Unsteady Aerodynamic Force Sensing from Measured Strain

Prepared For:
30th Congress of the International Council of the Aeronautical Science
Daejeon, South Korea, September 25-30, 2016

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Overview

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  - Technical features of new technology: velocity & acceleration (slide 7)
  - Technical features of expanding procedure (slide 8)
  - Technical features of new technology: unsteady aerodynamic loads (slide 9)

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- **Conclusions (slide 22)**
What the technology does

**Problem Statement**
- To improve **fuel efficiency** for an aircraft
  - Reducing weight or drag
    - Similar effect on fuel savings
  - Multidisciplinary design optimization (design phase) or active control (during flight)
- Real-time measurement of **structural responses and loads** during flight are **critical data**.
  - Active flexible motion control
  - Active induced drag control

**Objective**
- Compute **unsteady aerodynamic loads from unsteady strain measurements**
  - Structural responses (complete degrees of freedom) are essential quantities for load computations during flight.
  - Loads can be computed from the following governing equations of motion.
    \[
    [M] \{\ddot{q}(t)\} + [G] \{\dot{q}(t)\} + [K] \{q(t)\} = \{Q_a(Mach, \{q(t)\}, \{\dot{q}(t)\}, \{\ddot{q}(t)\})\}
    \]
    - Internal Loads: using finite element structure model
      - \([M] \{\ddot{q}(t)\}, [G] \{\dot{q}(t)\}, [K] \{q(t)\}\): Inertia, damping, and elastic loads
    - External Load: using unsteady aerodynamic model
      - \(\{Q_a(Mach, \{q(t)\}, \{\dot{q}(t)\}, \{\ddot{q}(t)\})\\): Aerodynamic load

**Issue**
- Traditionally, lift load over the wing are measured using a pressure gauge.
  - This conventional pressure gauge with associated piping and cabling would create **weight and space limitation** issues and pressure data will be available only at discrete gauge location. Therefore, a **new innovation** is needed.
  - **Fiber optic strain sensor (FOSS)** is an ideal choice for aerospace applications.
Previous technologies


  - NASA LRC; Application is limited for "beam"; static deflection & aerodynamic loads


  - JAXA; using inverse analysis. "Beam" application only; static deflection & loads


  - NASA AFRC; “sectional” bending moment, torsional moment, and shear force along the “beam”.


  - Oregon State University; Aerodynamic loads are estimated from measured strain using virtual strain sensor technique.
Steps used to compute aerodynamic load from measured strain

Measure unsteady strain

Assembler module

Strain

Motion analyzer

Expansion module

Deflection, velocity, & acceleration

Loading analysis

Flight controller

Drag and lift

Compute unsteady aerodynamic loads

{q}ₖ {q̈}ₖ

Expand wing deflection, velocity, & acceleration

{ε}ₖ

Fiber optic strain sensor

{Qₐ}ₖ

Z deflection, velocity, & acceleration along each fiber are model independent quantities

{qₘₑ}ₖ

Compute wing deflection, velocity, & acceleration

Model independent

Model dependent

Z deflection, velocity, & acceleration
First Step of two-step approach

- **Use piecewise least-squares method** to *minimize noise* in the measured strain data (strain/offset): **re-generate** strain data
- Obtain **cubic spline** (Akima spline) function using re-generated strain data points (assume small motion):
  \[
  \frac{d^2 \delta_k}{ds^2} = -\epsilon_k(s)/c(s)
  \]
- **Integrate fitted spline function** to get slope data:
  \[
  \frac{d\delta_k}{ds} = \theta_k(s)
  \]
- Obtain cubic spline (Akima spline) function using computed slope data
- **Integrate fitted spline function** to get deflection data: \( \delta_k(s) \)

---

A measured strain is fitted using a **piecewise least-squares method** together with the cubic spline technique.
Technical features of new technology: Velocity & Acceleration Computation

