominated Failure: R-Curve

\[ G = \frac{\pi \sigma^2 a}{E} \]

Crack propagates **stably** when driving force \( G(\sigma, a_0) > G_{\text{Init}} \)

Unstable propagation initiates at \( G_{\text{Init}} < G \leq G_c \)
Length of the Process Zone

Short Tensile Test
Lexan Polycarbonate

Cohesive elements

\[ l_c \approx 0.6 \frac{EG_c}{\sigma_c^2} = 3.4 \text{ mm} \]

A

Symmetry

B

Maximum Load

C

D

\[ \frac{h}{a_0} = 1 \]

\[ l_{pz} = 4.7 \text{ mm} \]

E

F
The use of cohesive laws to predict the fracture in complex stress fields is explored.

The bulk material is modeled as either elastic or elastic-plastic.

**Observations:**
LEFM overpredicts tests for \( h/a < 1 \)

- **Lexan Plexiglass tensile specimens (CT Sun)**

\( h/a=0.25 \) (long process zone)

\( h/a=1 \) (short process zone)

\( h/a = 0.25 \) (long process zone)

\( l_{cz} = 4.7 \text{ mm.} \)
Quasi-Brittle Response and Fracture

Before damage

\[ F = A\sigma = EA \frac{\Delta}{L + \frac{E}{K}} \]

After damage

\[ F = A\sigma = EA \frac{\Delta - \frac{2G_c}{\sigma_c}}{L - \frac{2EG_c}{\sigma_c^2} + \frac{E}{K}} \]

For stable fracture under \( \Delta \) control: \( \frac{\partial F}{\partial \Delta} \leq 0 \)

For “long” beams, the response is unstable, dynamic, and independent of \( G_c \)
• Strength or Toughness?
• Cohesive Laws
• Process Zones and Load Redistribution
• **Continuum Damage Models (CDM)**
• Transverse Matrix Cracks
• Summary
Failure Criteria and Material Degradation

Progressive Failure Analysis

Benefits

• Simplicity (no programming needed)
• Convergence of equilibrium iterations

Drawbacks

• Mesh dependence
• Dependence on load increment
• Ad-hoc property degradation
• Large strains can cause reloading
• Errors due to improper load redistributions
Failure Criteria and Material Degradation

Progressive Failure Analysis

- **Elastic property**
- **Failure criterion**
  - Residual = $E/100$

Progressive Damage Analysis – Regularized Softening Laws

- **Elastic property**
- **Failure criterion**
  - Increasing $I_{elem}$

- **Stress**

$\varepsilon/\varepsilon_0$
Critical Element size (snap-back):

\[ l_{elem} \leq \frac{2EG_c}{\sigma_c^2} \]
Advanced Failure Criteria for Laminated Composites

**Failure Criteria:** equations based on stresses and strengths that represent the initiation of damage mechanisms

LaRC04 Criteria
- In-situ matrix strength prediction
- Advanced fiber kinking criterion
- Prediction of angle of fracture

LaRC02 - Dávila, Rose et al., 2003
LaRC04 - Pinho, et al., 2005
Size Effect and Material Softening Laws

Damage Evolution Laws:

Each damage mode has its own softening response

Two material properties:

- $\sigma_c$ Strength
- $G_c$ Fracture toughness

Material length scale

$$l_c \approx 0.6 \frac{E G_c}{\sigma_c^2}$$
Continuum Damage Model (CDM) (Maimí/Camanho 2007)

Damage Modes:
- Tension $F^+$
- Compression $F^-$

Damage Evolution:
Thermodynamically-consistent material degradation takes into account energy release rate and element size for each mode.

LaRC04 Criteria
- In-situ matrix strength prediction
- Advanced fiber kinking criterion
- Prediction of angle of fracture (compression)
- Criteria used as activation functions within framework of damage mechanics

$$d_i = 1 - \frac{1}{f_i} \exp(A_i(1 - f_i))$$

$f_i$: LaRC04 failure criteria as activation functions

$$i = F^+; F^-; M^{y+}; M^{y-}; M^s$$

Bazant CBT:

$$A_i = \frac{2l^*X_i^2}{2E_iG_i - l^*X_i^2}$$

Critical (maximum) finite element size:

$$l^* \leq \frac{2E_iG_i}{X_i^2}$$
Prediction of size effects in notched composites

- Stress-based criteria predict no size effect.
- CDM damage model predicts scale effects w/out calibration.

(P. Camanho, 2007)
“Both energy and stress criteria are necessary conditions for fracture but neither one nor the other are sufficient.”

(Leguillon, E J of Mech and Solids, 2001)
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Transverse matrix cracks propagate in thickness direction and in the fiber direction.

Cracks shield the adjacent material and prevent new cracks.
Matrix Cracking – In Situ Effect

![Graph showing the transverse strength of 90° ply vs. inner 90° ply thickness]

- **T300/944**
  - [±25/90ₜ]ₙ
  - [25₂/−25₂/90₂]ₙ
  - [90ₜ]ₙ
  - Onset of delamination
  - [0/90ₜ/0]

- Potential crack plane, with crack nucleus
- Thin ply model
- Thick ply model
- Unidirectional

Data points:

- Thickness propagation
- Longitudinal propagation

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28/34
Initial crack density in a uniformly stressed laminate is strictly a function of material inhomogeneity.

- Strength scaled by $f$, Fracture toughness scaled by $f^2$
- Constant $f$ along each crack path

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Effect of Transverse Mesh Density on Crack Spacing

Analyses with 3 levels of mesh refinement
Transverse Matrix Cracks w/ One Element Per Ply

Multi-element model: correct crack evolution

Conventional single-element: no opening w/out delam.

Modified single-element: correct Energy Release Rate

\[ K \approx \frac{4E_2}{\pi^2t} \]

Van der Meer, F.P. & Dávila, C., JCM, 2013
Crack Initiation, Densification, and Saturation

\[ \sigma = 182 \text{ MPa} \]

\[ \sigma = 273 \text{ MPa} \]

\[ \sigma = 372 \text{ MPa} \]

\[ \sigma = 679 \text{ MPa} \]

Van der Meer, F.P. & Dávila, C., JCM, 2013
Comparison of X-FEM Single-Ply Model and Full 3-D

\[ \sigma = 372 \text{ MPa} \]

Van der Meer, F.P. & Dávila, C., JCM, 2013
Summary

- Strength and fracture issues are:
  - Interrelated
  - Subject to size effects

- Cohesive laws:
  - Account for strength and fracture toughness
  - More than 3 elements are required in the process zone

- Continuum damage models
  - Snap-back imposes maximum element size

- Transverse matrix cracks:
  - More than one element per ply is required

- Other mesh issues?