Wall-modeled Large Eddy Simulation of Flow Past a Wall-Mounted Hump

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FD-03: Turbulence Modeling 1: LES, DNS, Hybrid LES/RANS  
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Motivation

- Most RANS models work reasonably well for attached flows, but do not accurately predict flows involving separation.

- High fidelity DNS/ LES infeasible for realistic high $Re$ encountered in flight.

<table>
<thead>
<tr>
<th>DNS</th>
<th>LES</th>
<th>WMLES</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Re_{Lx}^{37/14}$</td>
<td>$Re_{Lx}^{13/7}$</td>
<td>$Re_{Lx}$</td>
</tr>
</tbody>
</table>

- Wall-Modeled LES (WMLES) and hybrid RANS-LES methods show good promise for high $Re$ complex turbulent flows.

1 Figure taken from: https://en.wikipedia.org/wiki/Boeing_787_Dreamliner
Wall-mounted hump geometry\textsuperscript{1}

- Standard test case for Revolutionary Computational Aerosciences (RCA) under Transformational Tools & Technologies (TTT) Project.

- Detailed experimental data available for validation.


- Up to 35\% error in bubble length for most RANS models.

\textsuperscript{1} Figures taken from: http://turbmodels.larc.nasa.gov/StandardTestCasesFinal6.pdf
Outline of Presentation

• Experiment and Simulation details

• Equilibrium Wall Model description

• Results
  → Inflow turbulence validation
  → Qualitative flow features
  → Comparisons to experiment

• Summary
Experiment Details

- $M_\infty = 0.1$, $Re_c = 929000$
- $Re_\theta \sim 7,200$ and $Re_x \sim 4 \times 10^6$ at $x/c = -2.14$
- Hump chord, $c = 420$ mm. Hump height, $h = 53.7$ mm
- Hump model was $584$ mm wide ($1.39$ $c$) with side-mounted end plates

Simulation Details

- $M_\infty = 0.1$, $Re_c = 936000$, $Re_\theta \sim 7,200$ at $x/c = -2.14$ matched with experiment.
- Simulations performed using the compressible *Charles* solver from Cascade Technologies.
- 2\textsuperscript{nd} order cell-centered, unstructured finite volume spatial discretization with explicit RK3 time stepping.
- Constant coefficient Vreman SGS model and Equilibrium Wall Model.
- Contoured top wall to account for effect of end plates.
- Small slot at $x/c \sim 0.65$ of width $4 \times 10^{-3} c$ not modeled in the current simulations.
- Synthetic method used to specify inflow turbulence.
Equilibrium Wall Model Equations

- Compressible equilibrium BL ODEs solved in WM region.

\[ \frac{d}{d\eta} \left( (\mu + \mu_{t,wm}) \frac{du_\parallel}{d\eta} \right) = 0, \]

\[ \frac{d}{d\eta} \left( (\mu + \mu_{t,wm}) u_\parallel \frac{du_\parallel}{d\eta} + (\lambda + \lambda_{t,wm}) \frac{dT}{d\eta} \right) = 0, \]

- Eddy viscosity obtained from mixing-length model and turbulent thermal conductivity obtained assuming constant Pr_t=0.9.

\[ \mu_{t,wm} = \kappa \eta \sqrt{\rho \tau_w} \left[ 1 - \exp \left( -\frac{\eta^+}{A^+} \right) \right]^2, \quad \text{with } A^+ = 17, \quad \kappa = 0.41. \]

1 Figure taken from: Bodart & Larsson, AIAA-2012-3022.
Equilibrium Wall Model


- Additional cost: ~10-30% for equilibrium WM. (100-150% for non-equilibrium WM, Park JCP 2016). Also, parallel efficiency sub-optimal for < 50,000 cells/core (Park, JCP 2016).

- Traditionally WMLES uses LES information from the 1st grid point away from the wall.

- Kawai & Larsson (PoF 2012) showed that this could lead to an erroneous wall shear stress prediction for a flat plate boundary layer and using information from the 3rd or away grid point significantly improves results.

