Hardware Demonstration:
Radiated Emissions as a Function of Common Mode Current
July 26, 2016
Acronyms

- Common mode conducted emissions (CMCE)
- Conducted emissions (CE)
- Electric field per unit current (E/I)
- Electromagnetic interference (EMI)
- Equipment under test (EUT)
- Radiated emissions (RE)
Due to the time- and resource-consuming nature of the test for radiated emissions, electric field (RE02/RE102), it is often prohibitive to perform early diagnostics by performing the test or even an abbreviated version of it.

Fortunately, it is possible to perform a much simpler, cheaper test – a common mode conducted emissions (CMCE) test – which will give useful predictions for much less expenditure of time and money.

The CMCE measurement can be easily performed in the hardware development lab in order to provide an early indication of whether the Equipment Under Test (EUT) will pass the RE102 test before the EUT is taken to a full EMI test facility.
At frequencies below 200 MHz, a significant portion of the radiated energy originates from uncontrolled common mode currents on cables connected to the unit.

In this demonstration, a controlled current is applied to a 1 m wire 5 cm above a ground plane, and the resulting electric field is measured.
The transfer function of electric field per unit current (E/I) is determined

- Presented as a tool for predicting radiated electric fields from a simple measurement with a clamp-on current probe before the product ever leaves the development laboratory
- \(|E/I|\) correspondence is evaluated at frequencies \(\geq 30\) MHz
- Per MIL-STD-461, RE102 below 30 MHz is measured with rod antenna, which responds to potential, not current

Product development engineers are encouraged to perform these measurements in order to facilitate diagnosis of potential problems as early as possible in the product's development cycle.
First, a bit of theory...

Electric Field from Elemental Electrical (Hertzian) Dipole

\[ E_\phi = 0 \]

\[ E_r = 2 \frac{Idl}{4\pi} \eta_0 \beta_0^2 \cos \theta \left( \frac{1}{\beta_0^2 r^2} - j \frac{1}{\beta_0^3 r^3} \right) e^{-j\beta_0 r} \]

\[ E_\theta = \frac{Idl}{4\pi} \eta_0 \beta_0^2 \sin \theta \left( j \frac{1}{\beta_0 r} + \frac{1}{\beta_0^2 r^2} - j \frac{1}{\beta_0^3 r^3} \right) e^{-j\beta_0 r} \]

Wavenumber:
\[ \beta_0 = \frac{2\pi}{\lambda} = \frac{2\pi f}{c} \]
• For a wire of any finite length, a precise calculation of the electric field at distance \( r \) would require a very complex integral

• Distance to measurement point must be varied with location of \( dl \) along the wire

• Relative phase of each field contribution must be considered

• Vector contributions of \( E_r \) and \( E_\theta \)

• Etc., etc., etc.
Simplifying Assumptions for $|E/I|$ Envelope

- Measurement is in traditional definition of “far field”
  - $\beta r > 1$ (f > 48 MHz for r = 1 m; generally OK for f > 30 MHz)
  - $1/\beta r$ term dominates
  - $1/(\beta r)^2$ and $1/(\beta r)^3$ terms may be neglected

- Cable behaves as a point source for estimating worst-case envelope
  - All current carrying elements are assumed to be at the same distance r from the measurement point
  - The electric field contributions from all current-carrying elements are assumed to be in phase at the measurement point

$$\left|\frac{E_{\text{max}}}{I}\right| \approx \frac{\mu_0 lf}{2r}$$
Transmission Line Current Model

- Transmission line current has horizontal and vertical components
- Resulting electric field will have horizontal and vertical components

\[ V(z) \]
\[ L \Delta z \]
\[ V(z + \Delta z) \]
\[ I(z) \]
\[ I(z + \Delta z) \]
\[ dI_d \Delta z \]
\[ C \Delta z \]
Math in backup charts, but here's the bottom line:

- **Horizontal polarization**
  - For cables of $l > 1 \text{ m}$ (typical on most spacecraft), $|E/I|$ is essentially independent of wire length above 48 MHz ($\beta l > 1$)
  - For most spacecraft, RE102 is mainly a concern for $f \geq 200 \text{ MHz}$
  - High frequency asymptote determined largely by interaction of cable with ground plane (details in backup slides)

