Assessment of Preconditioner for a USM3D Hierarchical Adaptive Nonlinear Iteration Method (HANIM) (Invited)

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Outline

• Background and Motivation
• Summary of Present Extensions
• Overview of USM3D Solution Methods
• Results
• Concluding Remarks
• Future Directions
Background and Motivation

- **Initial mixed-element USM3D version in 2012 (AIAA-2013-2541)**
  - extends USM3D tetrahedral grid CFD code to support various cell topologies
  - “preconditioner-alone” (PA) baseline solver technology to advance nonlinear solution in pseudo-time
  - preconditioner based on defect-correction scheme and point-implicit Gauss-Seidel solution method

- **Sluggish iterative convergence on finer grids**
  - 2D bump-in-channel finest grid solution after 500,000 nonlinear iterations
    - forces and moment almost converged, residuals several orders above machine-zero
Background and Motivation

- **USM3D mixed-element version enhanced in 2014 for improved iterative convergence and robustness (AIAA-2015-1747)**
  - Hierarchical Adaptive Nonlinear Iteration Method (HANIM)
    - similar to other recent approaches at NASA (FUN3D), Boeing (GGNS), etc.
    - convergence speedup factor of 1.4 to 13 relative to the baseline PA method
- **Diminished convergence acceleration on finer grids**
Summary of Present Extensions

- **Multicolor line-implicit preconditioner**
  - highly-anisotropic grids typically used for turbulent flow computations
  - line-implicit preconditioner can efficiently reduce high-frequency errors in the directions of small and large mesh spacing
  - point-implicit preconditioner can be inefficient on highly-anisotropic grids
  - line generation algorithm implemented
    - extracts a sequence of ordered cells that share a face
    - relies only on grid connectivity, does not use geometric/discretization information
    - currently limited to prismatic and hexahedral grids
  - Thomas algorithm for block tri-diagonal linear system
Summary of Present Extensions

• Improved discretization of turbulence model source terms
  – velocity gradients modified using a line-mapping method
    ➢ relies on line structure

• Discretely-consistent and general symmetry boundary condition
  – intersection of up to 3 symmetry boundary patches, angle between two
    symmetry boundary patches can be any divisor of 360°
  – new procedures for computing nodal averaging, fluxes, and flux linearization
    ➢ gradient, nodal averaging, and flux reconstruction stencils not biased

• Grid sequencing for solution initialization
  – currently limited to structured grids

• Preconditioner optimization
  – static residuals to monitor convergence of preconditioner
  – residual reduction target changed from 0.1 to 0.5
    ➢ improved efficiency observed for HANIM solutions using point-implicit
      preconditioner (up to factor 2.3)
Overview of USM3D Nonlinear Solution Methods

- System of nonlinear equations, $R(Q) = 0$
- Two different methods for nonlinear iterations
  - Preconditioner-Alone (PA) method
  - Hierarchical Adaptive Nonlinear Iteration Method (HANIM)
- PA method
  - baseline code technology with improved discretization and preconditioner
- HANIM
  - significant improvements over PA in robustness and iterative convergence
  - enhanced solver for the system of nonlinear equations
  - provides two additional hierarchies around the preconditioner of PA
    - matrix-free linear solver for the exact linearization of nonlinear RANS equations
    - nonlinear control of solution updates
  - CFL adaptation used as a comprehensive tool
PA Method

- Baseline code technology with improved discretization and preconditioner
  - first-order FDS scheme for mean flow approximate Jacobian
  - point- or line-implicit scheme for solving preconditioner equations
  - residual reduction targets for preconditioner

\[
\frac{V}{\Delta \tau} \Delta Q + \frac{\partial \hat{R}}{\partial Q} \Delta Q = -R \left( Q^n \right) \\
Q^{n+1} = Q^n + \Delta Q
\]
Hierarchical Adaptive Nonlinear Iteration Method (HANIM)

Preconditioner

GCR Solver

Realizability Check

Nonlinear Control

Nonlinear Update

Failure Handler

nonlinear iteration

nonlinear residuals
pseudo-time step

failed

failed

failed

HANIM Flowchart

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HANIM Modules

Preconditioner, GCR Solver, Realizability Check, Nonlinear Control

- **Preconditioner** generates a new search direction for GCR

- **GCR Solver** uses Generalized Conjugate Residual (GCR) method
  - matrix-free linear solver for
    \[ \frac{V}{\Delta \tau} \Delta Q + \frac{\partial R}{\partial Q} \Delta Q = - R(Q^n) \]
  - Frechet derivative used for \[ \frac{\partial R}{\partial Q} \Delta Q \]
  - suggests updates, \( \Delta Q \), for current nonlinear solution