- Effect of exchange location in separated flows needs to be examined.
Simulation Details

<table>
<thead>
<tr>
<th>Case</th>
<th>Grid</th>
<th>Grid points</th>
<th>Inflow plane</th>
<th>Wall Model applied at</th>
</tr>
</thead>
<tbody>
<tr>
<td>cWM1</td>
<td>Coarse</td>
<td>4.4 million</td>
<td>-2.29</td>
<td>1st grid point</td>
</tr>
<tr>
<td>cWM3</td>
<td>Coarse</td>
<td>4.4 million</td>
<td>-2.29</td>
<td>3rd grid point</td>
</tr>
<tr>
<td>fWM1</td>
<td>Fine</td>
<td>11 million</td>
<td>-3.0</td>
<td>1st grid point</td>
</tr>
<tr>
<td>fWM3</td>
<td>Fine</td>
<td>11 million</td>
<td>-3.0</td>
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</tr>
</tbody>
</table>

- 4 different simulations performed to study effect of grid resolution and exchange location.
- Spanwise width of the domain = 0.3 c.
- Simulations run for 12-15 c/u∞ after which statistics were collected for 15 c/u∞.
- Coarse grid simulations ran in ~ 6 days with 320 cores (Intel Westmere X5675) for 30 c/u∞ while fine grid took 14 days.
- 3rd grid point of fine grid roughly corresponds to the 1st grid point in the coarse grid.
- Wall-resolved LES calculation for the same domain size would require ~ 400 million grid points for Δx+ ~ 50, Δy_min+ ~ 1 and Δz+ ~ 25.
Grid Spacing

**Coarse grid**

- 2.29 < x/c < 0
  - 360, 72, 180
  - 0.1
  - 10, 32, 20
- 0 < x/c < 1.1
  - 514, 25, 260
  - 0.07
  - 7, 32, 14
- x/c > 1.1
  - 300, 100, 150
  - 0.2
  - 20, 32, 40

**Fine grid**

- -3 < x/c < 0
  - 360, 36, 180
  - 0.1
  - 10, 32, 20
- 0 < x/c < 1.1
  - 77, 10, 260
  - 0.07
  - 46, 32, 14
- x/c > 1.1
  - 300, 100, 150
  - 0.2
  - 20, 32, 40

- **Coarse grid**: 814, 90 and 60 points in x, y and z, 222 points on hump.
  - **Fine grid**: 1377, 140 and 60 points in x, y and z, 771 points on hump.

- n_x/δ ~ 5-32, n_y/δ ~ 16-32, n_z/δ ~ 15-32 commonly used in WMLES (Choi & Moin, PoF 2012).
Comparison to experiment at $x/c = -2.14$

Turbulence not fully developed at this location. Typically $\sim 20\delta$ needed to get fully developed turbulence for synthetic turbulence inflow.

Distance from inflow $\sim 2\delta$ for coarse grid (inflow at $x/c = -2.29$) and $\sim 13\delta$ for fine grid (inflow at $x/c = -3$), hence, better agreement for fine grid.
Inflow Turbulence Validation

- Incompressible turbulent boundary layer mean data at \( Re_\theta = 7000 \) from resolved LES of Eitel-Amor et al.\(^1\) specified at inflow of domain.

- Turbulence statistics at \( x/c = -0.5 \) compared to incompressible TBL data at \( x/c \sim -2.2 \).

- Reasonable agreement indicating turbulence is fully developed.

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• Iso-contours of Q-criterion colored by instantaneous streamwise velocity to depict vortices for fWM3.
• WMLES captures the hairpin-shaped eddies in the outer layer of the boundary layer.
• Instantaneous streamwise velocity contours for fWM3
• Fine scales seen over the hump at y/c=0.15
• Spanwise oscillation of the separated region at y/c=0.05
• Instantaneous streamwise velocity and spanwise vorticity contours for fWM3

• Vortices in the boundary layer of the flow over the hump clearly visible

• Flapping of the separation shear layer
Separation Bubble Characteristics

- Qualitatively similar to experiment
- Overall, fine grid simulations with LES information from 3rd grid point agrees best with experiment
Separation Bubble Characteristics

<table>
<thead>
<tr>
<th>Case</th>
<th>Separation location (x/c)</th>
<th>Reattachment location (x/c)</th>
<th>Bubble length (∆x/c)</th>
<th>Error in bubble length (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expt.</td>
<td>0.665 ± 0.005</td>
<td>1.10 ± 0.005</td>
<td>0.435</td>
<td>-</td>
</tr>
<tr>
<td>cWM1</td>
<td>0.615</td>
<td>1.08</td>
<td>0.465</td>
<td>6.9</td>
</tr>
<tr>
<td>cWM3</td>
<td>0.655</td>
<td>1.045</td>
<td>0.39</td>
<td>10.3</td>
</tr>
<tr>
<td>fWM1</td>
<td>0.637</td>
<td>1.035</td>
<td>0.398</td>
<td>8.5</td>
</tr>
<tr>
<td>fWM3</td>
<td>0.655</td>
<td>1.105</td>
<td>0.45</td>
<td>3.4</td>
</tr>
</tbody>
</table>

- Separation and reattachment locations reported based on mean streamlines
- fWM3 agrees within 3.4% of the experimental value for bubble length
Wall pressure comparisons

- Reference pressure chosen to match the experimental $C_p$ upstream of the hump.

