NDARC
NASA Design and Analysis of Rotorcraft

Theory

Appendix 1
Release 1.10
March 2016

Wayne Johnson
NASA Ames Research Center, Moffett Field, CA

March 2016
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Chapter 1

Introduction

The NASA Design and Analysis of Rotorcraft (NDARC) software is an aircraft system analysis tool that supports both conceptual design efforts and technology impact assessments. The principal tasks are to design (or size) a rotorcraft to meet specified requirements, including vertical takeoff and landing (VTOL) operation, and then analyze the performance of the aircraft for a set of conditions. For broad and lasting utility, it is important that the code have the capability to model general rotorcraft configurations, and estimate the performance and weights of advanced rotor concepts. The architecture of the NDARC code accommodates configuration flexibility, a hierarchy of models, and ultimately multidisciplinary design, analysis, and optimization. Initially the software is implemented with low-fidelity models, typically appropriate for the conceptual design environment.

An NDARC job consists of one or more cases, each case optionally performing design and analysis tasks. The design task involves sizing the rotorcraft to satisfy specified design conditions and missions. The analysis tasks can include off-design mission performance calculation, flight performance calculation for point operating conditions, and generation of subsystem or component performance maps. For analysis tasks, the aircraft description can come from the sizing task, from a previous case or a previous NDARC job, or be independently generated (typically the description of an existing aircraft).

The aircraft consists of a set of components, including fuselage, rotors, wings, tails, and propulsion. For each component, attributes such as performance, drag, and weight can be calculated; and the aircraft attributes are obtained from the sum of the component attributes. Description and analysis of conventional rotorcraft configurations is facilitated, while retaining the capability to model novel and advanced concepts. Specific rotorcraft configurations considered are single-main-rotor and tail-rotor helicopter, tandem helicopter, coaxial helicopter, and tiltrotor. The architecture of the code accommodates addition of new or higher-fidelity attribute models for a component, as well as addition of new components.

1–1 Background

The definition and development of NDARC requirements benefited substantially from the experiences and computer codes of the preliminary design team of the U.S. Army Aeroflightdynamics Directorate (AFDD) at Ames Research Center.

In the early 1970s, the codes SSP-1 and SSP-2 were developed by the Systems Research Integration Office (SRIO, in St. Louis) of the U.S. Army Air Mobility Research and Development Laboratory. SSP-1 performed preliminary design to meet specified mission requirements, and SSP-2 estimated the performance for known geometry and engine characteristics, both for single-main-rotor helicopter (ref. 1). Although similar tools were in use in the rotorcraft community, these computer programs were independently developed, to meet the requirements of government analysis. The Advanced Systems Research Office (ASRO, at Ames Research Center) of USAAMRDL produced in 1974 two Preliminary
Introduction

Systems Design Engineering (PSDE) studies (refs. 2 and 3) using SSP-1 and SSP-2. These two codes were combined into one code called PSDE by Ronald Shinn.

The MIT Flight Transportation Laboratory created design programs for helicopters (ref. 4) and tiltrotors (ref. 5). Michael Scully, who wrote the helicopter design program and was significantly involved in the development of the tiltrotor design program, joined ASRO in 1975, and ideas from the MIT programs began to be reflected in the continuing development of PSDE. An assessment of design trade-offs for the Advanced Scout Helicopter (ASH) used a highly modified version of PSDE (ref. 6).

A DoD Joint Study Group was formed in April 1975 to perform an Interservice Helicopter Commonality Study (HELCOM) for the Director of Defense Research and Engineering. The final HELCOM study report was published in March 1976 (ref. 7). A result of this study was an assessment by ASRO that PSDE needed substantial development, including better mathematical models and better technical substantiation, more flexible mission analysis, and improved productivity for both design and analysis tasks. Thus began an evolutionary improvement of the code, eventually named RASH (after the developer Ronald A. Shinn, as a consequence of the computer system identification of output by the first four characters of the user name). RASH included improvements in flight performance modeling, output depth, mission analysis, parametric weight estimation, design sensitivity studies, off-design cases, and coding style. The code was still only for single-main-rotor helicopters.

