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July 2016
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Acknowledgments

The author acknowledges funding for this work from the NASA Engineering Safety Center (NESC) Passive Thermal Technical Committee. The support provided by Mr. Steve Rickman, NESC Passive Thermal Technical Fellow and Mr. Daniel Nguyen, Deputy of the Passive Thermal Technical Discipline Team is greatly appreciated. Valuable comments and suggestions were also provided by Mr. Chris Kostyk, Mr. Larry Hudson, Mr. Andrew Holguin, and Mr. Jason Lechniak. Additional reviewers who also provided many valuable comments throughout the development include Ms. Ruth Amundsen, Mr. Jentung Ku, Mr. David Gilmore, Mr. Richard Wear, and Mr. Duane Beach.

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Abstract

Determining the intensity and spectral distribution of radiation emanating from a heated surface has applications in many areas of science and engineering. Areas of research in which the quantification of spectral radiation is used routinely include thermal radiation heat transfer, infrared signature analysis, and radiation thermometry. In the analysis of radiation, it is helpful to be able to predict the radiative intensity and the spectral distribution of the emitted energy. Presented in this report is a set of routines written in Microsoft Visual Basic for Applications® (VBA) (Microsoft Corporation, Redmond, Washington) and incorporating functions specific to Microsoft Excel® (Microsoft Corporation, Redmond, Washington) that are useful for predicting the radiative behavior of heated surfaces. These routines include functions for calculating quantities of primary importance to engineers and scientists. In addition, the routines also provide the capability to use such information to determine surface temperatures from spectral intensities and for calculating the sensitivity of the surface temperature measurements to unknowns in the input parameters.

Nomenclature

c  speed of light in a vacuum, $2.99792458 \times 10^8$ m/s
$C_1$  Planck’s first constant, $2\hbar c^2 = 1.191043 \times 10^8$ W·μm⁴/m²·sr
$C_2$  Planck’s second constant, $\hbar c/k = 14,387.77$ μm·K
$C_3$  Wien’s displacement law constant, 2,897.77 μm·K
$C_4$  Constant giving maximum spectral intensity at peak wavelength, $C_1/C_3^5 \left(e^{C_2/C_3} - 1\right) = 4.09567 \times 10^{-12}$ W/m²·μm·sr·K⁶
$D_i(\lambda)$  detector spectral response function for detector $i$
$e_b(T)$  total blackbody emissive power, W/m²
$e(T)$  total emissive power, W/m²
$\hbar$  Planck’s constant, 6.626070040 × 10⁻³⁴ J·s
$i_{b,\lambda}(\lambda, T)$  spectral emissive intensity of a perfect blackbody at wavelength $\lambda$ and temperature $T$, W/m²·sr·μm
$i_{\lambda}(\lambda, T)$  spectral emissive intensity of a non-blackbody at wavelength $\lambda$ and temperature $T$, W/m²·sr·μm
$i_b(T)$  total emissive intensity of a blackbody at temperature $T$, W/m²·sr
$i(T)$  total emissive intensity of a non-blackbody at temperature $T$, W/m²·sr
$I(\lambda_1, \lambda_u, T)$  general integrated spectral intensity function from wavelength $\lambda_1$ to $\lambda_u$ at temperature $T$
$I_0(\lambda_1, \lambda_u, T)$  integrated spectral intensity function from wavelength $\lambda_1$ to $\lambda_u$ at temperature $T$, W/m²·sr
$I_1(\lambda_1, \lambda_u, T)$  integrated spectral intensity derivative with respect to temperature from wavelength $\lambda_1$ to $\lambda_u$ at temperature $T$, W/m²·sr·K
$I_2(\lambda_1, \lambda_u, T)$  integrated first moment with respect to wavelength of the spectral intensity function from wavelength $\lambda_1$ to $\lambda_u$ at temperature $T$, W·μm/m²·sr
$I_3(\lambda_1, \lambda_u, T)$  integrated first moment with respect to wavelength of the spectral intensity derivative with respect to temperature from wavelength $\lambda_1$ to $\lambda_u$ at temperature $T$, W·μm/m²·sr·K
$k$  Boltzmann constant, 1.38064852 × 10⁻²³ J/K
$R$  universal gas constant, 8.3144598 J/mol·K
$T$  actual true surface temperature, K
$T_\lambda$ measured equivalent blackbody temperature at wavelength $\lambda$, assuming a perfect emitter, K

$\bar{\varepsilon}$ average emissivity across a waveband

$\varepsilon_\lambda(\lambda, T)$ monochromatic emissivity of a non-blackbody at wavelength $\lambda$ and temperature $T$

$\bar{\varepsilon}_i$ wavelength-averaged emissivity for detector $i$

$\varepsilon_r$ emissivity ratio at two wavelengths $\lambda_1$ and $\lambda_2$, $\varepsilon_2/\varepsilon_1$

$\bar{\varepsilon}_r$ wavelength averaged emissivity for detector 1 and 2, $\varepsilon_2/\varepsilon_1$

$\lambda_i$ wavelength of detector $i$

$\lambda_i$ lower wavelength on wide-band detector

$\lambda_u$ upper wavelength on wide-band detector

$\lambda_{il}$ lower wavelength on wide-band detector $i$

$\lambda_{iu}$ upper wavelength on wide-band detector $i$

$\Lambda$ equivalent wavelength, $\lambda_1\lambda_2/(\lambda_2 - \lambda_1)$, $\mu$m

$\sigma$ Stefan-Boltzmann constant, $5.670367 \times 10^{-8}$ W/m$^2$·K$^4$

$\Theta$ elevation or altitude angle, rad

$\phi$ circumferential or azimuthal angle, rad

$\Omega$ solid angle, sr

Subscripts

$b$ blackbody

$i$ for detector $i$

$\lambda$ at wavelength $\lambda$

Note: Values for physical constants were obtained from ref. 1.

List of Acronyms

InGaAs indium-gallium-arsenide

IR infrared

LWIR long-wavelength infrared

MWIR medium-wavelength infrared

NASA National Aeronautics and Space Administration

NESC NASA Engineering Safety Center

SWIR short-wavelength infrared

UV ultraviolet

UVA A-band ultraviolet

UVB B-band ultraviolet

UVC C-band ultraviolet

VBA Visual Basic for Applications®
**Routine Summary**

The seven tables presented immediately below summarize each function included in the blackbody function library. The full definition of each function is included in the Section “VBA Routine Summaries.”

### Narrow-band spectral functions

<table>
<thead>
<tr>
<th>Routine</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>bb_iibl</td>
<td>Calculate the blackbody emissive intensity at a given temperature and wavelength.</td>
</tr>
<tr>
<td>bb_tibl</td>
<td>Calculate the equivalent blackbody temperature at a specified wavelength for a given monochromatic blackbody emissive intensity.</td>
</tr>
<tr>
<td>bb_dibldt</td>
<td>Calculate the derivative of the blackbody emissive intensity with temperature.</td>
</tr>
<tr>
<td>bb_dlnibldint</td>
<td>Calculate the derivative of the logarithm of the blackbody emissive intensity with respect to the logarithm of temperature at a given temperature and wavelength.</td>
</tr>
<tr>
<td>bb_d2ibldt2</td>
<td>Calculate the second derivative of the blackbody emissive intensity with temperature.</td>
</tr>
<tr>
<td>bb_dibldl</td>
<td>Calculate the derivative of the blackbody emissive intensity with wavelength.</td>
</tr>
<tr>
<td>bb_dlnibldlnt</td>
<td>Calculate the derivative of the logarithm of the blackbody emissive intensity with respect to the logarithm of wavelength at a given temperature and wavelength.</td>
</tr>
<tr>
<td>bb_d2ibldl2</td>
<td>Calculate the second derivative of the blackbody emissive intensity with wavelength.</td>
</tr>
<tr>
<td>bb_iib</td>
<td>Calculate the total blackbody emissive intensity at a specified temperature.</td>
</tr>
<tr>
<td>bb_eb</td>
<td>Calculate the total blackbody emissive power at a specified temperature.</td>
</tr>
<tr>
<td>bb_lmax</td>
<td>Calculate the wavelength at the maximum spectral intensity for a given temperature using Wien’s displacement law.</td>
</tr>
<tr>
<td>bb_imax</td>
<td>Calculate the maximum spectral intensity for a given temperature.</td>
</tr>
<tr>
<td>bb_tmax</td>
<td>Calculate the temperature for the maximum spectral intensity.</td>
</tr>
</tbody>
</table>

### Spectral method functions

<table>
<thead>
<tr>
<th>Routine</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>bb_tsi</td>
<td>Calculate the true temperature for a non-blackbody given the measured equivalent blackbody spectral intensity at a specified wavelength and the spectral emissivity.</td>
</tr>
<tr>
<td>bb_tli</td>
<td>Calculate the equivalent blackbody temperature at a specified wavelength for a non-blackbody given the blackbody spectral intensity at the true temperature and the spectral emissivity.</td>
</tr>
<tr>
<td>bb_tst</td>
<td>Calculate the true temperature for a non-blackbody given the equivalent blackbody temperature at a specified wavelength and the spectral emissivity.</td>
</tr>
<tr>
<td>bb_tstw</td>
<td>Calculate the true temperature for a non-blackbody given the equivalent blackbody temperature at a specified wavelength and the spectral emissivity assuming Wien’s approximation is applicable. This routine can also be used to calculate the true temperature using the ratio method with the effective wavelength and the ratio temperature replacing the</td>
</tr>
</tbody>
</table>
wave length and blackbody temperature, again assuming Wien’s approximation is applicable.

<table>
<thead>
<tr>
<th>Routine</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>bb_tlt</td>
<td>Calculate the equivalent blackbody temperature at a specified wavelength for a non-blackbody given the true temperature and the spectral emissivity.</td>
</tr>
<tr>
<td>bb_dlntdlnl</td>
<td>Calculate the sensitivity of the true temperature with the spectral emissivity.</td>
</tr>
<tr>
<td>bb_dlntdlnl</td>
<td>Calculate the sensitivity of the equivalent blackbody temperature with the spectral emissivity.</td>
</tr>
<tr>
<td>bb_dlnlrdlnl</td>
<td>Calculate the sensitivity of the equivalent blackbody temperature with respect to the logarithm of the true temperature. The sensitivity is expressed as the derivative of the logarithm of the true temperature with respect to the logarithm of the equivalent blackbody temperature.</td>
</tr>
<tr>
<td>bb_emiss</td>
<td>Calculate the spectral emissivity at a specified wavelength given the equivalent blackbody temperature and the true temperature.</td>
</tr>
<tr>
<td>bb_dlnedlnl</td>
<td>Calculate the sensitivity of the emissivity at a given wavelength (2) with respect to the uncertainty in emissivity at another wavelength (1) which has been used to determine the true temperature. The sensitivity is expressed as the derivative of the logarithm of the emissivity at the second wavelength with respect to the logarithm of the emissivity at the first wavelength.</td>
</tr>
</tbody>
</table>

**Narrow-band ratio functions**

<table>
<thead>
<tr>
<th>Routine</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>bb_erlam</td>
<td>Calculate the effective wavelength for a two-band radiation thermometer.</td>
</tr>
<tr>
<td>bb_ertemp</td>
<td>Calculate the ratio temperature for a two-band radiation thermometer.</td>
</tr>
<tr>
<td>bb_tratio</td>
<td>Calculate the true temperature for a two-band radiation thermometer given the temperatures at two wavelengths and an emissivity ratio.</td>
</tr>
<tr>
<td>bb_emissr</td>
<td>Calculate the effective emissivity ratio for a two-band radiation thermometer given the temperatures at two wavelengths and the true temperature.</td>
</tr>
<tr>
<td>bb_dlnedlnl</td>
<td>Calculate the derivative of the logarithm of the emissivity ratio with respect to the logarithm of the true temperature for a two-band radiation thermometer given the temperatures at two wavelengths and emissivity ratio.</td>
</tr>
<tr>
<td>bb_dlnedlnl</td>
<td>Calculate the derivative of the logarithm of the emissivity ratio with respect to the logarithm of the equivalent blackbody temperature of the first detector for a two-band radiation thermometer given the given the detector wavelength and temperature.</td>
</tr>
<tr>
<td>bb_dlnlrdlnl1</td>
<td>Calculate the derivative of the logarithm of the ratio temperature for a two-band radiation thermometer with respect to the logarithm of the first spectral temperature.</td>
</tr>
<tr>
<td>bb_dlnlrdlnl1</td>
<td>Calculate the derivative of the logarithm of the equivalent wavelength for a two-band radiation thermometer with respect to the logarithm of the first wavelength.</td>
</tr>
</tbody>
</table>
### Wide-band functions

<table>
<thead>
<tr>
<th>Routine</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>bb_iibls</td>
<td>Calculate the integrated emissive intensity for a wide- or narrow-band</td>
</tr>
<tr>
<td></td>
<td>wavelength band using a series solution.</td>
</tr>
<tr>
<td>bb_fiibls</td>
<td>Calculate the spectral emissive intensity fraction between zero and specified</td>
</tr>
<tr>
<td></td>
<td>wavelength using a series solution.</td>
</tr>
<tr>
<td>bb_iibl</td>
<td>Calculate the total integrated emissive intensity for a wide- or narrow-band</td>
</tr>
<tr>
<td></td>
<td>wavelength band using a quadrature integration scheme. The routine provides</td>
</tr>
<tr>
<td></td>
<td>the integration of the emissive intensity, derivative of the emissive intensity</td>
</tr>
<tr>
<td></td>
<td>with temperature, the first moment of the emissive intensity, the first</td>
</tr>
<tr>
<td></td>
<td>moment of the emissive intensity, or the first moment of the derivative of</td>
</tr>
<tr>
<td></td>
<td>the emissive intensity with temperature. Optionally, the integrand can be</td>
</tr>
<tr>
<td></td>
<td>weighted by the emissivity, a detector response function, or both.</td>
</tr>
<tr>
<td>bb_tiiibl</td>
<td>Calculate the equivalent blackbody temperature across a waveband for a</td>
</tr>
<tr>
<td></td>
<td>non-blackbody with an emissivity as a function of wavelength for a specified</td>
</tr>
<tr>
<td></td>
<td>integrated intensity.</td>
</tr>
<tr>
<td>bb_ttiibl</td>
<td>Calculate the equivalent blackbody temperature across a waveband for a</td>
</tr>
<tr>
<td></td>
<td>non-blackbody with an emissivity as a function of wavelength for a specified</td>
</tr>
<tr>
<td></td>
<td>temperature.</td>
</tr>
</tbody>
</table>

### Wide-band ratio functions

<table>
<thead>
<tr>
<th>Routine</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>bb_itratio</td>
<td>Calculate the true temperature for a two-band radiation thermometer given</td>
</tr>
<tr>
<td></td>
<td>the temperatures at the two bands and an emissivity ratio.</td>
</tr>
<tr>
<td>bb_iemissr</td>
<td>Calculate the effective emissivity ratio from a two-band radiation</td>
</tr>
<tr>
<td></td>
<td>thermometer given the temperatures for the two bands and the true temperature.</td>
</tr>
<tr>
<td>bb_dlniedlnt</td>
<td>Calculate the derivative of the logarithm of the emissivity ratio with</td>
</tr>
<tr>
<td></td>
<td>respect to the logarithm of the true temperature for a two-band radiation</td>
</tr>
<tr>
<td></td>
<td>thermometer.</td>
</tr>
<tr>
<td>bb_dlniedlnt1</td>
<td>Calculate the derivative of the logarithm of the emissivity ratio with</td>
</tr>
<tr>
<td></td>
<td>respect to the logarithm of the first band temperature for a two-band</td>
</tr>
<tr>
<td></td>
<td>radiation thermometer.</td>
</tr>
</tbody>
</table>

### Constant functions (values from ref. 1)

<table>
<thead>
<tr>
<th>Routine</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>bb_C1</td>
<td>Return the value of $C_1 = 1.191043 \times 10^8$ W-μm$^4$/m$^2$-sr</td>
</tr>
<tr>
<td>bb_C2</td>
<td>Return the value of $C_2 = 14,387.77$ μm-K</td>
</tr>
<tr>
<td>bb_C3</td>
<td>Return the value of $C_3 = 2,897.77$ μm-K</td>
</tr>
<tr>
<td>bb_C4</td>
<td>Return the value of $C_4 = 4.09567 \times 10^{-12}$ W/m$^2$-sr-μm</td>
</tr>
<tr>
<td>bb_SIGMA</td>
<td>Return the value of the Stephan-Boltzmann constant, $\sigma = 5.670367 \times$</td>
</tr>
<tr>
<td></td>
<td>$10^{-8}$ W/m$^2$-K</td>
</tr>
<tr>
<td>bb_RCONST</td>
<td>Return the value of the universal gas constant $R = 8.3144598$ J/mol-K</td>
</tr>
<tr>
<td>bb_CLIGHT</td>
<td>Return the value of the speed of light in a vacuum, $c = 2.99792458 \times$</td>
</tr>
<tr>
<td></td>
<td>$10^8$ m/s</td>
</tr>
<tr>
<td>bb_HBAR</td>
<td>Return the value of Planck’s constant $\hbar = 6.62607004 \times 10^{-34}$ J/s</td>
</tr>
<tr>
<td>bb_KBOLTZ</td>
<td>Return the value of the Boltzmann constant $k = 1.38064852 \times 10^{-23}$ J/K</td>
</tr>
</tbody>
</table>
### Configuration control functions

<table>
<thead>
<tr>
<th>Routine</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>bb_version</td>
<td>Returns the version of the blackbody function library as a string.</td>
</tr>
</tbody>
</table>

### Introduction

Determining the intensity and spectral distribution of radiation emanating from a heated surface has applications in many areas of science and engineering. Areas of research in which the quantification of spectral radiation is routinely used include thermal radiation heat transfer, infrared signature analysis, and radiative thermometry. In these applications, the radiation is predominantly emitted across the entire electromagnetic spectrum (continuum radiation) as opposed to radiation emitted at specific wavelengths due to molecular or electronic transitions (line radiation).

In the analysis of radiation, it is helpful to be able to predict the heat transfer rate radiative intensity and the spectral distribution of the emitted energy. Presented in this report is a set of routines written in Microsoft Visual Basic for Applications (VBA) (Microsoft Corporation, Redmond, Washington) and incorporating functions specific to Microsoft Excel (Microsoft Corporation, Redmond, Washington) that are useful for predicting the behavior of heated surfaces. These routines include functions for calculating engineering quantities of primary importance to engineers and scientists. In addition, the routines also provide the capability to use such information to determine surface temperatures from spectral intensities and for calculating the sensitivity of these temperature measurements to unknowns in the input parameters.