- **Realizability Check** module checks for non-physical solution state using updates, \( \Delta Q \), from GCR Solver module

- **Nonlinear Control** reduces nonlinear residuals to a specified target \( Q = Q^n + \omega \Delta Q \)
  - finds optimal under-relaxation parameter, \( \omega \), for updates from GCR-Solver
  - quadratic search used for under-relaxation parameter

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Results

- **Steady-state Reynolds-averaged Navier-Stokes (RANS) solutions**
  - 2D bump-in-channel
  - 2D NACA 0012 airfoil
  - 3D bump-in-channel
  - 3D hemisphere-cylinder

- **Grids from NASA Turbulence Modeling Resource (TMR) website**
  - uniformly-refined nested grids

- **3 different solution sets**
  - HANIM solutions using line-implicit preconditioner (LI-HANIM)
  - HANIM solutions using point-implicit preconditioner (PI-HANIM)
  - PA method solutions using point-implicit preconditioner (PA)

- **Convergence of LI-HANIM assessed relative to PI-HANIM and PA**

- **Solution convergence criteria**
  - \( \text{rms norm of combined mean flow and turbulence model residuals} \leq 10^{-13} \)
  - aerodynamic coefficients converged to six significant digits
Key Solution Parameters

• Spalart-Allmaras (SA) model; negative variant
• Mean flow convective terms: second-order, Roe’s FDS
• SA model convective term: first-order
• Mean flow approximate Jacobian convective terms: first-order, FDS
• Preconditioner: maximum 500 G-S iterations, residual reduction target 0.5
• HANIM parameters
  – only 1 search direction for GCR Solver
  – linear residual reduction target for GCR Solver module: 0.96
  – nonlinear residual target for Nonlinear Control module: 0.92
  – adaptive CFL, 1 for the first nonlinear iteration
    ➢ If all modules declare success increase CFL by factor 2
    ➢ If any module declares failure, reduce CFL by factor 10
  – two different solutions using point-implicit preconditioner (PI-HANIM) and line-implicit preconditioner (LI-HANIM)
• PA parameters:
  – point-implicit preconditioner
  – prescribed CFL, ramped from 1 to 150 over 150 nonlinear iterations
2D NACA 0012 Airfoil

- Solutions computed using TMR Family II structured grid series
  - 2x113x33, 2x225x65, **2x449x129**, 2x897x257, 2x1793x513, **2x3585x1025**
  - grid points listed in spanwise, streamwise, and normal directions

Flow conditions: $M_\infty = 0.15$, $\alpha = 10^\circ$, $Re_c = 6\times10^6$

Solutions initialized using grid sequencing

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2D NACA 0012 Airfoil
Solution Convergence on 2x449x129 Grid

- Combined residuals
  - Mean flow and SA model

- Maximum eddy viscosity

- Log₁₀(rms of residuals)
  - PA
  - PI-HANIM
  - LI-HANIM

- CPU time, seconds

- Log₁₀(CFL number)

- Mean flow and SA model

- Total drag

- CPU time, seconds

- PA

- PI-HANIM

- LI-HANIM

- CPU time, seconds

- Log₁₀(CFL number)

- CPU time, seconds

- Log₁₀(rms of residuals)

- CPU time, seconds
2D NACA 0012 Airfoil
Solution Convergence on 2x3585x1025 Grid

$\mu_{t,\text{max}}$

CPU time, seconds

maximum eddy viscosity

Combined residuals

Log$_{10}$(rms of residuals)

CFL number

Log$_{10}$(CFL number)

CPU time, seconds

$H_{t,\text{max}}$

CPU time, seconds

$C_D$

CPU time, seconds

$C_D$

CPU time, seconds

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2D NACA 0012 Airfoil

LI-HANIM Speedup for Converged* Solutions

*combined residual rms $1.0 \times 10^{-13}$
force coefficients converge to six significant digits

