FEAMAC/CARES
Stochastic-Strength-Based Damage Simulation Tool for Ceramic Matrix Composites

Noel Nemeth\textsuperscript{1}, Brett Bednarcyk, Evan Pineda, Steven Arnold, Subodh Mital\textsuperscript{2}, Pappu Murthy, Ramakrishna Bhatt\textsuperscript{3}

Multiscale & Multiphysics Branch, NASA Glenn Research Center

United States Advanced Ceramics Association (USACA) 40\textsuperscript{th} Annual Conference on Composites, Materials, and Structures

January 25-29, 2016, Radisson Resort at the Port, Cocoa Beach, Florida

\textsuperscript{1}noel.n.nemeth@nasa.gov ; \textsuperscript{2}University of Toledo, Toledo, Ohio ; \textsuperscript{3}Ohio Aerospace Institute, Cleveland Ohio
Scope, Technical Challenge & Approach

- Predict the strength and service life of ceramic composite structures

- **Need to account for:**
  - Wide variability in the strength of individual components (probabilistic/stochastic strength)
  - How damage response changes with loading types (multiaxial loading, flexural loading, size effect)
  - How composite architecture effects strength/damage response
  - How strength degrades with time and fluctuating load

- **Approach → Combine two NASA developed codes:**
  - (MAC/GMC) : composite micromechanics analysis &
  - (CARES/Life) : probability of failure prediction of ceramic components coupled to commercial finite element analysis (Abaqus)
Outline

1. Overview: Describe the MAC, CARES, & FEAMAC/CARES codes
   - Batdorf Unit Sphere stochastic-strength failure criteria

2. Applying CARES to the MAC code to simulate stochastic damage progression in a ceramic matrix composite (CMC)
   - **Cellular Automaton**: Encouraging failure of adjacent elements - mimics crack-like growth

3. Stress-strain response of a SiC-RBSN laminate (literature circa 1990)
   - Off-axis loading
   - Double-notched tensile specimen

4. On-going work: Flexure-bar simulation – *Is there a Weibull size effect?*
MAC/GMC Micromechanics Analysis Code

**FEAMAC**: MAC/GMC embedded in FEA as constitutive material

- Repeating Unit Cell (RUC) of composite material
  - RUC made of material subcells
  - Multiscale capability

CARES/Life: Life Prediction Code For Advanced Ceramics

- Predicts the probability of failure of ceramic components under thermomechanical loading
- Combines Weibull & Weakest Link theory with concepts of linear elastic fracture mechanics (the Batdorf Unit Sphere model)
- CARES is a post-processor to FEA

**Component Reliability Analysis Capability:**
- Transient loads and temperatures
- Fast-Fracture Rupture
- Time-dependent (da/dt) crack growth
- Cycle-dependent (da/dn) crack growth
- Multiaxial stress failure models (*PIA & Unit Sphere & Tsai-Wu & Tsai-Hill*)
- Proof test
Approach for Life Prediction & Component Design of Composites

- **Combine CARES, MAC & FEA codes**

  Move CARES from the macroscopic scale of the structure to the microscale of the individual RUC material constituents

- **FEAMAC/CARES Capability:**
  - Individual constituent and component level probability of failure tracked (for failure initiation)
  - Individual & concurrent failure modes
  - Laminate level analysis capability
  - Progressive damage capability/simulation
  - Subcells killed at random failure thresholds

  *Debonding/crack path physics at constituent level not explicitly included*

FEAMAC/CARES

- Abaqus UMAT “User Material”
- Structural-Scale FEA
- Element/Integration Point
- Micromechanics Analysis

(MAC/GMC)

(CARES) Subroutine

Reliability analysis at the RUC level
Fracture-mechanics-based failure criteria to predict probability of failure/damage of a material constituent over time

- Crack shape
- Mixed-mode fracture criterion

- Two models for anisotropic strength response:
  - $K_{lc}$ / Critical strength
  - Flaw orientation bias

The unit sphere model is applied at the constituent level of the composite

Has been extended to anisotropy

Unit Sphere Probability Density Distribution For Orientation Of Critical Flaws for anisotropic material constituent

(Isotropic Material)