### Theoretical Background

This section presents the basic theory underlying radiative thermometry. The basic equations governing the emission of continuum radiation from a heated surface are presented first, followed by the derivation of the applicable equations for three common thermometry methods: 1) spectral method, 2) ratio method, and 3) multispectral method. The equations derived here are the basis for the routines described in the subsequent sections.

### Basic Equations

Every surface emits radiation when heated. The emission of radiant energy from an ideal heated emitter (known as a black surface or a blackbody) is governed by the Planck equation (ref. 1). The Planck equation relates the emitted radiation intensity for a perfect emitter \( i_{b,\lambda}(\lambda, T) \), in power per unit area of emitting surface per unit solid angle \( \Omega \) and per unit wavelength \( \lambda \), to the wavelength and the surface temperature \( T \), as shown in equation (1).

\[
i_{b,\lambda}(\lambda, T) = \frac{1}{\lambda^5} \cdot \frac{C_1}{(\text{e}^{C_2/\lambda T} - 1)}
\]

In equation (1), \( C_1 \) and \( C_2 \) are Planck's first and second constants \( 1.191043 \times 10^8 \text{ W} \cdot \mu \text{m}^4/\text{m}^2\cdot\text{sr} \) and 2,897.77 \( \mu \text{m} \cdot \text{K} \), respectively. A graphical representation of equation (1) is shown in figure 1.

---

1 Radiative thermometry is the technique for determining the temperature of a surface or a volume by measuring the electromagnetic radiation it emits.
Figure 1 illustrates that for a given temperature, the emitted intensity is distributed across an infinite range of wavelengths and for the high temperatures of interest to engineering (0 to 6,000 K), falls predominantly within the visible and infrared wavelength ranges. This range of electromagnetic energy, commonly referred to as thermal radiation, is divided into several subranges, as shown in figure 2.

The Planck equation has the following three important properties, as shown in figure 1:

- The radiant intensity increases with increasing temperature at all wavelengths and for all temperatures (that is, $dI_{b,\lambda}/dT$ is always greater than zero).
- For any given temperature, the radiant intensity reaches a maximum at a specific wavelength, and for radiant intensities below the maximum there are two wavelengths at which the radiant intensity is equal.
- The wavelength at the maximum radiant intensity decreases as the temperature increases (Wien's displacement law).
Figure 2. Electromagnetic spectrum showing ranges applicable to radiative thermometry.
The inversion of Equation 1 provides the black body temperature $T$ for a given spectral intensity $i_{b,\lambda}$ at wavelength $\lambda$ and is given in equation (2):

$$T = \frac{C_2}{\lambda} \cdot \frac{1}{\left(\ln\frac{C_1}{\lambda^5 i_{b,\lambda}} + 1\right)} \quad (2)$$

The wavelength of maximum spectral intensity $\lambda_{max}$ is given by Wien’s displacement law, as shown in equation (3):

$$T\lambda_{max} = C_3 = 2897.77 \, \mu m \cdot K \quad (3)$$

and varies from approximately 10 $\mu m$ at room temperature to 0.7 $\mu m$ at 6,000 K. According to equations (1) and (2), the maximum spectral intensity $i_{b,\lambda_{max}}$ at $\lambda_{max}$ is equal to (eq. (4)).

$$i_{b,\lambda_{max}} = \frac{T^5}{C_3^5} \cdot \frac{C_1}{(e^{C_2/C_3} - 1)} = C_4 T^5 \quad (4)$$

Real materials do not act as perfect emitters, but instead emit at a rate less than a perfect emitter. The ratio of the intensity emitted by a real surface to that of a perfect emitter (blackbody) is (eq. (5)):

$$\varepsilon_\lambda = i_\lambda(T, \lambda) / i_{b,\lambda}(\lambda, T) \quad (5)$$

and defines the spectral emissivity $\varepsilon_\lambda$. The spectral emissivity is temperature-, wavelength-, and angle-dependent, and can vary substantially as the surface conditions change. In the work described in this report, emissivity is assumed to be independent of angle.

The derivative of the Planck equation (eq. (1)) with temperature is presented in equation (6):

$$\frac{di_{b,\lambda}}{dT} = \frac{C_2}{\lambda^5 T^2} \cdot \frac{C_1 e^{C_2/\lambda T}}{(e^{C_2/\lambda T} - 1)^2} \quad (6)$$

while the derivative with wavelength is presented in equation (7).

$$\frac{di_{b,\lambda}}{d\lambda} = \frac{1}{\lambda^6} \cdot \frac{C_1}{(e^{C_2/\lambda T} - 1)} \left(\frac{C_2}{\lambda T} \cdot \frac{e^{C_2/\lambda T}}{(e^{C_2/\lambda T} - 1)} - 5\right) \quad (7)$$

The derivatives expressed in equations (6) and (7) are useful in determining the uncertainty in radiative measurements. The derivatives can be more usefully represented in terms of sensitivities, which equal the fractional change in the intensity with fractional change in temperature or wavelength, as shown in equations (8) and (9):

$$\frac{T}{i_{b,\lambda}} \frac{di_{b,\lambda}}{dT} = \frac{d\ln i_{b,\lambda}}{d\ln T} = \frac{C_2}{\lambda T} \cdot \frac{e^{C_2/\lambda T}}{(e^{C_2/\lambda T} - 1)} \quad (8)$$
\[
\frac{\lambda}{i_{b,\lambda}} \frac{d}{d\lambda} = \frac{d \ln i_{b,\lambda}}{d \ln \lambda} = \frac{C_2 e^{C_2/\lambda T}}{\lambda^4 T^4 (e^{C_2/\lambda T} - 1)^2} - 5
\]  
(9)

Derivatives discussed later in this report will be presented predominantly in this form.

The second derivatives of the Planck equation with respect to wavelength and temperature are useful for numerical analysis involving solutions for derivative values. The second derivative of intensity with temperature is identified in equation (10).

\[
\frac{d^2 i_{b,\lambda}}{dT^2} = \frac{C_2}{\lambda^2} \cdot \frac{C_1 e^{C_2/\lambda T}}{\lambda^4} \cdot \frac{C_2 e^{C_2/\lambda T}}{\lambda^4} \cdot \left( \frac{2 e^{C_2/\lambda T}}{\lambda^4} \left( e^{C_2/\lambda T} - 1 \right) - 1 \right) - 2
\]  
(10)

The second derivative with respect to wavelength is given in equation (11).

\[
\frac{d^2 i_{b,\lambda}}{d\lambda^2} = \frac{C_1}{\lambda^2} \cdot \frac{1}{\lambda^4} \cdot \frac{C_2}{\lambda^2} \cdot \frac{e^{C_2/\lambda T}}{\lambda^4} \cdot \frac{C_2 e^{C_2/\lambda T}}{\lambda^4} \cdot \left( 2 \frac{C_2 e^{C_2/\lambda T}}{\lambda^4} \left( e^{C_2/\lambda T} - 1 \right) - 12 \frac{C_2}{\lambda^4} \right) + 30
\]  
(11)

Often the cumulative intensity across a wavelength band is of interest, because many practical detectors measure across a wide spectral band. The cumulative intensity can be determined by integrating the Planck equation across the band bounded by the lower and upper wavelengths with the solution shown in equation (12).

\[
\int_{\lambda_l}^{\lambda_u} i_{b,\lambda}(\lambda, T) \, d\lambda = \int_{\lambda_l}^{\lambda_u} \frac{C_1}{\lambda^5} \frac{C_2 e^{C_2/\lambda T}}{\lambda^4} \left( e^{C_2/\lambda T} - 1 \right) \, d\lambda = I(\lambda_l, \lambda_u, T)
\]  
(12)

No closed-form solution to equation (12) exists, however, so that the integration must be performed numerically, for example, by polynomial formulae (for example, Simpson's rule); Gaussian quadrature; or series solution.

A useful expression for the fraction of emitted intensity between zero and a specified wavelength is illustrated in eqs. (13) and (14) (ref. 2):

\[
F_{0 \rightarrow \lambda T} = \frac{15}{\pi^4} \sum_{n=1}^{\infty} \frac{e^{-n \xi}}{n} \left( \xi^3 + 3 \frac{\xi^2}{n} + 6 \frac{\xi}{n^2} + 6 \frac{1}{n^3} \right)
\]  
(13)

where \( \xi = \frac{C_2}{\lambda T} \) and \( F_{0 \rightarrow \lambda T} = \frac{e_{0 \rightarrow \lambda T}}{\sigma T^4} \)  
(14)

so that the fractional intensity between two wavelengths \( \lambda_1 \) and \( \lambda_2 \) is as shown in equation (15).

\[
F_{\lambda_1 T \rightarrow \lambda_2 T} = F_{0 \rightarrow \lambda_2 T} - F_{0 \rightarrow \lambda_1 T}
\]  
(15)

Equation (14) is included in the blackbody function library and within a higher level routine that integrates the energy across a finite energy band with an arbitrary starting wavelength. The blackbody function library described herein includes a VBA function to integrate the basic Planck equation (eq. 1) using Gaussian quadrature (eq. (16)).
\[
I_0(\lambda_l, \lambda_u, T) = \int_{\lambda_l}^{\lambda_u} D(\lambda) \varepsilon_\lambda(\lambda) i_{b,\lambda} \, d\lambda = \int_{\lambda_l}^{\lambda_u} D(\lambda) \varepsilon_\lambda(\lambda) \frac{1}{\lambda^5} \cdot \frac{C_1}{(e^{C_2/\lambda T} - 1)} \, d\lambda,
\]

(16)

The library function also includes the ability to integrate three additional integrands derived from the Planck equation, as seen in equations (17)-(19).

1) The derivative of the Planck equation with temperature (eq. (5)):
\[
I_1(\lambda_l, \lambda_u, T) = \int_{\lambda_l}^{\lambda_u} D(\lambda) \varepsilon_\lambda(\lambda) \frac{di_{b,\lambda}}{dT} \, d\lambda = \int_{\lambda_l}^{\lambda_u} D(\lambda) \varepsilon_\lambda(\lambda) \frac{C_2}{\lambda^5 T^2} \cdot \frac{C_1 e^{C_2/\lambda T}}{(e^{C_2/\lambda T} - 1)^2} \, d\lambda
\]

(17)

2) The first moment with respect to wavelength of the Planck equation:
\[
I_2(\lambda_l, \lambda_u, T) = \int_{\lambda_l}^{\lambda_u} D(\lambda) \varepsilon_\lambda(\lambda) i_{b,\lambda} \lambda d\lambda = \int_{\lambda_l}^{\lambda_u} D(\lambda) \varepsilon_\lambda(\lambda) \frac{1}{\lambda^4} \cdot \frac{C_1}{(e^{C_2/\lambda T} - 1)} \, d\lambda
\]

(18)

3) The first moment with respect to wavelength of the derivative of the Planck equation with temperature:
\[
I_3(\lambda_l, \lambda_u, T) = \int_{\lambda_l}^{\lambda_u} D(\lambda) \varepsilon_\lambda(\lambda) \frac{di_{b,\lambda}}{dT} \lambda d\lambda = \int_{\lambda_l}^{\lambda_u} D(\lambda) \varepsilon_\lambda(\lambda) \frac{C_2}{\lambda^5 T^2} \cdot \frac{C_1 e^{C_2/\lambda T}}{(e^{C_2/\lambda T} - 1)^2} \, d\lambda
\]

(19)

Equations (16)-(19) also incorporate weighting response functions for the detector sensitivity \(D(\lambda)\) and the surface emissivity \(\varepsilon_\lambda(\lambda)\). The inclusion of the response functions accounts for the wavelength varying response of a detector or the surface emissivity, if required.

Integrating over all wavelengths for a perfect emitter provides the total intensity of the emitted radiation and is shown in equation (20).
\[
\int_0^\infty \frac{1}{\lambda^5} \cdot \frac{C_1}{(e^{C_2/\lambda T} - 1)} \, d\lambda = i_b = \frac{\sigma}{\pi} T^4
\]

(20)

The quantity \(\sigma\) is the Stefan-Boltzmann constant and is equal to 5.6704 \times 10^{-8} \text{W/m}^2\cdot\text{K}^4.

To calculate the radiant power of a planar surface requires the integration of an arbitrary emitted ray into a hemispherical cap over the surface, as shown in figure 3 and equation (21) (ref. 3):
\[
e_{b,\lambda}(\lambda, T) = i_{b,\lambda}(\lambda, T) \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} \cos \theta \sin \theta d\theta d\phi = \pi i_{b,\lambda}(\lambda, T)
\]

(21)

The quantity \(e_{b,\lambda}(\lambda, T)\) is known as the spectral blackbody emissive power. The integrated spectral emissive intensity across all wavelengths is the total emissive power and is given by equation (22):
\[
e_b = \pi \int_0^\infty i_{b,\lambda}(\lambda, T) d\lambda = \sigma T^4
\]

(22)
Figure 3. Integration of the spectral intensity through a cap to determine the emissive power.

The basic challenge in radiation thermometry, aside from actually measuring the emitted radiant intensity, is reducing the data to meaningful temperature and emissivity values. For any real surface, the emitted spectral intensity depends on the surface temperature and the spectral emissivity. The intensity measured from a calibrated detector can be readily converted to temperature using unity emissivity and the calibrated response function of signal voltage versus temperature. This temperature, known as the “brightness temperature” or “equivalent blackbody temperature”, does not correspond to the actual or “true” surface temperature, since the surface of a real emitting object never behaves as a perfect emitter.

The brightness temperature or equivalent blackbody temperature corresponds to the lower limit on the true or actual surface temperature, since by definition, the emissivity of all real surfaces must be less than or equal to one. Using the definition of spectral emissivity in equation (4) along with the Planck equation from equation (1), the general expression for the conversion of measured spectral intensity to true surface temperature for a non-blackbody is presented in equation (23).

\[
T = \frac{C_2}{\lambda} \cdot \frac{1}{\left(\ln\left(\frac{\varepsilon_\lambda C_1}{\lambda^5 I_\lambda}\right) + 1\right)}
\] (23)

Equivalently, the intensity measurement can be related to an equivalent blackbody temperature at the specified wavelength, so the surface temperature can be determined from equation (24).

\[
T = \frac{C_2}{\lambda} \cdot \frac{1}{\ln[\varepsilon_\lambda (e^{C_2/\lambda T\lambda} - 1) + 1]}
\] (24)
If the value of \( \exp(C_2/\lambda T) \) is much greater than 1 (Wien’s approximation, ref. 3), then equation (24) can be simplified to the form shown in equation (25).

\[
\frac{1}{T} = \frac{1}{T_\lambda} + \frac{\lambda}{C_2} \ln \varepsilon_\lambda
\]  

(25)

It is not possible to decouple or separate the effect of temperature and emissivity from only a single intensity measurement. Two or more measurements must be analyzed together to give both temperature and emissivity results. The methods used to decouple the effect of temperature and emissivity from the measured radiometric signals have been the subject of extensive analysis. Several methods are available, each utilizing different assumptions and requiring different levels of effort. Three of the current methods that have been used to reduce data taken from measurements are discussed: the spectral method; the ratio method; and multispectral methods (ref. 4 and 5).

### Spectral Method

In the spectral method, the temperature of the emitting surface is inferred from the measured spectral intensity or equivalent blackbody temperature at a single wavelength by assuming a value for the surface emissivity using equation (24) or (25). Equations (24) and (25) can be used in principle to calculate the surface temperature exactly if the assumed emissivity is known precisely.

Generally, however, the emissivity is not known exactly, so there will be some uncertainty in the emissivity value, which will result in a corresponding error in the calculated temperature. The sensitivity of the inferred true temperature due to uncertainties in the assumed emissivity can be computed by differentiating equation (24) with respect to the surface emissivity to obtain equation (26).

\[
\frac{d \ln T}{d \ln \varepsilon_\lambda} = -\frac{\lambda T}{C_2} \cdot \left( \frac{e^{C_2/\lambda T} - 1}{e^{C_2/\lambda T}} \right)
\]  

(26)

If the wavelength is short, such that the quantity \( \exp(C_2/\lambda T) \) is much greater than 1 (Wien’s approximation), the sensitivity of equation (26) simplifies to equation (27).

\[
\frac{d \ln T}{d \ln \varepsilon_\lambda} \approx -\frac{\lambda T}{C_2}
\]  

(27)

Equation 27 shows that the magnitude of the uncertainty in the inferred true surface temperature decreases with decreasing wavelength and that short wavelength detectors have a lower uncertainty than longer wavelength detectors when used to determine the surface temperature.

Once the corrected blackbody temperature is determined, the resulting surface emissivities at longer wavelengths can be computed from the basic definition of emissivity (eq. (5)) and the Planck equation (eq. (1)), as shown in equation (28).

\[
\varepsilon_\lambda = \frac{\left( e^{C_2/\lambda T} - 1 \right)}{\left( e^{C_2/\lambda T_\lambda} - 1 \right)}
\]  

(28)
Equations (26) and (28) can be differentiated to obtain an expression for the uncertainty in the calculated emissivities at other wavelengths, \( \varepsilon_2 \), with respect to the uncertainty in the emissivity at the reference wavelength \( \varepsilon_1 \), as in equation (29).

\[
\frac{d \ln \varepsilon_2}{d \ln \varepsilon_1} = \frac{\varepsilon_1}{\varepsilon_2} \frac{d \varepsilon_2}{d \varepsilon_1} = \frac{\lambda_1}{\lambda_2} \left( \frac{e^{-c_2/\lambda_1 T} - 1}{e^{-c_2/\lambda_2 T} - 1} \right)
\]  

(29)

If Wien’s approximation is appropriate for both wavelengths, then equation (29) reduces to equation (30):

\[
\frac{d \ln \varepsilon_2}{d \ln \varepsilon_1} = \frac{\varepsilon_1}{\varepsilon_2} \frac{d \varepsilon_2}{d \varepsilon_1} = \frac{\lambda_1}{\lambda_2}
\]  

(30)

In the simplest terms, when the exponential terms are small the resulting uncertainty in the calculated emissivity at wavelength \( \lambda_2 \) from a temperature measurement at wavelength \( \lambda_1 \) is attenuated by the ratio \( \lambda_1/\lambda_2 \). The resulting uncertainty in the emissivity wavelength \( \lambda_2 \) is the ratio of the two measurement wavelengths multiplied by the relative uncertainty in the assumed emissivity at wavelength \( \lambda_1 \).

