A COLLISION AVOIDANCE STRATEGY FOR A POTENTIAL NATURAL SATELLITE AROUND THE ASTEROID BENNU FOR THE OSIRIS-REX MISSION

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AGENDA

• Introduction to the OSIRIS-REx mission
• Proximity Operations Concept
• Current status of Natural Satellites
• Approaches to Collision Avoidance
• Wald Sequential Probability Ratio Test (WSPRT)
• Conjunction analysis Example 1: 8 hr prediction
• Conjunction analysis Example 2: 3 hr prediction
• Summary
The OSIRIS-REx mission launched on Sept 8\textsuperscript{th} 2016 at Cape Canaveral, FL onwards to the asteroid Bennu.

Bennu is a carbonaceous asteroid and a potentially hazardous asteroid with a probability of impacting the Earth in the late 22\textsuperscript{nd} century. The determination of Bennu’s physical and chemical properties are of key importance in the event an impact mitigation mission will be required.

OSIRIS-REx’s key science objectives include\textsuperscript{(1)}:

- Return and analyze a sample of Bennu’s surface
- Map the asteroid
- Document the sample site
- Measure the orbit deviation cause by non-gravitational forces (the Yarkovsky effect)
- Compare observations at the asteroid to ground-based observations

\textsuperscript{(1)} www.asteroidmission.org
INTRODUCTION TO THE OSIRIS-REx MISSION

BENNU FACTS

- Equatorial Diameter: \(~500\,m\)
- Polar Diameter: \(~510\,m\)
- Average Speed: \(63,000\,\text{mph}\)
- Rotation Period: \(4.3\,\text{hrs}\)
- Orbital Period: \(1.2\,\text{yrs}\)
- Orbital Inclination: \(6\,\text{degrees}\)
- Earth Approach: Bennu comes close to Earth every 6 yrs
PROXIMITY OPERATIONS

- There are various phases of the OSIRIS-REx mission proximity operations in which specific scientific campaigns at specified cadences are in place.
- Eventually, there is a safe home orbit in which OSIRIS-REx remains in a terminator orbit as the staging point for all subsequent activities.
- The terminator orbit is a plane that is perpendicular to the sun vector:
  - For OSIRIS-REx, minimizes solar radiation pressure (SRP) perturbations
  - Relatively large perturbation due to the small size of Bennu
**Operations Concept Flowchart**

1. **Determine Cadence of object observations (OpNav)**
   1. How does this interfere prox. Ops in the off-terminator orbit (Recon, TAG)?
   2. Implement Collision avoidance scheme

2. **Characterize object with predictions for Prelim. Survey (7km) etc**

**Is the object in an orbit < 2km radius?**

- **No**
  - Characterize object with predictions for Prelim. Survey (7km) etc

- **Yes**
  - **Is the orbit a terminator orbit?**
    - **No**
      - Does the object pose a threat to the S/C?
        - **No**
          - Continue with Preliminary Survey
        - **Yes**
          - Determine Cadence of object observations (OpNav)
            1. How does this interfere mission ops
            2. Implement Collision avoidance scheme

    - **Yes**
      - Determine Cadence of object observations (OpNav)
        1. How does this interfere prox. Ops (Orbit A and B, Recon phasing, TAG timings etc.)
        2. Implement Collision avoidance scheme

**Is object in the vicinity of Proximity Ops < 10km?**

- **No**
  - Proceed with cadence of operations with situational awareness**

- **Yes**
  - Characterize satellite ephemeris and physical properties*

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* Based on the Hill Sphere, proceed with caution of potential objects cruising within vicinity

**Based on the Hill Sphere, proceed with caution of potential objects cruising within vicinity**

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(2) D.S. Lauretta et al., "The OSIRIS-Rex target asteroid (101955) Bennu: constraints on its physical, geological, and dynamical nature from astronomical observations" Meteoritics & Planetary Science, Vol 50, No 4, pg 834-849
CURRENT STATUS OF BENNU NATURAL SATELLITES

• The presence of natural satellites depend on the rotation rates of the primary body. Bennu’s rotation rate is 4.29 hrs for a Bennu sidereal day\(^{(3)}\).
• Most NEA of spheroidal shapes and rapid rotation rates have been found to be primaries of a binary system.
• About 16% of Near Earth Asteroids (NEA) with diameters larger than 200m may belong to binary systems.\(^{(4)}\)

<table>
<thead>
<tr>
<th>Potential Stable Natural Satellites size</th>
<th>Bennu’s Hill Sphere</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameters 1m</td>
<td>Out to 26 km</td>
</tr>
<tr>
<td>Diameters 10cm</td>
<td>Out to 16 km</td>
</tr>
<tr>
<td>Diameters 1cm</td>
<td>Out to 5km</td>
</tr>
</tbody>
</table>