Most probable angle (normal to flaw plane)

(Uniaxial tension)

(Equibiaxial tension)

(Shear: shear-sensitive flaws)

(Uniaxial compression: Compression criterion)

(Transversely isotropic ($K_{lc}$) Material)

For a 15° offset uniaxial load
Random Element Failure vs: Neighbor Influenced Failure
(Cellular Automaton Enhancement)

Encourage more abrupt failure and “crack-like” damage growth patterns

failure probability thresholds of elements adjacent to failed elements adjusted to promote a biased damage direction according to rules defined for a cellular automaton

Random element failure
➢ simulates stochastic toughening

Example: 0° Ply uniaxial ramp load
25x25 FEA mesh of shell elements

With cellular automaton Rules
➢ “crack-like” growth patterns
Example: SiC/RBSN Laminated Composite in *On-Axis & Off-Axis* Loading

- Tested by Bhatt & Phillips (1990)

- displays key mechanisms/features for model material

- SCS-6 fiber/Reaction Bonded Silicon Nitride matrix composite examined in detail by NASA ➢ several papers published

- Laminated CMCs of interest to industry and less complex than woven composites
  ➢ failure modes are not conflicted with complex fiber architecture

- [0] & [0/90] laminates display nonlinearity due to matrix failure, followed by fiber failure.

- Remaining ply orientations display sudden brittle failure.

---

Rectangular specimens under uniaxial tensile loading
25.4 mm x 12.7 mm x 1.2 mm
30% fiber volume fraction

Unreinforced RBSN
[45]_8
[90]_8

[0]_8

[10]_8

[0/90]_8

[+45/-45]_8

Matrix cracking

Full details of stress-strain response not available

Experimental Results

Bhatt & Phillips (1990)

SiC/RBSN

Strain, %

Strain, %

Stress (Mpa)
SiC/RBSN Example Procedure & Setup

Stochastic strength analysis:
(from individual trials / simulations / realizations)

- Use CARES Unit Sphere failure criterion
  - assume isotropic material constituent strength
    – for simplicity and initial testing

1) Cool down from stress-free temperature of 550° to room temperature 23°
2) apply fixed-displacement ramp load

Residual stresses in constituents

a) Calibration of model:
  Correlate Weibull parameters and “stress-free” temperature to experimental results for 0° tensile specimen

b) Prediction of damage response for off-axis plys and laminates

- Interface strength made large: Encourage matrix to fail before interface

Abaqus FEA
S4 Shell elements
Fixed-displacement ramp load

Fiber direction
0°
Loading Direction

(10x20 mesh)
Model Calibration: Effect of matrix Weibull modulus on stress-strain response

0° single ply tensile specimen

Calibrating to experimental data

Matrix damage

Stress (MPa)

0 50 100 150 200 250

Strain

0 0.0005 0.001 0.0015 0.002 0.0025

$m_V$

- 5
- 10
- 20
- 60
Model Calibration: Constituent properties of SiC/RBSN with anisotropic thermal expansion coefficients

<table>
<thead>
<tr>
<th>Constituent</th>
<th>Modulus, GPa</th>
<th>Poisson ratio</th>
<th>Longitudinal coefficient of thermal expansion, $\alpha_L$ (m/m/°C)</th>
<th>Transverse coefficient of thermal expansion, $\alpha_T$ (m/m/°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fiber</td>
<td>390</td>
<td>0.17</td>
<td>4.1\times10^{-6}</td>
<td>1.84\times10^{-6}</td>
</tr>
<tr>
<td>Matrix</td>
<td>110</td>
<td>0.22</td>
<td>2.2\times10^{-6}</td>
<td>2.2\times10^{-6}</td>
</tr>
<tr>
<td>Interface</td>
<td>1.8</td>
<td>0.22</td>
<td>2.0\times10^{-6}</td>
<td>2.0\times10^{-6}</td>
</tr>
</tbody>
</table>

Assumed Weibull Parameters:

- **Fiber**: $m_V = 20$, $\sigma_{oV} = 2875$ Mpa $\cdot$ m$^{3/20}$
- **Matrix**: $m_V = 5.0$, $\sigma_{oV} = 150$ Mpa $\cdot$ m$^{3/5}$
- **Interface**: $m_V = 5.0$, $\sigma_{oV} = 80$ Mpa $\cdot$ m$^{3/5}$
**Model Calibration: 0° Single Ply**