LI-HANIM speedup = method time to solution/LI-HANIM time to solution

Speedup relative to PA method

Speedup relative to PI-HANIM

LI-HANIM converged on all grids

Speedup vs. characteristic grid spacing, $h$

PA solution yet to converge

PI-HANIM solution yet to converge

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3D Bump-in-Channel

- Solutions computed using structured grid series
  - 3x45x21, 5x89x41, 9x177x81, 17x353x161, **33x705x321**, 65x1409x641
  - grid points listed in spanwise, streamwise, and normal directions

Flow conditions: $M_\infty = 0.2$, $\alpha = 0^\circ$, $Re_L = 3 \times 10^6$

Solutions initialized using grid sequencing

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3D Bump-in-Channel
Solution Convergence on 33x705x321 Grid

combined residuals

Log$_{10}$(rms of residuals)

CPU time, seconds

PA
PI-HANIM
LI-HANIM

CFL number

Log$_{10}$(CFL number)

CPU time, seconds

C$_D$

total drag

CPU time, seconds

μ$_{t,max}$

maximum eddy viscosity

μ$_{t,max}$

CPU time, seconds

CD

CPU time, seconds

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LI-HANIM speedup = method’s time to solution/LI-HANIM time to solution

*combined residual rms 1.0x10^{-13}
force coefficients converge to six significant digits

LI-HANIM converged on all grids

PA solution failed
PA solution not attempted

PI-HANIM solution not attempted

Residuals
Drag

PA solution
not attempted

characteristic grid spacing, h

characteristic grid spacing, h

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3D Hemisphere-Cylinder

- Solutions computed using mixed-element unstructured grid series
  - 355,200 cells in coarse grid, 2,841,600 cells in medium grid, and 22,732,800 cells in fine grid
  - 60° circumferential domain

Flow conditions: $M_\infty = 0.6$, $\alpha = 0^\circ$, $Re_L = 0.35 \times 10^6$

Solutions initialized using freestream conditions
3D Hemisphere-Cylinder
LI-HANIM Speedup for Converged* Solutions

*combined residual rms 1.0x10^{-13}
force coefficients converge to six significant digits

LI-HANIM speedup = method’s time to solution/LI-HANIM time to solution

Speedup relative to PA method

Speedup relative to PI-HANIM

Residuals

Drag

characteristic grid spacing, h

characteristic grid spacing, h
HANIM Convergence
Time to Target Level* Residuals for Four Cases

*combined residual rms $1.0 \times 10^{-13}$

- 2D bump-in-channel
- 2D NACA 0012 airfoil
- 3D bump-in-channel
- 3D hemisphere-cylinder

(characteristic grid spacing, $h$)

(time to solution, normalized by DoF)

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Concluding Remarks

- Enhancements for mixed-element USM3D to further improve efficiency and accuracy of CFD solutions
  - multicolor line-implicit preconditioner
  - discretely-consistent and general symmetry boundary condition
  - improved discretization of turbulence model source term using line-mapping method

- Iterative convergence of line-implicit HANIM assessed relative to point-implicit HANIM and PA method on four turbulent flow cases
  - 2D bump in a channel, 2D NACA 0012 airfoil, 3D bump in a channel, 3D hemisphere cylinder

- Only line-implicit HANIM met convergence targets for all cases
  - rms norm of combined mean flow and turbulence model residuals ≤ $10^{-13}$
  - aerodynamic coefficients converged to six significant digits
Concluding Remarks

- **Line-implicit HANIM relative to point-implicit HANIM**
  - at least factor 2.1 speedup for bump cases and NACA 0012 airfoil on finer grids, speedup higher than 10 on many grids
  - negligible speedup or even minor slowdown on coarse grids
  - slowdown for 3D hemisphere cylinder case, more competitive in grid refinement
  - less case-to-case variations in performance
  - less degradation in performance with grid refinement

- **Line-implicit HANIM relative to PA method**
  - more efficient across all cases
  - at least factor 6.2 speedup for bump cases and NACA 0012 airfoil on finer grids, speedup higher than 10 on many grids

- **Discretely-consistent and general symmetry boundary condition enabled efficient simulation of 3D hemisphere cylinder case**
  - one sixth of the grid for the full computational domain
Future Directions

- Parallelize current improvements
- Develop line generation algorithm for general unstructured grids
- Assess line-implicit HANIM on more 3D cases
  - transonic/supersonic flows
  - tetrahedral grids
- Seek **grid-independent convergence rate**
  - agglomeration scheme for grid sequencing and multigrid
  - linear multigrid for preconditioner
  - nonlinear multigrid solver

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