The equation (30) example indicates that the error in the temperature measurement is minimized as the detector wavelength decreases, while the uncertainty in the emissivity decreases as the wavelength increases. This reasoning suggests that to minimize the error in the temperature measurement, a detector with a very short wavelength should be selected, while to achieve a precise emissivity measurement, a detector with a very long wavelength should be selected.

There are, however, limitations on this method. When measuring temperature, the available detector signal decreases rapidly as the wavelength decreases. Therefore, there is a practical lower limit to the wavelength based on detector sensitivity, even though the theoretical uncertainty always decreases with wavelength. At the other extreme, while the relative error in the calculated emissivity does indeed decrease with increasing wavelength, the emissive properties of many materials vary significantly over a wide wavelength band. Thus, assurance should be made that the longer wavelength detectors are chosen in the spectral region wherein the emissivity is desired, or at the least, in a region wherein the emissivity is expected to be close to the emissivity of the desired wavelength.

For a given emissivity, the sensitivity of the true surface with respect to the equivalent blackbody temperature is as given by equation (31).

\[
\frac{d \ln T}{d \ln T_\lambda} = \frac{T}{T_\lambda} e^{\frac{c_2}{T_\lambda} \left( \frac{1}{T_\lambda} - \frac{1}{T} \right)}
\]  

(31)

Additionally, the effects of signal-to-noise ratios propagate in the same manner as those of the emissivities. The measured signal being directly proportional to the emissivity, the signal sensitivity is equal to the calculated emissivity sensitivity. That is, uncertainties in the measured detector signals affect the resulting temperature and computed emissivities at the long wavelengths in the same way as the assumed emissivity at the short wavelength.

**Ratio Method**

The ratio method attempts to simultaneously determine a single temperature and emissivity ratio that satisfies the measurements from two detectors at two independent wavelengths. The expression for the ratio method is obtained from the definition of the spectral emissivity in equation (5) for two wavelengths. Taking the ratio of the two emissivities at the two wavelengths results in
the following relationship between the two equivalent blackbody temperatures \( T_{\lambda_1} \) and \( T_{\lambda_2} \) and the true surface temperature \( T \) (eq. (32)):

\[
\frac{e^{C_2/\lambda_2 T_{\lambda_2}} - 1}{(e^{C_2/\lambda_2 T_{\lambda_2}} - 1)} \cdot \frac{(e^{C_2/\lambda_1 T} - 1) - \varepsilon_r}{(e^{C_2/\lambda_1 T_{\lambda_1}} - 1)} = 0 \tag{32}
\]

where (eq. (33)):

\[
\varepsilon_r = \frac{\varepsilon_1}{\varepsilon_2} \tag{33}
\]

Given the two measured equivalent blackbody temperatures \( T_{\lambda_1} \) and \( T_{\lambda_2} \) at wavelengths \( \lambda_1 \) and \( \lambda_2 \), the surface temperature is found that satisfies equation (32).

There is no analytic solution to equation (32), so the solution must be found numerically by iteration. If, however, Wien’s approximation is applicable, then equation (32) can be reduced to equation (34):

\[
\frac{1}{T} = \frac{1}{T_r} + \frac{\Lambda}{C_2} \ln \varepsilon_r \tag{34}
\]

where \( T_r \) is the ratio temperature given by equation (35):

\[
\frac{1}{T_r} = \Lambda \left[ \frac{1}{\lambda_1 T_{\lambda_1}} - \frac{1}{\lambda_2 T_{\lambda_2}} \right] \tag{35}
\]

and \( \Lambda \) is an effective wavelength (eq. (36)).

\[
\Lambda = \frac{\lambda_1 \lambda_2}{\lambda_2 - \lambda_1} \tag{36}
\]

The ratio temperature is the temperature of an equivalent blackbody having the same ratio of spectral intensities at two specified wavelengths as that of the target material.

Equation (34) is the same form as equation (25) for the spectral method except that the emissivity ratio is used instead of the spectral emissivity ratio, the ratio temperature replaces the equivalent blackbody temperature, and the effective wavelength replaces the wavelength.

The uncertainty in the surface temperature with respect to uncertainty in the emissivity ratio can be determined by the reciprocal of equation (37).

\[
\frac{d \ln \varepsilon_r}{d \ln T} = \frac{1}{T} \cdot \frac{(e^{C_2/\lambda_2 T} - 1) \cdot \left( \frac{C_2}{\lambda_2 T} \cdot \frac{e^{C_2/\lambda_1 T} (e^{C_2/\lambda_1 T} - 1) - \left( \frac{C_2}{\lambda_1 T} \right) \cdot (e^{C_2/\lambda_2 T} - 1)}{(e^{C_2/\lambda_1 T} - 1)^2} \right)}{(e^{C_2/\lambda_1 T} - 1)} \tag{37}
\]

If Wien’s approximation is applicable, then the sensitivity can be approximated by the analog of equation (26), seen as equation (38).

\[
\frac{d \ln T}{d \ln \varepsilon_r} = -\frac{\Delta T}{C_2} \tag{38}
\]
Note that the equivalent wavelength $\Lambda$ can be many times larger than either of the two individual wavelengths if the two wavelengths are closely spaced. The key concern is whether the decrease in emissivity uncertainty is more significant than the increase in the effective wavelength as the wavelengths are chosen closer together. Otherwise, the large increase in effective wavelength can render the ratio method highly inaccurate. Conversely, the advantage of the ratio method is that it may be possible to more accurately estimate the emissivity ratio over a small wavelength difference, and this method may reduce the uncertainty in the temperature measurement overall.

Many commercial systems that claim to measure “true” temperature contain two independent detectors operating at two distinct wavelengths between 0.5 and 1.0 $\mu$m. The devices also include a “ratio” setting that adjusts the measurements for variations in the assumed emissivity ratio of the surface. Obviously such a device can produce an accurate reading only if the emissivity ratio at the two wavelengths approaches a well-characterized value.

Once the surface temperature is determined, the emissivities at the two wavelengths can be calculated using equation (28), similar to the spectral method.

Other useful derivatives that quantify error analysis include the sensitivity of the emissivity ratio to the first equivalent blackbody temperature $T_{\lambda_1}$ shown in equation (39):

$$\frac{d \ln \varepsilon_r}{d \ln T_{\lambda_1}} = \frac{C_2}{\lambda_1 T_{\lambda_1}^n} \frac{e^{C_2/\lambda_1 T_{\lambda_1}}}{e^{C_2/\lambda_1 T_{\lambda_1}} - 1} \quad (39)$$

and the derivative of the logarithm of the ratio temperature $T_r$ with respect to the logarithm of the first equivalent blackbody temperature $T_{\lambda_1}$ shown in equation (40):

$$\frac{d \ln T_r}{d \ln T_{\lambda_1}} = \frac{\Lambda}{\lambda_1} \frac{T_r}{T_{\lambda_1}} \quad (40)$$

and the derivative of the logarithm of the equivalent wavelength $\Lambda$ with respect to the logarithm of the wavelength of the first detector $\lambda_1$ for a two-band radiation thermometer shown in equation (41).

$$\frac{d \ln \Lambda}{d \ln \lambda_1} = 1 + \frac{\lambda_1}{\lambda_2 - \lambda_1} \quad (41)$$

**Multispectral Methods**

Multispectral methods are an extension of the ratio method; measurements from a larger number of detectors are used. Multiple measurements are then used to perform a best fit to single temperature and wavelength relationship. The emissivity relationship can be a constant value or some other more complex function, such as a polynomial. A least-squares minimization of the data is then performed to determine the temperature and emissivity relationship.

In addition to providing a continuous emissivity across a wide wavelength range, another advantage of multispectral methods is that the large number of detectors and measurements reduces the noise and averages errors inherent in measurements. This method requires more complex hardware, however, and the use of multiple detectors increases the data collection and processing requirements. One shortcoming of the method is that it may not provide an identifiably unique solution unless a large number of measurements is made.
Although the routines described herein do not include any specific capability directly applicable to analyzing multi-spectral measurements, the routines along with the general capabilities of Microsoft Excel® can be used to implement this method. Example 12, located in the Example section demonstrates one variation of this method.

### Wide-Band Detectors

A wide-band detector is one that covers a range of wavelengths and the range is sufficiently broad that spectral intensity cannot be assumed to be constant over the wavelength band. Additionally, for certain detectors, the sensitivity of the detector is not constant with wavelength, and, therefore, the detector signal will represent a weighted average across the waveband. The averaged signal can be represented mathematically as an integral over the waveband consisting of the product of the Planck equation, the detector response function, and, for a non-blackbody, the spectral emissivity.

For the given true surface temperature $T$, the equivalent blackbody temperatures $T_{\lambda_i}$ that a given detector should measure can be calculated by solving equation (42).

$$
\int_{\lambda_{il}}^{\lambda_{iu}} \varepsilon_{\lambda_i}(\lambda) D_{\lambda}(\lambda) i_{b,\lambda}(T) \, d\lambda = \int_{\lambda_{il}}^{\lambda_{iu}} D_{\lambda}(\lambda) i_{b,\lambda}(T_{\lambda_i}) \, d\lambda
$$

(42)

The equivalent blackbody temperature provides the same detector signal from a blackbody source at the equivalent blackbody temperature as the real surface at the given true temperature.

Following the same concept as the ratio method for a set of narrow-band detectors, an equivalent method can be employed, except that measurements at the discrete wavelengths are replaced by integrated measurements across the bands, as seen in equation (43).

$$
\bar{\varepsilon}_r = \frac{\bar{\varepsilon}_1}{\bar{\varepsilon}_2} = \frac{\int_{\lambda_{il}}^{\lambda_{iu}} D_{\lambda}(\lambda) i_{b,\lambda}(\lambda, T_{\lambda_1}) \, d\lambda}{\int_{\lambda_{il}}^{\lambda_{iu}} D_{\lambda}(\lambda) i_{b,\lambda}(\lambda, T_{\lambda_2}) \, d\lambda} \cdot \frac{\int_{\lambda_{il}}^{\lambda_{iu}} D_{\lambda}(\lambda) i_{b,\lambda}(\lambda, T) \, d\lambda}{\int_{\lambda_{il}}^{\lambda_{iu}} D_{\lambda}(\lambda) i_{b,\lambda}(\lambda, T) \, d\lambda}
$$

(43)

Here, $\bar{\varepsilon}_r$ is a weighted average emissivity ratio. Again, since the response of most detectors is not uniform across the band, the integrals include the detector response function $D_{\lambda}(\lambda)$, which accounts for this variation. Similar to the wavelength ratio for discrete wavelengths, equation (42) is solved by iteration for the true temperature.

The sensitivity of the emissivity ratio to the true surface temperature is therefore as shown in equation (44):

$$
\frac{d \ln \bar{\varepsilon}_r}{d \ln T} = T \frac{\int_{\lambda_{il}}^{\lambda_{iu}} D_{\lambda}(\lambda) i_{b,\lambda}(\lambda, T_{\lambda_1}) \, d\lambda}{\int_{\lambda_{il}}^{\lambda_{iu}} D_{\lambda}(\lambda) i_{b,\lambda}(\lambda, T_{\lambda_2}) \, d\lambda} \cdot \left[ \int_{\lambda_{il}}^{\lambda_{iu}} D_{\lambda}(\lambda) i_{b,\lambda} \, d\lambda \cdot \frac{\int_{\lambda_{il}}^{\lambda_{iu}} D_{\lambda}(\lambda) \, d\lambda}{\int_{\lambda_{il}}^{\lambda_{iu}} D_{\lambda}(\lambda) \, d\lambda} \right]^2
$$

(44)

and the sensitivity of the emissivity ratio to the first equivalent blackbody temperature is as shown in equation (45).
\[
\frac{d \ln \bar{\varepsilon}_r}{d \ln T_{\lambda_1}} = T_{\lambda_1} \frac{\int_{\lambda_{\text{hl}}}^{\lambda_{\text{hi}}} D_1(\lambda) \, di_{b,\lambda}(T_{\lambda_1}) / dT_1 \, d\lambda}{\int_{\lambda_{\text{hl}}}^{\lambda_{\text{hi}}} D_1(\lambda) \, i_{b,\lambda}(T_{\lambda_1}) \, d\lambda} \tag{45}
\]

Alternatively, wideband measurements are sometimes converted to equivalent narrow-band measurements assuming that the discrete wavelength is selected at some intermediate point (usually the mean wavelength) in the band. The determination of the equivalent surface temperature and emissivities then reduces to solving an equivalent form of equation (31), seen here as equation (46):

\[
\bar{\varepsilon}_r = \left( \frac{e^{-c_2/\lambda_2 T_{\lambda_2}} - 1}{e^{-c_2/\lambda_2 T} - 1} \right) \left( \frac{e^{-c_2/\lambda_1 T} - 1}{e^{-c_2/\lambda_1 T_{\lambda_1}} - 1} \right)
\tag{46}
\]

where \( \lambda_1 \) and \( \lambda_2 \) are some equivalent wavelengths.

Note that the accuracy of this method can be highly variable and especially inaccurate if the detector bandwidths are wide or are widely separated in wavelength. The derived emissivities and surface temperatures can result in an ambiguous interpretation for the following reasons: 1) the detector response function varies with detector type; 2) the detector response function is often asymmetrical around the equivalent wavelength; or 3) the Planck equation is also asymmetric around the equivalent wavelength. The asymmetrical shape of the detector response and Planck function combined with a spectrally non-uniform emissivity can result in distortions that cannot be corrected through calibrations. For these reasons, the use of wideband detectors is highly discouraged unless there are other more compelling reasons for them to be used. Example 13 in the Examples portion of the document will demonstrate this point.

**VBA Routine Summaries**

The following section contains detailed descriptions of the individual blackbody function library functions and the calling conventions. For each routine, a description is given of the calling parameters and their type. The return value is also described. Note that each function only returns one value.

For all functions, a value of zero is returned if there is a calculation error or if one of the parameters is out of range. Typically, the input values for all temperatures, spectral intensities, and emissivities must be greater than zero.

**Basic Single Wavelength Functions**

**Function** \( \text{bb}_\text{ibl}(\lambda \text{ As Double, temp As Double) As Double} \)

Calculate the blackbody emissive intensity \( i_{b,\lambda}(\lambda,T) \) at a given temperature \( T \) and wavelength \( \lambda \) (eq. (1)).

\[
i_{b,\lambda}(\lambda,T) = \frac{1}{\lambda^5} \cdot \frac{C_1}{(e^{c_2/\lambda T} - 1)}
\tag{1}
\]
Description of Parameters:

- \textit{lambda}  \quad \textit{wavelength} \, \lambda, \, \mu m
- \textit{temp}  \quad \textit{temperature} \, T, \, K

Returns:

Monochromatic blackbody emissive intensity for a given wavelength and temperature \( i_{b,\lambda}(\lambda, T) \) in W/m\(^2\)-sr-\(\mu m\).

\textbf{Function bb_tibl(lambda As Double, ibl As Double) As Double}

Calculate the equivalent blackbody temperature \( T \) at a specified wavelength \( \lambda \) for a given monochromatic blackbody emissive intensity \( i_{b,\lambda} \) (eq. (2)).

\[ T = \frac{C_2}{\lambda} \cdot \frac{1}{\ln \left( \frac{C_1}{\lambda^5 i_{b,\lambda}} + 1 \right)} \]  

(2)

Description of Parameters:

- \textit{lambda}  \quad \textit{wavelength} \, \lambda, \, \mu m
- \textit{ibl}  \quad \textit{blackbody emissive intensity} \, i_{b,\lambda}, \, W/m^2\text{-sr}\cdot\mu m

Returns:

Equivalent blackbody temperature \( T \) in K.

\textbf{Function bb_dibldt(lambda As Double, temp As Double) As Double}

Calculate the derivative of the blackbody emissive intensity with respect to temperature \( di_{b,\lambda}/dT \) at a given temperature \( T \) and wavelength \( \lambda \) (eq. (6)).

\[ \frac{di_{b,\lambda}}{dT} = \frac{C_2}{\lambda^6 T^2} \cdot \frac{C_1 e^{C_2/\lambda T}}{(e^{C_2/\lambda T} - 1)^2} \]  

(6)

Description of Parameters:

- \textit{lambda}  \quad \textit{wavelength} \, \lambda, \, \mu m
- \textit{temp}  \quad \textit{temperature} \, T, \, K

Returns:

The derivative of the monochromatic blackbody emissive intensity with temperature \( di_{b,\lambda}/dT \) in W/m\(^2\)-sr-\(\mu m\)-K.
Function bb_dlnibldInt(lambda As Double, temp As Double) As Double

Calculate the derivative of the logarithm of the blackbody emissive intensity with respect to the logarithm of temperature $d \ln i_{b,\lambda}/d \ln T$ at a given temperature $T$ and wavelength $\lambda$ (eq. (8)).

$$\frac{d \ln i_{b,\lambda}}{d \ln T} = \frac{C_2}{\lambda T} \cdot \frac{e^{C_2/\lambda T}}{(e^{C_2/\lambda T} - 1)}$$ \hspace{1cm} (8)

Description of Parameters:
- $\text{lambda}$: wavelength $\lambda$, $\mu$m
- $\text{temp}$: temperature $T$, K

Returns:

The derivative of the logarithm of the monochromatic blackbody emissive intensity with the logarithm of temperature $d \ln i_{b,\lambda}/d \ln T$.

Function bb_dibldl(lambda As Double, temp As Double) As Double

Calculate the derivative of the blackbody emissive intensity with respect to wavelength $di_{b,\lambda}/d\lambda$ at a given temperature $T$ and wavelength $\lambda$ (eq. (7)).

$$\frac{di_{b,\lambda}}{d\lambda} = \frac{1}{\lambda^6} \cdot \frac{C_1}{\lambda T} \cdot \left( \frac{C_2}{\lambda T} \cdot \frac{e^{C_2/\lambda T}}{(e^{C_2/\lambda T} - 1)} - 5 \right)$$ \hspace{1cm} (7)

Description of Parameters:
- $\text{lambda}$: wavelength $\lambda$, $\mu$m
- $\text{temp}$: temperature $T$, K

Returns:

The derivative of the monochromatic blackbody emissive intensity with wavelength $di_{b,\lambda}/d \ln \lambda$ in W/m$^2$-sr-$\mu$m$^2$.