- **Vertical polarization**
  - $|E/I|$ is essentially independent of wire length for $f \geq 30 \text{ MHz}$
  - Equivalent to $|E/I|$ for horizontal polarization for wire of $l = 1 \text{ m}$
  - High frequency asymptote determined by monopole formed between cable and ground plane (details in backup slides)
To be presented by John McCloskey and video recorded at the 2016 IEEE International Symposium on Electromagnetic Compatibility, Ottawa, Canada, July 26, 2016.
**Current Measurement Helpful Hint**

- **For** $f > 30 \text{ MHz}$, **cables of** $l > 1 \text{ m}$ **will exhibit standing waves**
  - Current will NOT be constant along length
  - Note that $|E/I|$ is based on integrated, i.e. average, current along cable

- **Traditional approach**
  - Place spectrum analyzer in max hold mode
  - Physically scan probe along length of cable
  - Relatively quick and easy, but this will give the peak current on cable, not average
  - Will overestimate current and electric field

- **Recommended approach**
  - Measure current at many locations along wire
  - Take average
  - If measuring in dB, convert to numeric first
Summary

- Radiated emissions and conducted emissions "joined at the hip"
- \(|E/I|\) transfer function envelope provides useful tool for predicting radiated emissions early in product development cycle
- The current probe is your friend; measure those common mode currents early and often
  - Easy and useful measurement to make in hardware development lab before taking product to EMI facility
  - Don't have to fight RF background, room resonances, etc.
  - Identify specific sources of potential problems as early as possible
- Control those common mode currents
  - Provide the desired low impedance path (e.g. shield, ground plane, etc.) and make them flow where you want them to
  - Let nature do the work
- When you control common mode currents, you go a long way toward controlling radiated emissions and most other EMI problems
Backup
Electric Field as Function of Current
(Electrically Short Cable, Hertzian Dipole Model)

\[ E_\theta = \frac{Idl}{4\pi} \eta_0 \beta_0^2 \sin \theta \left( j \frac{1}{\beta_0 r} + \frac{1}{\beta_0^2 r^2} - j \frac{1}{\beta_0^3 r^3} \right) e^{-j\beta_0 r} \]

\[ |E_\theta|_{\text{max}} = \frac{dl}{4\pi} \eta_0 \beta_0^2 \sqrt{\left( \frac{1}{\beta_0^2 r^2} \right)^2 + \left( \frac{1}{\beta_0 r} - \frac{1}{\beta_0^3 r^3} \right)^2} \]

**Far field approximation** \((\beta_0 r > 1)\):

\[ |E_\theta|_{\text{max}} \approx \frac{l}{4\pi} \eta_0 \beta_0^2 \sqrt{\left( \frac{1}{\beta_0 r} \right)^2} = \frac{l}{4\pi r} \eta_0 \beta_0 \]

But: \( \beta_0 = \frac{2\pi}{\lambda} = \frac{2\pi f}{c} = 2\pi f \sqrt{\mu_0 \varepsilon_0} \) and \( \eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} \rightarrow \eta_0 \beta_0 = 2\pi f \sqrt{\mu_0 \varepsilon_0} \cdot \sqrt{\frac{\mu_0}{\varepsilon_0}} = 2\pi f \mu_0 \)

\[ |E_\theta|_{\text{max}} \approx \frac{l}{4\pi r} 2\pi f \mu_0 \]

\[ \frac{|E_\theta|_{\text{max}}}{I} \approx \frac{\mu_0 l}{2r} f \]
Transmission Line Currents

Horizontal:

\[ I_H = \frac{1}{l} \int_0^l I_0 \cos \beta z \, dz = \frac{I_0}{\beta l} \sin \beta l \Bigg|_0^l = I_0 \frac{\sin \beta l}{\beta l} \]

- \( \beta l \leq 1 \): \[ \left| \frac{I_H}{I_0} \right| \Bigg|_0^l = 1 \]
- \( \beta l \geq 1 \): \[ \left| \frac{I_H}{I_0} \right| \Bigg|_0^l = \frac{1}{\beta l} \]

Vertical:

\[ dI_d (z) \Delta z = I(z) - I(z + \Delta z) \]

\[ dI_d (z) = \frac{I(z) - I(z + \Delta z)}{\Delta z} = \frac{dI(z)}{dz} = -\beta I_0 \sin \beta z \]

\[ I_V = -\beta I_0 \int_0^l \sin \beta z \, dz = I_0 \cos \beta l \Bigg|_0^l = I_0 (\cos \beta l - 1) \]

\[ \left| \frac{I_V}{I_0} \right| \Bigg|_0^l = 2 \]
Vertical (Displacement) Current

