Applications of Displacement Transfer Functions to Deformed Shape Predictions of the GIII Swept-Wing Structure

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Outline

- Motivations
- Background
- GIII wing load calibration test
- Finite element model correlation
- Displacement Transfer Functions
- DTFs Application
- Results comparisons
- Conclusion
Motivations

- Validate the accuracy of the displacement transfer functions (DTFs) when applied to the swept-wing structure
- Evaluate real time shape sensing possibility and efficiency to support future flight testing activities for the GIII aircraft
- Evaluate the accuracy of the wing deflection estimation when changing the number of strain stations
In June 2003, Helios broke up during flight test due to pitching oscillation under large wing dihedral bending. Therefore, real time wing deformed shape monitoring during flight is needed.

In 2007, Ko et al developed the Displacement Transfer Functions for transforming surface strain into deflections for wing deformed shape estimations.

Displacement Transfer Functions have been applied to wing shape predictions of Ikhana and Global Hawk successfully.


The current AFRC project utilizing the G-III airplane is the Adaptive Compliant Trailing Edge [ACTE] flap experiment. These unconventional adaptive compliant flap structures developed by FlexSys Inc. (Ann Arbor, Michigan) replaced the conventional Fowler flaps.

Due to the modification of the control surfaces, extensive ground load tests have been done on the GIII aircraft for the wing load calibration.
Due to differences between the ACTE structure and the original Fowler flaps with respect to weight, geometry, and flight-testing conditions, the aerodynamic and inertial loads were expected to be different.

In order to protect the wing structure during flight, load equations were developed using strains loads data from a ground load calibration test. These load equations were integrated in the Mission Control Room for real-time monitoring of the aerodynamic loads during flight. Wing deflected shape under load was also characterized and used to tune existing FEM models of the G-III wing structure.

<table>
<thead>
<tr>
<th>Load case</th>
<th>Type of loading</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Shot bags</td>
<td>Outboard loading</td>
</tr>
<tr>
<td>3</td>
<td>Combined</td>
<td>Forward shot and aft hydraulic loading</td>
</tr>
<tr>
<td>6</td>
<td>Combined</td>
<td>Aft shot and forward hydraulic loading</td>
</tr>
<tr>
<td>24</td>
<td>Hydraulic</td>
<td>Maximum loading</td>
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Finite Element Model Correlations

- Two finite element models
  - Model 1 built from CAD (Top)
  - Model 2 built from Stress Report (Bottom)

Table 1. Finite element model correlations for load case 1.

<table>
<thead>
<tr>
<th>String pot</th>
<th>Measured deflection</th>
<th>Wing box model 1</th>
<th>Wing box model 2</th>
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<tr>
<td></td>
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<td>Difference, %</td>
</tr>
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<td>-0.96</td>
<td>-4</td>
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<tr>
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<td>-4</td>
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<td>-0.44</td>
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<tr>
<td>6</td>
<td>-0.21</td>
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Table 2. Finite element model correlations for load case 3.

<table>
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<tr>
<th>LRT</th>
<th>Measured deflection</th>
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<td>0.65</td>
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</tr>
<tr>
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<td>0.20</td>
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Table 3. Finite element model correlations for load case 6.

<table>
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Table 4. Finite element model correlations for load case 24.

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<tr>
<td>7</td>
<td>0.17</td>
<td>0.18</td>
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</tr>
</tbody>
</table>
Displacement Theory

- Shifted Lagrangian curvature equation:

\[
\frac{1}{R(x)} = \frac{d^2 y}{dx^2} \sqrt{1 - \left(\frac{dy}{dx}\right)^2}
\]

Increased to maintain same \(\varepsilon(s)/c(s)\)

\[
\frac{d^2 y}{dx^2} = \frac{\varepsilon(s)}{c(s)}
\]

(1)

- Piece-wise representations:

\[
c(x) = c_{i-1} + (c_i - c_{i-1}) \frac{x - x_{i-1}}{l} \quad (x_{i-1} \leq x \leq x_i)
\]

\[
x = x_i + (x_{i-1} - x_i) \frac{x - x_{i-1}}{l} \quad (x_{i-1} \leq x \leq x_i)
\]

Slope [integration of (1)]:

\[
\tan \theta(x) = \frac{dy}{dx} = \frac{x}{x_{i-1}} \frac{c(x)}{c(x)} dx + \tan \theta_{i-1}
\]

(2)

Deflection [integration of (2)]:

\[
y(x) = \int_{x_{i-1}}^{x} \tan \theta(x) dx + y_{i-1}
\]

Integration of slope

Deflection at \(x_{i-1}\)

