Unexpected Nonlinear Effects in Superconducting Transition-Edge Sensors

Jack Sadleir

Code 553

NASA Goddard Space Flight Center
6 keV

est. $\Delta E = 1.6$ eV

World Record Energy Resolution TES Fabricated in the DDL at NASA GSFC
$6 \text{ keV}$

$\text{est. } \Delta E = 1.6 \text{ eV}$
6 keV

TES Fabricated in the DDL at NASA GSFC (But not specifically designed for this application)
6 keV

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est. $\Delta E = 1.6$ eV

![Graph showing the relationship between bias current and estimated energy difference. The graph includes a trend line with data points and a horizontal line indicating the estimated energy difference.]
6 keV

![Graph showing the relationship between bias current and estimated energy change (ΔE). The graph indicates that the estimated energy change (ΔE) is approximately 1.6 eV. There is a note that MTES has a negative β value (β < 0).]
CAUTION:
- **If a Linear detector...**
- *(Estimated)*

\[
est. \Delta E = true \Delta E
\]
What is “nonlinearity”? the “nonlinear-model”… field “nonlinear dynamics”

Nonlinear $\Rightarrow$ Everything Else (a very $\infty$ set ;-)

• **Nonlinearity ubiquitous** in nature:
  – e.g. 60 Hz harmonics pickup at 120 Hz, 180 Hz,.... requires nonlinearity in the system.

• **Linearity ubiquitous** in our mathematical description of nature:
  – often a good approximation to real physical systems.
  – It is mathematically easier
  – Linear Tools:
    • Superposition Principle
    • Transform methods, transfer functions etc.
What is “nonlinearity”? the “nonlinear-model”...
field “nonlinear dynamics”
Nonlinear $\rightarrow$ Everything Else (a very $\infty$ set ;-)

• If our thermistor TES sensor...
  – If $R(T) \rightarrow R(T,J)$.
  – Then $\textbf{Nonlinear}$ system of Diff Eqs.
  – (Can approximate by a linearized system of Diff Eqs.)

• “$\textbf{Nonlinear}$” if
  – $R(T) \rightarrow$ anything other than a single straight line
  – $R(T,J) \rightarrow \text{“} \text{“} \text{“} \text{“}$ a single plane.
  – $R(T,J,B) \rightarrow \text{“} \text{“} \text{“} \text{“}$ a single hyperplane.
Superconductivity: Resistive R Transition Surface in Temperature T and Current I

Normal State

Superconducting State
$R(T)_{|I}$

contour
Stationary Noise:
Noise at the bias point

(R_0, T_0, I_0)
\[ R(T, I) \approx R_0 + \frac{\partial R}{\partial T} \delta T + \frac{\partial R}{\partial I} \delta I \]
AND NonStationary Noise: 
*In the pulse trajectory*

X-ray pulse trajectory 
$T(t), I(t)$
X-ray pulse trajectory $T(t), I(t)$

AND
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**Known or Expected Nonlinearities**
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\textbf{Known or Expected Nonlinearities}

\textbf{“Unexpected Nonlinearities”}

In steady state with standard DC biased TES operation. The TES current is wildly oscillating in time at high frequencies and we only measure (and are only aware of) the time averaged current in the TES.
Conclusions: ...Paradigm Shifting

• In steady state with standard DC biased TES operation. The TES current is wildly oscillating in time at high frequencies and we only measure (and are only aware of) the time averaged current in the TES.

• The equations governing this time response is nonlinear time-dependent diff eqs.
  
  – **KEY POINT:** This time dependent current exists when the TES is only DC biased, NO time dependent inputs what so ever. It comes about from the intrinsic physics governing the superconducting state of the TES.

  – Circuit and TES parameters can have values such that these time dependent solutions become multivalued in time. What does that mean? Can mean excess noise or fine structure of the time averaged resistive transition surface.

  – The biased TES waiting to detect a photon (μ-calorimeter) or flux of photons (μ-bolometers) can itself act as a radiation source.
  
  – *Tricky* when an array of exquisitely sensitive micro-wave radiation detectors are themselves sources of microwave radiation.
  
  – This can lead to radiation resistance and fine structure in the resistive transition. Possible cross talk between pixels in an array.
  
  – The time dependence of the current can take on pure sin waves and also very nonsinusoidal forms (variable harmonic content).

