Mesh Convergence Requirements for Composite Damage Models

Carlos G. Dávila
Structural Mechanics and Concepts Branch
NASA Langley Research Center
Hampton, VA
Outline

• Strength or Toughness?
• Cohesive Laws
• Process Zones and Load Redistribution
• Continuum Damage Models
• Transverse Matrix Cracks
• Summary
“Physics” of failure:

- At each scale, damage is described by different physical observations
Effect of Material on Structural Response

- Quasi-isotropic laminates, three-dimensional (3-D) scaling
  - \([45_m/90_m/-45_m/0_m]_{ns}, m, n = 1, 2, 4, 8\)
  - \(d = \frac{1}{8}\) in., \(\frac{1}{4}\) in., \(\frac{1}{2}\) in.

*Green and Wisnom, 2007

Identical material, different modes of failure!
Scaling: the Effect of Structure Size on Strength

Strength of Materials
(da Vinci, 1505) (Galileo, 1638)

Fracture Mechanics
“weakness is due to the presence of flaws” (Griffith, 1921)

\[
\sigma_u = \sqrt{\frac{E G_c}{\pi a}}
\]

Statistical Theory of Size Effect
(Mariotte, 1686) (Weibull, 1939)
Strength versus toughness:

- Strength and toughness is the result of the interplay between a number of individual mechanisms originating at different length scales.
- Some damage mechanisms inhibit crack propagation.
Size Effect - the Issue of Scale

Scaling from test coupon to structure

Scaling Laws
(Z. Bažant)

log $\sigma_n$

Yield or Strength Criteria

Linear Elastic Fracture Mechanics

$\sigma_u = \sqrt{\frac{EG_c}{\pi a}}$

Structural Size, in.

Number of Tests

Range of main practical interest for size effect

Extrapolation that must be anchored in a theory

86% of data
Cohesive Laws

Two material properties:
- $G_c$ Fracture toughness
- $\sigma_c$ Strength

Bilinear Traction-Displacement Law

$$\int_0^{\delta_c} \sigma(\delta) \, d\delta = G_c$$

Characteristic Length:

$$l_c = \gamma \frac{E G_c}{\sigma_c^2}$$
Mesh Convergence Studies

$$l_c = \frac{E G_c}{\sigma_c^2}$$

- **Non-converged meshes**
- **Converged meshes**

Graph showing the relationship between Applied Force (N) and Applied displacement ($\Delta$, mm) for different mesh elements ($l_{elem}$):
- $l_{elem} = 2l_c$
- $l_{elem} = l_c$
- $l_{elem} = l_c/3$
- $l_{elem} = l_c/5$

Analytical solution and converged meshes are compared.
Crack Length and Process Zone

Force, $F$

LEFM analytical solution

$G_c = \text{constant}$

LEFM error

Quasi-brittle

Steady-state

Initiation

Applied displacement, $\Delta$

$$l_p = \gamma \frac{E G_c}{\sigma_c^2}$$

$F, \Delta$

$a_0$
Crack Length and Process Zone

Brittle: 
\[ a_0 > 100 \ l_p \]

Quasi-brittle: 
\[ 100 \ l_p \geq a_0 \geq 5l_p \]

Ductile: 
\[ 5 \ l_p > a_0 \]
Fracture-Dominated Failure

\[ G = \frac{\pi \sigma^2 a}{E} \]

Crack propagates unstably once driving force \( G(\sigma, a_0) \) reaches \( G_{lc} \).
Fracture-Dominated Failure: R-Curve

\[ G = \frac{\pi \sigma^2 a}{E} \]

Fracture-dominated failure

\[ \sigma \]

\[ 2(a+\Delta a) \]

\[ 2a \]

Crack propagates **stably** when driving force \( G(\sigma, a_0) > G_{\text{Init}} \)

**Unstable** propagation initiates at \( G_{\text{Init}} < G \leq G_c \)

Unstable propagation initiates at \( G_{\text{Init}} < G \leq G_c \)
Length of the Process Zone

Short Tensile Test
Lexan Polycarbonate

Cohesive elements

\[ l_c \approx 0.6 \frac{EG_c}{\sigma_c^2} = 3.4 \text{ mm} \]

Symmetry

A

B

C

D

h/a_o = 1

E

F

Maximum Load

l_{pz} = 4.7 \text{ mm}
• The use of cohesive laws to predict the fracture in complex stress fields is explored
• The bulk material is modeled as either elastic or elastic-plastic

**Observations:**
• LEFM overpredicts tests for h/a<1

**Graph:**
- **Fmax, N vs h/a**
- **LEFM**
- **Test (CT Sun)**
- **Cohesive**

**Lexan Plexiglass tensile specimens (CT Sun)**

$h/a = 0.25$ (long process zone)

$h/a = 0.25$ (long process zone)

$l_{cz} = 4.7$ mm.
Quasi-Brittle Response and Fracture

Before damage

After damage

For stable fracture under $\Delta$ control:

For “long” beams, the response is unstable, dynamic, and independent of $Gc$
• Strength or Toughness?
• Cohesive Laws
• Process Zones and Load Redistribution
• **Continuum Damage Models (CDM)**
• Transverse Matrix Cracks
• Summary
Failure Criteria and Material Degradation