(3)
Displacement Transfer Functions

Slope equation (recursive form):

\[
\tan \theta_i = (\Delta l)_i \left[ \frac{\varepsilon_{i-1} - \varepsilon_i}{c_{i-1} - c_i} + \frac{\varepsilon_{i-1} c_i - \varepsilon_i c_{i-1}}{(c_{i-1} - c_i)^2} \log \frac{c_i}{c_{i-1}} \right] + \tan \theta_{i-1}
\]

\[\text{Slightly nonuniform} \quad \frac{(c_{i-1} \approx c_i)}{2c_{i-1}} \rightarrow \frac{(\Delta l)_i}{2c} \left[ (2 - \frac{c_i}{c_{i-1}}) \varepsilon_{i-1} + \varepsilon_i \right] + \tan \theta_{i-1}
\]

\[\text{Uniform} \quad \frac{(c_{i-1} = c_i = c)}{2c} \rightarrow \frac{(\Delta l)_i}{2c} (\varepsilon_{i-1} + \varepsilon_i) + \tan \theta_{i-1}
\]

Deflection equation (recursive form):

\[
y_i = (\Delta l)_i^2 \left[ \frac{\varepsilon_{i-1} - \varepsilon_i}{2(c_{i-1} - c_i)} - \frac{\varepsilon_{i-1} c_i - \varepsilon_i c_{i-1}}{(c_{i-1} - c_i)^3} \left( c_i \log \frac{c_i}{c_{i-1}} + (c_{i-1} - c_i) \right) \right] + y_{i-1} + (\Delta l)_i \tan \theta_{i-1}
\]

\[\text{Slightly nonuniform} \quad \frac{(c_{i-1} \approx c_i)}{6c_{i-1}} \rightarrow \frac{(\Delta l)_i^2}{6c_{i-1}} \left[ 3 - \frac{c_i}{c_{i-1}} \right] \varepsilon_{i-1} + \varepsilon_i \right] + y_{i-1} + (\Delta l)_i \tan \theta_{i-1}
\]

\[\text{Uniform} \quad \frac{(c_{i-1} = c_i = c)}{6c} \rightarrow \frac{(\Delta l)_i^2}{6c} (2\varepsilon_{i-1} + \varepsilon_i) + y_{i-1} + (\Delta l)_i \tan \theta_{i-1}
\]
Deformed Shape Visualization

- Structure deformed shape visualization Procedure

\[ y_i = f(\varepsilon_0 \sim \varepsilon_i) \]
\[ y'_i = f(\varepsilon'_0 \sim \varepsilon'_i) \]

Input
Surface
Strains
\( \varepsilon_i, \varepsilon'_i \)

Displacement
Transfer Functions

Output
Lateral
Deflections
\( y, y' \)

Computer Program for
Deformed Shape
Visualizations
\( y_0 \sim y_n, y'_0 \sim y'_n \)
\( \phi_0 \sim \phi_n \)

Control
Feedback

<table>
<thead>
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<th>38</th>
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<td>85</td>
<td>110</td>
</tr>
</tbody>
</table>

Structural Dynamics Group

Shun-Fat Lung-10
**DTFs Application**

- Use four strain-sensing lines (use two strain-sensing lines if \( c_i \) is known).
- Discretize the beam into \( n \) domains
- Determine the neutral axis (depth factor, \( c_i \))

\[
c_i = \frac{\varepsilon_i}{\varepsilon_i + \overline{\varepsilon}_i} h_i \\
c'_i = \frac{\varepsilon'_i}{\varepsilon'_i + \overline{\varepsilon}'_i} h'_i
\]

- Use equation (4) to calculate slope \( \tan \theta_i \)
- Use equation (5) to calculate deflection \( y_i \)
- Calculate the cross sectional twisted angle

\[
\phi_i = \sin^{-1} \left( \frac{y_i - y'_i}{d_i} \right) \quad (i = 0, 1, 2, 3, \ldots, n)
\]
Surface Strains for Load Case 24

- Load case 24
- Strains output from FEM model 2

Rear strain lines

Front strain lines

Normalized Strain

Wing Spanwise Locations
Deflection Comparison

- Use equation (4) to calculate deflection

**Rear strain lines**

- Normalized Deflection vs Wing Spanwise Location

**Front strain lines**

- Normalized Deflection vs Wing Spanwise Location
Twist Angle Comparison

- Twist angle calculated from \[ \phi_i = \sin^{-1}\left(\frac{y_i - y_i'}{d_i}\right) \]
Convergent Study

- Wing deflection based on different number of strain stations
Wing Tip Deflection Error

- Wing tip deflection error from 12% with 5 strain stations reduces to 1.6% with 17 strain stations
- Further increase number of strain stations will increase the error percentage
Conclusion

- The displacement transfer functions (DTFs) were applied to the GIII swept wing for the deformed shape prediction.
- The calculated deformed shapes are very close to the correlated finite element results as well as the measured data.
- The convergence study showed that using 17 strain stations, the wing-tip displacement prediction error was 1.6 percent, and that there is no need to use a large number of strain stations for G-III wing shape predictions.
Questions