  – The fundamental frequency of these oscillations changes with bias voltage. Makes the prospects of FDM TES arrays with an AC-biased TES challenging. Structure, sensitivity, and cross-talk.
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• The measured resistive transition surface is not simply a function of R(T,J,B). But is also a function of the electric circuit. R(T,J,B,L,Rsh).
  – → Problem compartmentalizing: → Same TES measured in slightly different circuits will appear to have a different resistive transition surface. TEST in real setup early!!!
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DONE
ELECTROTHERMAL MODEL
ELECTROThERMAL MODEL
ELECTROTHERMAL MODEL

Thermal Circuit

Independent Variables: $T_b, V_b$

Electrical Circuit

$V_b$

$R_{in}$

$R_{sh}$

$R(T,I)$

$C$

$T$

$G$

$T_b$
ELECTROTHERMAL MODEL
TES

Thermal Circuit

Independent Variables:

\( T_b, V_b \)

Electrical Circuit

Measure: I

ELECTROTHERMAL MODEL
TES on a: ... (1) solid substrate (2) thin isolated membrane (3) thin perforated membrane (4) island suspended with long thin legs

**ELECTROTHERMAL MODEL**

Thermal Circuit

Independent Variables: $T_b$, $V_b$

Electrical Circuit

Measure: $I$
TES on a: ...
(1) solid substrate
(2) thin isolated membrane
(3) thin perforated membrane
(4) island suspended with long thin legs

Thermal power balance

$$I^2 R = \frac{G}{nT(n-1)} \left( T^n - T_b^n \right)$$

Electrical steady state

$$I = \frac{V_b}{R_{in}} \frac{R_{sh}}{R + R_{sh}}$$

IN STEADY STATE
TES on a: 

1. solid substrate 
2. thin isolated membrane 
3. thin perforated membrane 
4. island suspended with long thin legs

Thermal power balance

\[ I^2 R = \frac{G}{n T^{(n-1)}} (T^n - T_b^n) \]

Electrical steady state

\[ I = \frac{V_b}{R_{in}} \frac{R_{sh}}{R + R_{sh}} \]

Independent Variables:

\[ T_b, V_b \]
TES on a: ... (1) solid substrate (2) thin isolated membrane (3) thin perforated membrane (4) island suspended with long thin legs

Thermal power balance

\[ I^2 R = \frac{G}{nT^{(n-1)}} (T^n - T_b^n) \]

Electrical steady state

\[ I = \frac{V_b}{R_{sh}} \frac{R_{sh}}{R + R_{sh}} \]

\[ R = \frac{V_b}{I R_{in} R + R_{sh}} \]
Different G’s

Thermal Surfaces
Different $T_b$’s

Thermal Surfaces
Intersection point = "bias point"
Different Vb’s

I vs Vb

“IV curve”
Bias Circuit

TES

Separable?

R_{sh}, L

R(T,j,B)
Does changing $R_{sh}$ and $L$ have any direct impact on the TESs $R(T,j,B)$?

Of course changing $R_{sh}$ and $L$ impacts:
- bias stability conditions
- time constants
- the $j_{in}$ needed to get the the same $R$ etc.

Can we separate the TES transition from the bias circuit?
Does changing $R_{sh}$ and $L$ have no direct impact on the TESs $R(T,j,B)$?

Of course changing $R_{sh}$ and $L$ impacts:
- bias stability conditions
- time constants
- the $j_{in}$ needed to get the the same $R$ etc.

Can we separate the TES transition from the bias circuit?  
ANSWER: generally NO.
ELECTROTHERMAL MODEL

Thermal Circuit

Independent Variables: $T_b, V_b$

Electrical Circuit

Measure: I

Simply drop in TESs $R(T,I)$
ELECTROTHERMAL MODEL

Can we separate the TES transition from the bias circuit?

ANSWER: generally NO.
ELECTROTHERMAL MODEL

Can we separate the TES transition from the bias circuit?

ANSWER: generally NO.
Can we separate the TES transition from the bias circuit?

**ANSWER:** generally NO.
(1) \( j(t) = ? \)
   
   *What is the time dependence of the current \( j \)?*

(2) \( \langle j(t) \rangle \ vs \ j_{in} = ? \)

*What is the shape of the time averaged IV curve?*
J-TES model

Time rescaled so we can compare shape in time (harmonic content).

L = 0 limit

Increasing bias \( jjin = jin/jc \)
J-TES model

**L = 0 limit**

<table>
<thead>
<tr>
<th>jjin</th>
<th>1.00001</th>
</tr>
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<tbody>
<tr>
<td>javg</td>
<td>0.995538</td>
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**Green**: TES current versus time

**Red Dashed**: time averaged TES current

Movie evolves with increasing bias current **jjin**.