Function bb_dlnibldlnl(lambda As Double, temp As Double) As Double

Calculate the derivative of the logarithm of the blackbody emissive intensity with respect to the logarithm of wavelength $d \ln i_{b,\lambda}/d \ln \lambda$ at a given temperature $T$ and wavelength $\lambda$ (eq. (9)).

$$\frac{d \ln i_{b,\lambda}}{d \ln \lambda} = \frac{C_2}{\lambda T} \cdot \frac{e^{C_2/\lambda T}}{(e^{C_2/\lambda T} - 1)} - 5$$ \hspace{1cm} (9)

Description of Parameters:
- $\text{lambda}$: wavelength $\lambda$, $\mu$m
- $\text{temp}$: temperature $T$, K
Returns:

The derivative of the logarithm of the monochromatic blackbody emissive intensity with the logarithm of the wavelength $d \ln i_{b, \lambda} / d \ln \lambda$.

**Function bb_d2ibldt2(lambda As Double, temp As Double) As Double**

Calculate the second derivative of the blackbody emissive intensity with respect to temperature $d^2 i_{b, \lambda} / dT^2$ at a given temperature $T$ and wavelength $\lambda$ (eq. (10)).

$$
\frac{d^2 i_{b, \lambda}}{dT^2} = \frac{C_2}{\lambda^6 T^3} \cdot \frac{C_1 e^{C_2/\lambda T}}{(e^{C_2/\lambda T} - 1)^2} \left[ \frac{C_2}{\lambda T} \left( \frac{2 e^{C_2/\lambda T}}{(e^{C_2/\lambda T} - 1)} - 1 \right) - 2 \right] \tag{10}
$$

**Description of Parameters:**

- **lambda**: wavelength $\lambda$, $\mu$m
- **temp**: temperature $T$, K

Returns:

The second derivative of the monochromatic blackbody emissive intensity with temperature $d^2 i_{b, \lambda} / dT^2$ in W/m²-sr-$\mu$m-K².

**Function bb_d2ibldl2(lambda As Double, temp As Double) As Double**

Calculate the second derivative with respect to wavelength of the blackbody emissive intensity $d^2 i_{b, \lambda} / d\lambda^2$ at a given temperature $T$ and wavelength $\lambda$ (eq. (11)).

$$
\frac{d^2 i_{b, \lambda}}{d\lambda^2} = \frac{C_1}{\lambda^7} \cdot \frac{1}{(e^{C_2/\lambda T} - 1)} \left[ \frac{C_2}{\lambda T} \cdot \frac{e^{C_2/\lambda T}}{(e^{C_2/\lambda T} - 1)} \cdot \frac{2 C_2}{\lambda T} \cdot \frac{e^{C_2/\lambda T}}{(e^{C_2/\lambda T} - 1)} - 12 - \frac{C_2}{\lambda T} \right] + 30 \tag{11}
$$

**Description of Parameters:**

- **lambda**: wavelength $\lambda$, $\mu$m
- **temp**: temperature $T$, K

Returns:

The second derivative of the monochromatic blackbody emissive intensity with wavelength in $d^2 i_{b, \lambda} / d\lambda^2$ in W/m²-sr-$\mu$m³.

**Function bb_ib(temp As Double) As Double**

Calculate the total blackbody emissive intensity $i_b$ in at a specified temperature $T$ (eq. (20)).

$$
i_b = \frac{\sigma}{\pi} T^4 \tag{20}
$$
Description of Parameters:

\[ \text{temp} \quad \text{temperature} \ T, \ K \]

Returns:

The total blackbody emissive intensity \( i_b \) in W/m\(^2\)-sr.

**Function bb\_eb(temp As Double) As Double**

Calculate the total blackbody emissive power \( e_b \) in at a specified temperature \( T \) (eq. (22)).

\[
e_b = \sigma T^4
\]  
(22)

Description of Parameters:

\[ \text{temp} \quad \text{temperature} \ T, \ K \]

Returns:

The total blackbody emissive power \( e_b \) in W/m\(^2\).

**Function bb\_imax(temp As Double)**

Calculate the wavelength at the maximum spectral intensity \( \lambda_{max} \) for a given temperature \( T \) using Wien’s displacement law where (eq. (3)).

\[
T \lambda_{max} = C_3 = 2897.77 \ \mu \text{m-K}
\]  
(3)

Description of Parameters:

\[ \text{temp} \quad \text{temperature} \ T, \ K \]

Returns:

Wavelength at maximum spectral intensity \( \lambda_{max} \) in \( \mu \text{m} \).

**Function bb\_imax(temp As Double)**

Calculate the maximum spectral intensity \( i_{b, \lambda_{max}} \) for a given temperature \( T \). The maximum spectral intensity occurs at the wavelength \( \lambda_{max} \) according to Wien’s displacement law where (eq. (3)):

\[
T \lambda_{max} = C_3
\]  
(3)

and the maximum spectral intensity is therefore (eq. (4)).
\[
i_{b,\lambda_{\text{max}}} = \frac{T^5}{C_3^5} \cdot \frac{C_1}{(e^{C_2/C_3} - 1)} = C_4 T^5
\]  

(4)

Description of Parameters:

\textit{temp} \quad \textit{temperature} \ T, \ K

Returns:

Maximum spectral intensity \( i_{b,\lambda_{\text{max}}} \) in W/m\(^2\)-sr-\(\mu\)m.

**Function bb\_tmax(ibl As Double)**

Calculate the temperature \( T \) at the maximum spectral intensity. The maximum spectral intensity occurs at the wavelength \( \lambda_{\text{max}} \) according to Wien’s displacement law where (eq. (3)):

\[
T\lambda_{\text{max}} = C_3
\]

(3)

and the maximum spectral intensity is therefore (eq. (4)).

\[
i_{b,\lambda_{\text{max}}} = \frac{T^5}{C_3^5} \cdot \frac{C_1}{(e^{C_2/C_3} - 1)} = C_4 T^5
\]

(4)

The temperature at the maximum spectral intensity is (eq. (47)).

\[
T = \left( \frac{i_{b,\lambda_{\text{max}}}}{C_4} \right)^{1/5}
\]

(47)

Description of Parameters:

\textit{ibl} \quad \textit{maximum spectral intensity at the given temperature} \( i_{b,\lambda_{\text{max}}} \) in W/m\(^2\)-sr-\(\mu\)m

Returns:

Temperature at the maximum spectral intensity \( T \) in K.

**Spectral Method Functions**

**Function bb\_tsi(lambda As Double, ilambda As Double, emiss As Double) As Double**

Calculate the true surface temperature \( T \) for a non-black surface given the measured spectral intensity \( i_\lambda \) at a specified wavelength and the spectral emissivity (eq. (23)):

\[
T = \frac{C_2}{\lambda} \cdot \frac{1}{\left( \ln \frac{\varepsilon \lambda C_1}{\lambda^5 i_\lambda} + 1 \right)}
\]

(23)
Description of Parameters:

- \( \lambda \): Wavelength, \( \mu m \)
- \( \lambda \): Measured spectral intensity, \( W/m^2\cdot sr\cdot \mu m \)
- \( \varepsilon \): Spectral emissivity, \( \varepsilon \)

Returns:

The true surface temperature \( T \) in K.

**Function bb_tli(lambda As Double, ibl As Double, emiss As Double) As Double**

Calculate the equivalent blackbody temperature \( T_\lambda \) at a specified wavelength \( \lambda \) for a non-blackbody given the blackbody spectral intensity at the true surface temperature \( i_{b,\lambda} \) and the spectral emissivity \( \varepsilon_{\lambda} \) (eq. (48)).

\[
T_\lambda = \frac{C_2}{\lambda} \cdot \frac{1}{\left( \ln \left( \frac{C_1}{\lambda^5 \varepsilon_{\lambda} i_{b,\lambda}} + 1 \right) \right)}
\]  

(48)

Description of Parameters:

- \( \lambda \): Wavelength, \( \mu m \)
- \( i_{b,\lambda} \): Blackbody spectral intensity at true surface temperature \( i_{b,\lambda} \), \( W/m^2\cdot sr\cdot \mu m \)
- \( \varepsilon_{\lambda} \): Spectral emissivity, \( \varepsilon_{\lambda} \)

Returns:

The effective blackbody temperature at a specific wavelength \( T_\lambda \) in K.

**Function bb_tst(lambda As Double, tlambda As Double, emiss As Double) As Double**

Calculate the true temperature \( T \) for a non-blackbody given the equivalent blackbody temperature \( T_\lambda \) at a specified wavelength \( \lambda \) and the spectral emissivity \( \varepsilon_{\lambda} \) (eq. (24)).

\[
T = \frac{C_2}{\lambda} \cdot \frac{1}{\ln \left( \varepsilon_{\lambda} (e^{C_2/\lambda T_\lambda} - 1) + 1 \right)}
\]  

(24)

Description of Parameters:

- \( \lambda \): Wavelength, \( \mu m \)
- \( T_\lambda \): Equivalent blackbody temperature at the given wavelength \( T_\lambda \), K
- \( \varepsilon_{\lambda} \): Spectral emissivity, \( \varepsilon_{\lambda} \)

Returns:

The true surface temperature \( T \) in K.
Function bb_tstw(lambda As Double, tlambda As Double, emiss As Double) As Double

Calculate the true temperature $T$ for a non-blackbody given the equivalent blackbody temperature $T_\lambda$ at a specified wavelength $\lambda$ and the spectral emissivity $\varepsilon_\lambda$ assuming Wien’s approximation is applicable (eq. (25)).

$$\frac{1}{T} = \frac{1}{T_\lambda} + \frac{\lambda}{C_2} \ln \varepsilon_\lambda$$  \hspace{1cm} (25)

This routine also can be used to calculate the true temperature using the ratio method according to equation (34):

$$\frac{1}{T} = \frac{1}{T_r} + \frac{\Lambda}{C_2} \ln \varepsilon_r$$  \hspace{1cm} (34)

where $T_r$ is the ratio temperature given by equation (35):

$$\frac{1}{T_r} = \Lambda \left[ \frac{1}{\lambda_1 T_\lambda} - \frac{1}{\lambda_2 T_{\lambda_2}} \right]$$  \hspace{1cm} (35)

and $\Lambda$ is an effective wavelength, shown in equation (36).

$$\Lambda = \frac{\lambda_1 \lambda_2}{\lambda_2 - \lambda_1}$$  \hspace{1cm} (36)

Description of Parameters:

- lambda: wavelength $\lambda$ or effective wavelength $\Lambda$, $\mu$m
- tlambda: equivalent blackbody temperature at the given wavelength $T_\lambda$ or ratio temperature $T_r$, K
- emiss: spectral emissivity $\varepsilon_r$

Returns:

The true temperature $T$ in K.

Function bb_tlt(lambda As Double, temp As Double, emiss As Double) As Double

Calculate the equivalent blackbody temperature $T_\lambda$ at a specified wavelength $\lambda$ for a non-blackbody given the true temperature $T$ and the spectral emissivity $\varepsilon_\lambda$ (eq. (49)).

$$T_\lambda = \frac{C_2}{\lambda} \cdot \frac{1}{\ln[1/\varepsilon_\lambda(e^{C_2/\lambda T} - 1) + 1]}$$  \hspace{1cm} (49)
Description of Parameters:

- \texttt{lambda} \quad \text{wavelength} \; \lambda, \; \mu m
- \texttt{temp} \quad \text{true temperature} \; T, \; K
- \texttt{emiss} \quad \text{spectral emissivity} \; \varepsilon_\lambda

Returns:

The equivalent blackbody temperature $T_\lambda$ in K.

**Function bb\_dlntdlne(lambda As Double, temp As Double) As Double**

Calculate the sensitivity of the true temperature $T$ with the spectral emissivity $\varepsilon_\lambda$. The sensitivity is expressed as the derivative of the logarithm of the true temperature with respect to the logarithm of the spectral emissivity and is expressed by equation (26).

\[
\frac{d \ln T}{d \ln \varepsilon_\lambda} = -\frac{\lambda T}{C_2} \cdot \left( e^{C_2/\lambda T} - 1 \right)
\]  

(26)

Description of Parameters:

- \texttt{lambda} \quad \text{wavelength} \; \lambda, \; \mu m
- \texttt{tlambda} \quad \text{equivalent blackbody temperature at the given wavelength} \; T, \; K

Returns:

The derivative of the logarithm of the equivalent blackbody temperature with the logarithm of the spectral emissivity $d \ln T_\lambda / d \ln \varepsilon_\lambda$.

**Function bb\_dlntltdlne(lambda As Double, tlambda As Double) As Double**

Calculate the sensitivity of the equivalent blackbody temperature $T_\lambda$ with the spectral emissivity $\varepsilon_\lambda$. The sensitivity is expressed as the derivative of the logarithm of the true temperature with respect to the logarithm of the spectral emissivity and is expressed by equation (50):

\[
\frac{d \ln T_\lambda}{d \ln \varepsilon_\lambda} = \frac{\lambda T_\lambda}{C_2} \cdot \left( e^{C_2/\lambda T_\lambda} - 1 \right)
\]  

(50)

Description of Parameters:

- \texttt{lambda} \quad \text{wavelength} \; \lambda, \; \mu m
- \texttt{tlambda} \quad \text{equivalent blackbody temperature at the given wavelength} \; T, \; K

Returns:

The derivative of the logarithm of the equivalent blackbody temperature with the logarithm of the spectral emissivity $d \ln T_\lambda / d \ln \varepsilon_\lambda$. 
Function bb_dlnTdInt(temp As Double, tlambda As Double, emiss As Double) As Double

Calculate the sensitivity of the true temperature $T$ to the equivalent blackbody temperature $T_{\lambda}$. The sensitivity is expressed as the derivative of the logarithm of the true temperature with respect to the logarithm of the equivalent blackbody temperature as shown in equation (31):

$$
\frac{d \ln T}{d \ln T_{\lambda}} = \frac{\epsilon_{\lambda}}{T_{\lambda}} \frac{C_2}{T} \left( \frac{1}{T_{\lambda}} - 1 \right)
$$

(31)

Description of Parameters:

- **temp**: true temperature $T$, K
- **tlambda**: equivalent blackbody temperature at the given wavelength $T_{\lambda}$, K
- **emiss**: spectral emissivity $\epsilon_{\lambda}$

Returns:

The derivative of the logarithm of the true temperature with respect to the logarithm of the equivalent blackbody temperature $d \ln T / d \ln T_{\lambda}$.

Function bb_dlnldlnT(temp As Double, tlambda As Double, emiss As Double) As Double

Calculate the sensitivity of the equivalent blackbody temperature $T_{\lambda}$ with respect to the true temperature $T$. The sensitivity is expressed as the derivative of the logarithm equivalent blackbody temperature with respect to the logarithm of the true temperature (eq. (51)):

$$
\frac{d \ln T_{\lambda}}{d \ln T} = \frac{1}{\epsilon_{\lambda}} \frac{C_2}{T} \left( \frac{1}{T_{\lambda}} - 1 \right)
$$

(51)

Description of Parameters:

- **temp**: true temperature $T$, K
- **tlambda**: equivalent blackbody temperature at the given wavelength $T_{\lambda}$, K
- **emiss**: spectral emissivity $\epsilon_{\lambda}$

Returns:

The derivative of the logarithm of the equivalent blackbody temperature with respect to the logarithm of the true temperature $d \ln T_{\lambda} / d \ln T$.

Function bb_emiss(lambda As Double, tlambda As Double, temp As Double) as Double

Calculate the spectral emissivity at a specified wavelength $\epsilon_{\lambda}$ given the equivalent blackbody temperature and the true temperature $T$ (eq. (28)):

$$
\epsilon_{\lambda} = \frac{e^{C_2 / \lambda T} - 1}{(e^{C_2 / \lambda T_{\lambda}} - 1)}
$$

(28)
Description of Parameters:

\[
\begin{align*}
\text{lambda} & : \text{wavelength } \lambda, \mu m \\
\text{tlambda} & : \text{equivalent blackbody temperature at the given wavelength } T_\lambda, K \\
\text{temp} & : \text{true temperature } T, K
\end{align*}
\]

Returns:

The spectral emissivity at the given wavelength \( \varepsilon_\lambda \).

**Function bb_dlne2dlne1(lambda1 As Double, lambda2 As Double, temp As Double) As Double**

Calculate the sensitivity of the emissivity at a given wavelength \( \varepsilon_2 \) with respect to the uncertainty in emissivity at another wavelength \( \varepsilon_1 \) which has been used to determine the true temperature \( T \). The sensitivity is expressed as the derivative of the logarithm of the emissivity at the second wavelength with respect to the logarithm of the emissivity at the first wavelength (eq. (29)).

\[
\frac{d \ln \varepsilon_2}{d \ln \varepsilon_1} = \frac{\varepsilon_1}{\varepsilon_2} \frac{d \varepsilon_2}{d \varepsilon_1} = \frac{\lambda_1}{\lambda_2} \left( \frac{e^{-c_2/\lambda_1T} - 1}{e^{-c_2/\lambda_2T} - 1} \right)
\]

(29)

If Wien's approximation is appropriate for both wavelengths, then equation (31) reduces to equation (30).

\[
\frac{d \ln \varepsilon_2}{d \ln \varepsilon_1} = \frac{\varepsilon_1}{\varepsilon_2} \frac{d \varepsilon_2}{d \varepsilon_1} = \frac{\lambda_1}{\lambda_2}
\]

(30)

Description of Parameters:

\[
\begin{align*}
\text{lambda1} & : \text{first detector wavelength } \lambda_1, \mu m \\
\text{lambda2} & : \text{second detector wavelength } \lambda_2, \mu m \\
\text{temp} & : \text{true temperature } T, K
\end{align*}
\]

Returns:

The sensitivity of emissivity 2 \( \varepsilon_2 \) with respect to emissivity 1 \( \varepsilon_1 \) or \( d \ln \varepsilon_2/d \ln \varepsilon_1 \).

**Ratio Method Functions**

**Function bb_erlam(lambda1 As Double, lambda2 As Double) As Double**

Calculate the effective wavelength \( \Lambda \) for a two-band radiation thermometer given the wavelengths of the two detectors \( \lambda_1 \) and \( \lambda_2 \) (eq. (36)).

\[
\Lambda = \frac{\lambda_1 \lambda_2}{\lambda_2 - \lambda_1}
\]

(36)
Note that it is assumed that the second wavelength is greater than the first which will give positive values for the effective wavelength. If the two wavelengths are the same, then the result is undefined and the function returns zero.

Description of Parameters:

\( \text{lambda1} \)  
first wavelength \( \lambda_1, \mu m \)

\( \text{lambda2} \)  
second wavelength \( \lambda_2, \mu m \)

Returns:

The effective wavelength \( \Lambda \) in \( \mu m \).