• Based on radar albedo of Bennu and a tidally locked rotation period, the largest undetected satellite within 300km of Bennu is 2m.\(^{(1)}\)

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(1) D.S. Lauretta et al., “The OSIRIS-Rex target asteroid (101955) Bennu: constraints on its physical, geological, and dynamical nature from astronomical observations” Meteoritics & Planetary Science, Vol 50, No 4, pg 834-849
APOLLO ASTEROIDS SIMILAR TO BENNU

- Apollo asteroids are Earth-crossing asteroids with
  - Semi-major axes, $a >$ Earth’s semi major axis (1 AU)
  - Perihelion distances $< $ Earth’s aphelion (1.017 AU)

- There are 55 known NEAs with moons
  - (14 Amor, 34 Apollo, and 7 Aten) with a total of 57 moons

<table>
<thead>
<tr>
<th>Name of Asteroid</th>
<th>Diameter (km)</th>
<th>Name of Moon</th>
<th>Diameter (km)</th>
<th>Separation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1999 DJ4</td>
<td>0.43+/- 0.08</td>
<td>S/2004</td>
<td>0.21+/- 0.05</td>
<td>0.8</td>
</tr>
<tr>
<td>2002 AM31</td>
<td>0.45+/- 0.05</td>
<td>S/2012</td>
<td>0.11</td>
<td>1.5</td>
</tr>
<tr>
<td>2004 DC</td>
<td>0.36</td>
<td>S/2006</td>
<td>0.07</td>
<td>0.75+/- 0.045</td>
</tr>
</tbody>
</table>


* Note: No rotation rate information is included here
**STANDARD APPROACH TO COLLISION AVOIDANCE (CA)**

Collision Probability, $P_c$

- Compute $P_c$ (might be hard)
- Compare to some threshold

Potential Issues with $P_c^{(5)}$

- Often integrating PDF in the tail region
- Must project PDF into future
- Is PDF even Gaussian?
- $P_c$ might “roll-off” – when to decide?

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**Wald Sequential Probability Ratio Test (WSPRT) for CA**

- Test for true miss distance at time of closest approach: \( r_* = r_{ca} \)
- Given set of observations at times \( t_k \) prior to \( t_* = t_{ca} \):
  - \( H_0 \): fixed* hypothesis that true miss distance is unsafe (\( ||r_*|| \leq R \), hard body radius)
  - \( H_1 \): fixed* hypothesis that true miss distance is safe
- Form ratio of conditional PDF’s:
  \[
  \Lambda_k = \frac{p(\mathbb{Y}_{1:k} | H_1)}{p(\mathbb{Y}_{1:k} | H_0)} = \frac{p(\mathbb{Y}_{1:k} | ||r_*|| > R)}{p(\mathbb{Y}_{1:k} | ||r_*|| \leq R)}
  \]
- Compare ratio to decision limits \( A \) & \( B \):
  - If \( \Lambda_k \geq A \), reject \( H_0 \) and dismiss conjunction
  - If \( \Lambda_k \leq B \), accept \( H_0 \) and maneuver
  - Otherwise, if possible, seek another observation

* Fixed hypotheses imply that there are no random disturbances, e.g. process noise, that can change a hit into a miss; if this can occur, must use a different test, such as Shirayeyev SPRT

Targeted \( P_{md} \) (missed detection) and targeted \( P_{fa} \) (false alarm) are values that need to be pre-determined based on apriori statistics and/or Monte Carlo analysis

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The conditional probabilities are calculated as:

\[
\Lambda_k = \frac{p(Y_{1:k} | H_1)}{p(Y_{1:k} | H_0)} = \frac{p(Y_{1:k} \mid \|r_*\| > \mathcal{R})}{p(Y_{1:k} \mid \|r_*\| \leq \mathcal{R})}
\]

Previously:

\[
\Lambda_k = \frac{1 - P_{c|k}}{P_{c|k}} \frac{P_{c|o}}{1 - P_{c|o}}
\]

Likelihood ratio of WSPRT:

\[
p(Y_{k:1} | r_* \in \mathbb{B}) = \frac{\prod_{i=1}^{k} \left(\frac{1}{(2\pi)^{n/2} \sqrt{|\hat{P}_{*|i}|}} \sqrt{\frac{|\hat{P}_{*|k}|}{|P_{*|o}|}} e^{-\frac{1}{2} \alpha} P_{c|k}\right)}{P_{c|o}}
\]

\[
p(Y_{k:1} | r_* \notin \mathbb{B}) = \frac{\prod_{i=1}^{k} \left(\frac{1}{(2\pi)^{n/2} \sqrt{|\hat{P}_{*|i}|}} \sqrt{\frac{|\hat{P}_{*|k}|}{|P_{*|o}|}} e^{-\frac{1}{2} \alpha} (1 - P_{c|k})\right)}{1 - P_{c|o}}
\]