In the early 1980s, tool development evolved in two separate directions with the Preliminary Design Team at ASRO. RASH was developed into the HELO (or PDPAC) code, for conventional and compound single-main-rotor helicopters. With the addition of conversion models and wing weight estimation methods (refs. 8 and 9), RASH became the TR code, for tiltrotor aircraft. The JVX Joint Technology Assessment of 1982 utilized the HELO and TR codes. A special version called PDABC, including a weight estimation model for lift-offset rotors (ref. 10), was used to analyze the Advancing Blade Concept. The JVX JTA report (ref. 11) documented the methodology implemented in these codes.

Work in support of the LHX program from 1983 on led to a requirement for maneuver analysis of helicopters and tiltrotors, implemented in the MPP (Maneuver Performance Program) code by John Davis. The core aircraft model in MPP was similar to that in TR and HELO, but the trim strategy in particular was new. A design code does not require extensive maneuver analysis capability, but MPP had an impact on the design code development, with the MPP performance and trim methods incorporated into TR87. The sizing analysis of TR88 and the aircraft flight model from MPP were combined into the VAMP (VSTOL Design and Maneuver Program) code. VAMP combined the capability to analyze helicopters and tiltrotors in a single tool, although the capability of HELO to analyze compound helicopters was not replicated.

In the early 1990s, the RC (RotorCraft) code emerged from the evolution of VAMP, with John Preston as the lead developer (refs. 12 and 13). Some maneuver analysis capabilities from MPP were added, and the analysis capability extended to helicopters. The models were confirmed by comparison with results from TR and HELO. RC was operational by 1994, although HELO and TR continued to be used into the mid-1990s. RC97 was a major version, unifying the tiltrotor and helicopter analyses. The RC code introduced new features and capabilities, and productivity enhancements, as well as coding standards and software configuration control. Special versions of RC were routinely produced to meet the unique requirements of individual projects (such as ref. 14).

NASA, with support from the U.S. Army, in 2005 conducted the design and in-depth analysis of rotorcraft configurations that could satisfy the Vehicle Systems Program technology goals (ref. 15). These
technology goals and accompanying mission were intended to identify enabling technology for civil application of heavy-lift rotorcraft. The emphasis was on efficient cruise and hover, efficient structures, and low noise. The mission specified was to carry 120 passengers for 1200 nm, at a speed of 350 knots and 30,000 ft altitude. The configurations investigated were a Large Civil Tiltrotor (LCTR), a Large Civil Tandem Compound (LCTC), and a Large Advancing Blade Concept (LABC). The results of the NASA Heavy Lift Rotorcraft Systems Investigation subsequently helped define the content and direction of the Subsonic Rotary Wing project in the NASA Fundamental Aeronautics program. The design tool used was the AFDD RC code. This investigation was an example of the role of a rotorcraft sizing code within NASA. The investigation also illustrated the difficulties involved in adapting or modifying RC for configurations other than conventional helicopters and tiltrotors, supporting the requirement for a new tool.

1–2 Requirements

Out of this history, the development of NDARC began in early 2007. NDARC is entirely new software, built on a new architecture for the design and analysis of rotorcraft. From the RC theoretical basis, the parametric weight equations and the Referred Parameter Turboshaft Engine Model were used with only minor changes. Use was also made of the RC component aerodynamic models and rotor performance model. The current users of RC, informed by past and recent applications, contributed significantly to the requirements definition.

The principal tasks are to design (size) rotorcraft to meet specified requirements, and then analyze the performance of the aircraft for a set of flight conditions and missions. Multiple design requirements, from specific flight conditions and various missions, must be used in the sizing task. The aircraft performance analysis must cover the entire spectrum of aircraft capabilities, and allow general and flexible definition of conditions and missions.

For government applications and to support research, it is important to have the capability to model general rotorcraft configurations, including estimates of the performance and weights of advanced rotor concepts. In such an environment, software extensions and modifications are routinely required to meet the unique requirements of individual projects, including introduction of special weight and performance models for particular concepts.

Thus the code architecture must accommodate configuration flexibility and alternate models, including a hierarchy of model fidelity. Although initially implemented with low-fidelity models, typical of the conceptual design environment, ultimately the architecture must allow multidisciplinary design, analysis, and optimization. The component performance and engine models must cover all operating conditions. The software design and architecture must facilitate extension and modification of the software.