Progressive Failure Analysis

Elastic property

Failure criterion

Residual=E/100

Stress

Strain

Benefits
- Simplicity (no programming needed)
- Convergence of equilibrium iterations

Drawbacks
- Mesh dependence
- Dependence on load increment
- Ad-hoc property degradation
- Large strains can cause reloading
- Errors due to improper load redistributions
Failure Criteria and Material Degradation

Progressive Failure Analysis

Elastic property

E

Failure criterion

Residual=E/100

Progressive Damage Analysis – Regularized Softening Laws

Elastic property

E

Failure criterion

Increasing $l_{elem}$

Increasing $l_{elem}$

Stress

Strain

$\varepsilon/\varepsilon_0$

$\varepsilon/\varepsilon_0$
Critical Element size (snap-back):

\[ l_{elem} \leq \frac{2EG_c}{\sigma_c^2} \]
Advanced Failure Criteria for Laminated Composites

**Failure Criteria:** equations based on stresses and strengths that represent the initiation of damage mechanisms

**LaRC04 Criteria**
- In-situ matrix strength prediction
- Advanced fiber kinking criterion
- Prediction of angle of fracture

LaRC02  - Dávila, Rose et al., 2003
LaRC04  - Pinho, et al., 2005
Size Effect and Material Softening Laws

Damage Evolution Laws:
Each damage mode has its own softening response

Two material properties:
- $\sigma_c$: Strength
- $G_c$: Fracture toughness

Material length scale

$$l_c \approx 0.6 \frac{E G_c}{\sigma_c^2}$$
Continuum Damage Model (CDM)  (Maimi/Cammanho 2007)

**Damage Modes:**
- Tension $F^+$
- Compression $F^−$

**Damage Evolution:**
Thermodynamically-consistent material degradation takes into account energy release rate and element size for each mode.

**LaRC04 Criteria**
- In-situ matrix strength prediction
- Advanced fiber kinking criterion
- Prediction of angle of fracture (compression)
- Criteria used as activation functions within framework of damage mechanics

\[ d_i = 1 - \frac{1}{f_i} \exp \left(A_i \left(1 - f_i \right) \right) \]

\[ f_i: \text{LaRC04 failure criteria as activation functions} \]
\[ i = F^+; F^-; M^{y+}; M^{y-}; M^s \]

**Bazant CBT:**
\[ A_i = \frac{2l^* X_i^2}{2E_i G_i - l^* X_i^2} \]

Critical (maximum) finite element size:
\[ l^* \leq \frac{2E_i G_i}{X_i^2} \]
Prediction of size effects in notched composites

- Stress-based criteria predict no size effect.
- CDM damage model predicts scale effects w/out calibration.
  (P. Camanho, 2007)
Process Zone and Scale Effect in OHT

"Both energy and stress criteria are necessary conditions for fracture but neither one nor the other are sufficient."

(Leguillon, E J of Mech and Solids, 2001)
Outline

• Strength or Toughness?
• Cohesive Laws
• Process Zones and Load Redistribution
• Continuum Damage Models
• Transverse Matrix Cracks
• Summary
Transverse matrix cracks propagate in thickness direction and in the fiber direction.

Cracks shield the adjacent material and prevent new cracks.

Shielded area
Matrix Cracking – In Situ Effect

The graph shows the transverse strength of 90° ply as a function of inner 90° ply thickness. The data points represent different orientations and types of materials, such as [±25/90]s, [25_2/-25_2/90_2]s, [90_8]s, and [0/90,0/0]. The thin ply model and thick ply model are indicated, along with the unidirectional behavior.

Potential crack plane, with crack nucleus:

- Thickness propagation
- Longitudinal propagation

Initial crack density in a uniformly stressed laminate is strictly a function of material inhomogeneity.

- Strength scaled by $f$, Fracture toughness scaled by $f^2$
- Constant $f$ along each crack path

$\sigma_{\text{init}} \quad \sigma_{\text{sat}}$

**Inhomogeneity applied to 3 levels of mesh refinement**

- 10 elts.
- 2 elts.
- 1 elt.

F Leone, 2015
Effect of Transverse Mesh Density on Crack Spacing

Analyses with 3 levels of mesh refinement
Transverse Matrix Cracks w/ One Element Per Ply

Multi-element model: correct crack evolution

Conventional single-element: no opening w/out delam.

Modified single-element: correct Energy Release Rate

Uncracked

Cracked

\[ K \approx \frac{4E_2}{\pi^2t} \]

Van der Meer, F.P. & Dávila, C., JCM, 2013
Crack Initiation, Densification, and Saturation

$$\sigma = 182 \text{ MPa}$$

$$\sigma = 273 \text{ MPa}$$

$$\sigma = 372 \text{ MPa}$$

$$\sigma = 679 \text{ MPa}$$

Van der Meer, F.P. & Dávila, C., JCM, 2013
Comparison of X-FEM Single-Ply Model and Full 3-D

\[ \sigma = 372 \text{ MPa} \]
Summary

- Strength and fracture issues are:
  - Interrelated
  - Subject to size effects

- Cohesive laws:
  - Account for strength and fracture toughness
  - More than 3 elements are required in the process zone

- Continuum damage models
  - Snap-back imposes maximum element size

- Transverse matrix cracks:
  - More than one element per ply is required

- Other mesh issues?