where

\[
P_{c|k} = \frac{1}{(2\pi)^{n/2} \sqrt{|\hat{P}_{*|k}|}} \int_{\mathbb{B}} \exp \left(-\frac{1}{2} (\hat{r}_{*|k} - r_*)^\top \hat{P}_{*|k}^{-1} (\hat{r}_{*|k} - r_*)\right) \, dr_*
\]

\[
P_{c|o} = \frac{1}{(2\pi)^{n/2} \sqrt{|\hat{P}_{*|o}|}} \int_{\mathbb{B}} \exp \left(-\frac{1}{2} (r_* - \hat{r}_{*|o})^\top \hat{P}_{*|o}^{-1} (r_* - \hat{r}_{*|o})\right) \, dr_*
\]

The time series relative position vector and its covariance:

\[
\hat{P}_{*|k} = \left(\hat{P}_{*|o}^{-1} + \sum_{i=1}^{k} \hat{P}_{*|i}^{-1}\right)^{-1}
\]

\[
\hat{r}_{*|k} = \hat{P}_{*|k} \left(\hat{P}_{*|o}^{-1} \hat{r}_{*|o} + \sum_{i=1}^{k} \hat{P}_{*|i}^{-1} \hat{r}_{*|i}\right)
\]

\[
\alpha = \hat{r}_{*|o}^\top \hat{P}_{*|o}^{-1} \hat{r}_{*|o} + \sum_{i=1}^{k} \hat{r}_{*|i}^\top \hat{P}_{*|i}^{-1} \hat{r}_{*|i} - \hat{r}_{*|k}^\top \hat{P}_{*|k}^{-1} \hat{r}_{*|k}
\]
CONJUNCTION ANALYSIS EXAMPLE 1: 8 HR PREDICTION

- Simulated range, azimuth and elevation measurements for 4 hrs
- Epoch 18 Feb 2019 12:00:00.000 UTC
- 500 Monte Carlo runs
- Natural Satellite
  - $P_0 = \left[\sigma_{xx}^2 = (2/3 \text{ HBR})^2 \text{km}^2, \sigma_{vv}^2 = (10e-6)^2 \text{km}^2/\text{s}^2\right]$
  - $R = 0.01\text{km}$
  - $P_{c|0} = 0.052075$

FAR (False Alarm Rate)
MDR (Missed Detection Rate)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>(a) $P_{f_0,0.2} P_{md,0.2}$</th>
<th>(b) $P_{f_0,0.01} P_{md,0.01}$</th>
<th>(c) $P_{f_0,0.2} P_{md,0.01}$</th>
<th>(d) $P_{f_0,0.01} P_{md,0.2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alarm Limit $P_{alarm}$</td>
<td>0.180155</td>
<td>0.844687</td>
<td>0.213704</td>
<td>0.814638</td>
</tr>
<tr>
<td>Dismissal Limit $P_{c}$</td>
<td>0.013548</td>
<td>0.000555</td>
<td>0.000686</td>
<td>0.010976</td>
</tr>
<tr>
<td>Alarms</td>
<td>234</td>
<td>77</td>
<td>242</td>
<td>75</td>
</tr>
<tr>
<td>Dismissals</td>
<td>263</td>
<td>286</td>
<td>234</td>
<td>344</td>
</tr>
<tr>
<td>Hits</td>
<td>19</td>
<td>19</td>
<td>19</td>
<td>19</td>
</tr>
<tr>
<td>Misses</td>
<td>481</td>
<td>481</td>
<td>481</td>
<td>481</td>
</tr>
<tr>
<td>True Alarms</td>
<td>17</td>
<td>11</td>
<td>17</td>
<td>11</td>
</tr>
<tr>
<td>False Alarms</td>
<td>217</td>
<td>66</td>
<td>225</td>
<td>61</td>
</tr>
<tr>
<td>True Dismissals</td>
<td>261</td>
<td>284</td>
<td>232</td>
<td>341</td>
</tr>
<tr>
<td>False Dismissals</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>No Decisions</td>
<td>3</td>
<td>137</td>
<td>24</td>
<td>81</td>
</tr>
<tr>
<td>False Alarm Rate</td>
<td>45.11%</td>
<td>13.72%</td>
<td>46.76%</td>
<td>13.31%</td>
</tr>
<tr>
<td>Missed Detection Rate</td>
<td>10.53%</td>
<td>10.53%</td>
<td>10.53%</td>
<td>15.79%</td>
</tr>
</tbody>
</table>

Low $P_{fa}$
- High Alarm Limit
- Low Alarms
- Low False Alarms

Hi $P_{md}$
- Hi Dismissal Limit
- Affects No Decisions ($f(P_{fa})$)

How to decide based on one parameter only or two?
**Conjunction Analysis Example 1: 8 HR Prediction**

- The decision is made depending on the values of the targeted $P_{fa}$ and $P_{md}$ and calculated $P_{c|k}$.