**Function bb_ertemp**(*lambda1* As Double, *lambda2* As Double, *temp1* As Double, *temp2* As Double) as Double

Calculate the ratio temperature \( T_r \) for a two-band radiation thermometer given the two equivalent blackbody temperatures \( T_{\lambda_1} \) and \( T_{\lambda_2} \) at the two detector wavelengths \( \lambda_1 \) and \( \lambda_2 \).

The definition for the ratio temperature \( T_r \) is seen in equation (35).

\[
\frac{1}{T_r} = \Lambda \left[ \frac{1}{\lambda_1 T_{\lambda_1}} - \frac{1}{\lambda_2 T_{\lambda_2}} \right] \quad (35)
\]

Description of Parameters:

\( \text{lambda1} \)  
first wavelength \( \lambda_1, \mu m \)

\( \text{lambda2} \)  
second wavelength \( \lambda_2, \mu m \)

\( \text{temp1} \)  
equivalent blackbody temperature at first wavelength \( T_{\lambda_1} \), K

\( \text{temp2} \)  
equivalent blackbody temperature at second wavelength \( T_{\lambda_2} \), K

Returns:

The effective ratio temperature \( T_r \) in K.


Calculate the true temperature \( T \) for a two-band radiation thermometer given the equivalent blackbody temperatures \( T_{\lambda_1} \) and \( T_{\lambda_2} \) at the two detector wavelengths \( \lambda_1, \lambda_2 \) and an emissivity ratio \( \varepsilon_r \).

The true temperature satisfies equation (32):

\[
\frac{(e^{C_2/\lambda_2 T_{\lambda_2}} - 1)}{(e^{C_2/\lambda_2 T} - 1)} \cdot \frac{(e^{C_2/\lambda_1 T_{\lambda_1}} - 1)}{(e^{C_2/\lambda_1 T} - 1)} - \varepsilon_r = 0 
\]

(32)
The solution cannot be represented analytically but can only be solved numerically by iteration. The true temperature is determined by a Newton-Raphson iteration scheme.

Note that if the solution returns zero, then the iteration has probably failed. Using a better value for the starting guess, increasing the maximum number of iterations, or increasing the tolerance will most likely result in convergence.

Description of Parameters:

\n
\begin{itemize}
\item \texttt{lambda1} \quad \textbf{first detector wavelength} \( \lambda_1, \ \mu m \)
\item \texttt{lambda2} \quad \textbf{second detector wavelength} \( \lambda_2, \ \mu m \)
\item \texttt{temp1} \quad \textbf{equivalent blackbody temperature of first detector} \( T_{\lambda_1} \), \ K
\item \texttt{temp2} \quad \textbf{equivalent blackbody temperature of second detector} \( T_{\lambda_2} \), \ K
\item \texttt{emissr} \quad \textbf{emissivity ratio} \( \epsilon_r \)
\item \texttt{temp0} \quad \textbf{starting guess on true temperature} \( K \)
\item \texttt{tol} \quad \textbf{tolerance on temperature}
\item \( > 0 \) \textbf{relative tolerance}
\item \( < 0 \) \textbf{absolute tolerance} \( K \)
\item \texttt{nmax} \quad \textbf{maximum number of iterations}
\end{itemize}

Returns:

The true temperature \( T \) in K.

\textbf{Function bb_emissr(lambda1 As Double, lambda2 As Double, temp1 As Double, temp2 As Double, temp As Double) As Double}

Calculate the effective emissivity ratio \( \epsilon_r \) from a two-band radiation thermometer given the equivalent blackbody temperatures \( T_{\lambda_1} \) and \( T_{\lambda_2} \) at the two detector wavelengths \( \lambda_1, \lambda_2 \) and the true temperature \( T \) (eq. (52) derived from eq. (32)):

\[
\epsilon_r = \left( \frac{e^{C_2/\lambda_2 T_{\lambda_2}} - 1}{e^{C_2/\lambda_2 T} - 1} \right) \left( \frac{e^{C_2/\lambda_1 T} - 1}{e^{C_2/\lambda_1 T_{\lambda_1}} - 1} \right)
\]  \hspace{1cm} (52)

Description of Parameters:

\begin{itemize}
\item \texttt{lambda1} \quad \textbf{first detector wavelength} \( \lambda_1, \ \mu m \)
\item \texttt{lambda2} \quad \textbf{second detector wavelength} \( \lambda_2, \ \mu m \)
\item \texttt{temp1} \quad \textbf{equivalent blackbody temperature of first detector} \( T_{\lambda_1} \), \ K
\item \texttt{temp2} \quad \textbf{equivalent blackbody temperature of second detector} \( T_{\lambda_2} \), \ K
\item \texttt{temp} \quad \textbf{true temperature} \( T \), \ K
\end{itemize}

Returns:

The emissivity ratio \( \epsilon_r \) from the two-band temperatures.
Function bb_dlnerdInt(lambda1 As Double, lambda2 As Double, temp As Double) As Double

Calculate the derivative of the logarithm of the emissivity ratio $\varepsilon_r$ with respect to the logarithm of the true temperature $T$ for a two-band radiation thermometer given the temperatures at two wavelengths and true temperature (eq. (37)):

$$\frac{d \ln \varepsilon_r}{d \ln T} = \frac{1}{T} \left( \frac{C_2}{C_2/\lambda_2 T} - 1 \right) \left[ \frac{C_2}{\lambda_2^2} \left( e^{C_2/\lambda_2 T} - 1 \right) - \frac{C_2}{\lambda_1 T} \left( e^{C_2/\lambda_1 T} - 1 \right)^2 \right]$$

(37)

Description of Parameters:

- lambda1: first detector wavelength $\lambda_1$, $\mu m$
- lambda2: second detector wavelength $\lambda_2$, $\mu m$
- temp: true temperature $T$, K

Returns:

The derivative of the logarithm of the emissivity ratio with respect to the logarithm of the true temperature $d \ln \varepsilon_r / d \ln T$.

Function bb_dlnerdInt1(lambda1 As Double, temp1 As Double) As Double

Calculate the derivative of the logarithm of the emissivity ratio $\varepsilon_r$ with respect to the logarithm of the equivalent blackbody temperature of the first detector $T_{1}$ for a two-band radiation thermometer given the detector wavelength and equivalent blackbody temperature (eq. (39)):

$$\frac{d \ln \varepsilon_r}{d \ln T_{1}} = \frac{C_2}{\lambda_1 T} \left( e^{C_2/\lambda_1 T_{1}} - 1 \right)$$

(39)

Description of Parameters:

- lambda1: first detector wavelength $\lambda_1$, $\mu m$
- temp1: equivalent blackbody temperature of first detector $T_{1}$, K

Returns:

The derivative of the logarithm of the emissivity ratio with respect to the logarithm of the equivalent blackbody temperature of the first detector $d \ln \varepsilon_r / d \ln T_{1}$.

Function bb_dlntrdInt1(lambda1 As Double, lambda2 As Double, temp1 As Double, temp2 As Double) as Double

Calculate the derivative of the logarithm of the ratio temperature $T_r$ with respect to the logarithm of the equivalent blackbody temperature of the first detector $T_{1}$ for a two-band radiation thermometer as shown in equation (40):
\[
\frac{d \ln T_r}{d \ln T_{\lambda_1}} = \frac{\Lambda}{\lambda_1 T_{\lambda_1}} T_r
\]

where \( T_r \) is defined as such in equation (35):

\[
\frac{1}{T_r} = \Lambda \left[ \frac{1}{\lambda_1 T_{\lambda_1}} - \frac{1}{\lambda_2 T_{\lambda_2}} \right]
\]

and the equivalent wavelength is shown in equation (36).

\[
\Lambda = \frac{\lambda_1 \lambda_2}{\lambda_2 - \lambda_1}
\]

Description of Parameters:

- \textit{lambda1} first detector wavelength \( \lambda_1, \mu m \)
- \textit{lambda2} second detector wavelength \( \lambda_2, \mu m \)
- \textit{temp1} equivalent blackbody temperature of first detector \( T_{\lambda_1}, K \)
- \textit{temp2} equivalent blackbody temperature of second detector \( T_{\lambda_2}, K \)

Returns:

The derivative of the logarithm of the ratio temperature with respect to the logarithm of the first temperature \( d \ln \varepsilon_r/d \ln T_r \).

\textbf{Function bb_dlnrdlnl1(lambda1 As Double, lambda2 As Double) as Double}

Calculate the derivative of the logarithm of the equivalent wavelength \( \Lambda \) with respect to the logarithm of the wavelength of the first detector \( \lambda_1 \) for a two-band radiation thermometer (eq. (41)):

\[
\frac{d \ln \Lambda}{d \ln \lambda_1} = 1 + \frac{\lambda_1}{\lambda_2 - \lambda_1}
\]

in which the equivalent wavelength \( \Lambda \) is defined in equation (36).

\[
\Lambda = \frac{\lambda_1 \lambda_2}{\lambda_2 - \lambda_1}
\]

Description of Parameters:

- \textit{lambda1} first detector wavelength \( \lambda_1, \mu m \)
- \textit{lambda2} second detector wavelength \( \lambda_2, \mu m \)

Returns:
The derivative of the logarithm of the equivalent wavelength with respect to the logarithm of the first detector wavelength $d \ln \Lambda / d \ln \lambda_1$.

**Wide-Band Functions**

**Function iibls(llow As Double, lhigh As Double, temp As Double, tol As Double, nmax As Long) As Double**

Calculate the integrated emissive intensity for a wide or narrow-band using a series solution as described in ref. 3. For a wide-band calculation, the upper and lower wavelengths are specified. For a narrow-band calculation, the same value is input for the upper and lower wavelengths.

For a wide-band, the calculated integral is shown in equation (12):

$$I(\lambda_1, \lambda_u, T) = \int_{\lambda_1}^{\lambda_u} \frac{1}{\lambda^8} \frac{C_1}{e^{C_2/\lambda T} - 1} d\lambda$$

which can be written as equation (53):

$$I(\lambda_1, \lambda_u, T) = \int_{\lambda_1}^{\lambda_u} \frac{C_1}{\lambda^8} \frac{1}{e^{C_2/\lambda T} - 1} d\lambda - \int_{0}^{\lambda_1} \frac{C_1}{\lambda^8} \frac{1}{e^{C_2/\lambda T} - 1} d\lambda$$

or using equation 14 and 15 as equation (54):

$$\sigma T^4 \left( F_{0 \rightarrow \lambda_u T} - F_{0 \rightarrow \lambda_1 T} \right)$$

Now $F_{0 \rightarrow \lambda T}$ can be represented by an integrated series as shown in equation in (13):

$$F_{0 \rightarrow \lambda T} = \frac{15}{\pi^4} \sum_{n=1}^{\infty} \frac{e^{-n\xi}}{n} \left( \xi^3 + 3 \frac{\xi^2}{n} + 6 \frac{\xi}{n^2} + 6 \frac{1}{n^3} \right)$$

where (eq. (14)):

$$\xi = \frac{C_2}{\lambda T}$$

For a narrow-band, where $\lambda_1 = \lambda_2 = \lambda$, no integration is performed, but the spectral radiant intensity at the single wavelength is returned (eq. (1)):

$$i_{b,\lambda}(\lambda, T) = \frac{1}{\lambda^8} \frac{C_1}{(e^{C_2/\lambda T} - 1)}$$
Description of Parameters:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>llow</td>
<td>lower wavelength ( \lambda_l, \mu m )</td>
</tr>
<tr>
<td>lhigh</td>
<td>upper wavelength ( \lambda_u, \mu m )</td>
</tr>
<tr>
<td>temp</td>
<td>Temperature ( T, K )</td>
</tr>
<tr>
<td>tol</td>
<td>tolerance on series term</td>
</tr>
<tr>
<td></td>
<td>&gt; 0 absolute tolerance</td>
</tr>
<tr>
<td></td>
<td>&lt; 0 relative tolerance</td>
</tr>
<tr>
<td>nmax</td>
<td>maximum number of series terms</td>
</tr>
</tbody>
</table>

Note that the integration is carried out until the value of the series term falls below the tolerance value or until the maximum number of terms is reached.

Returns:

The integrated emissive intensity \( I \) in W/m\(^2\)-sr between two wavelengths for a given temperature if different values are given for the upper and lower wavelengths. If the same value is input for the upper and lower wavelength, then the spectral emissive intensity \( i_\lambda \) in W/m\(^2\)-sr-\( \mu m \) is computed.

**Function bb_fiibls(lamt As Double, tol As Double, nmax As Long) As Double**

Calculate the spectral emissive intensity fraction between zero and specified wavelength \( F_{0\rightarrow\lambda T} \) using a series solution as described in ref. 3.

The fraction is calculated using a series solution according to equation (13):

\[
F_{0\rightarrow\lambda T} = \frac{15}{\pi^4} \sum_{n=1}^{\infty} \frac{e^{-n\xi}}{n} \left( \xi^3 + \frac{3\xi^2}{n} + \frac{6\xi}{n^2} + \frac{6}{n^3} \right)
\]

where (eq. (14)):

\[
\xi = \frac{C_2}{\lambda T}
\]

Description of Parameters:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>lamt</td>
<td>product of temperature and wavelength ( \lambda T, \mu m - K )</td>
</tr>
<tr>
<td>tol</td>
<td>tolerance on series term</td>
</tr>
<tr>
<td></td>
<td>&gt; 0 absolute tolerance</td>
</tr>
<tr>
<td></td>
<td>&lt; 0 relative tolerance</td>
</tr>
<tr>
<td>nmax</td>
<td>maximum number of series terms</td>
</tr>
</tbody>
</table>

Note that the integration is carried out until the value of the series term falls below the tolerance value or until the maximum number of terms is reached.

Returns:

The fraction of emissive intensity between zero and a specified wavelength at a given temperature \( F_{0\rightarrow\lambda T} \).
Function bb_iibl(DRange As range, ERange As range, llow As Double, lhigh As Double, temp As Double, tol As Double, maxdbl As Long, flag As Long) As Double

Calculate the total integrated emissive intensity \( I(\lambda_1, \lambda_u, T) \) for a wide- or narrow-width band using a quadrature integration scheme. For a wide-band calculation, the upper and lower wavelengths are specified. For a narrow-band calculation, the same value is input for the upper and lower wavelengths. This routine also allows the spectral distribution to be weighted by a detector response function \( D(\lambda) \) and a spectral emissivity \( \varepsilon_\lambda(\lambda) \). These weighting functions are input in tabular forms as two dimensional Excel® Ranges.

For options 0 through 3, the equation evaluated is the basic Planck equation (eq. (16)).

\[
I_0(\lambda_1, \lambda_u, T) = \int_{\lambda_1}^{\lambda_u} D(\lambda) \varepsilon_\lambda(\lambda) i_{b,\lambda} d\lambda = \int_{\lambda_1}^{\lambda_u} D(\lambda) \varepsilon_\lambda(\lambda) \frac{1}{\lambda^5} \cdot \frac{C_1}{(e^{C_2/\lambda T} - 1)} d\lambda
\]  

(16)

For options 4 through 7, the equation evaluated is the derivative of the Planck equation with temperature (eq. (17)).

\[
I_1(\lambda_1, \lambda_u, T) = \int_{\lambda_1}^{\lambda_u} \frac{d}{dT} D(\lambda) \varepsilon_\lambda(\lambda) \frac{d}{dT} i_{b,\lambda} \lambda d\lambda = \int_{\lambda_1}^{\lambda_u} D(\lambda) \varepsilon_\lambda(\lambda) \frac{e^{C_2/\lambda T}}{\lambda^6 T^2} \cdot \frac{C_1}{(e^{C_2/\lambda T} - 1)^2} d\lambda
\]  

(17)

For options 10 through 13, the equation evaluated is the first moment with respect to wavelength of the Planck equation (eq. (18)).

\[
I_2(\lambda_1, \lambda_u, T) = \int_{\lambda_1}^{\lambda_u} D(\lambda) \varepsilon_\lambda(\lambda) i_{b,\lambda} \lambda d\lambda = \int_{\lambda_1}^{\lambda_u} D(\lambda) \varepsilon_\lambda(\lambda) \frac{1}{\lambda^4} \cdot \frac{C_1}{(e^{C_2/\lambda T} - 1)} d\lambda
\]  

(18)

Finally, for options 14 through 17, the equation (eq. (19)) evaluated is the first moment with respect to wavelength of the derivative of the Planck equation with temperature:

\[
I_3(\lambda_1, \lambda_u, T) = \int_{\lambda_1}^{\lambda_u} D(\lambda) \varepsilon_\lambda(\lambda) \frac{d}{dT} i_{b,\lambda} \lambda d\lambda = \int_{\lambda_1}^{\lambda_u} D(\lambda) \varepsilon_\lambda(\lambda) \frac{C_2}{\lambda^5 T^2} \cdot \frac{C_1 \lambda e^{C_2/\lambda T}}{(e^{C_2/\lambda T} - 1)^2} d\lambda
\]  

(19)

Description of Parameters:

DRange Excel Range object containing the wavelength and detector response function \( D(\lambda) \). The wavelength is contained in the first column and the detector response value is contained in the last column. In the integration, values for the detector response function are interpolated linearly from the table of data. Note that the wavelengths in the table must be in ascending order and duplicate wavelength entries are not allowed.

