A COLLISION AVOIDANCE STRATEGY FOR A POTENTIAL NATURAL SATELLITE AROUND THE ASTEROID BENNU FOR THE OSIRIS-REX MISSION

ALINDA MASHIKU*
RUSSELL CARPENTER**
*NASA GSFC – Flight Dynamics Team
**NASA GSFC - Navigation Advisory Group

AIAA/AAS Astrodynamics Specialist Conference
Long Beach, CA
SEPTEMBER 16, 2016
• Introduction to the OSIRIS-REx mission
• Proximity Operations Concept
• Current status of Natural Satellites
• Approaches to Collision Avoidance
• Wald Sequential Probability Ratio Test (WSPRT)
• Conjunction analysis Example 1: 8 hr prediction
• Conjunction analysis Example 2: 3 hr prediction
• Summary
The OSIRIS-REx mission launched on Sept 8th 2016 at Cape Canaveral, FL onwards to the asteroid Bennu.

Bennu is a carbonaceous asteroid and a potentially hazardous asteroid with a probability of impacting the Earth in the late 22nd century. The determination of Bennu’s physical and chemical properties are of key importance in the event an impact mitigation mission will be required.

OSIRIS-REx’s key science objectives include

1. Return and analyze a sample of Bennu’s surface
2. Map the asteroid
3. Document the sample site
4. Measure the orbit deviation cause by non-gravitational forces (the Yarkovsky effect)
5. Compare observations at the asteroid to ground-based observations

(1) www.asteroidmission.org
INTRODUCTION TO THE OSIRIS-REx MISSION

BENNU FACTS

- Equatorial Diameter: \(~500\) m
- Polar Diameter: \(~510\) m
- Average Speed: \(63,000\) mph
- Rotation Period: \(4.3\) hrs
- Orbital Period: \(1.2\) yrs
- Orbital Inclination: \(6\) degrees
- Earth Approach: Bennu comes close to Earth every 6 yrs
PROXIMITY OPERATIONS

- There are various phases of the OSIRIS-REx mission proximity operations in which specific scientific campaigns at specified cadences are in place.
- Eventually, there is a safe home orbit in which OSIRIS-REx remains in a terminator orbit as the staging point for all subsequent activities.
- The terminator orbit is a plane that is perpendicular to the sun vector:
  - For OSIRIS-REx, minimizes solar radiation pressure (SRP) perturbations
  - Relatively large perturbation due to the small size of Bennu
**OPERATIONS CONCEPT FLOWCHART**

- Characterize satellite ephemeris and physical properties*
  - Is object in the vicinity of Proximity Ops < 10km?
    - Yes: Proceed with cadence of operations with situational awareness**
    - No: Characterize object with predictions for Prelim. Survey (7km) etc.
  - Is the object in an orbit < 2km radius?
    - Yes: Does the object pose a threat to the S/C?
      - No: Continue with Preliminary Survey
      - Yes: Determine Cadence of object observations (OpNav)
        1. How does this interfere prox. Ops (Orbit A and B, Recon phasing, TAG timings etc.)
        2. Implement Collision avoidance scheme
    - No: Characterize object with predictions for Prelim. Survey (7km) etc.
  - Is the orbit a terminator orbit?
    - Yes: Determine Cadence of object observations (OpNav)
      1. How does this interfere prox. Ops (Orbit A and B, Recon phasing, TAG timings etc.)
      2. Implement Collision avoidance scheme
    - No: Proceed with cadence of operations with situational awareness**

---

*Based on the Hill Sphere, proceed with caution of potential objects cruising within vicinity

**Based on the Hill Sphere, proceed with caution of potential objects cruising within vicinity

---

1. Determine Cadence of object observations (OpNav)
   1. How does this interfere prox. Ops in the off-terminator orbit (Recon, TAG)?
2. Implement Collision avoidance scheme