Use low pass filter, ARMA model, on-line parameter estimator, and least-squares curve fitting method to obtain velocity and acceleration.

least-squares curve fitting

\[
\{q_{Me}(t)\} = \{\tilde{q}_{Me}\} + \sum_{i=1}^{nm} e^{-\sigma_i t} \left\{ A_i \cos(\omega_{di} t) + B_i \sin(\omega_{di} t) \right\}
\]

\[
\{\dot{q}_{Me}(t)\} = \frac{d}{dt} \{q_{Me}(t)\}
\]

\[
\{\ddot{q}_{Me}(t)\} = \frac{d^2}{dt^2} \{q_{Me}(t)\}
\]

\[
\{q_{Me}\}_k \quad \{\tilde{q}_{Me}\}_k \quad \{\dot{q}_{Me}\}_k \quad \{\ddot{q}_{Me}\}_k
\]

Use sine, cosine, and exponential functions in least squares fitting with respect to time.
Technical features of expanding procedure

- Second step of two step approach: Based on General Transformation
  - Definition of the generalized coordinates vector \( \{q\}_k \) and the orthonormalized coordinates vector \( \{\eta\}_k \) at discrete time \( k \)
  \[
  \{q\}_k = \{q_M\}_k = [\Phi][\eta]_k = [\Phi_M][\eta]_k
  \]
  - For all model reduction/expansion techniques, there is a relationship between the master (measured or tested) degrees of freedom and the slave (deleted or omitted) degrees of freedom which can be written in general terms as
  \[
  \{q_M\}_k = [\Phi_M][\eta]_k \quad \{q_S\}_k = [\Phi_S][\eta]_k
  \]
  - Changing master DOF at discrete time \( k \) \( \{q_M\}_k \) to the corresponding measured values \( \{q_{Me}\}_k \)
  \[
  \{q_{Me}\}_k = [\Phi_M][\eta]_k
  \[
  [\Phi_M]^T\{q_{Me}\}_k = [\Phi_M]^T[\Phi_M][\eta]_k
  \[
  \{\eta\}_k = \left( [\Phi_M]^T[\Phi_M] \right)^{-1}[\Phi_M]^T\{q_{Me}\}_k
  \]
  - Expansion of displacement using SEREP: kinds of least-squares surface fitting: most accurate reduction-expansion technique
    - \( \{q_{Me}\}_k \): master DOF at discrete time \( k \); deflection along the fiber "computed from the first step"
    - \( \{q_M\}_k = [\Phi_M]\left([\Phi_M]^T[\Phi_M]\right)^{-1}[\Phi_M]^T\{q_{Me}\}_k \): smoothed master DOF
    - \( \{q_S\}_k = [\Phi_S]\left([\Phi_M]^T[\Phi_M]\right)^{-1}[\Phi_M]^T\{q_{Me}\}_k \): deflection and slope all over the structure

\[
\{\eta\}_k = ([\Phi_M]^T[\Phi_M])^{-1}[\Phi_M]^T\{q_{Me}\}_k
\]

\[
\{\dot{\eta}\}_k = ([\Phi_M]^T[\Phi_M])^{-1}[\Phi_M]^T\{\dot{q}_{Me}\}_k
\]

\[
\{\ddot{\eta}\}_k = ([\Phi_M]^T[\Phi_M])^{-1}[\Phi_M]^T\{\ddot{q}_{Me}\}_k
\]

\[
\{\eta\}_k = ([\Phi_M]^T[\Phi_M])^{-1}[\Phi_M]^T\{q_{Me}\}_k
\]

\[
[M]\{\ddot{q}(t)\} + [G]\{\dot{q}(t)\} + [K]\{q(t)\} = \{Q_a(t)\}
\]

\[
[M]\{\ddot{q}(s)\} + [G]\{\dot{q}(s)\} + [K]\{q(s)\} = \{Q_a(s)\}
\]

\[
[M]\{\ddot{q}(s)\} + [G]\{\dot{q}(s)\} + [K]\{q(s)\} = \{Q_a(s)\}
\]