Complete and thorough documentation of the theory and its software implementation is essential, to support development and maintenance, and to enable effective use and modification. Most of the history described above supports this requirement by the difficulties encountered in the absence of good documentation. Documentation of the methodology was often prompted only by the need to substantiate conclusions of major technology assessments, and occasionally by the introduction of new users and developers. For a new software implementation of a new architectures, documentation is required from the beginning of the development.
The NDARC code performs design and analysis tasks. The design task involves sizing the rotorcraft to satisfy specified design conditions and missions. The analysis tasks can include off-design mission performance analysis, flight performance calculation for point operating conditions, and generation of subsystem or component performance maps. Figure 1-1 illustrates the tasks. The principal tasks (sizing, mission analysis, and flight performance analysis) are shown in the figure as boxes with heavy borders. Heavy black arrows show control of subordinate tasks.

The aircraft description (fig. 1-1) consists of all the information, input and derived, that defines the aircraft. The aircraft consists of a set of components, including fuselage, rotors, wings, tails, and propulsion. This information can be the result of the sizing task; can come entirely from input, for a fixed model; or can come from the sizing task in a previous case or previous job. The aircraft description information is available to all tasks and all solutions (indicated by light green arrows).
The sizing task determines the dimensions, power, and weight of a rotorcraft that can perform a specified set of design conditions and missions. The aircraft size is characterized by parameters such as design gross weight, weight empty, rotor radius, and engine power available. The relationships between dimensions, power, and weight generally require an iterative solution. From the design flight conditions and missions, the task can determine the total engine power or the rotor radius (or both power and radius can be fixed), as well as the design gross weight, maximum takeoff weight, drive system torque limit, and fuel tank capacity. For each propulsion group, the engine power or the rotor radius can be sized.

Missions are defined for the sizing task and for the mission performance analysis. A mission consists of a number of mission segments, for which time, distance, and fuel burn are evaluated. For the sizing task, certain missions are designated to be used for engine sizing, for design gross weight calculations, for transmission sizing, and for fuel tank sizing. The mission parameters include mission takeoff gross weight and useful load. For specified takeoff fuel weight with adjustable segments, the mission time or distance is adjusted so the fuel required for the mission (burned plus reserve) equals the takeoff fuel weight. The mission iteration is on fuel weight or energy.

Flight conditions are specified for the sizing task and for the flight performance analysis. For the sizing task, certain flight conditions are designated to be used for engine sizing, for design gross weight calculations, for transmission sizing, for maximum takeoff weight calculations, and for antitorque or auxiliary-thrust rotor sizing. The flight condition parameters include gross weight and useful load. For flight conditions and mission takeoff, the gross weight can be maximized, such that the power required equals the power available.

A flight state is defined for each mission segment and each flight condition. The aircraft performance can be analyzed for the specified state, or a maximum effort performance can be identified. The maximum effort is specified in terms of a quantity such as best endurance or best range, and a variable such as speed, rate of climb, or altitude. The aircraft must be trimmed, by solving for the controls and motion that produce equilibrium in the specified flight state. Different trim solution definitions are required for various flight states. Evaluating the rotor hub forces may require solution of the blade flap equations of motion.

1–4 Terminology

The following terminology is introduced as part of the development of the NDARC theory and software. Relationships among these terms are reflected in figure 1-1.

a) Job: An NDARC job consists of one or more cases.

b) Case: Each case performs design and/or analysis tasks. The analysis tasks can include off-design mission performance calculation, flight performance calculation for point operating conditions, and generation of airframe aerodynamics or engine performance maps.

c) Design Task: Size rotorcraft to satisfy specified set of design flight conditions and/or design missions. Key aircraft design variables are adjusted until all criteria are met. The resulting aircraft description can be basis for the mission analysis and flight performance analysis tasks.

d) Mission Analysis Task: Calculate aircraft performance for one off-design mission.

e) Flight Performance Analysis Task: Calculate aircraft performance for point operating condition.
f) Mission: Ordered set of mission segments, for which time, distance, and fuel burn are evaluated. Gross weight and useful load are specified for the beginning of the mission, and adjusted for fuel burn and useful load changes at each segment. Missions are defined for the sizing task and for the mission performance analysis.

g) Flight Condition: Point operating condition, with specified gross weight and useful load. Flight conditions are specified for the sizing task and for the flight performance analysis.

h) Flight State: Aircraft flight condition, part of definition of each flight condition and each mission segment. Flight state solution involves rotor blade motion, aircraft trim, and perhaps a maximum-effort calculation.

i) Component: The aircraft consists of a set of components, including fuselage, rotors, wings, tails, and propulsion. For each component, attributes such as performance, drag, and weight are calculated.

j) Propulsion: A propulsion group is a set of components and engine groups, connected by a drive system. An engine group consists of one or more engines of a specific type. The components define the power required. The engine groups define the power available. A jet group consists of one or more systems that produce a force on the aircraft. A charge group consists of systems that generate energy for the aircraft. Fuel tank systems are associated with the engine groups, jet groups, and charge groups. Fuel quantity is measured as either weight or energy.