- Comparison looks reasonable except in the separation region.

- cWM3 closer to experiment but still needs improvement.
Wall pressure comparisons

- **Coarse grid**
- **Fine grid**

- fWM3 agrees well with experiment.
• All simulations qualitatively agree with experimental data with fWM3 showing good quantitative agreement with experiment.

• Pressure fluctuations higher near separation. Appears to be unphysical based on data from other separated flows. Possible reasons: Grid smoothness.
All simulations qualitatively agree with experimental data with fWM3 showing good quantitative agreement with experiment.

Pressure fluctuations higher near separation. Appears to be unphysical based on data from other separated flows. Possible reasons: Grid smoothness.
Smoothness of hump geometry

- Non-smoothness of curvature at $x/c\sim0.65$ could be responsible for the peak in $C_p$.

- Similar peak but lower magnitude also observed in resolved LES simulations of Uzun$^1$.

$^1$Dr. Ali Uzun, NASA Langley Research Center, Personal Communication.
Wall skin friction comparisons

- $C_f$ obtained by averaging wall shear stress obtained from the wall model.

- Reasonable agreement in the attached regions but significant differences in the separated region

- Equilibrium wall model does not account for streamwise gradients.

- Negligible differences in $C_f$ between applying the wall model at the 1st and 3rd grid point for the coarse grid.
Wall skin friction comparisons

- Reasonable agreement in the attached regions but significant differences in the separated region for fWM3
Wall skin friction comparisons

- Reasonable agreement in the attached regions but significant differences in the separated region for fWM3.
- fWM1 qualitatively captures the double peak at x/c~0.15 and minimum $C_f$ at x/c~0.8 but severely underpredicts $C_f$ over the hump.
- fWM1 predicts early reattachment as compared to experiment.
Separation region: Velocity comparisons

- **Legend**: Solid red: cWM1, Dashed red: cWM3, Solid blue: fWM1, Dashed blue: fWM3
- Note the larger unsteadiness near separation in the simulations.
- Overall, fWM3 agrees best with experiment.
Separation region: Stress comparisons

\[ \frac{u'u'}{u_\infty^2} \]

Legend: Solid red: cWM1, Dashed red: cWM3, Solid blue: fWM1, Dashed blue: fWM3

- \( \overline{u'u'} \) overpredicted by a factor of 5-6 at \( x/c=0.65 \)

- Resolved LES of Morgan, Rizzetta & Visbal (2007)\(^1\) also overpredict by a factor of 6 for a lower \( Re_c = 200,000 \)

- Consistent with peak in \( C_p' \) at \( x/c=0.65 \)

Reasonable agreement overall

fWM3 agrees better with experiment near wall while fWM1 agrees better in the upper half of the separation bubble.
Summary

• Fine grid with exchange location away from the wall gave best agreement with experiments.

• Length of separation bubble predicted within 3.4% of the experimental value.

• Results suggest that WMLES is a promising technique to study high $Re$ turbulent flows involving separation at reasonable cost.

• **Future Work:** Apply WMLES to other problems such as Axisymmetric transonic bump and Juncture flow.
Velocity Profiles on and downstream of the Hump

On hump

Downstream of hump
Comparisons to other WMLES data at x/c=-0.81

- Comparison to incompressible WMLES of Avdis et al. at x/c=-0.81
- Good agreement between fine grid and Avdis data
- Coarse grid turbulence still not fully developed.
- Some of the differences can be attributed to compressibility effects and the different wall models used.
Possible reasons for discrepancies

- Mean SGS viscosity shown on left indicating regions that are least resolved.
- May require a higher resolution in the wake where SGS viscosity is high.

Less resolved regions

- Grid too coarse in $x$.
- Boundary layer on the hump not sufficiently captured in $y$.

Other considerations (apart from grid refinement):

1. VREMAN model (should we use DSM which might perform better in separated regions?)
2. Inflow turbulence. Is the digital filtering technique good enough?
3. Validity of equilibrium assumption used in the wall model in highly separated regions.
4. Currently using convective BCs at outflow by prescribing inflow pressure. Change pressure or use sponge BCs at the outflow?