Augmenting Conceptual Design Trajectory Tradespace Exploration with Graph Theory

AIAA Space 2016

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Motivation

Acquisition Timeline

<table>
<thead>
<tr>
<th>Phase</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-milestone 0</td>
<td>Determination of Mission Need and Deficiencies</td>
</tr>
<tr>
<td>Phase 0</td>
<td>Concept Exploration</td>
</tr>
<tr>
<td>Phase I</td>
<td>Program Definition and Risk Reduction</td>
</tr>
<tr>
<td>Phase II</td>
<td>Engineering &amp; Manufacturing Development</td>
</tr>
<tr>
<td>Phase III</td>
<td>Production, Deployment, and Operation Support</td>
</tr>
</tbody>
</table>

Design Timeline

- Knowledge becomes available when time to make decision
- Design Freedom
- Cost Committed

Today
Future
Motivation
Motivation

Ease of Use

Fidelity

POST2
Motivation
Augmenting Conceptual Design Trajectory Tradespace Exploration with Graph Theory
Repetitions Method

- Repetitions are generated by pairing a single case from the Vehicle-Level DOE with a number of randomly selected Steering-Level input vectors.

Vehicle Level: $\hat{v} = \begin{bmatrix} masses \\ engines \\ enviro. \end{bmatrix}$

Control Level: $\hat{u} = \begin{bmatrix} pitch rates \\ launch azi. \\ etc. \end{bmatrix}$
Repetitions Method

- \( p_2 = 0 \) Failure Limit
- \( p_2 \neq 0 \)
- \( p_2 \geq 1 \) Feasibility Limit
- \( p_2 < 1 \)
Chaining

\[ p_2 = 0 \]

Failure Limit

\[ p_2 \neq 0 \]

Feasibility Limit

\[ p_2 \geq 1 \]

\[ p_2 < 1 \]
Graph Method

Node

Edge
Graph Method

\[ d_{ij} = \left( \sum_{x_i} (u_i - v_i)^2 \right)^{1/2} \]
Graph Method

\[ d_{ij} = \frac{\text{Kruskal's Algorithm} \cdot (v_i)^2}{n - 1} \]

\[ n(n-1) \]

\[ n-1 \]
Graph Method

\[ n(n - 1) \quad \text{to} \quad n - 1 \]

Kruskal’s Algorithm

National Aeronautics and Space Administration
Graph Method

Graph Method

Failure Limit
Halfway Nodes

Failure Limit
Comparison Setup

- Comparison Metrics
  - Time to gather data
  - Surrogate fit from data – 2\textsuperscript{nd} Order RSE
    - Coefficient of Determination - \( R^2 \)
    - Root Mean Squared Error - \( RMSE \)

- Repetitions
  - Number of available processors
  - Required completions to call a case ‘done’

- Graph
  - Number of available processors
  - Number of seed points initially included

<table>
<thead>
<tr>
<th>Trial</th>
<th>Processors</th>
<th>Required Completions</th>
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</thead>
<tbody>
<tr>
<td>R1</td>
<td>71</td>
<td>1</td>
</tr>
<tr>
<td>R2</td>
<td>71</td>
<td>5</td>
</tr>
<tr>
<td>R3</td>
<td>71</td>
<td>10</td>
</tr>
<tr>
<td>R4</td>
<td>32</td>
<td>1</td>
</tr>
<tr>
<td>R5</td>
<td>32</td>
<td>5</td>
</tr>
<tr>
<td>R6</td>
<td>32</td>
<td>10</td>
</tr>
<tr>
<td>R7</td>
<td>16</td>
<td>1</td>
</tr>
<tr>
<td>R8</td>
<td>16</td>
<td>5</td>
</tr>
<tr>
<td>R9</td>
<td>16</td>
<td>10</td>
</tr>
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<tr>
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<th>Seed Points</th>
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</thead>
<tbody>
<tr>
<td>G1</td>
<td>71</td>
<td>10</td>
</tr>
<tr>
<td>G2</td>
<td>71</td>
<td>15</td>
</tr>
<tr>
<td>G3</td>
<td>71</td>
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</table>
Comparison Example Problem

◆ Single-Stage-To-Orbit Mars Ascent Vehicle

Rendezvous Orbit

Parking Orbit

ΔV Reserve

Throttle Down

Variable | Constant
---|---
Number of Engines | 3
Minimum Throttle | 20%

Variable | Range
---|---
Parking Orbit Perigee | +/- 10%
Parking Orbit Apogee | +/- 10%
Rendezvous Orbit ΔV | +/- 10%
Engine Isp | +/- 10%
Engine Thrust | +/- 10%
Propellant Mass | +/- 10%
Burnout Mass | +/- 10%
Repetitions Trials

- As available processors increases
  - Time required decreases
  - Repetitions submitted increases
- As required completions increases
  - Time required increases
  - Repetitions submitted increases
- Time history of Trial R3
  - Representative of Repetitions trials
  - “Easy” cases handled first, “Hard” cases require more repetitions, drag down convergence rate
Repetitions Results

- As the amount of data available for surrogate fitting increases, the fit improves
- Surrogate
  - Best trial: $R^2 = 0.9999$, RMSE = 45.50
  - Worst trial: $R^2 = 0.95214$, RMSE = 1024.41
- In the best case, the model averages at 0.02% error
As available processors increases
- Time required decreases
- DOE coverage decreases

As number of seeds increases
- Time required increases
- DOE coverage increases

Graph Trials

- As available processors increases
  - Time required decreases
  - DOE coverage decreases
- As number of seeds increases
  - Time required increases
  - DOE coverage increases
Graph Trials

- **Time history of trials**
  - Total data acquisition proceeds similarly to Repetitions
  - Optimal data acquisition sees a bump in rate around halfway through via the creation of halfway nodes
  - Increasing the number of seeds lengthens the process as a finite number of runs can be performed simultaneously
Graph Results

- Fastest graph trial returned over 2x the data returned by the fastest repetitions trial in 2/3 the time

- Surrogate
  - Best trial: $R^2 = 0.999986$, RMSE = 15.61
  - Worst trial: $R^2 = 0.999977$, RMSE = 19.90

- Best trial has average error of 0.00008%
  - Worst Graph trial outperforms best Repetitions trial
Comparison

- Repetitions
  - Produces more data per case on average
  - Output data is rough

- Graph
  - Produced data over 3x faster
  - Worst trial outperforms best Repetitions trial
Conclusion

◆ Repetitions
  ◆ Virtually no upper limit to concurrent executions
  ◆ Advantageous with a small number of points where very little is known

◆ Graph
  ◆ Finite number of chainings that can occur simultaneously
  ◆ Advantageous for filling in transition regions for better surrogate fit

◆ In the end, both are necessary for large-scale trade studies
Questions?