---

(2) D.S. Lauretta et al., "The OSIRIS-Rex target asteroid (101955) Bennu: constraints on its physical, geological, and dynamical nature from astronomical observations" Meteoritics & Planetary Science, Vol 50, No 4, pg 834-849
Current Status of Bennu Natural Satellites

- The presence of natural satellites depend on the rotation rates of the primary body. Bennu’s rotation rate is 4.29 hrs for a Bennu sidereal day $^{(3)}$
- Most NEA of spheroidal shapes and rapid rotation rates have been found to be primaries of a binary system.
- About 16% of Near Earth Asteroids (NEA) with diameters larger than 200m may belong to binary systems.$^{(4)}$

<table>
<thead>
<tr>
<th>Potential Stable Natural Satellites size</th>
<th>Bennu’s Hill Sphere</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameters 1m</td>
<td>Out to 26 km</td>
</tr>
<tr>
<td>Diameters 10cm</td>
<td>Out to 16 km</td>
</tr>
<tr>
<td>Diameters 1cm</td>
<td>Out to 5km</td>
</tr>
</tbody>
</table>

- Based on radar albedo of Bennu and a tidally locked rotation period, the largest undetected satellite within 300km of Bennu is 2m.$^{(1)}$

---

$^{(1)}$ D.S. Lauretta et al., “The OSIRIS-Rex target asteroid (101955) Bennu: constraints on its physical, geological, and dynamical nature from astronomical observations” Meteoritics & Planetary Science, Vol 50, No 4, pg 834-849


APOLLO ASTEROIDS SIMILAR TO BENNU

- Apollo asteroids are Earth-crossing asteroids with
  - Semi-major axes, $a >$ Earth’s semi major axis (1 AU)
  - Perihelion distances < Earth’s aphelion (1.017 AU)

- There are 55 known NEAs with moons
  - (14 Amor, 34 Apollo, and 7 Aten) with a total of 57 moons

<table>
<thead>
<tr>
<th>Name of Asteroid</th>
<th>Diameter (km)</th>
<th>Name of Moon</th>
<th>Diameter (km)</th>
<th>Separation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1999 DJ4</td>
<td>0.43 +/- 0.08</td>
<td>S/2004</td>
<td>0.21 +/- 0.05</td>
<td>0.8</td>
</tr>
<tr>
<td>2002 AM31</td>
<td>0.45 +/- 0.05</td>
<td>S/2012</td>
<td>0.11</td>
<td>1.5</td>
</tr>
<tr>
<td>2004 DC</td>
<td>0.36</td>
<td>S/2006</td>
<td>0.07</td>
<td>0.75 +/- 0.045</td>
</tr>
</tbody>
</table>

* Note: No rotation rate information is included here

STANDARD APPROACH TO COLLISION AVOIDANCE (CA)

Collision Probability, $P_c$

- Compute $P_c$ (might be hard)
- Compare to some threshold

Potential Issues with $P_c^{(5)}$

- Often integrating PDF in the tail region
- Must project PDF into future
- Is PDF even Gaussian?
- $P_c$ might “roll-off” – when to decide?

WALD SEQUENTIAL PROBABILITY RATIO TEST (WSPRT) FOR CA

- Test for true miss distance at time of closest approach: $r_\ast = r_{ca}$
- Given set of observations at times $t_k$ prior to $t_\ast = t_{ca}: Y_{1:k}$
  - $H_0$: fixed* hypothesis that true miss distance is unsafe ($\|r_\ast\| \leq R$, hard body radius)
  - $H_1$: fixed* hypothesis that true miss distance is safe
- Form ratio of conditional PDF’s:
  \[
  \Lambda_k = \frac{p(Y_{1:k} | H_1)}{p(Y_{1:k} | H_0)} = \frac{p(Y_{1:k} | \|r_\ast\| > R)}{p(Y_{1:k} | \|r_\ast\| \leq R)}
  \]
- Compare ratio to decision limits $A$ & $B$:
  - If $\Lambda_k \geq A$, reject $H_0$ and dismiss conjunction
  - If $\Lambda_k \leq B$, accept $H_0$ and maneuver
  - Otherwise, if possible, seek another observation