1–5 Analysis Units

The code can use either English or SI units for input, output, and internal calculations. A consistent mass-length-time-temperature system is used, except for weight and power:

<table>
<thead>
<tr>
<th></th>
<th>length</th>
<th>mass</th>
<th>time</th>
<th>temperature</th>
<th>weight</th>
<th>power</th>
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<tbody>
<tr>
<td>English:</td>
<td>foot</td>
<td>slug</td>
<td>second</td>
<td>°F</td>
<td>pound</td>
<td>horsepower</td>
</tr>
<tr>
<td>SI:</td>
<td>meter</td>
<td>kilogram</td>
<td>second</td>
<td>°C</td>
<td>kilogram</td>
<td>kiloWatt</td>
</tr>
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</table>

Weight in the design description is actually mass, with pounds converted to slugs using the reference gravitational acceleration (9.80665 m/sec²). Gravitational force is the product of the mass and the actual acceleration due to gravity. In addition, the default units for flight conditions and missions are: speed in knots, time in minutes, distance in nautical miles, and rate of climb in feet-per-minute. The user can specify alternate units for these and other quantities.

1–6 Outline of Report

This document provides a complete description of the NDARC theoretical basis and architecture. Chapters 3–5 describe the tasks and solution procedures, the cost model is described in chapter 6, the emissions model is described in chapter 7, and chapters 8–20 present the models for the aircraft and its components. The propulsion system models are described in chapters 15–20. The engine, jet, and charger models are described in chapters 21–28; and the weight models in chapters 29 and 30. The accompanying NDARC Input Manual describes the use of the code.
1–7 References


The nomenclature for geometry and rotations employs the following conventions. A vector $x$ is a column matrix of three elements, measuring the vector relative to a particular basis (or axes, or frame). The basis is indicated as follows:

a) $x^A$ is a vector measured in axes A;
b) $x^{EF/A}$ is a vector from point F to point E, measured in axes A.

A rotation matrix $C$ is a three-by-three matrix that transforms vectors from one basis to another:

c) $C^{BA}$ transforms vectors from basis A to basis B, so $x^B = C^{BA}x^A$.

The matrix $C^{BA}$ defines the orientation of basis B relative to basis A, so it also may be viewed as rotating the axes from A to B. For a vector $u$, a cross-product matrix $\tilde{u}$ is defined as follows:

$$\tilde{u} = \begin{bmatrix}
0 & -u_3 & u_2 \\
u_3 & 0 & -u_1 \\
-u_2 & u_1 & 0
\end{bmatrix}$$

such that $\tilde{u}v$ is equivalent to the vector cross-product $u \times v$. The cross-product matrix enters the relationship between angular velocity and the time derivative of a rotation matrix:

$$\dot{C}^{AB} = -\tilde{\omega}^{AB/A}C^{AB} = C^{AB}\tilde{\omega}^{BA/B}$$

(the Poisson equations). For rotation by an angle $\alpha$ about the $x$, $y$, or $z$ axis (1, 2, or 3 axis), the following notation is used:

$$X_\alpha = \begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \alpha & \sin \alpha \\
0 & -\sin \alpha & \cos \alpha
\end{bmatrix}$$

$$Y_\alpha = \begin{bmatrix}
\cos \alpha & 0 & -\sin \alpha \\
0 & 1 & 0 \\
\sin \alpha & 0 & \cos \alpha
\end{bmatrix}$$

$$Z_\alpha = \begin{bmatrix}
\cos \alpha & \sin \alpha & 0 \\
-\sin \alpha & \cos \alpha & 0 \\
0 & 0 & 1
\end{bmatrix}$$

Thus for example, $C^{BA} = X_\phi Y_\theta Z_\psi$ means that the axes B are located relative to the axes A by first rotating by angle $\psi$ about the $z$-axis, then by angle $\theta$ about the $y$-axis, and finally by angle $\phi$ about the $x$-axis.
Nomenclature

