Evan Haas and Frank De Luccia, The Aerospace Corporation, El Segundo, CA, USA

Introductions
- Perfectly navigated or registered images having somewhat different scene content, when passed through an image correlator, will generate false misregistrations.
- Scene content differences affect the evaluation of key GOES-R Advanced Baseline Imager (ABI) absolute and relative navigation metrics:
  - Truth images used to evaluate absolute navigation accuracy (NAV) of an ABI image are collected at different times under different illumination, atmospheric and ground conditions.
  - Pairs of ABI images used to evaluate channel-to-channel registration accuracy (CCR) differ due to differing spatial content.
  - Pairs of ABI images used to evaluate frame-to-frame registration accuracy (FFR) differ due to temporal effects such as cloud motion.
- For image pairs that are not perfectly navigated and registered, their misregistration will consist of both a true misregistration and a false one:
  $$d_R(x, y) = R(x, y) - \mu$$
  where:
  - $R(x, y)$ is the radiance at location $(i, j)$
  - $\mu$ is the mean radiance over the image.
- The amount of spatial structure in an image is key for determining $d_R$, so we describe images in terms of their deviation from the mean radiance and normalize by the mean radiance:
  $$V(i, j) = \frac{[R(i, j) - \mu]}{\mu}$$
  with the fixed grid angles used by ABI, translations of $V(i, j)$ span a two dimensional subspace of the N x M dimensional image space.
- This subspace can be represented by:
  $$P(x, y) = V(i, j) + (x - j) \cdot T_x + (y - i) \cdot T_y$$
  $T_x$ and $T_y$ are tangent vectors which can be defined as one pixel shifts in the x and y directions:
  $$T_x = V(i, j + 1) - V(i, j) \quad T_y = V(i + 1, j) - V(i, j)$$
  The essence of MU estimation is to consider typical perturbations of an image that are not associated with translation and to estimate the misregistrations that the correlator would generate when ingesting the original and perturbed images.
  - Consider a perturbed image $P(i, j)$, rescaled as $\frac{V(i, j)}{\mu}$
  - Define the 2-norm of the difference vector as:
    $$D = \|V[R(i, j) - V(i, j)]\|
  - Consider the set of perturbed images having a common magnitude of the image difference relative to an unperturbed image vector.
  - The tips of the arrows of the perturbed image vectors all lie on a sphere of radius $D$.
  - Let $SR_k$ denote the length of the projection of a vector on this sphere onto the tangent vector $T_x$ (see red arrow in picture below).
  - The associated value of the false misregistration $\delta x$ in the x direction is obtained by dividing by the radiance change per pixel for a translation in the x direction.
  - Assuming the perturbed vectors are uniformly distributed, we can write the RMS expected false registration in the x direction as:
    $$\langle \delta_x^2 \rangle_{false}^{1/2} = \frac{1}{\|T_x\|} \int_{-1/2}^{1/2} \int_{-1/2}^{1/2} x^2 dx$$
  - Solving the integral yields:
    $$\langle \delta_x^2 \rangle_{false}^{1/2} = \frac{1}{\|T_x\| \sqrt{NM}}D$$

MU Uncertainty Estimation Concept
- Image difference not due to misregistration
- Image differences due to misregistration
- For the ABI, requirements are specified as bounds on the 99.73% percentile of the magnitudes of per pixel navigation and registration errors.
- Measurement uncertainty inherent in the use of image registration techniques tend to broaden the dispersion in measured local navigation and registration errors due to false misregistrations noted above, masking the true performance of the ABI system.
- We have devised an analytic method of estimating the magnitude of measurement uncertainty (aMU).
- These measurement uncertainty estimates can be used to filter measurements of local navigation and registration errors to allow only high quality image pairs for inclusion in statistics.
- This reduction in dispersion allows for a better approximation of the true performance of the ABI.

Analytic Measurement Uncertainty Formulation
- Consider images to be N x M arrays of radiances, where we tag the radiances by their location within the image, i.e. $R(i, j)$ is the radiance at location $(i, j)$.
- Measurements with small false misregistrations.

Validation
- The basic premise of validation is to show good agreement between aMU and some measure of dispersion (standard deviation, median absolute deviation (MAD)) of misregistrations across a large number of correlated image pairs.
- Substantial validation performed by correlating a reference image with many corrupted images from the same reference image are created to produce dispersion statistics.

Filtering Application
- In validating runs with representative ABI images, aMU scales with dispersion, but it is not a good absolute measure of misregistration dispersion.
- Further work, taking into account spatial correlations in images, is underway to reconcile this aMU formulation with misregistration dispersions in ABI/ABI images.
- Despite this, aMU is still a good measure for filtering out image pairs with large false misregistrations.
- Image pairs are shifted relative to one another, correlated, and misregistration is measured as the location of the peak Pearson correlation coefficient value, with some additional refinements.
- Filtering by aMU allows for a better measure of the true performance of the ABI.
- For CCR and FFR, aMU filtering greatly reduces dispersion in misregistrations.
- For the plots below, large dispersions indicative of false misregistrations dominate on the left, while dispersions are quite small moving to the right as 1/aMU is increased.

Summary
- aMU is a simple metric with great benefit as a relative measure of registration quality for a correlated image pair.
- This approach has been validated under the conditions for which it was derived, and is being extended to account for spatial correlations.