*Non-linear (graceful) failure behavior*

Calibrated to experimental data for 24% fiber volume fraction

**Calibrated to 550° C stress-free temperature**

**Experimental Data:**
- Matrix Frac. Str.: 227 ± 41 MPa
- Ultimate Str: 682 ± 150 MPa

**Predicted response for 30% fiber volume fraction**

**Individual simulation**

**Fiber** $m_V = 20 \quad \sigma_{OV} = 2875 \text{ Mpa} \cdot m^{3/20}$

**Matrix** $m_V = 5.0 \quad \sigma_{OV} = 150 \text{ Mpa} \cdot m^{3/5}$

**Interface** $m_V = 5.0 \quad \sigma_{OV} = 80 \text{ Mpa} \cdot m^{3/5}$
0° Single Ply

Strength scatter prediction from simulations for proportional limit strain offset

- 0.005% strain offset
- 0.01% strain offset

Linear elastic

Intersection of simulation trial with strain offset

1-of-10 Individual simulation trials

FEAMAC/CARES:
Mean = 238.2 MPa
Std. Dev. = 10.28 MPa

Experimental Data:
Matrix Frac. Str.: 227 ± 41 Mpa

**PLS is defined as the stress at 0.005% strain offset:**

Model Calibration & Prediction: Ten Trials for 90° Fiber Orientation

Assuming matrix and interface are isotropic strength materials

FEAMAC/CARES analysis was for a single ply to speed computation

\[ [90]_8 \]

Brittle behavior

Note: very few specimens were tested which means the range of uncertainty (the confidence bounds) for \( m \) is large!

Experimental Data:
Fracture Stress: 27 ±3 MPa

1-of-10 Individual simulation trials

CARES calculated 50% matrix failure probability prior to any damage initiation
CARES calculated 50% matrix failure probability prior to any damage initiation.

FEAMAC/CARES analysis was for a single ply to speed computation.

Predictions for Ten Trials for 10° and 45° Fiber Orientations Assuming matrix and interface are isotropic strength materials.

10° Fiber Orientation

45° Fiber Orientation

Experimental Data:
Frac. Str.: 162 MPa

Experimental Data:
Frac. Str.: 43 MPa

1-of-10 Individual simulation trials

CARES calculated 50% matrix failure probability prior to any damage initiation.
Prediction for Ten Trials for $[+45_2/-45_2]_s$ Fiber Orientation
Assuming matrix and interface are isotropic strength materials

FEAMAC/CARES analysis was for four plys ($+45/-45/-45/+45$) to speed computation

$[+45/-45]_s$

Matrix cracks approx. normal to loading direction

Note: very few specimens were tested which means the range of uncertainty (the confidence bounds) for $m$ is large!

Experimental Data:
Matrix Frac. Str.: $75 \pm 10$ MPa; $m \approx 8.8$
Ultimate Str: $88 \pm 16$ MPa; $m \approx 6.3$

Neither graceful or brittle behavior

CARES calculated 50% matrix failure probability prior to any damage initiation

1-of-10 Individual simulation trials
FEAMAC/CARES analysis was for four plys (0/90/90/0) to speed computation. Note: very few specimens were tested which means the range of uncertainty (the confidence bounds) for $m$ is large!

Experimental Data:
Matrix Frac. Str.: $127 \pm 26$ MPa
Ultimate Str: $294 \pm 87$ MPa

1-of-10 Individual simulation trials

Non-linear (graceful) failure behavior
0° Double-Notched Tensile Specimen
Failure mode showed axial splitting of matrix

FEAMAC/CARES analysis was for a single ply to speed computation

Experiment

Fiber Direction

Axial splitting

Loading Direction
0° Double-Notched vs: Central-Hole Tensile Specimen

Loading Direction

Early matrix damage

Matrix damage progression

- **Matrix failure**
- **Adjacent to failed matrix**
- **Fiber failure**
- **Adjacent to failed fiber**
- **No failure**
**Objective**
- Finite element models using solid elements of 3-point bending flexure, 4-point bending flexure, and a tensile specimen have been prepared.