ERange Excel Range object containing the wavelength and spectral emissivity function \( \varepsilon(\lambda) \). The wavelength is contained in the first column of the range variable and the spectral emissivity is contained in the last column. In the integration, values for the detector response function are interpolated linearly from the table of data. Note that the wavelengths in the table must be in ascending order and duplicate wavelength entries are not allowed.
| **llow** | lower wavelength $\lambda_l$, $\mu$m |
| **lhigh** | upper wavelength $\lambda_u$, $\mu$m |
| **temp** | Temperature $T$, K |
| **tol** | tolerance on integration |
|          | $> 0$ absolute tolerance |
|          | $< 0$ relative tolerance |
| **maxdbl** | maximum number of interval doublings in integration |
| **flag** | flag that determines the integrand function |
|          | $= 0$ Blackbody spectral intensity function only |
|          | $= 1$ Blackbody spectral intensity function multiplied by spectral detector response function |
|          | $= 2$ Blackbody spectral intensity function multiplied by spectral emissivity |
|          | $= 3$ Blackbody spectral intensity function, detector response, and spectral emissivity spectral intensity function multiplied together |
|          | $= 4$ Derivative of blackbody spectral intensity function with temperature only |
|          | $= 5$ Derivative of blackbody spectral intensity function with temperature multiplied by spectral detector response function |
|          | $= 6$ Derivative of blackbody spectral intensity function with temperature multiplied by spectral emissivity |
|          | $= 7$ Derivative of blackbody spectral intensity function with temperature, detector response, and spectral emissivity function multiplied together |
|          | $= 8$ Reserved |
|          | $= 9$ Reserved |
|          | $= 10$ First moment of blackbody spectral intensity function only |
|          | $= 11$ First moment of blackbody spectral intensity function multiplied by spectral detector response function |
|          | $= 12$ First moment of blackbody spectral intensity function multiplied by spectral emissivity |
|          | $= 13$ First moment of blackbody spectral intensity function, detector response, and spectral emissivity function multiplied together |
|          | $= 14$ First moment of derivative of blackbody spectral intensity function with temperature only |
|          | $= 15$ First moment of derivative of blackbody spectral intensity function with temperature multiplied by spectral detector response function |
|          | $= 16$ First moment of derivative of blackbody spectral intensity function with temperature multiplied by spectral emissivity |
|          | $= 17$ First moment of derivative of blackbody spectral intensity function with temperature, detector response, and spectral emissivity function multiplied together |

**Returns:**

The detector and/or emissive weighted average of the integrand which can be: 1) wide-band emissive intensity in W/m²-sr; 2) derivative of the weighted average wide-band emissive intensity with temperature in W/m²-sr-K; 3) first moment with respect to wavelength of the Planck equation in W-µm/m²-sr; or 4) first moment with respect to wavelength of the derivative of the Planck equation with temperature in function in W-µm/m²-sr-K. If different values are given for the upper
and lower wavelengths, the integral is computed. If the same value is input for the upper and lower wavelength, then the value of the integrand at the particular wavelength is computed.

Function `bb_tiiibl(DRange As range, ERange As range, lambda1 As Double, lambda2 As Double, iibln As Double, temp0 As Double, toli As Double, toln As Double, nmaxdbl As Long, nmax As Long) As Double`

Calculate the equivalent blackbody temperature $T_\lambda$ across a waveband for a non-blackbody with a spectral emissivity as a function of wavelength $\varepsilon(\lambda)$ given the total integrated intensity across the waveband $I(\lambda_l, \lambda_u, T)$ and detector response as a function of wavelength $D(\lambda)$. The blackbody temperature is calculated iteratively using a Newton-Raphson iteration scheme that solves equation (55).

$$\int_{\lambda_l}^{\lambda_u} D(\lambda) \varepsilon(\lambda) i_{b,\lambda}(\lambda, T) d\lambda = I(\lambda_l, \lambda_u, T) = \int_{\lambda_l}^{\lambda_u} D(\lambda) i_{b,\lambda}(\lambda, T_\lambda) d\lambda$$

(55)

Note that if the solution returns zero, then the iteration has probably failed. Using a better value for the starting guess, increasing the maximum number of iterations, or increasing the tolerance will most likely result in convergence.

Description of Parameters:

- **DRange** Excel Range object containing the wavelength and detector response function $D(\lambda)$. The wavelength is contained in the first column and the detector response value is contained in the last column. In the integration, values for the detector response function are interpolated linearly from the table of data. Note that the wavelengths in the table must be in ascending order and duplicate wavelength entries are not allowed.

- **ERange** Excel Range object containing the wavelength and spectral emissivity function $\varepsilon(\lambda)$. The wavelength is contained in the first column of the range variable and the spectral emissivity is contained in the last column. In the integration, values for the detector response function are interpolated linearly from the table of data. Note that the wavelengths in the table must be in ascending order and duplicate wavelength entries are not allowed.

- **ilow** lower wavelength $\lambda_l$, $\mu$m

- **ilhigh** upper wavelength $\lambda_u$, $\mu$m

- **iibln** integrated intensity at the true temperature, W/m$^2$

- **temp0** starting guess on equivalent blackbody temperature, K

- **toli**
  - $>0$ relative tolerance
  - $<0$ absolute tolerance, K

- **toln**
  - tolerance on Newton-Raphson temperature iteration
  - $>0$ relative tolerance
  - $<0$ absolute tolerance, K

- **nmaxdbl** maximum number of doublings during integration

- **nmax** maximum number of iterations

Returns:
The equivalent blackbody temperature $T_λ$ in K.

Function `bb_ttiibl(DRange As range, ERange As range, lambda1 As Double, lambda2 As Double, temp As Double, temp0 As Double, toli As Double, toln As Double, nmaxdbl As Long, nmax As Long) As Double`

Calculate the equivalent blackbody temperature $T_λ$ across a waveband for a non-blackbody with an emissivity as a function of wavelength $\varepsilon_λ(λ)$ given the true temperature $T'$ and detector response as a function of wavelength $D(λ)$. The blackbody temperature is calculated iteratively using a Newton-Raphson iteration scheme that solves equation (56).

\[
\int_{λ_l}^{λ_u} D(λ) \varepsilon_λ(λ) i_{b,λ}(λ, T) dλ = \int_{λ_l}^{λ_u} D(λ) i_{b,λ}(λ, T_λ) dλ \tag{56}
\]

Note that if the solution returns zero, then the iteration has probably failed. Using a better value for the starting guess, increasing the maximum number of iterations, or increasing the tolerance will most likely result in convergence.

Description of Parameters:

- **DRange** Excel Range object containing the wavelength and detector response function $D(λ)$. The wavelength is contained in the first column and the detector response value is contained in the last column. In the integration, values for the detector response function are interpolated linearly from the table of data. Note that the wavelengths in the table must be in ascending order and duplicate wavelength entries are not allowed.

- **ERange** Excel Range object containing the wavelength and spectral emissivity function $\varepsilon(λ)$. The wavelength is contained in the first column of the range variable and the spectral emissivity is contained in the last column. In the integration, values for the detector response function are interpolated linearly from the table of data. Note that the wavelengths in the table must be in ascending order and duplicate wavelength entries are not allowed.

- **llow** lower wavelength $λ_l$, μm
- **lhigh** upper wavelength $λ_u$, μm
- **temp** true temperature $T$, K
- **temp0** starting guess on equivalent blackbody temperature, K
- **toli** tolerance on integration
- > 0 relative tolerance
- < 0 absolute tolerance, K
- **toln** tolerance on Newton-Raphson temperature iteration
- > 0 relative tolerance
- < 0 absolute tolerance, K
- **nmaxdbl** maximum number of doublings during integration
- **nmax** maximum number of iterations

Returns:

The equivalent blackbody temperature $T_λ$ in K.
Wide-Band Ratio Functions

Function bb_itratio(DRange1 As range, DRange2 As range, lambda1l As Double, lambda1u As Double, lambda2l As Double, lambda2u As Double, temp1 As Double, temp2 As Double, emissr As Double, temp0 As Double, toli As Double, tolN As Double, nmaxdbl As Long, nmax As Long) As Double

Calculate the true temperature $T$ for a wide, two-band, radiation thermometer given the equivalent blackbody temperatures for the two detectors $T_{\lambda_1}$ and $T_{\lambda_2}$ and a band-averaged emissivity ratio $\bar{\varepsilon}_r$. The true temperature satisfies equation (43):

$$\bar{\varepsilon}_r = \frac{\int_{\lambda_{1l}}^{\lambda_{1u}} D_1(\lambda) i_{b,\lambda}(\lambda, T) \, d\lambda}{\int_{\lambda_{1l}}^{\lambda_{1u}} D_1(\lambda) \, d\lambda} = \frac{\int_{\lambda_{2l}}^{\lambda_{2u}} D_2(\lambda) i_{b,\lambda}(\lambda, T) \, d\lambda}{\int_{\lambda_{2l}}^{\lambda_{2u}} D_2(\lambda) \, d\lambda}$$  \hspace{1cm} (43)

where $D_1(\lambda)$ and $D_2(\lambda)$ are the detector response functions that are functions of wavelength. The true temperature is calculated iteratively using a Newton-Raphson iteration scheme.

Note that if the solution returns zero, then the iteration has probably failed. Using a better value for the starting guess, increasing the maximum number of iterations, or increasing the tolerance will most likely result in convergence.

Description of Parameters:

**DRange** Excel Range object containing the wavelength and detector response function $D(\lambda)$. The wavelength is contained in the first column and the detector response value is contained in the last column. In the integration, values for the detector response function are interpolated linearly from the table of data. Note that the wavelengths in the table must be in ascending order and duplicate wavelength entries are not allowed.

**ERange** Excel Range object containing the wavelength and spectral emissivity function $\varepsilon(\lambda)$. The wavelength is contained in the first column of the range variable and the spectral emissivity is contained in the last column. In the integration, values for the detector response function are interpolated linearly from the table of data. Note that the wavelengths in the table must be in ascending order and duplicate wavelength entries are not allowed.

lambda1l lower wavelength for detector 1 $\lambda_{1l}$, $\mu$m
lambda1u upper wavelength for detector 1 $\lambda_{1u}$, $\mu$m
lambda2l lower wavelength for detector 2 $\lambda_{2l}$, $\mu$m
lambda2u upper wavelength for detector 2 $\lambda_{2u}$, $\mu$m
temp1 equivalent blackbody temperature for detector 1 $T_{\lambda_1}$
temp2 equivalent blackbody temperature for detector 2 $T_{\lambda_2}$
emissr emissivity ratio $\bar{\varepsilon}_r$
temp0 starting guess on true temperature, K
toli tolerance on temperature integration
< 0 absolute tolerance, K
toln  
tolerance on Newton-Raphson integration
  > 0 relative tolerance
  < 0 absolute tolerance, K
nmaxdb  
maximum number of integration doublings
nmax  
maximum number of iterations

Returns:

The true temperature $T$ in K.

Function bb_iemissr(DRange1 As range, DRange2 As range, lambda1l As Double,
lambda1u As Double, lambda2l As Double, lambda2u As Double, temp1 As Double,
temp2 As Double, temp As Double, tol As Double, nmaxdbl As Long) As Double

Calculate the effective emissivity ratio $\bar{\varepsilon}$ from a wide, two-band radiation thermometer given
the equivalent blackbody temperatures for the two detectors $T_{\lambda_1}$ and $T_{\lambda_2}$ and the true
temperature $T$.

The emissivity ratio is calculated from equation (43).

$$
\bar{\varepsilon} = \frac{\varepsilon_1}{\varepsilon_2} = \frac{\int_{\lambda_1l}^{\lambda_1u} D_1(\lambda) \ i_{b,\lambda}(T_{\lambda_1}) \ d\lambda}{\int_{\lambda_1l}^{\lambda_1u} D_1(\lambda) \ i_{b,\lambda}(\lambda) \ d\lambda} \cdot \frac{\int_{\lambda_2l}^{\lambda_2u} D_2(\lambda) \ i_{b,\lambda}(T_{\lambda_2}) \ d\lambda}{\int_{\lambda_2l}^{\lambda_2u} D_2(\lambda) \ i_{b,\lambda}(T) \ d\lambda}
$$

(43)

Description of Parameters:

**DRange1**  
Excel Range object containing the wavelength and detector response function
for detector 1 $D_1(\lambda)$. The wavelength is contained in the first column and the
detector response value is contained in the last column. In the integration,
values for the detector response function are interpolated linearly from the
table of data. Note that the wavelengths in the table must be in ascending order
and duplicate wavelength entries are not allowed.

**DRange2**  
Excel Range object containing the wavelength and detector response function
for detector 2 $D_2(\lambda)$. The wavelength is contained in the first column and the
detector response value is contained in the last column. In the integration,
values for the detector response function are interpolated linearly from the
table of data. Note that the wavelengths in the table must be in ascending order
and duplicate wavelength entries are not allowed.

**lambda1l**  
lower wavelength for detector 1 $\lambda_{1l}$, $\mu$m

**lambda1u**  
upper wavelength for detector 1 $\lambda_{1u}$, $\mu$m

**lambda2l**  
lower wavelength for detector 2 $\lambda_{2l}$, $\mu$m

**lambda2u**  
upper wavelength for detector 2 $\lambda_{2u}$, $\mu$m

**temp1**  
equivalent blackbody temperature for detector 1 $T_{\lambda_1}$

**temp2**  
equivalent blackbody temperature for detector 2 $T_{\lambda_2}$

**temp**  
true temperature $T$, K

**tol**  
tolerance on temperature integration
  > 0 relative tolerance
  < 0 absolute tolerance, K

**nmaxdbl**  
maximum number of doublings
Returns:

The effective emissivity ratio \( \bar{\varepsilon} \).

**Function bb_dlniedlnt**

\[
\text{Calculate the derivative of the logarithm of the emissivity ratio } \bar{\varepsilon} \text{ with respect to the true temperature } T \text{ for a wide, two-band radiation thermometer (eq. (44)):}
\]

\[
\frac{d \ln \bar{\varepsilon}}{d \ln T} = T \frac{\varepsilon_2 \int_{\lambda_{1L}}^{\lambda_{1U}} D_1(\lambda) i_{b,\lambda}(T) d\lambda}{\varepsilon_1 \int_{\lambda_{2L}}^{\lambda_{2U}} D_2(\lambda) i_{b,\lambda}(T) d\lambda}.
\]

\[
\left[ \int_{\lambda_{1L}}^{\lambda_{1U}} D_1(\lambda) i_{b,\lambda}(T) d\lambda \cdot \int_{\lambda_{2L}}^{\lambda_{2U}} D_2(\lambda) i_{b,\lambda}(T) d\lambda \right] \cdot \left[ \frac{\int_{\lambda_{1L}}^{\lambda_{1U}} D_1(\lambda) i_{b,\lambda}(T) d\lambda}{\int_{\lambda_{1L}}^{\lambda_{1U}} D_1(\lambda) i_{b,\lambda}(T) d\lambda} - \frac{\int_{\lambda_{2L}}^{\lambda_{2U}} D_2(\lambda) i_{b,\lambda}(T) d\lambda}{\int_{\lambda_{2L}}^{\lambda_{2U}} D_2(\lambda) i_{b,\lambda}(T) d\lambda} \right]
\]

\[
(44)
\]

**Description of Parameters:**

**DRange1** Excel Range object containing the wavelength and detector response function for detector 1 \( D_1(\lambda) \). The wavelength is contained in the first column and the detector response value is contained in the last column. In the integration, values for the detector response function are interpolated linearly from the table of data. Note that the wavelengths in the table must be in ascending order and duplicate wavelength entries are not allowed.

**DRange2** Excel Range object containing the wavelength and detector response function for detector 2 \( D_2(\lambda) \). The wavelength is contained in the first column and the detector response value is contained in the last column. In the integration, values for the detector response function are interpolated linearly from the table of data. Note that the wavelengths in the table must be in ascending order and duplicate wavelength entries are not allowed.

**lambda1L** lower wavelength for detector 1 \( \lambda_{1L}, \mu m \)

**lambda1u** upper wavelength for detector 1 \( \lambda_{1U}, \mu m \)

**lambda2L** lower wavelength for detector 2 \( \lambda_{2L}, \mu m \)

**lambda2u** upper wavelength for detector 2 \( \lambda_{2U}, \mu m \)

**temp1** equivalent blackbody temperature for detector 1 \( T_{\lambda_1} \)

**temp2** equivalent blackbody temperature for detector 2 \( T_{\lambda_2} \)

**temp** true temperature \( T, K \)

**tol** tolerance on temperature integration

- > 0 relative tolerance
- < 0 absolute tolerance, K

**nmaxdbl** maximum number of doublings
Returns:

The derivative of the logarithm of the emissivity ratio with respect to the true temperature for a two-band radiation thermometer $d \ln \varepsilon_r / d \ln T$.

**Function bb_dlniedlnt1(DRange1 As range, lambda1l As Double, lambda1u As Double, temp1 As Double, tol As Double, nmaxdbl As Long) As Double**

Calculate the derivative of the logarithm of the emissivity ratio $\varepsilon_r$ with respect to the first band temperature $T_{\lambda_1}$ for a two-band radiation thermometer (eq. (45)).

\[
\frac{d \ln \varepsilon_r}{d \ln T_{\lambda_1}} = T_{\lambda_1} \frac{\int_{\lambda_{1l}}^{\lambda_{1u}} D_1(\lambda) \, di_{b\lambda}(T_{\lambda_1}) / dT_1 \, d\lambda}{\int_{\lambda_{1l}}^{\lambda_{1u}} D_1(\lambda) \, i_{b\lambda}(T_{\lambda_1}) \, d\lambda}
\]  

*(45)*

Description of Parameters:

- **DRange1** Excel Range object containing the wavelength and detector response function for detector 1 $D_1(\lambda)$. The wavelength is contained in the first column and the detector response value is contained in the last column. In the integration, values for the detector response function are interpolated linearly from the table of data. Note that the wavelengths in the table must be in ascending order and duplicate wavelength entries are not allowed.
- **lambda1l** lower wavelength for detector 1 $\lambda_{1l}$, $\mu$m
- **lambda1u** upper wavelength for detector 1 $\lambda_{1u}$, $\mu$m
- **temp1** equivalent blackbody temperature for detector 1 $T_{\lambda_1}$
- **tol** tolerance on temperature integration
  - > 0 relative tolerance
  - < 0 absolute tolerance, K
- **nmaxdbl** maximum number of doublings

Returns:

The derivative of the logarithm of the emissivity ratio with respect to the first band temperature $\ln \varepsilon_r / d \ln T_{\lambda_1}$.

**Constant Functions**

Note: Values for physical constants were obtained from ref. 1.

**Function bb_C1() As Double**

Returns the value of $C_1 = 1.191043 \times 10^8 \, W \cdot \mu m^4 / m^2 \cdot sr$.

**Function bb_C2() As Double**

Returns the value of $C_2 = 14,387.77 \, \mu m \cdot K$. 
Function bb_C3() As Double

    Returns the value of \( C_3 = 2,897.77 \ \mu m \cdot K \).

Function bb_C4() As Double

    Returns the value of \( C_4 = 4.09567 \times 10^{-12} \ \text{W/m}^2\cdot \mu m \cdot \text{sr} \cdot \text{K}^5 \).

Function bb_SIGMA() As Double

    Returns the value of the Stephan-Boltzmann constant \( \sigma = 5.670367 \times 10^{-8} \ \text{W/m}^2\cdot \text{K}^4 \).

Function bb_RCONST() As Double

    Returns the value of the universal gas constant \( R = 8.3144598 \ \text{J/mol} \cdot \text{K} \).