* Fixed hypotheses imply that there are no random disturbances, e.g. process noise, that can change a hit into a miss; if this can occur, must use a different test, such as Shirayeyev SPRT

- Targeted $P_{md}$ (missed detection) and targeted $P_{fa}$ (false alarm) are values that need to be pre-determined based on apriori statistics and/or Monte Carlo analysis

WSPRT Algorithm

\[
\Lambda_k = \frac{p(Y_{1:k} | \mathcal{H}_1)}{p(Y_{1:k} | \mathcal{H}_0)} = \frac{p(Y_{1:k} \mid \|r_*\| > \mathcal{R})}{p(Y_{1:k} \mid \|r_*\| \leq \mathcal{R})}
\]

The likelihood ratio of WSPRT:

The conditional probabilities are calculated as:

\[
p(Y_{k:1} \mid r_* \in \mathbb{B}) = \frac{\prod_{i=1}^{k} \left( \frac{1}{(2\pi)^{\frac{n}{2}} \sqrt{|\hat{\mathbf{P}}_{*|k}|}} \right) \sqrt{\frac{|\mathbf{P}_{*|k}|}{|\mathbf{P}_{*|o}|}} e^{-\frac{1}{2} \alpha} P_{c|k}}{P_{c|o}}
\]

\[
p(Y_{k:1} \mid r_* \notin \mathbb{B}) = \frac{\prod_{i=1}^{k} \left( \frac{1}{(2\pi)^{\frac{n}{2}} \sqrt{|\hat{\mathbf{P}}_{*|i}|}} \right) \sqrt{\frac{|\mathbf{P}_{*|k}|}{|\mathbf{P}_{*|o}|}} e^{-\frac{1}{2} \alpha (1 - P_{c|k})}}{1 - P_{c|o}}
\]

The time series relative position vector and its covariance:

Therefore:

\[
\Lambda_k = \frac{p(Y_{1:k} | \mathcal{H}_1)}{p(Y_{1:k} | \mathcal{H}_0)} = \frac{p(Y_{1:k} \mid \|r_*\| > \mathcal{R})}{p(Y_{1:k} \mid \|r_*\| \leq \mathcal{R})}
\]

Simplifies to:

\[
\Lambda_k = \frac{1 - P_{c|k}}{P_{c|k}} \frac{P_{c|o}}{1 - P_{c|o}}
\]
Conjunction analysis Example 1: 8 HR Prediction

- Simulated range, azimuth and elevation measurements for 4 hrs
- Epoch 18 Feb 2019 12:00:00.000 UTC
- 500 Monte Carlo runs
- Natural Satellite
  - $P_o = [\sigma_{xx}^2=(2/3 \text{ HBR})^2\text{km}^2, \sigma_{vv}^2=(10e-6)^2\text{km}^2/\text{s}^2]$
  - $R = 0.01\text{km}$
  - $P_{c|0} = 0.052075$