![Graph showing the relationship between frequency and vertical (displacement) current for different lengths of the discharge path.](image)

- *Envelope* curve
- $l = 2 \text{ m}$
- $l = 1 \text{ m}$
- $l = 50 \text{ cm}$
- $l = 25 \text{ cm}$

The graph plots the dimensionless vertical current $I_v/I_0$ against frequency (MHz) for various lengths of the discharge path.
Response and Envelope of $\frac{\sin(x)}{x}$

Response of $|\frac{\sin(x)}{x}|$:
- $0$ for $x = n\pi$
- $1/x$ for $x = (n+1)\pi/2$

Envelope of $|\frac{\sin(x)}{x}|$:
- $1$ for $x \leq 1$
- $1/x$ for $x \geq 1$
Horizontal E-Field (No Ground Plane Attenuation)

\[
\frac{E_{W}}{I_0}_{ENV} = \frac{\mu_0lf}{2r} \cdot \frac{1}{\beta l} = \frac{\mu_0lf}{2r} \cdot \frac{1}{2\pi f \sqrt{\mu_0 \varepsilon_0}} = \frac{1}{4\pi r} \sqrt{\frac{\mu_0}{\varepsilon_0}} = \frac{120\pi}{4\pi r}
\]

\[
\frac{E_{W}}{I_0}_{ENV} = \frac{30}{(1m)} = 30 \cdot \Omega / m = 30 \cdot dB\Omega / m
\]

\[
\frac{E_{W}}{I_0}_{ENV} = 4\pi \times 10^{-7} \cdot \frac{f}{2(1m)} = \left(4\pi \times 10^{-7}\right)lf
\]

\[
\frac{E_{W}}{I_0}_{ENV, dB} = 20\log_{10} f_{MHz} + 20\log_{10} l - 4dB
\]
Ground Plane Attenuation Horizontal Polarization Only

\[ E_H = \frac{\mu_0 I_f}{2r} I_H e^{-j\beta r} - \frac{\mu_0 I_f}{2(r + 2h \cos \theta)} I_H e^{-j\beta(r + 2h \cos \theta)} \]

\[ E_H = E_W \left[ e^{-j\beta r} - \left( \frac{r}{r + 2h \cos \theta} \right) e^{-j\beta(r + 2h \cos \theta)} \right] \]

\[ \left| \frac{E_H}{E_W} \right| = \left| \frac{2h}{r + 2h} \cos \beta \right| \theta \cos \left( \frac{1}{2} \beta h \right) \approx 20 \text{dB} \]

\[ \left| \frac{E_H}{E_W} \right| \approx \left| \cos \beta r - \cos \beta \theta \right| \]

\[ \cos A - \cos B = -2 \sin \left( \frac{A + B}{2} \right) \sin \left( \frac{A - B}{2} \right) \]

\[ \left| \frac{E_H}{E_W} \right| \approx 2 \sin \left( \beta h \right) \]

2\( \beta h \leq 1 \):

2\( \beta h \geq 1 \):
Ground Plane Attenuation (Gain, Really)

Full Calculation
Envelope 1
Envelope 2

Frequency (MHz)

$E_H/E_W$ (dB)
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Electric field from tuned dipole:

\[
E_{\text{MAX}} = \frac{60I}{r} = \frac{60I}{1m} = 60I \\
\left| \frac{E_V}{I_0} \right|_{\text{ENV,db}} = 36 \cdot dB\Omega/m
\]

\[
\left| \frac{E_V}{I_0} \right|_{\text{ENV}} = \frac{\mu_0 hf}{2r} \\
\left| \frac{E_V}{I_0} \right|_{\text{ENV}} = \frac{\mu_0 hf}{r} \\
\left| \frac{E_V}{I_0} \right|_{\text{ENV,db}} = 20 \log_{10} f_{\text{MHz}} - 24 dB
\]
MIL-STD-461 RE102 Limits

Fixed Wing Internal, ≥25 meters Nose to Tail

Fixed Wing Internal, < 25 meters Nose to Tail

Fixed Wing External (2MHz to 18 GHz) and Helicopters

Frequency (Hz)

Limit Level (dBuV/m)

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Equivalent CMCE Limits