Acronyms
AFDD U.S. Army Aeroflightdynamics Directorate
ASM available seat mile
CAS calibrated airspeed
CG charge group
CPI consumer price index
CTM Cost Too Much (cost model)
EG engine group
GW gross weight
IGE in ground effect
IRP intermediate rated power
IRS infrared suppressor
ISA International Standard Atmosphere
ISO International Organization for Standardization
JG jet group
MCP maximum continuous power
MCT maximum continuous thrust
MJ Mega-Joule
MRP maximum rated power
NDARC NASA Design and Analysis of Rotorcraft
OEI one engine inoperative
OGE out of ground effect
PG propulsion group
RPJEM referred parameter jet engine model
RPTEM referred parameter turboshaft engine model
SDGW structural design gross weight
SI Système International d’Unités (International System of Units)
SLS sea level standard
TAS true airspeed
WMTO maximum takeoff weight

Weights
\[ W_D \] design gross weight
\[ W_E \] empty weight
\[ W_{MTO} \] maximum takeoff weight
\[ W_{SD} \] structural design gross weight
\[ W_G \] gross weight, \( W_G = W_E + W_{UL} = W_O + W_{pay} + W_{fuel} \)
\[ W_O \] operating weight, \( W_O = W_E + W_{FUL} \)
\[ W_{UL} \] useful load, \( W_{UL} = W_{FUL} + W_{pay} + W_{fuel} \)
\[ W_{pay} \] payload
\[ W_{fuel} \] fuel weight
\[ W_{FUL} \] fixed useful load
\[ W_{burn} \] mission fuel burn
\[ W_{vib} \] vibration control weight
\[ W_{cont} \] contingency weight
\[ \chi \] technology factor
Nomenclature

Fuel Tanks

\( W_\text{fuel-cap} \) fuel capacity, maximum usable fuel weight
\( E_\text{fuel-cap} \) fuel capacity, maximum usable fuel energy
\( V_\text{fuel-cap} \) fuel capacity, volume
\( N_\text{auxtank} \) number of auxiliary fuel tanks
\( W_\text{aux-cap} \) auxiliary fuel tank capacity (weight)
\( E_\text{aux-cap} \) auxiliary fuel tank capacity (energy)

Power

\( P_{\text{reqPG}} \) power required, propulsion group; \( P_{\text{comp}} + P_{\text{xmsn}} + P_{\text{acc}} \)
\( P_{\text{reqEG}} \) power required, engine group
\( P_{\text{reqCG}} \) power required, charge group
\( P_{\text{avPG}} \) power available, propulsion group; \( \min(\sum f_P P_{\text{avEG}}, (\Omega_{\text{prim}}/\Omega_{\text{ref}}) P_{\text{DSlimit}}) \)
\( P_{\text{avEG}} \) power available, engine group; \( (N_{\text{eng}} - N_{\text{inop}}) P_{\text{av}} \)
\( P_{\text{avCG}} \) power available, charge group; \( (N_{\text{chrg}} - N_{\text{inop}}) P_{\text{av}} \)
\( P_{\text{comp}} \) component power required
\( P_{\text{xmsn}} \) transmission losses
\( P_{\text{acc}} \) accessory power
\( N_{\text{inop}} \) number of inoperative systems, engine group or jet group or charge group
\( P_{\text{DSlimit}} \) drive system torque limit (specified as power limit at reference rotor speed)
\( P_{\text{ESlimit}} \) engine shaft limit
\( P_{\text{RSlimit}} \) rotor shaft limit

Engine

\( P_{\text{eng}} \) sea level static power available per engine at specified takeoff rating
\( N_{\text{eng}} \) number of engines in engine group
\( P_{\text{av}} \) power available, installed; \( \min(P_a - P_{\text{loss}}, P_{\text{mech}}) \)
\( P_a \) power available, uninstalled
\( P_{\text{req}} \) power required, installed; \( P_q - P_{\text{loss}} \)
\( P_q \) power required, uninstalled
\( P_{\text{loss}} \) installation losses
\( P_{\text{mech}} \) mechanical power limit
\( S_P \) specific power, \( P/\dot{m} \) (conventional units)
\( \text{sfc} \) specific fuel consumption, \( \dot{w}/P \) (conventional units)
\( \dot{m} \) mass flow (conventional units)
\( \dot{w} \) fuel flow (conventional units)
\( E \) energy flow
\( F_N \) net jet thrust
\( D_{\text{aux}} \) momentum drag
\( N \) specification turbine speed
\( SW \) specific weight, \( P/W \)
**Jet**