FAR (False Alarm Rate)
MDR (Missed Detection Rate)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>(a) $P_{fa0.2, pmd0.2}$</th>
<th>(b) $P_{fa0.1, pmd0.31}$</th>
<th>(c) $P_{fa0.2, pmd0.01}$</th>
<th>(d) $P_{fa0.1, pmd0.2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alarm Limit $P_{Alarm}$</td>
<td>0.180155</td>
<td>0.844687</td>
<td>0.213704</td>
<td>0.814638</td>
</tr>
<tr>
<td>Dismissal Limit $P_{c</td>
<td>0}$</td>
<td>0.013548</td>
<td>0.000555</td>
<td>0.000686</td>
</tr>
<tr>
<td>Alarms</td>
<td>234</td>
<td>77</td>
<td>242</td>
<td>75</td>
</tr>
<tr>
<td>Dismissals</td>
<td>263</td>
<td>286</td>
<td>234</td>
<td>344</td>
</tr>
<tr>
<td>Hits</td>
<td>19</td>
<td>19</td>
<td>19</td>
<td>19</td>
</tr>
<tr>
<td>Misses</td>
<td>481</td>
<td>481</td>
<td>481</td>
<td>481</td>
</tr>
<tr>
<td>True Alarms</td>
<td>17</td>
<td>11</td>
<td>17</td>
<td>11</td>
</tr>
<tr>
<td>False Alarms</td>
<td>217</td>
<td>66</td>
<td>225</td>
<td>64</td>
</tr>
<tr>
<td>True Dismissals</td>
<td>261</td>
<td>284</td>
<td>232</td>
<td>341</td>
</tr>
<tr>
<td>False Dismissals</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>No Decisions</td>
<td>3</td>
<td>137</td>
<td>24</td>
<td>81</td>
</tr>
<tr>
<td>False Alarm Rate</td>
<td>45.11%</td>
<td>13.72%</td>
<td>46.76%</td>
<td>13.31%</td>
</tr>
<tr>
<td>Missed Detection Rate</td>
<td>10.53%</td>
<td>10.53%</td>
<td>10.53%</td>
<td>15.79%</td>
</tr>
</tbody>
</table>

Low $P_{fa}$ - High Alarm Limit
- Low Alarms
- Low False Alarms

Hi $P_{md}$ - Hi Dismissal Limit
- Affects No Decisions ($f(P_{fa})$)

How to decide based on one parameter only or two?
CONJUNCTION ANALYSIS EXAMPLE 1: 8 HR PREDICTION

- The decision is made depending on the values of the targeted $P_{fa}$ and $P_{md}$ and calculated $P_{c|k}$.
- The required balance is a quick decision vs accuracy requirements.
- This can be implemented dependent on the mission phase.

\[
P_{c|k} \geq P_c \quad \text{Alarm} \\
P_{c, \text{Alarm}} = P_{c|0} (B + (1-B)P_{c|0}^{-1})
\]

\[
P_{c|k} < P_c \quad \text{Dismiss} \\
P_{c, \text{Dismiss}} = P_{c|0} (A + (1-A)P_{c|0}^{-1})
\]
**Conjunction analysis Example 2: 3 HR Prediction**

- Simulated range, azimuth and elevation measurements for 4 hrs + Prediction for 3 hrs
- Epoch 18 Feb 2019 12:00:00.000 UTC
- 500 Monte Carlo runs
- Natural Satellite
  - $P_o = \left[\sigma_{xx}^2 = (2/3 \text{ HBR})^2 \text{km}^2, \sigma_{vv}^2 = (10e^{-6})^2 \text{km}^2/\text{s}^2\right]$
  - $R = 0.01\text{km}$
  - $P_{c|0} = 0.052075$

**Table:**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>(a) $P_{f_{FA,2}}$</th>
<th>(b) $P_{f_{LA,0}}$</th>
<th>(c) $P_{f_{FA,2}P_{p_{MD,0}}}$</th>
<th>(d) $P_{f_{LA,0}P_{p_{MD,0}}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alarm Limit ($P_{Alarm}^c$)</td>
<td>0.180155</td>
<td>0.844687</td>
<td>0.213794</td>
<td>0.814638</td>
</tr>
<tr>
<td>Dismissal Limit ($P_{Dismissal}^c$)</td>
<td>0.013548</td>
<td>0.000555</td>
<td>0.000686</td>
<td>0.0010976</td>
</tr>
<tr>
<td>Alarms</td>
<td>198</td>
<td>91</td>
<td>195</td>
<td>96</td>
</tr>
<tr>
<td>Dismissals</td>
<td>273</td>
<td>236</td>
<td>229</td>
<td>282</td>
</tr>
<tr>
<td>Hits</td>
<td>19</td>
<td>19</td>
<td>19</td>
<td>19</td>
</tr>
<tr>
<td>Misses</td>
<td>481</td>
<td>481</td>
<td>481</td>
<td>481</td>
</tr>
<tr>
<td>True Alarms</td>
<td>18</td>
<td>14</td>
<td>18</td>
<td>18</td>
</tr>
<tr>
<td>False Alarms</td>
<td>180</td>
<td>77</td>
<td>177</td>
<td>81</td>
</tr>
<tr>
<td>True Dismissals</td>
<td>272</td>
<td>235</td>
<td>228</td>
<td>281</td>
</tr>
<tr>
<td>False Dismissals</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>No Decisions</td>
<td>29</td>
<td>173</td>
<td>76</td>
<td>122</td>
</tr>
<tr>
<td>False Alarm Rate</td>
<td>37.42%</td>
<td>16.01%</td>
<td>36.80%</td>
<td>16.84%</td>
</tr>
<tr>
<td>Missed Detection Rate</td>
<td>5.26%</td>
<td>5.26%</td>
<td>5.26%</td>
<td>5.26%</td>
</tr>
</tbody>
</table>