- Rational function approximation: Select Roger's Approximation
- Time marching algorithm:

\[
\{N\}_k = q_D([D_0]\{\eta\}_k + [D_1]\{\dot{\eta}\}_k + [D_2]\{\ddot{\eta}\}_k + [C]\{x\}_k)
\]

\[
\{x\}_k = \{E\}\{x\}_{k-1} + \theta [B]\frac{\{\dot{\eta}\}_k + \{\ddot{\eta}\}_{k-1}}{2}
\]

\[
[\Phi]^T\{Q_a\}_k = \{N\}_k
\]

\[
\{Q_a\}_k = ([\Phi]^T)^{-1}\{N\}_k
\]

\[
[E] = e^{[A]T_a} \quad [\theta] = \int_0^{T_a} e^{[A](T_a-\tau)}d\tau \quad [C] = [C_1 \ C_2 \ \cdots \ C_{LT}] \quad [A] = \\
\begin{bmatrix}
-\Omega_1 & 0 & \cdots & 0 \\
0 & -\Omega_2 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & -\Omega_{LT}
\end{bmatrix} \quad [B] = \\
\begin{bmatrix}
1 \\
\vdots \\
1
\end{bmatrix} \quad \{x\}_k = \begin{bmatrix}
x_1 \\
x_2 \\
\vdots \\
x_{LT}
\end{bmatrix}_k
\]

A rectangular matrix $[\Phi]^T$ can be inverted using a singular value decomposition technique.

Modal Aerodynamic Influence Coefficient Matrix $\rightarrow \{A(s)\} = [D_0] + s[D_1] + s^2[D_2] + \sum_{j=1}^{LT} \frac{s[C_j]}{s + \Omega_j}$
Computational Validation

Cantilevered rectangular wing model
Structural Model & Results from Modal Analysis

- Configuration of a wind tunnel test article
  - Has **aluminum insert** (thickness = 0.065 in) covered with 6% **circular arc** cross-sectional shape (plastic foam)
  - Lumped mass weight are computed based on 6% circular-arc cross-sectional shape.
    - Use structural dynamic model tuning technique
  - 300 beam elements for fictitious FOSS (50 per each fiber). Zero stiffness and zero weight.

- Modal analysis
  - NASTRAN sol. 103

---

### Measured and computed natural frequencies

<table>
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<tr>
<th>Mode</th>
<th>Measured (Hz)</th>
<th>Computed (Hz)</th>
<th>% Error</th>
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<tbody>
<tr>
<td>1</td>
<td>14.29</td>
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<td>0.0</td>
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<tr>
<td>2</td>
<td>80.41</td>
<td>80.17</td>
<td>-0.3</td>
</tr>
<tr>
<td>3</td>
<td>89.80</td>
<td>89.04</td>
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*Configuration of a wind tunnel test article*

- Has **aluminum insert** (thickness = 0.065 in) covered with 6% **circular arc** cross-sectional shape (plastic foam)
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CFL3D Model & Aerodynamic Analysis using CFL3D/NASTRAN

- **CFL3D code** is used to generate unsteady aerodynamic loads.
  - Compute aerodynamic load vector at structural grid points.
  - The CFD grid is a multi-block (97 \times 73 \times 57) grid with H-H topology.
  - M=0.714 selected. Delta t = 0.000060515 sec. 10240 time steps
  - The first three flexible modes are used.
  - Computes **deflections** and **velocities**. (compare with NASTRAN results)

- **MSC/NASTRAN sol 112: to compute unsteady strain**
  - Modal transient response analysis with 1024 time steps, Delta t = 0.00060515 sec.
  - Force cards are obtained from CFL3D code. Available @ CFD center points.
  - Computes strain (assume measured value), deflection, velocity, & acceleration (target)