Function bb_CLIGHT() As Double

    Returns the value of the speed of light in a vacuum \( c = 2.99792458 \times 10^8 \ \text{m/s} \).

Function bb_HBAR() As Double

    Returns the value of Planck’s constant \( \hbar = 6.62607004 \times 10^{-34} \ \text{J/s} \).

Function bb_KBOLTZ() As Double

    Returns the value of the Boltzmann constant \( k = 1.38064852 \times 10^{-23} \ \text{J/K} \).

Configuration Control Functions

Function bb_version() As String

    Returns the version of the blackbody function library as a string.

Examples

This section contains example problems using the routines defined previously. Each example contains a brief problem statement followed by a reference to the applicable equations, VBA routine names, and the numerical solution.

Example 1

Example 1: What is the emitted spectral intensity for a blackbody at 3,000 K measured with a detector at a wavelength of 0.5 \( \mu m \)?

Answer 1: The spectral intensity is calculated from equation (1) using routine bb_ibl, as shown in equation (57):
\[ i_{b, \lambda}(\lambda, T) = \frac{C_1}{\lambda^2 \left( e^{\frac{2}{\lambda T}} - 1 \right)} = \frac{1.191043 \times 10^8}{0.5^5 (e^{14.387.75/0.5 \cdot 3000} - 1)} = 2.60 \times 10^5 \text{ W/m}^2\text{-sr-\mu m} \] (57)

**Example 2**

Example 2: What is the emitted power across a 10-nm (0.01-\mu m) waveband centered at 0.5 \mu m and at 3,000 K?

Answer 2: Since the bandwidth is small, the total power can be evaluated using the spectral intensity at the given wavelength calculated using routine `bb_ibl` and then multiplied by the wavelength band, as shown in equation (58):

\[ I_{b, \lambda}(\lambda_l, \lambda_u, T) \equiv i_{b, \lambda}(\lambda, T) \cdot \Delta \lambda = 2.60 \times 10^5 \text{ W/m}^2\text{-sr-\mu m \cdot 0.01 \mu m} = 2.60 \times 10^3 \text{ W/m}^2\text{-sr} \] (58)

**Example 3**

Example 3: What is the total emitted flux for a body at 1,500 K? What is the emitted flux between the wavelengths of 1 to 3 \mu m?

Answer 3: The total emitted flux is given by equation (15) and calculated using `bb_eb`, as shown in equation (59):

\[ e_b = \sigma T^4 = 5.67 \times 10^{-8} \cdot 1,500^4 = 2.87 \times 10^5 \text{ W/m}^2 \] (59)

The wavelength-temperature products for 1 and 3 \mu m at 1,500 K are (eq. (60)):

\[ \lambda_l T = 1,500 \mu m - K \text{ and } \lambda_u T = 4,500 \mu m - K \] (60)

Therefore, using the series solution described in equations (5) and (6) and the routine `bb_fiibls`, the fraction of flux is as shown in equation (61):

\[ F_{0 \rightarrow \lambda_l T} = 0.01285 \text{ and } F_{0 \rightarrow \lambda_u T} = 0.56430 \] (61)

So, (eq. (62)):

\[ e_{\lambda_1 T \rightarrow \lambda_u T} = F_{\lambda_l T \rightarrow \lambda_u T} \cdot \sigma T^4 = (F_{0 \rightarrow \lambda_u T} - F_{0 \rightarrow \lambda_l T}) \cdot \sigma T^4 \] (62)

and the flux in band is (eq. (63)):

\[ = (0.56430 - 0.01285) \cdot 2.87 \times 10^5 \text{ W/m}^2 = 1.58 \times 10^5 \text{ W/m}^2 \] (63)

A graphical representation of the integration is shown in figure 4.
Figure 4. Integration of the Planck equation over the range of the wavelength temperature product of 1,500 to 4,500 μm-K.

Alternatively, the solution could be found directly using routines bb_iibls or bb_iibl.

**Example 4**

Example 4: What is the true temperature when a surface is measured with a 0.5-μm wavelength detector and indicates an equivalent blackbody temperature of 3,820 K? Assume an emissivity of 0.8.

Answer 4: The wavelength temperature product is 3,820 K times 0.5 μm or 1,910 μm-K. This product is much less than \( C_2 \) or 14,388 μm-K. So, Wien’s approximation is valid (as discussed in the Basic Equations subsection of the Background section) and equation (24) using routine bb_tstw can be used, as shown in equations (64) and (65):

\[
\frac{1}{T} = \frac{1}{T_\lambda} + \frac{\lambda}{C_2} \ln \varepsilon_\lambda = \frac{1}{3,820 \ K} + \frac{0.5 \ \mu m}{14,388 \ \mu m-K} \ln(0.8)
\]  

(64)

\[
T = 3,937 \ K
\]  

(65)

**Example 5**

Example 5: Repeat Example 4, using an 8-μm detector. What is the true temperature?

Answer 5: The wavelength temperature product is 3,820 K times 8 μm or 30,560 μm-K. This product is greater than \( C_2 \) or 14,388 μm-K. So, the expression derived from the full Planck equation (eq. (24)) must be used, using routine bb_tst as shown in equation (66):
\[ T = \frac{C_2/\lambda}{\ln(\varepsilon_\lambda \cdot (e^{C_2/\lambda T_\lambda} - 1) + 1)} \]  

(24)

\[ T = \frac{14,388 \ \mu m \cdot K/8 \ \mu m}{\ln(0.8 \cdot (e^{14,388 \ \mu m \cdot K/8 \ \mu m} 3820 K - 1) + 1)} = 4,579 \ \text{K} \]  

(66)

If Wien’s approximation had been used, the result obtained for the calculated true temperature would have been over 7,200 K (and inaccurate by more than 2,600 K).

**Example 6**

Example 6: If the spectral emissivity of graphitic material at 0.53 \( \mu m \) is estimated to be 0.8 with an estimated uncertainty of 20%, what is the estimated uncertainty in the true temperature if a detector at this wavelength indicates an equivalent blackbody temperature of 2,950 K?

Answer 6: The wavelength temperature product is 2,950 K times 0.53 \( \mu m \) or 1,563.5 \( \mu m \cdot K \). This product is much less than \( C_2 \) or 14,388 \( \mu m \cdot K \). Thus, Wien’s approximation is valid and equation (24) and routine bb_tstw can be used as shown in equations (67)-(69):

\[ \frac{1}{T} = \frac{1}{T_\lambda} + \frac{\lambda}{C_2} \ln \varepsilon_\lambda = \frac{1}{2,950 \ \text{K}} + \frac{0.52 \ \mu m}{14,388 \ \mu m \cdot K} \ln(0.8) = 0.0003308 \]

(67)

\[ T = 3,023 \ \text{K} \]

(68)

\[ \frac{d \ln T}{d \ln \varepsilon_\lambda} = \frac{-\lambda T}{C_2} = \frac{-0.53 \mu m \cdot 3,023 \ \text{K}}{14,388 \ \mu m \cdot K} = -11\% \]

(69)

With the emissivity uncertainty estimated to be 20%, the resulting uncertainty in the true temperature is 20% times -11\%, or -2.2\%. For a temperature of 3,023 K, the absolute uncertainty is -67 K.

**Example 7**

Example 7: Repeat example 6 for a detector with a wavelength of 5.8 \( \mu m \).

Answer 7: The wavelength temperature product is 2,950 K times 5.8 \( \mu m \) or 17,110 \( \mu m \cdot K \). This product is greater than \( C_2 \) or 14,388 \( \mu m \cdot K \), so, Wien’s approximation is not valid and the full Planck equation (equation (1)) and routine bb_tst must be used.

The estimated “true” temperature with the nominal emissivity is calculated from equation (24) and routine bb_tst, as shown in equation (70):

\[ T = \frac{14,388 \ \mu m \cdot K/5.8 \ \mu m}{\ln(0.8 \cdot (e^{14,388 \ \mu m \cdot K/5.8 \ \mu m} 2,950 \ \text{K} - 1) + 1)} = 3,445 \ \text{K} \]

(70)

The estimated uncertainty in the emissivity is determined from equation (25) and routine dlntdlne and is shown in equation (71):
\[
\frac{d \ln T}{d \ln \varepsilon_{\lambda_1}} = -\frac{\lambda_1 T}{C_2} \cdot \frac{e^{C_2/\lambda_1 T \lambda_1} - 1}{e^{C_2/\lambda_1 T \lambda_1}} = \frac{5.8 \mu m \cdot 3,445 K}{14,388 \mu m \cdot K} \cdot \frac{e^{5.8 \mu m \cdot 3,445 K/14,388 \mu m \cdot K} - 1}{e^{5.8 \mu m \cdot 3,445 K/14,388 \mu m \cdot K}} = -71\% \quad (71)
\]

With the emissivity uncertainty estimated to be 20%, the resulting uncertainty in temperature is 20% times 71%, or 14%. For a temperature of 3,445 K, the absolute uncertainty is approximately 491 K.

**Example 8**

Example 8: A 0.53-μm wavelength detector is used to determine the true temperature. The measured equivalent blackbody temperature from this detector is 1,950 K and the spectral emissivity is estimated to be 0.6 with an estimated uncertainty of 25%. What are the inferred emissivities and uncertainties at 1.0 μm and 5.8 μm if detectors at both wavelengths indicate equivalent blackbody temperatures of 1,740 K?

Answer 8: The true temperature using equation (24) and routine bb_tst is obtained as shown in equation (72):

\[
T = \frac{14,388 \mu m \cdot K/0.53 \mu m}{\ln(0.8 \cdot (e^{14,388 \mu m \cdot K/0.53 \mu m \cdot 1,950 K} - 1) + 1)} = 2,024 K
\]

The emissivity at 1.0 μm using equation (28) and routine bb_emiss is obtained as shown in equation (73):

\[
\varepsilon_{\lambda_2} = \frac{e^{14,388 \mu m \cdot K/1.0 \mu m \cdot 2,024 K} - 1}{e^{14,388 \mu m \cdot K/1.0 \mu m \cdot 1,740 K} - 1} = 0.313
\]

The sensitivity to the emissivity at wavelength 2 based on the assumed emissivity at wavelength 1 using equation (28) and routine bb_dlnedlne1 is obtained as shown in equation (74):

\[
\frac{d \ln \varepsilon_{\lambda_2}}{d \ln \varepsilon_{\lambda_1}} = \frac{0.53 \mu m}{1.0 \mu m} \cdot \frac{e^{-14,388 \mu m \cdot K/0.53 \mu m \cdot 2,024 K} - 1}{e^{-14,388 \mu m \cdot K/1.0 \mu m \cdot 2,024 K} - 1} = 0.530
\]

The relative uncertainty in the emissivity is obtained as shown in equation (75):

\[
\frac{\Delta \varepsilon_{\lambda_2}}{\varepsilon_{\lambda_2}} = \frac{d \ln \varepsilon_{\lambda_2}}{d \ln \varepsilon_{\lambda_1}} \cdot \frac{\Delta \varepsilon_{\lambda_2}}{\varepsilon_{\lambda_2}} = 0.530 \cdot 25\% = 13.3\%
\]

The emissivity at 5.8 μm using routine bb_emiss is obtained as shown in equation (76):

\[
\varepsilon_{\lambda_2} = \frac{e^{14,388 \mu m \cdot K/5.8 \mu m \cdot 2,204 K} - 1}{e^{14,388 \mu m \cdot K/5.8 \mu m \cdot 1,740 K} - 1} = 0.761
\]

The sensitivity to the emissivity at wavelength 2 based on the assumed emissivity at wavelength 1 using routine bb_dlnedlne1 is obtained as shown in equation (77):
Finally, the relative uncertainty in the emissivity is obtained as shown in equation (78):

\[
\frac{\Delta \varepsilon_{\lambda_2}}{\varepsilon_{\lambda_2}} = \frac{d \ln \varepsilon_{\lambda_2}}{d \ln \varepsilon_{\lambda_1}} \cdot \frac{\Delta \varepsilon_{\lambda_2}}{\varepsilon_{\lambda_2}} = 0.129 \cdot 25\% = 3.2\%
\] (78)

Note that the emissivity uncertainty at 5.8 \( \mu \)m is four times less than at 1 \( \mu \)m.

**Example 9**

Example 9: For a two-band radiation thermometer with detector wavelengths at 0.5 and 0.6 \( \mu \)m, what is the true temperature and the spectral emissivities when the measured equivalent blackbody temperatures at the two wavelengths are 2,800 and 2,750 K, respectively and an emissivity ratio of 0.9 is assumed?

Answer 9: In this case, Wien’s approximation is valid, so that using the approximate method from equation (33) and calculating the effective wavelength \( \Lambda \) from equation (36) using routine `bb_erlam` is as presented in equation (79):

\[
\Lambda = \frac{0.5 \, \mu m \cdot 0.6 \, \mu m}{0.6 \, \mu m - 0.5 \, \mu m} = 3.0 \, \mu m
\] (79)

Using equation (34), the ratio temperature \( T_r \) from routine `bb_ertemp` is as presented in equation (80):

\[
T_r = \left( \frac{3.0 \, \mu m}{0.5 \, \mu m \cdot 2,800 \, K - 0.6 \, \mu m \cdot 2,750 \, K} \right)^{-1} = 3,080 \, K
\] (80)

As well, using equation (33) and routine `bb_tstw`, the true temperature is as presented in equation (81):

\[
T = \left( \frac{1}{3,080 \, K} - \frac{3.0 \, \mu m}{14,388 \, \mu m \cdot K \cdot \ln 0.9} \right)^{-1} = 3,304 \, K
\] (81)

Knowing the individual measured detector temperatures and the true temperature, the emissivities are determined using equation (29) and routine `bb_emiss` as shown in equations (82) and (83):

\[
\varepsilon_{\lambda_1} = \frac{e^{14,388 \, \mu m \cdot K / 0.5 \, \mu m \cdot 3,304 \, K} - 1}{e^{14,388 \, \mu m \cdot K / 0.5 \, \mu m \cdot 2,800 \, K} - 1} = 0.209
\] (82)

\[
\varepsilon_{\lambda_2} = \frac{e^{14,388 \, \mu m \cdot K / 0.6 \, \mu m \cdot 3,304 \, K} - 1}{e^{14,388 \, \mu m \cdot K / 0.6 \, \mu m \cdot 2,750 \, K} - 1} = 0.232
\] (83)
It is seen that $\varepsilon_{\lambda_1}/\varepsilon_{\lambda_2} = 0.9$, which is consistent with the initial assumption.

**Example 10**

Example 10: For the same conditions as used in Example 9, what would the error in the true temperature be if an assumed emissivity ratio of 1.0 was used but the actual ratio was 0.9?

Answer 10: This problem can be solved two ways: directly or using sensitivities. Sensitivities are presented first.

In the sensitivities case, Wien’s approximation is valid, since both of the wavelength-temperature products are small than $C_2$. The sensitivity from equation (27) is as shown in equation (84):

$$\frac{d \ln T}{d \ln \varepsilon_{\lambda i}} = -\frac{3.0 \mu m \cdot 3,304 K}{14,388 \mu m - K} = -0.688$$

(84)

The estimated error in temperature is as shown in equation (85):

$$-0.688 \cdot 0.1 = -6.9\% \text{ or } -228 K$$

(85)

The difference can also be calculated directly, rather than by using sensitivities.

For an emissivity of 1.0, the true temperature is simply the ratio temperature, or 3,080 K. The error, therefore, is as obtained by equation (86):

$$3080 K - 3,304 K = 224 K$$

(86)

This result is almost exactly equal to the linear approximation that was obtained using sensitivities.

**Example 11**

Example 11: For a two-band radiation thermometer with detector wavelengths at 4.0 and 8.0 $\mu$m, what is the error introduced in the temperature measurement when an emissivity ratio of 1.0 is assumed, when in fact the emissivity ratio of the sample has a ratio of 0.9? Assume the measured spectral temperatures at the two wavelengths are 2,800 and 2,750 K, as in Example 9.

Answer 11: Since the products of $T_\lambda$ and $\lambda$ for both wavelengths are not less than $C_2$ or 14,388 $\mu$m-K, the full Planck equation must be used. Similar to Example 10, this problem can be solved two ways: directly or by using sensitivities. Again, the solution using sensitivities is presented first. The true temperature is calculated using equation (31) and routine bb_ratio as shown in equation (87):

$$\left(\frac{e^{C_2/\lambda_2T_2} - 1}{e^{C_2/\lambda_2T_2} - 1}\right) \cdot \left(\frac{e^{C_2/\lambda_1T_1} - 1}{e^{C_2/\lambda_1T_1} - 1}\right) = \varepsilon_r = 0$$

(87)

The calculated values for true temperatures are for the two emissivity ratios presented as equations (88) and (89):

$$\varepsilon_r = 0.9, \ T = 4,118 K$$

(88)
\( \varepsilon_r = 1.0, \ T = 2,974 \text{ K} \)  

The difference is -1,144 K!

This example points out the large error using long wavelengths and emphasizes the need to use short wavelengths whenever possible.

The sensitivity of the emissivity ratio to true temperature is (equation 37) as shown in equation (90):

\[
\frac{d \ln \varepsilon_r}{d \ln T} = \frac{1}{T} \cdot \frac{\left(e^{\frac{C_2}{\lambda_2 T}} - 1\right)}{\left(e^{\frac{C_2}{\lambda_1 T}} - 1\right)} \cdot \left[ \frac{(C_2)}{(\lambda_2 T)} e^{\frac{C_2}{\lambda_2 T}} \left(\frac{e^{\frac{C_2}{\lambda_1 T}} - 1}{(e^{\frac{C_2}{\lambda_2 T}} - 1)^2} - (C_2) e^{\frac{C_2}{\lambda_1 T}}\right) \left(\frac{e^{\frac{C_2}{\lambda_2 T}} - 1}{(e^{\frac{C_2}{\lambda_2 T}} - 1)^2}ight)^2 - (C_2) e^{\frac{C_2}{\lambda_1 T}}\right]
\]

The temperature change is so great that linear sensitivities extrapolated from one point will not be accurate, so instead an average of temperatures and emissivity is used.