**Approach**
- Demonstrate that FEAMAC/CARES is functional with Abaqus solid elements (not been verified previously)
- Progressive damage simulations of a unidirectional fiber oriented CMC are on-going.

**Next Steps**
- The models will be interrogated regarding predictions on Weibull size effect comparing the three specimen geometries
Cut view of 3-pt bend bar showing damage at mid-plane

Damage propagates through the thickness on the tensile side but on the compression side it propagates laterally across the width. The compression failure criterion was not active, so failure is coming from shear failure on the unit sphere in the tensile domain.

Matrix failure  Adjacent to failed matrix
Fiber failure   Adjacent to failed fiber
No failure
Conclusions

• Progressive damage simulation of composite structures incorporating probabilistic material strength models is possible with the FEAMAC/CARES code

• The Unit Sphere multiaxial model was used to predict the strength response of a SiC-RBSN composite for various fiber orientations under uniaxial tension

• Reasonable correlation to matrix cracking strength experimental data was achieved assuming the matrix was an isotropic material with $m \approx 5$, and assuming residual stresses from thermal processing were present

• Brittle behavior vs: non-brittle failure (*graceful failure*) demonstrated

• Localized damage modes at stress concentration features shown

• *Component level probability of first damage initiation event tracked at each load increment*

*Future work:* Try to simulate EBC failure modes?

Acknowledgement

This work was funded by the NASA Transformative Tools and Technologies Program

noel.n.nemeth@nasa.gov
Extra Material
Abstract:
Reported here is a coupling of two NASA developed codes: CARES (Ceramics Analysis and Reliability Evaluation of Structures) with the MAC/GMC (Micromechanics Analysis Code/ Generalized Method of Cells) composite material analysis code. The resulting code is called FEAMAC/CARES and is constructed as an Abaqus finite element analysis UMAT (user defined material). Here we describe the FEAMAC/CARES code and an example problem (taken from the open literature) of a laminated CMC in off-axis loading is shown. FEAMAC/CARES performs stochastic-strength-based damage simulation response of a CMC under multiaxial loading using elastic stiffness reduction of the failed elements.
Some References:


Progressive Damage Criterion

Calculate failure probability, $P_f$, for each material constituent of the RUC associated with an element integration point.

- **CARES** calculated $P_f^{(CARES)}$ of RUC
- Random number generated $P_f^{(Random)}$ of RUC

**If** $P_f^{(CARES)} \geq P_f^{(Random)}$ 
- **Yes**
  - Fail all material constituent subcells
  - Kill elastic modulus
- **No**
  - Don’t fail subcells

Encourages more rapid damage propagation than failing individual subcells.
Residual matrix stresses after cool-down from temperature

- Effect of anisotropic fiber-thermal-expansion-coefficient, $\alpha_f$ on RUC

"The fiber is orthotropic, with different coefficients of thermal expansion along and perpendicular to the fiber axis."

0° Single Ply

**Actual stress-strain curve from:**
Chulya, A., Gyekenyesi, J. P., and Bhatt, R. (1991);
“Mechanical Behavior of Fiber Reinforced SiC/RBSN Ceramic Matrix Composites: Theory and Experiment. NASA TM 103688;

**Feamac/CaresCalibrated**
to 550° C stress-free temperature & 24% fiber volume fraction, V_f

**Calibration (24% V_f)**

**Prediction (30% V_f)**

**Approx. exp. Curve (30% V_f)**

- CARES calculated 50% matrix failure probability prior to any damage initiation
- Exp. 24% fiber vol.
- Exp. 30% fiber vol.
- 50% Fail Prob 24%
- 50% Fail. Prob. 30%
- FEAMAC/CARES 24%
- FEAMAC/CARES 30%

Effect of matrix fragments on fiber response not modeled.
**0° single ply tensile specimen**

Progression of damage in FE model of a unidirectional ply under longitudinal loading

**Not Adjusted**

- **a**: Early diffuse damage
- **b**: Damage clustering
- **c**: Final fiber failure