**Graph:**

- Low $P_{fa}$
  - High Alarm Limit
  - Low Alarms
  - Low False Alarms
- Hi $P_{md}$
  - Hi Dismissal Limit
  - Affects No Decisions ($f(P_{fa})$)

- Similar value patterns to 8 hr
- Noticeably reduced FAR and MDR
CONJUNCTION ANALYSIS EXAMPLE 2: 3 HR PREDICTION

The decision is made depending on the values of the targeted $P_{fa}$ and $P_{md}$ and calculated $P_{c|k}$.

- With 19 hits, still notice $P_{c|k} < P_{c}$ Dismiss
  $P_{c}$ Dismiss = $P_{c|0}(A + (1-A)P_{c|0})^{-1}$
- $P_{c|k} ≥ P_{c}$ Alarm
  $P_{c}$ Alarm = $P_{c|0}(B + (1-B)P_{c|0})^{-1}$

(a) $P_{fa} = 0.2$ and $P_{md} = 0.2$
(b) $P_{fa} = 0.01$ and $P_{md} = 0.01$
(c) $P_{fa} = 0.2$ and $P_{md} = 0.01$
(d) $P_{fa} = 0.01$ and $P_{md} = 0.2$
SUMMARY

There exists a trade in False Alarm Rates and Missed Detection Rates accuracies with the prediction duration to the time of Closest approach (TCA).

Desired $P_{fa}$ and $P_{md}$ can be tailored based on the mission phase and the available time of prediction to TCA.

This preliminary study will be useful in determining the correct approaches for each mission phase during proximity operations.

WORK TO GO:

- Complete build of generating a range of targeted $P_{fa}$ and $P_{md}$ using a representative number of Monte Carlo runs.
- Run examples of specific mission phase scenarios.

<table>
<thead>
<tr>
<th></th>
<th>Hi $P_{fa}$ /Hi $P_{md}$</th>
<th>Low $P_{fa}$ /Low $P_{md}$</th>
<th>Hi $P_{fa}$ /Low $P_{md}$</th>
<th>Low $P_{fa}$ /Hi $P_{md}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>FAR</strong></td>
<td>45.11%</td>
<td>13.72%</td>
<td>46.78%</td>
<td>13.31%</td>
</tr>
<tr>
<td><strong>MDR</strong></td>
<td>10.53%</td>
<td>10.53%</td>
<td>10.53%</td>
<td>15.79%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Hi $P_{fa}$ /Hi $P_{md}$</th>
<th>Low $P_{fa}$ /Low $P_{md}$</th>
<th>Hi $P_{fa}$ /Low $P_{md}$</th>
<th>Low $P_{fa}$ /Hi $P_{md}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>FAR</strong></td>
<td>37.42%</td>
<td>16.01%</td>
<td>36.80%</td>
<td>16.84%</td>
</tr>
<tr>
<td><strong>MDR</strong></td>
<td>5.26%</td>
<td>5.26%</td>
<td>5.26%</td>
<td>5.26%</td>
</tr>
</tbody>
</table>
THANK YOU