- **Splines between CFL3D and NASTRAN**
  - Develop new approach.
  - Use interpolation element, RBE3, between FE grids and CFD grids & center points.
    - CFD grids: pressure
    - CFD center points: aerodynamic load vector
CFL3D vs. NASTRAN: deflection & velocity

$q_d = 1.455$

- (a) Deflection
- (b) Velocity

$q_{df} = 1.4561$: Dynamic pressure for wing flutter condition
Time Histories of Strain under Different Levels of Random White Noise

- Strain is measured at the leading-edge of wing root section (upper surface).
- $SNR \equiv 20 \times \log_{10} \frac{\epsilon_{rms}}{n_{rms}}$
  - $\epsilon_{rms}$ root-mean-squared level of strain
  - $n_{rms}$ root-mean-squared level of noise
- SNR value is correct near 0.33 sec.

- $LSNR \equiv 20 \times \log_{10} \frac{\epsilon_{max}}{n_{rms}}$
  - $\epsilon_{max}$ local maximum strain
  - $n_{rms}$ root-mean-squared level of noise
- LSNR @ 0.035 sec
  - $20\log_{10}(8.97/3.28) = 8.74$ dB
- LSNR @ 0.24 sec
  - $20\log_{10}(3.95/3.28) = 1.61$ dB
- LSNR @ 0.33 sec
  - $20\log_{10}(3.28/3.28) = 0$ dB
- LSNR @ 0.59 sec
  - $20\log_{10}(1.06/3.28) = -9.83$ dB

(a) Without noise

(b) SNR = 10 dB

(c) SNR = 6 dB

(d) SNR = 0 dB
Z deflection is computed at the leading-edge of wing tip section (upper surface).

- Time interval: 0 – 0.2414 sec
  - Learning period for on-line parameter estimator.
  - Effect of piecewise least squares method can be observed. (first step of two step approach)

- Time interval: 0.2141 sec – 0.6 sec
  - Least-squares curve fitting method is on.
  - Working even with “SNR = 0 dB”

Effect of SEREP transformation can be observed.
  - SEREP transformation is a kind of least-squares surface fitting approach.
  - Noise in the signal after the first step of the two step approach is further filtered using SEREP transformation.

\[
\{q_{Me}(t)\} = \{\bar{q}_{Me}\} + \sum_{i=1}^{nm} e^{-\sigma t_i} \{A_i \cos(\omega_i t) + B_i \sin(\omega_i t)\}
\]

\[
\{q\}_k = \begin{bmatrix} \Phi_M \\ \Phi_S \end{bmatrix} \{\eta\}_k \\
\{\eta\}_k = \left( [\Phi_M]^T [\Phi_M] \right)^{-1} [\Phi_M]^T \{q_{Me}\}_k
\]
The diagram illustrates the time histories of Z velocity with SNR = 0 dB. The Z velocity is computed at the leading-edge of the wing tip section (upper surface).

- **Learning period**: 0 sec to 0.2414 sec
  - Learning period for on-line parameter estimator.
  - Velocities are not computed during this period.
- **Time interval**: 0.2141 sec to 0.6 sec
  - Least-squares curve fitting method is on.
  - Working even with “SNR = 0 dB”

Mathematical equations for Z velocity are given as:

\[
\{q_{Me}(t)\} = \frac{d}{dt}\{q_{Me}(t)\}
\]

\[
\{q_{Me}(t)\} = \{\tilde{q}_{Me}\} + \sum_{i=1}^{nm} e^{-\sigma_it} \{A_i \cos(\omega_{di}t) + B_i \sin(\omega_{di}t)\}
\]

\[
\{\dot{q}\}_k = \begin{bmatrix} \Phi_M \\ \Phi_S \end{bmatrix} \{\dot{\eta}\}_k \\
\{\ddot{\eta}\}_k = (\Phi_M^T \Phi_M)^{-1} \Phi_M^T \{\dot{q}_{Me}\}_k
\]
Z acceleration is computed at the leading-edge of wing tip section (upper surface).