**Automaton Adjusted**

- **d**: Early crack-like damage
- **e**: Matrix damage more organized
- **f**: Final fiber failure

**Legend**

- Yellow: Matrix failure
- Green: Adjacent to failed matrix
- Red: Fiber failure
- Orange: Adjacent to failed fiber
- Blue: No failure

*(a) and (d)*; early matrix damage

*(b) and (e)*; progression to substantial matrix damage

*(c) and (f)*; final composite failure (fiber failure)
Damage progression of 0º tensile specimen - two trials (undeformed plot)
90° single ply tensile specimen

Progression of damage in FE model of a unidirectional ply under *transverse* loading

**Not Adjusted**
- **a:** Early diffuse damage
- **b:** Final matrix failure

**Automaton Adjusted**
- **c:** Adjacent elements encouraged to fail in early damage stages
- **d:** Final matrix failure

- **Matrix failure**
- **Adjacent to failed matrix**
- **No failure**

**(a) and (c) ; early matrix damage**
**(b) and (d) ; final composite failure (matrix failure)**
90° Tensile specimen *at final failure for 10 trials* – Undeformed plots

Final specimen failure from matrix damage

Loading Direction

1 2 3 4 5

6 7 8 9 10
10° off-axis tensile specimen; 10 trials at final (matrix) failure; deformed plots

- Edges are allowed to freely deform (warp) on cool-down
- After cool-down; bottom edge fixed in loading direction when displacement load applied
- After cool-down; single node along top edge (middle) fixed in direction perpendicular to displacement direct.
For $[+45/-45]_s$ Fiber Orientation; 10 trials at final (matrix) failure; deformed plots

FEAMAC/CARES analysis was for four plys ($+45/-45/-45/+45$) to speed computation
Prediction for Ten Trials for $[0_2/90_2]_s$ Fiber Orientation

Strength scatter from proportional limit strain offset

0.005% strain offset

0.01% strain offset

Linear elastic

Intersection of simulation trial with strain offset

1-of-10 Individual simulation trials

Experimental Data:
Matrix Frac. Str.: 127 ±26 MPa

FEAMAC/CARES:
Mean = 133.3 MPa
Std. Dev. = 8.11 MPa
0° single ply tensile specimen (Load parallel to fiber axis)

Mesh effect & time step sensitivity

CARES calculated 50% matrix failure probability prior to any damage initiation

Trade off: Mesh sensitivity vs: localization of damage
Unit Sphere Multiaxial (Batdorf) Model: Puts linear elastic fracture mechanics into Weibull weakest-link theory

- **Incremental failure probability is the product of two probabilities:**

\[ \Delta P_f = P_1 \cdot P_2 \]

- **Component failure probability:**

\[ P_f = 1 - \exp \left\{ - \int_V \int_0^{\sigma_c} P_1(\sigma_c) P_2(\sigma_c) d\sigma_c dV \right\} \]

- **Mixed-Mode Fracture Criteria:**
  - Normal stress (shear-insensitive cracks)
  - Maximum tensile stress
  - Total coplanar strain energy release rate
  - Noncoplanar (Shetty)

- **Flaw Shapes:**
  - Griffith crack
  - Penny-shaped crack

**Notes:**
- \( P_2 \) involves integration of an equivalent stress \( \sigma_e \), where \( \sigma_e \geq \sigma_c \), over the surface of a unit radius sphere (all possible flaw orientations) divided by the total surface area of the unit radius sphere.
- Component failure probability is the product of two probabilities.
- \( P_1 \) is the probability of the existence of a crack having a critical strength between \( \sigma_c \) and \( \sigma_c + \Delta\sigma_c \) in the incremental volume \( \Delta V \).
- \( P_2 \) is the probability a crack having a critical strength of \( \sigma_c \) will be oriented in a direction such that it will fail under the applied multiaxial stress state.
Unit Sphere Stochastic-Strength Multiaxial Failure Criterion model

- Two models for transverse isotropy

  - Flaw / Fracture-Plane Orientation Anisotropy

  - Strength Orientation Anisotropy


Failure probability ➔ surface area of a unit radius sphere (all possible flaw orientations), where equivalent mode I stress ($\sigma_{Ieq}$) exceeds critical mode I strength ($\sigma_{Ic}$), divided by the total surface area of the unit radius sphere.
Anisotropic Unit Sphere model defined in a material coordinate system reference frame