Movie rescales time as the bias is increased so two periods are contained in time

**Bias **jjin**:

**Small**:
slow spikes (+jc to -jc)

**Large**:
fast sinusoidal (+jc to -jc)
J-TES model
Finite Inductance Effect

*Closed form solution found!

Large $\j_{\text{in}}$

<table>
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<tr>
<th>$\j_{\text{in}} = \frac{\j_{\text{in}}}{\j_{\text{c}}}$</th>
<th>3.33333</th>
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<td>${L_{\text{min}}, L_{\text{max}}, L_{\text{step}}}$</td>
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Increasing $L$
J-TES model
Finite Inductance Effect

Large $jjin$

\[
\begin{align*}
jjin &= \frac{j_{in}}{j_c} \\
3.33333 & \\
\{L_{min}, L_{max}, L_{step}\} & \{0, 0.7, 0.1\}
\end{align*}
\]

Increasing $L$

Small $jjin$

\[
\begin{align*}
jjin &= \frac{j_{in}}{j_c} \\
1.25 & \\
\{L_{min}, L_{max}, L_{step}\} & \{0, 0.7, 0.1\}
\end{align*}
\]

Increasing $L$

Rescaled time [periods/2]
J-TES model
Finite Inductance Effect

Large $jj_{in}$

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Increasing $L$

Multivalued $j[t]$
Current becomes multivalued with sufficiently large L
J-TES model
Large L behavior

$Large \ L \rightarrow \sim \ Sawtooth \ j[t]$
Test my claim

Quantum wave function of the superconducting condensate

\[ \Psi = |\Psi| \ e^{i \phi(r,t)} \]

First Josephson Equation:

\[ J = J_c \sin \phi(t) \]

Second Josephson Equation:

\[ \phi'(t) = \frac{2\pi V}{\Phi_0} \]

Shapiro voltage steps:

\[ V_n = n \ \Phi_0 \ f \]

(voltage to frequency transducer)

Fundamental:

1. Gauge Invariance
$V_b = V_{b \text{ DC}}$
\[ V_b = V_{b\text{ DC}} + v \sin (2\pi f t) \]

Phase Locking

Voltage Steps Observed from 
\( f=200 \text{ kHz} \) to 
\( f=30 \text{ MHz} \)
\[ V_n = n \Phi_0 f \]

- \( f = 200\text{kHz} \)
- \( T_b = 102\text{mK} \)
- Green circle: measured IV curve
- Red plus sign: Shapiro step position
- Purple dashed line: fit to the superconducting state

Graph showing the relationship between \( V_{fb} \) and \( V_b \) with marked steps at \( n = 1, n = 2, n = 3, n = 4 \).
Measuring a 200 μOhm resistor to better than 1/1000!

Rsh vs current is flat down to 5μA → Ohmic!
Time Averaged IV curves

VRSJ-TES model

Time averaged IV curve.

Only changing the circuit Inductor value
Everything else constant.
Time Averaged IV curves
VRSJ-TES model

Time averaged IV curve.

Only changing the circuit Inductor value
Everything else constant.
Time Averaged IV curves
VRSJ-TES model

Time averaged IV curve.

Only changing the circuit Inductor value
Everything else constant.
jj oscillation amplitude

jj oscillation range at $vv=0.01$ for each $\beta_L$ value.

$v v$ voltage

$v v=0.01$

$v v=0.1$

All similar in size
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THE

END
$\beta_L$ vs $j_j(t)$ for different $\eta$:

- $\eta = 0.001$
- $\eta = 0.01$
- $\eta = 0.1$
- $\eta = 0.5$
- $\eta = 1.0$
- $\eta = 2.0$
- $\eta = 10.0$

$v = \{0.01, 0.1, 1, 5\}$

Sawtooth and Sin waveforms are illustrated for $v$.
Power for relationship holds for: $\lambda \gg \text{antenna length}$

**Blue:** $\Delta \ell / \lambda < 0.1$
(satisfied)

**Red:** $\Delta \ell / \lambda > 0.1$
Actual radiated power is larger than red surface

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<th>$R_{\text{sh}}$</th>
<th>$\frac{1}{100}$</th>
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<td>$\log [L]$</td>
<td>$-8$</td>
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<td>$\log [j_c]$ {min, max, step}</td>
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approximating the waveform as a pure sinusoid for the purposes of this