- Time interval: 0 – 0.2414 sec
  - Learning period for on-line parameter estimator.
  - Accelerations are not computed during this period.
- Time interval: 0.2141 sec – 0.6 sec
  - Least-squares curve fitting method is on.
  - Working even with “SNR = 0 dB”

\[
\begin{align*}
\{\ddot{q}_{Me}(t)\} &= \frac{d^2}{dt^2} \{q_{Me}(t)\} \\
\{q_{Me}(t)\} &= \{\ddot{q}_{Me}\} + \sum_{i=1}^{nm} e^{-\sigma_i t} \{A_i \cos(\omega_i t) + B_i \sin(\omega_i t)\} \\
\{\ddot{q}\}_k &= [\Phi_M] [ij]_k \\
\{ij\}_k &= ([\Phi_M]^T [\Phi_M])^{-1} [\Phi_M]^T \{\ddot{q}_{Me}\}_k
\end{align*}
\]
Time Histories of Total Induced Drag Load under Different Levels of Random White Noise

- Time interval: 0 – 0.2414 sec
  - Learning period for on-line parameter estimator.
  - Load computations are based on wing deflection only.

- Time interval: 0.2141 sec – 0.6 sec
  - Least-squares curve fitting method is on.
  - Big difference before and after the proposed method is on.
  - Working even with “SNR = 0 dB”

CFL3D calculation
  - Subtracted 0.0353 (thickness effect)
Time Histories of Total Spanwise Load under Different Levels of Random White Noise

- **Time interval: 0 – 0.2414 sec**
  - Learning period for on-line parameter estimator.
  - Load computations are based on wing deflection only.

- **Time interval: 0.2141 sec – 0.6 sec**
  - Least-squares curve fitting method is on.
  - Big difference before and after the proposed method is on.
  - Working even with “SNR = 0 dB”

- **CFL3D calculation**
  - Subtracted 0.0961 (thickness effect)

(a) Without noise
(b) SNR = 10 dB
(c) SNR = 6 dB
(d) SNR = 0 dB
Time Histories of Total Lift Load under Different Levels of Random White Noise

− Learning period for on-line parameter estimator.
− Load computations are based on wing deflection only.

− Least-squares curve fitting method is on.
− Big difference before and after the proposed method is on.
− Working even with “SNR = 0 dB”
Updating aerodynamic forces using scaling factor

Scaling factor = 1.2649


- Scaling factors for the ATW2 wing were 1.2579 and 1.2719.
  - Scaling between flight test and ZAERO code based linear panel theory.
  - Use average of 1.2579 & 1.2719 for updating the unsteady aerodynamic forces.

- Scaling between CFL3D code based Euler theory and ZAERO code based linear panel theory.
Conclusions

- Unsteady aerodynamic loads are computed using simulated measured strain data.
  - Unsteady structural deflections are computed using the **two-step approach**.
  - Unsteady velocities and accelerations are computed using the ARMA model, on-line parameter estimator, low pass filter, and a least-squares curve fitting method together with an analytical derivatives with respect to time.
  - The deflections, velocities, and accelerations at each sensor location is independent of structural and aerodynamic models.
  - The distributed strain data together with the current proposed approaches can be used as a distributed deflection, velocity, and acceleration sensors.

- Induced drag loads, spanwise loads, and lift loads are obtained from the orthonormalized deflection, velocity, and acceleration together with the following approaches.
  - The modal AIC matrices are fitted in Laplace-domain using Roger's approximation.
  - Laplace-domain aerodynamics are converted to the time-domain using time-marching algorithm.
  - Orthonormalized aerodynamic load vectors are transformed to the general coordinates using pseudo matrix inversion based on singular value decomposition.
  - Normal vectors to the oscillating wing surface are used to compute drag and spanwise loads.
  - An active induced drag control system can be designed using these two computed aerodynamic loads, induced drag and lift, to improve the fuel efficiency of an aircraft.