All possible vectors normal to the flaw plane of the fiber or tow or unidirectional lamina

Width of belt defines extent of misalignment

Similar to Puck’s composite failure criterion except in a probabilistic framework

When represented on a unit radius sphere of all possible flaw plane orientations the flaw plane normals define an *equatorial belt* distribution
Example of a biaxial failure envelope at 50% probability of failure ($P_f$) for a PMC unit cell

FE model of a fiber-in-matrix unit cell
(Sampled points indicated with ☒

Data from WWFE

Tracking individual failure modes at sampled points for the matrix
(What mode is critical, where, and when)

Biaxial failure envelope at 50% $P_f$ for a composite unit cell for all sampled points and failure modes *(with and without thermal residual stresses)*

- **Isotropic matrix**
- **Anisotropic $K_{lc}$ matrix**

- Isotropic matrix not suitable/adjustable to predict longitudinal strength
- Anisotropy in unit sphere
  - Compares Tsai-Wu & Tsai-Hill
- Validates approach taken

Failure mode normalized to stress axis:
- Tension
- Compression

Experiment Data
- No Thermal
- No Thermal - Adj.
- With Thermal
- With Thermal - Adj.

Tsai/Wu/Hill curves come from the applied stresses on the composite

Predicted response between calibrated points
Time-Dependent Life Prediction Theory - Slow Crack Growth and Cyclic Fatigue Crack Growth Laws

**Power Law:** - Slow Crack Growth (SCG)

\[
\frac{da}{dt} = AK_{N_{\text{eq}}}
\]

**Combined Power Law & Walker Law:** SCG and Cyclic Fatigue

\[
\frac{da}{dt} = A_1 g K_{N_{\text{eq}}} + A_2 f_c (1 - R)^Q K_{N_{\text{eq}}}
\]
Modeling individual time steps in the life prediction methodology enables simulating transient events such as turbine start-up/shut-down or atmospheric re-entry. A computationally efficient methodology has been developed that can extrapolate the reliability calculation for an arbitrary number of $Z$ cycles – where each cycle is described by $k$ number of time steps. This conceivably allows the coupling of other effects such as stiffness degradation and oxidation effects on the individual time steps and this can be accounted for interactively within the transient finite element and micromechanics analysis.
Transient Life Prediction Theory - Power Law SCG

Reliability formula for $k$ discrete time steps over $Z$ cycles:

$$P_{SV}(t_k) = \exp\left\{-\sum_{i=1}^{n} \frac{V_i}{4\pi} \int_{\Omega} \left[ \cdots \left[ \left( \frac{\sigma_{Ieq,k,T_{\max}}}{\sigma_{0BV,k}} \right)^{N_{V,k}-2} + \frac{\sigma_{Ieq,k} Z \Delta t_k}{N_{V,k}-2} \sum_{j} \frac{m_{V,k}(N_{V,k}-2)}{N_{V,k}-2} \right] + \frac{\sigma_{Ieq,j} Z \Delta t_j}{N_{V,j}-2} \sum_{j} \frac{m_{V,j}(N_{V,j}-2)}{N_{V,j}-2} \right] \cdots + \frac{\sigma_{Ieq,2} Z \Delta t_2}{N_{V,2}-2} \sum_{j} \frac{m_{V,2}(N_{V,2}-2)}{N_{V,2}-2} \right] \right] \left[ \frac{m_{V,1}(N_{V,1}-2)}{N_{V,1}-2} \right] \left[ N_{V,1}-2 \right] \right) d\Omega \right\}$$

**Individual time step:** Each time step can have different loading, Weibull, and fatigue parameters. Compatibility of failure probability is maintained between the individual time steps.
Time-dependent Failure Example: Static Loading
(Matrix Damage Accumulation From Slow Crack Growth)

Strain response for applied static tensile load over time

Service life prediction
Longitudinal stress applied to a 0° SiC/RBSN ply
10 time increments per time magnitude

Slow Crack Growth
Power Law:
\[
\frac{da}{dt} = AK_{\text{eq}}^N
\]