The calculated sensitivities are as obtained in equations (81) and (92):

\[
\varepsilon_r = 0.9, \ \frac{d \ln T}{d \ln \varepsilon_r} = -3.77
\]

\[
\varepsilon_r = 1.0, \ \frac{d \ln T}{d \ln \varepsilon_r} = -2.56
\]

The temperature error can be estimated using a centered average of the temperature and sensitivity, as shown in equation (93):

\[
\Delta T = -\left(4,118 \text{ K} + 2,974 \text{ K}\right) \cdot \frac{\left(3.77 + 2.56\right)}{4} \cdot 0.1 = -1,122 \text{ K}
\]

**Example 12**

Example 12: Given the data presented in figure 5 from a 32-channel silicon array detector, calculate the spectral emissivity and true temperature if the spectral emissivity is assumed to follow a second-order polynomial with wavelength.
Figure 5. Spectral intensity versus wavelength for a 32-detector measurement.

Answer 12: The solution requires that values be found for $T$, $a_0$, $a_1$, and $a_2$ that minimize the function $f$ (eq. (94)):

$$f = \sum_{i=1}^{n} \left( \varepsilon_{\lambda_i} \cdot i_{b,\lambda_i}(T) - i_{\lambda_i}(T_{\lambda_i}) \right)^2$$

where equation (95) is shown as:

$$\varepsilon_{\lambda} = a_2 \lambda^2 + a_1 \lambda + a_0$$

Using routine bb_ibl to calculate $i_{b,\lambda_i}(T)$ along with the multi-variable minimization capability “Solver” in Excel®, the solution is as shown in equations (96) and (97):

$$T = 3,802 \text{ K}$$

$$a_2 = -0.06222, \ a_1 = 0.0610, \text{ and } a_0 = 0.7347$$

The solution is shown in figure 6.
Figure 6. Calculated best-fit spectral intensity and emissivity using least squares minimization of 32-detector measurement.

**Example 13**

Example 13: What is the non-linearity introduced by averaging the response of a detector covering the range of 1 to 4 \( \mu \text{m} \) at temperatures in between 600 to 1,100 K? Assume a uniform spectral response function.

Answer 13: The problem requires a comparison (eq. (98)):

\[
I(\lambda_l, \lambda_u, T) = \int_{\lambda_l}^{\lambda_u} i_{b,\lambda}(\lambda, T) d\lambda
\]  

(98)

using routine bb_iibl for each temperature \( T \) to the quantity calculated at the average wavelength (eq. (99)):

\[
I(\bar{\lambda}, \Delta \lambda, T) = i_{b,\lambda}(\bar{\lambda}, T) \Delta \lambda
\]  

(99)

using routine bb_ibl, where (eq. (100)):

\[
\bar{\lambda} = \frac{1}{2}(\lambda_l + \lambda_u) \text{ and } \Delta \lambda = \lambda_u - \lambda_l
\]  

(100)

so that \( \bar{\lambda} = 2.5 \mu \text{m} \) and \( \Delta \lambda = 3.0 \mu \text{m} \). The comparison is shown in figure 7.
Figure 7. Comparison of signals calculated with exact and approximate methods for a wide-band detector with wavelengths between 1 and 4 μm and a uniform detector response across the wavelength band.

Note the non-linearity in the calibration and the high error at low intensities as discussed in the section pertaining to wide-band detectors, found in the Basic Equations Chapter.

Example 14

Example 14: What is the non-linearity introduced by averaging the response of a detector covering the range of 0.25 to 1.05 μm at temperatures in between 2,000 to 3,000 K? Assume a linearly increasing detector spectral response function, as shown in figure 8.
Figure 8. Assumed detector response function for a 0.25- to 1.05-μm detector showing a linearly increasing detector response across the wavelength band.

Answer 14: The problem requires that we compare the quantity calculated by the integral in eq. 101:

$$I(\lambda_l, \lambda_u, T) = \int_{\lambda_l}^{\lambda_u} D(\lambda) \cdot i_b(\lambda, T) d\lambda$$

(101)

using routine bb_iibl for each temperature $T$, to the quantity calculated at the average wavelength by eq. 102:

$$I(\bar{\lambda}, \Delta \lambda, T) = D(\bar{\lambda}) \cdot i_b(\bar{\lambda}, T) \Delta \lambda$$

(102)

using routine bb_ibl, where (eq. (103)):

$$\bar{\lambda} = \frac{1}{2}(\lambda_l + \lambda_u) \text{ and } \Delta \lambda = \lambda_u - \lambda_l$$

(103)

such that $\bar{\lambda} = 0.65$ μm and $\Delta \lambda = 0.8$ μm. The comparison is shown in figure 9.
Figure 9. Comparison of signals calculated with exact and approximate methods for a wide-band detector with wavelengths between 0.25 to 1.05 μm and a linearly increasing detector response across the wavelength band.

The result is similar to Example 13. Again note the non-linearity in the calibration and the high error at low intensities.

Example 15

Example 15: A commercial Silicon/InGaAs sandwich detector has the detector response shown in figure 10. If the Silicon detector measures 3,013 K and the InGaAs measures 2,909 K equivalent blackbody temperatures, what are the average spectral emissivities for the two wavelengths and the true temperature, assuming an emissivity ratio of 1.0?

Figure 10. Detector response function for a Silicon/InGaAs sandwich detector.

Answer 15: The detector response function is modeled by the line segments shown in figure 11.
To ensure that the problem is self-consistent, already calculated are the equivalent blackbody temperatures $T_\lambda$ from integration of the true temperature $T$ across the wavelength band. The calculation uses the detector response functions as shown previously with an emissivity $\bar{\varepsilon}$ uniform over the wavelength range and equal to 0.7 from equation (104):

$$
\bar{\varepsilon} \int_{\lambda_1}^{\lambda_u} D(\lambda) \ i_{b,\lambda}(T) \ d\lambda = \int_{\lambda_1}^{\lambda_u} D(\lambda) \ i_{b,\lambda}(T_\lambda) \ d\lambda
$$

for $T = 3,200 \text{ K}, T_{\lambda_1} = 3,013 \text{ K}$ and $T_{\lambda_2} = 2,909 \text{ K}$. The problem requires solving equation (43) for a wide-band, or equation (46) for an equivalent narrow-band, with $\bar{\varepsilon}_r = 0.9$ and $T_1 = 3,013 \text{ K}$ and $T_2 = 2,909 \text{ K}$, as shown in equations (105) and (106):

$$
\bar{\varepsilon}_r = \frac{\bar{\varepsilon}_1}{\bar{\varepsilon}_2} = \frac{\int_{\lambda_1}^{\lambda_u} D_1(\lambda) i_{b,\lambda}(T_\lambda) \ d\lambda}{\int_{\lambda_1}^{\lambda_u} D_1(\lambda) i_{b,\lambda}(T) \ d\lambda} \cdot \frac{\int_{\lambda_1}^{\lambda_u} D_2(\lambda) i_{b,\lambda}(T) \ d\lambda}{\int_{\lambda_1}^{\lambda_u} D_2(\lambda) i_{b,\lambda}(T_\lambda) \ d\lambda}
$$

$$
\bar{\varepsilon}_r = \frac{\left( e^{-c_2/\lambda_2 T_{\lambda_2}} - 1 \right)}{\left( e^{-c_2/\lambda_2 T} - 1 \right)} \cdot \frac{\left( e^{-c_2/\lambda_1 T_{\lambda_1}} - 1 \right)}{\left( e^{-c_2/\lambda_1 T} - 1 \right)}
$$

The results are summarized in table 1.
Table 1. Solution results for Example 14 with $\bar{\varepsilon} = 0.9$.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Integrated band</th>
<th>Averaged band</th>
<th>Difference, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$, K</td>
<td>3,200</td>
<td>3,134</td>
<td>2</td>
</tr>
<tr>
<td>$\varepsilon_1$</td>
<td>0.70</td>
<td>0.77</td>
<td>10</td>
</tr>
<tr>
<td>$\varepsilon_2$</td>
<td>0.70</td>
<td>0.76</td>
<td>9</td>
</tr>
</tbody>
</table>

**Example 16**

Example 16: Consider a dual-band instrument with spectral bands covering 1 to 4.5 $\mu$m and 2 to 13.5 $\mu$m. The detector response functions are shown in figure 12. The average detector wavelengths are 2.75 and 7 $\mu$m. Using the average detector wavelengths, what is the true temperature and what is the inferred emissivity assuming an emissivity ratio of 1?

![Detector response function two infrared detectors.](image)

Figure 12. Detector response function two infrared detectors.

Answer 16: The example assumes a gray material, which means that the spectral emissivity is independent of wavelength. Assume that the emissivity is equal to 0.65, as shown in figure 13, and that the true temperature is 3,000 K.
Figure 13. Assumed spectral emissivity as a function of wavelength for Example 16.

Using the function $bb\_tiibl$, calculate what the measured equivalent blackbody temperatures $T_\lambda$ would be by solving equation (107) given the detector response functions $D(\lambda)$, the emissivity $\varepsilon$ (independent of wavelength), and the spectral blackbody radiant intensity.

$$\bar{\varepsilon} \int_{\lambda_1}^{\lambda_u} D(\lambda) \, i_{b,\lambda}(T) \, d\lambda = \int_{\lambda_1}^{\lambda_u} D(\lambda) \, i_{b,\lambda}(T) \, d\lambda$$ (107)

The calculated blackbody temperatures for detectors 1 and 2 are 2,589 and 2,426 K, respectively.

Using the average wavelengths, calculate the equivalent blackbody temperature $T$ and emissivity for an emissivity ratio of 1.0 from the two detector equivalent blackbody temperatures $T_1$ and $T_2$. This problem is solved using the function $bb\_ratio$, shown in equation (108):

$$\frac{(e^{-C_2/\lambda_2 T_2} - 1)}{(e^{-C_2/\lambda_2 T} - 1)} \cdot \frac{(e^{-C_2/\lambda_3 T} - 1)}{(e^{-C_2/\lambda_3 T_1} - 1)} - \varepsilon_r = 0$$ (108)

For $\varepsilon_r = 1$, the result is (eq. (109)):

$$T = 2,879 \text{ K and } \bar{\varepsilon} = 0.79$$ (109)

versus the correct values of $T = 3,000 \text{ K and } \varepsilon = 0.65$, a 4% and 18% error, respectively.

This solution demonstrates the uncertainty introduced using wide-bandwidth detectors when assuming a single, average wavelength for each detector.
Example 17

Example 17: The final wide-band detector problem demonstrates not only the complexity in reducing data from wide-band detectors, but also the difficulty in interpreting the data.

First, consider a dual-band instrument with spectral bands covering 1 to 4.5 \( \mu \text{m} \) and 2 to 13.5 \( \mu \text{m} \). The applicable detector response functions are shown in figure 12.

The material to be measured has a spectral emissivity decreasing with wavelength as shown in figure 14. The true temperature is assumed to be 3,000 K. Using equation (41) for the given detector response functions and the spectral emissivity relationship, the calculated equivalent blackbody temperatures for detectors 1 and 2 are 2,941 and 2,841 K, respectively.

If an emissivity ratio equal to the values at the center wavelength is used, what is the calculated true temperature and the individual spectral emissivities? The exact emissivity behavior with wavelength will generally be unknown, so repeat the problem when the emissivity ratio is naturally assumed to be equal 1.

![Figure 14. Assumed spectral emissivity as a function of wavelength for Example 17.](image)

Answer 17: The centers of the two bands are 2.75 and 7.75 \( \mu \text{m} \). The emissivities at the center wavelengths from figure 14 are 0.931 and 0.806, resulting in an emissivity ratio of 1.155. Using equation (45) and routine bb_ratio for an equivalent narrow-band calculation, the calculated true temperature is 2,677 K with emissivities for detector 1 and detector 2 being 1.29 and 1.12, respectively. Neither the temperature nor the calculated emissivities are close to that of the actual conditions and material properties.

Using the equivalent wideband equation (equation (43)) and routine bb_itratio with an emissivity ratio \( \varepsilon_r \) equal to 1.16 results in a calculated true temperature of 2,706 K and emissivities for detector 1 and 2 of 1.27 and 1.10. While the temperature is closer, the accuracy is still poor and the emissivities are quite different and non-physical (both greater than 1).
If an emissivity ratio $\varepsilon_r$ equal to 1 is used, then the calculated true temperature is 3,179 K for the wideband calculation with resulting emissivities for the two detectors equal to 0.81. The calculated true temperature is also quite far from the true value. As can be seen, over such a wide range of wavelengths, it is difficult to choose the proper emissivity ratio since the wavelength dependence on spectral emissivity generally is not known.

Table 2. Solution results for Example 17.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\varepsilon_r = 1.0$</th>
<th>$\varepsilon_r = 1.13$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Integrated band</td>
<td>Averaged band</td>
</tr>
<tr>
<td>$T$, K</td>
<td>3,179</td>
<td>3,127</td>
</tr>
<tr>
<td>$\bar{\varepsilon}_1$</td>
<td>0.81</td>
<td>0.88</td>
</tr>
<tr>
<td>$\bar{\varepsilon}_2$</td>
<td>0.81</td>
<td>0.88</td>
</tr>
</tbody>
</table>

Using wide-band detectors centered at long wavelengths and that span a wide range of wavelengths do not give realistic results for both emissivity and temperature for any assumption.

**Example 18**

Example 18: A blackbody source is estimated to have an uncertainty of 10 K and is used to perform a single-point calibration on two detectors, 1) a short-wavelength, 0.5-$\mu$m detector and 2) a longer-wavelength 3-$\mu$m detector. An unknown material is then measured at the same temperature with these detectors. The equivalent blackbody temperatures of the material from the two detectors are measured to be 1,600 and 1,500 K, respectively. What is the value and the uncertainty in the derived emissivity at 3 $\mu$m, assuming the material has an emissivity of 0.8 ±0.1 at 0.5 $\mu$m?

Answer 18: The equivalent blackbody temperature from the measurement of the 0.5-$\mu$m detector is for $T_{\lambda_1} = 1,600$K and $\varepsilon_{\lambda_1} = 0.8$ using equation (23) and routine bb_tst is as shown in equation (110):

$$T = \frac{C_2}{\lambda_1} \cdot \frac{1}{\ln \left[ \varepsilon_{\lambda_1} \left( e^{C_2/A_\lambda_1} - 1 \right) + 1 \right]} = 1,620 \text{ K} \quad (110)$$

The calculated emissivity at 3 $\mu$m using routine bb_emiss is as shown in equation (111):

$$\varepsilon_{\lambda} = \frac{\left( e^{C_2/A_\lambda} - 1 \right)}{\left( e^{C_2/A_T} - 1 \right)} = 0.78 \quad (111)$$
Assuming the temperature errors in the two calibration measurements are uncorrelated and random, the total uncertainty in the emissivity will be as shown in equation (112):

\[
\Delta \varepsilon_{\lambda_2} = \sqrt{\left(\frac{\partial \varepsilon_{\lambda_2}}{\partial T_{\lambda_1}} \Delta T_{\lambda_1}\right)^2 + \left(\frac{\partial \varepsilon_{\lambda_2}}{\partial T_{\lambda_2}} \Delta T_{\lambda_2}\right)^2 + \left(\frac{\partial \varepsilon_{\lambda_2}}{\partial \varepsilon_{\lambda_1}} \Delta \varepsilon_{\lambda_1}\right)^2}
\]

(112)

where \(\Delta T_{\lambda_1} = 10\text{K}\) and \(\Delta \varepsilon_{\lambda_1} = 0.1\); or, in terms of sensitivities (eq. (113)):

\[
\frac{\Delta \varepsilon_{\lambda_2}}{\varepsilon_{\lambda_2}} = \sqrt{\left(\frac{\partial \ln \varepsilon_{\lambda_2}}{\partial \ln \varepsilon_{\lambda_1}} \frac{\Delta \varepsilon_{\lambda_1}}{\varepsilon_{\lambda_1}}\right)^2 + \left(\frac{\partial \ln \varepsilon_{\lambda_2}}{\partial \ln T_{\lambda_1}} \frac{\Delta T_{\lambda_1}}{T_{\lambda_1}}\right)^2 + \left(\frac{\partial \ln \varepsilon_{\lambda_2}}{\partial \ln T_{\lambda_2}} \frac{\Delta T_{\lambda_2}}{T_{\lambda_2}}\right)^2 + \left(\frac{\partial \ln \varepsilon_{\lambda_2}}{\partial \ln \varepsilon_{\lambda_1}} \frac{\Delta \varepsilon_{\lambda_1}}{\varepsilon_{\lambda_1}}\right)^2}
\]

(113)

Evaluating terms gives the following calculations, seen in equations (114)-(118):

\[
\frac{d \ln \varepsilon_{\lambda_1}}{d \ln T_{\lambda_1}} = \frac{C_2}{\lambda_1 T_{\lambda_1}} \cdot \frac{e^{C_2/\lambda_1 T_{\lambda_1}}}{e^{C_2/\lambda_1 T_{\lambda_1}} - 1} = 18.0
\]

(114)

\[
\frac{d \ln \varepsilon_{\lambda_2}}{d \ln T_{\lambda_2}} = \frac{C_2}{\lambda_2 T_{\lambda_2}} \cdot \frac{e^{C_2/\lambda_2 T_{\lambda_2}}}{e^{C_2/\lambda_2 T_{\lambda_2}} - 1} = 3.33
\]

(115)

\[
\frac{d \ln \varepsilon_{\lambda_2}}{d \ln \varepsilon_{\lambda_1}} = \frac{\lambda_1}{\lambda_2} \cdot \frac{e^{-C_2/\lambda_1 T} - 1}{e^{-C_2/\lambda_2 T} - 1} = 0.176
\]

(116)

\[
\frac{\Delta \varepsilon_{\lambda_2}}{\varepsilon_{\lambda_2}} = \sqrt{0.176 \cdot 18.0 \cdot \frac{10}{1,600}^2 + 3.33 \cdot \frac{10}{1,500}^2 + 0.176 \cdot 0.1 \cdot \frac{10}{0.8}^2}
\]

(117)

\[
\frac{\Delta \varepsilon_{\lambda_2}}{\varepsilon_{\lambda_2}} = \sqrt{3.94 \times 10^{-4} + 4.94 \times 10^{-4} + 4.83 \times 10^{-4} + 3.7\%}
\]

(118)
and, therefore, (eqs. (119) and (120)):

\[ \Delta \varepsilon_{\lambda_2} = 0.78 \cdot 0.037 = 0.029 \]  \hspace{1cm} (119)

\[ \varepsilon_{\lambda_2} = 0.78 \pm 0.029 \]  \hspace{1cm} (120)

Note that the uncertainty resulting from the calibration is on the same order as the estimation of the emissivity at 0.5 \( \mu \)m, 0.8 \( \pm \)0.1.
References