- Interpolation elements (RBE3 in MSC/NASTRAN terminology) between structural FE grids and the CFD grids are successfully incorporated with the unsteady aeroelastic computation scheme.
  - The numerical issues often associated with the Harder and Desmarais surface splines technique are bypassed through the use of the current technique with RBE3 elements.

- The deflection, velocity, and acceleration computation based on the proposed least-squares curve fitting method are validated with respect to the unsteady strain with SNR of 10dB, 6dB, & 0dB (LSNR of 8.7dB to -9.8dB).

- The most critical technology for the success of the proposed approach is the robust on-line parameter estimator since the least-squares curve fitting method depends heavily on aeroelastic system frequencies and damping factors.
Questions?

Unsteady Strain

Assembler module

Strain

Motion analyzer

Expansion module

Deflection, velocity, & acceleration

Loading analysis

Flight controller

Drag and lift

Unsteady Motion

Unsteady Aerodynamic Loads
Time Histories of Z Deflection

- NASTRAN
- Current Method
Time Histories of Z Velocity

- : NASTRAN
- : Current Method

Z velocity (inch/sec)

Time (sec)
Time Histories of Z Acceleration

- NASTRAN
- Current Method

Z acceleration (inch/sec^2)

Time (sec)
Time Histories of Strain under 0 dB Random White Noise
Time Histories of Total Induced Drag Load under 0 dB Random White Noise

- CFL3D calculation
- Subtracted 0.0353 (thickness effect)
Aeroelastic System Frequencies

From AIC computations
- 0 Hz
- 4.006 Hz
- 10.02 Hz
- 23.37 Hz
- 53.42 Hz
- 86.80 Hz
- 173.6 Hz

Aerodynamic lag terms
- 11.81 Hz
- 47.22 Hz
- 106.2 Hz
- 188.9 Hz

<table>
<thead>
<tr>
<th>Mode</th>
<th>Natural frequency (Hz)</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>14.29</td>
</tr>
<tr>
<td>2</td>
<td>80.17</td>
</tr>
<tr>
<td>3</td>
<td>89.04</td>
</tr>
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</table>
Roger’s Approximation

$k_1 = 0.0$

$k_2 = 0.006$

$k_3 = 0.015$

$k_4 = 0.035$

$k_5 = 0.08$

$k_6 = 0.13$

$k_7 = 0.26$

- Imaginary part of AIC(1,1)
- Real part of AIC(1,1)

○ : AIC data
--- : RFA
On-line parameter estimation with and without noise

**First three damped aeroelastic frequencies**

<table>
<thead>
<tr>
<th>Mode</th>
<th>Without noise</th>
<th>With noise, SNR=10dB</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>13.81</td>
<td>-10.02</td>
</tr>
<tr>
<td></td>
<td>13.88</td>
<td>-11.11</td>
</tr>
<tr>
<td>2</td>
<td>63.15</td>
<td>-25.46</td>
</tr>
<tr>
<td></td>
<td>62.73</td>
<td>-25.92</td>
</tr>
<tr>
<td>3</td>
<td>89.82</td>
<td>-4.187</td>
</tr>
<tr>
<td></td>
<td>89.44</td>
<td>-5.740</td>
</tr>
</tbody>
</table>

FFT (Hz) | On-line parameter estimator
---|---
Damp. factor | Freq. (Hz)
Damp. factor | Freq. (Hz)

\[ SNR \equiv 20 \times \log_{10} \frac{\epsilon_{max}}{n_{rms}} \]

- \( \epsilon_{max} \): maximum unsteady strain after 0.1 second.
- \( n_{rms} \): root-mean-squared level of noise.

On-line parameter estimator is applied to the unsteady strain data.