Weibull Parameters
\( m = 7 \) (Weibull slope)
\( \sigma_o = 106 \text{ Mpa} \cdot \text{mm}^{3/7} \)

Fatigue Parameters
\( N = 20 \) (fatigue slope)
\( B = 1.0E9 \text{ MPa}^2 \cdot \text{sec} \)

Damage increases with time and load

Applied Static Load (MPa)

- 170
- 180 (CARES 50%)
- 190
- 200

Effect of N

Note: Parameter “B” is related to parameter “A”
**MAC/GMC Methodology:** Generalized Method of Cells (GMC) & High-Fidelity Generalized Method of Cells (HFGMC)

- *Micromechanics links the size scales & provides the composite response based on the composite constituent materials*

**FEAMAC:** MAC/GMC embedded in FEA as constitutive material

### GMC (1990s)
- 1st order displacement field in subcells
- Stresses and strains piecewise constant
- Number of linear algebraic equations function of number of subcells
- Local inelasticity/damage
- No shear coupling
- No “subcell mesh” sensitivity

### HFGMC (2000s)
- 2nd order displacement field in subcells
- Elastic stresses and strains piecewise linear
- Number of linear algebraic equations is rather large
- Local inelasticity/damage
- Has shear coupling
- Has “subcell mesh” sensitivity

**Repeating Unit Cell (RUC) of composite material**
- RUC made subcells
- Multiscale capability

---

**Macromechanics:**

- Micromechanics links the size scales & provides the composite response based on the composite constituent materials.

**Aboudi, J.; Arnold, S.M.; and Bednarcyk, B.A. (2013):**


**Aboudi, J.; Pindera, M.J.; and Arnold, S.M. (2003):**

**CARES: Ceramics Analysis and Reliability Evaluation of Structures**

**Life Prediction & Component Design Code For Advanced Ceramics**

- Developed to predict the probability of failure of ceramic components under complex thermomechanical loading.

- Combines Weibull & Weakest Link theory with concepts of linear elastic fracture mechanics (the *Batdorf Unit Sphere* model).

**Component Reliability Analysis Capability:**

- Transient loads and temperatures
- Fast-Fracture Rupture
- Time-dependent \((da/dt)\) crack growth
- Cycle-dependent \((da/dn)\) crack growth
- Multiaxial stress failure models (*PIA & Unit Sphere & Tsai-Wu & Tsai-Hill*)
- Proof test

---

*CARES is a post-processor to FEA*

*Operates at the macro scale of the material*

---

CARES: Life Prediction & Component Design Tools For Advanced Ceramics

Software (Ceramics Analysis and Reliability Evaluation of Structures)

Predicts the probability of failure of ceramic components under load

- Silicon nitride, silicon carbide, alumina
- Ultra high temperature ceramics
- MEMS materials – silicon; SiC
- Glass

Utilized worldwide for life prediction of brittle material components.

- Aerospace
- Automotive
- Electronic
- Energy
- Glass
- Medical
- Power

Applications

MEMS
- Microturbine (a)
- Microrocket (b)
- Pressure sensor (c)

Biomedical
- Hip joint (d)
- Dental Bridge (e)
- MEMS implants (f)

Aerospace
- Turbine blade (g)
- Rocket Nozzle (h)
- Mars Aeroshell (i)

Fuel Cell (SOFC)
- Power generation (j)
- Propulsion (k)
Ceramic Matrix Composites (CMCs)

- Ceramic matrix composites (CMC) are being developed for hot section of advanced turbine engines (2700° F) and other uses.
- Specific mechanisms governing response of ceramic matrix composites need to account for in analysis approach:
  - Brittle material response
  - Weak, compliant interface
  - Residual stresses present due to processing
- Robust, efficient analysis tools required to analyze deformation, failure and life of these materials.

⇒ CMCs are designed to have “graceful failure” (non-linear stress-strain response) as opposed to brittle failure.
Work in Progress: Weibull Size Effect Demo

4-Point Flexure Specimen

Damage Simulation

Cut-view of damage at various points along the specimen length

Initial effort intended for Weibull size effect demo of 3-point flexure, 4-point flexure, and tensile specimen