- The required balance is a quick decision vs accuracy requirements.

- This can be implemented dependent on the mission phase.

\[
P_{c|k} \geq P_c \quad \text{Alarm} \\
P_{c \text{ Alarm}} = P_{c|0}(B + (1-B)P_{c|0})^{-1}
\]

\[
P_{c|k} < P_c \quad \text{Dismiss} \\
P_{c \text{ Dismiss}} = P_{c|0}(A + (1-A)P_{c|0})^{-1}
\]
CONJUNCTION ANALYSIS EXAMPLE 2: 3 HR PREDICTION

- Simulated range, azimuth and elevation measurements for 4 hrs + Prediction for 3 hrs
- Epoch 18 Feb 2019 12:00:00.000 UTC
- 500 Monte Carlo runs
- Natural Satellite
  - \( P_o = \left[ \sigma_{xx}^2 = (2/3 \text{ HBR})^2 \text{km}^2, \sigma_{vv}^2 = (10e-6)^2 \text{km}^2/\text{s}^2 \right] \)
  - \( R = 0.01 \text{km} \)
  - \( P_{c|0} = 0.052075 \)

**Parameter** | \( P_{f_{\text{fa}a,2}} \) | \( P_{f_{\text{fa}a,2}} \) | \( P_{f_{\text{fa}a,2}} \) | \( P_{f_{\text{fa}a,2}} \) |
--- | --- | --- | --- | --- |
Alarm Limit (\( P_{\text{Alarm}} \)) | 0.180155 | 0.844687 | 0.213794 | 0.814638 |
Dismissal Limit (\( P_{\text{Disc}} \)) | 0.013548 | 0.000555 | 0.000686 | 0.010976 |
Alarms | 198 | 91 | 195 | 96 |
Dismissals | 273 | 236 | 229 | 282 |
Hits | 19 | 19 | 19 | 19 |
Misses | 481 | 481 | 481 | 481 |
True Alarms | 18 | 14 | 18 | 15 |
False Alarms | 180 | 77 | 177 | 81 |
True Dismissals | 272 | 235 | 228 | 281 |
False Dismissals | 1 | 1 | 1 | 1 |
No Decisions | 29 | 173 | 76 | 122 |
False Alarm Rate | 37.42\% | 16.01\% | 36.80\% | 16.84\% |
Missed Detection Rate | 5.26\% | 5.26\% | 5.26\% | 5.26\% |

**Low \( P_{fa} \) :**
- High Alarm Limit
- Low Alarms
- Low False Alarms

**Hi \( P_{md} \) :**
- Hi Dismissal Limit
- Affects No Decisions \( f(P_{fa}) \)

-Similar value patterns to 8 hr
- Noticeably reduced FAR and MDR
**Conjunction Analysis Example 2: 3 HR Prediction**

- The decision is made depending on the values of the targeted $P_{fa}$ and $P_{md}$ and calculated $P_{c|k}$.
- With 19 hits, still notice $P_{c|k} < P_c$ Dismiss $P_c$ Dismiss = $P_{c|0} (A + (1-A)P_{c|0})^{-1}$

\[
P_{c|k} \geq P_c \text{ Alarm} \\
P_c \text{ Alarm} = P_{c|0} (B + (1-B)P_{c|0})^{-1}
\]

\[
P_{c|k} < P_c \text{ Dismiss} \\
P_c \text{ Dismiss} = P_{c|0} (A + (1-A)P_{c|0})^{-1}
\]
• There exists a trade in False Alarm Rates and Missed Detection Rates accuracies with the prediction duration to the time of Closest approach (TCA).
• Desired $P_{fa}$ and $P_{md}$ can be tailored based on the mission phase and the available time of prediction to TCA.
• This preliminary study will be useful in determining the correct approaches for each mission phase during proximity operations.

**WORK TO GO:**
• Complete build of generating a range of targeted $P_{fa}$ and $P_{md}$ using a representative number of Monte Carlo runs.
• Run examples of specific mission phase scenarios.

### SUMMARY

<table>
<thead>
<tr>
<th></th>
<th>8 hrs Prediction to Close approach</th>
<th>3 hrs Prediction to Close approach</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Hi $P_{fa}$ /Hi $P_{md}$</td>
<td>Low $P_{fa}$ /Low $P_{md}$</td>
</tr>
<tr>
<td><strong>FAR</strong></td>
<td>45.11%</td>
<td>13.72%</td>
</tr>
<tr>
<td><strong>MDR</strong></td>
<td>10.53%</td>
<td>10.53%</td>
</tr>
</tbody>
</table>