- $T_{\text{eng}}$: sea level static thrust available per jet at specified takeoff rating
- $N_{\text{jet}}$: number of jets in jet group
- $T_{\text{avg}CG}$: thrust available, jet group; $(N_{\text{jet}} - N_{\text{inop}})T_{\text{av}}$
- $T_{\text{av}}$: thrust available, installed; min$(T_{\text{a}}\eta, T_{\text{mech}})$
- $T_{\text{a}}$: thrust available, uninstalled
- $T_{\text{req}JG}$: thrust required, jet group
- $T_{\text{req}}$: thrust required, installed; $T_{\text{a}}\eta$
- $T_{\text{q}}$: thrust required, uninstalled
- $\eta$: installation losses (efficiency)
- $T_{\text{mech}}$: mechanical thrust limit
- $ST$: specific thrust, $T/\dot{m}$ (conventional units)
- $\text{sfc}$: specific fuel consumption, $\dot{w}/T$ (conventional units)
- $\dot{m}$: mass flow (conventional units)
- $\dot{w}$: fuel flow (conventional units)
- $D_{\text{aux}}$: momentum drag
- $SW$: specific weight, $T/W$

**Charger**

- $P_{\text{chrg}}$: sea level static power available per charger at specified takeoff rating
- $N_{\text{chrg}}$: number of chargers in charge group
- $\eta$: installation losses (efficiency)

**Tip Speed and Rotation**

- $V_{\text{tip-ref}}$: reference tip speed, propulsion group primary rotor; each drive state
- $r$: gear ratio; $\Omega_{\text{dep}}/\Omega_{\text{prim}}$ for rotor, $\Omega_{\text{spec}}/\Omega_{\text{prim}}$ for engine
- $\Omega_{\text{prim}}$: primary rotor rotational speed, $\Omega = V_{\text{tip-ref}}/R$
- $\Omega_{\text{dep}}$: dependent rotor rotational speed, $\Omega = V_{\text{tip-ref}}/R$
- $\Omega_{\text{spec}}$: specification engine turbine speed
- $N_{\text{spec}}$: specification engine turbine speed (rpm)

**Mission**

- $T$: mission segment time
- $D$: mission segment distance
- $dR$: mission segment range contribution
- $E$: endurance
- $R$: range
- $\dot{w}$: fuel flow
### Environment
- $g$: gravitational acceleration
- $h$: altitude
- $c_s$: speed of sound
- $\rho$: density
- $\nu$: kinematic viscosity
- $\mu$: viscosity
- $T$: temperature, °R or °K
- $\tau$: temperature, °F or °C
- $V_w$: wind speed

### Axis Systems
- $I$: inertial
- $F$: aircraft
- $A$: component aerodynamic
- $B$: component
- $V$: velocity

### Geometry
- SL, BL, WL: fixed input position (station line, buttline, waterline)
- $x/L, y/L, z/L$: scaled input position; positive aft, right, up; arbitrary origin
- $L$: reference length (fuselage length, rotor radius, or wing span)
- $x, y, z$: calculated position, aircraft axes; positive forward, right, down; origin at reference point for geometry, origin at center of gravity for motion and loads
- $z^F$: component position vector, in aircraft axes, relative reference point
- $\ell$: length
- $S_{\text{wet}}$: wetted area
Motion
\( \phi_F, \theta_F, \psi_F \) roll, pitch, yaw angles; orientation airframe axes F relative inertial axes
\( \dot{\psi}_F \) turn rate
\( \theta_V, \psi_V \) climb, sideslip angles; orientation velocity axes V relative inertial axes
\( v_{AC}^F \) aircraft velocity
\( \omega_{AC}^F \) aircraft angular velocity
\( a_{AC}^F \) aircraft linear acceleration
\( n \) load factor
\( V \) aircraft velocity magnitude
\( V_h \) horizontal velocity
\( V_f \) forward velocity
\( V_s \) sideward velocity
\( V_c \) climb velocity
\( V_{cal} \) calibrated airspeed

Aerodynamics and Loads
\( v \) component velocity relative air (including interference)
\( q \) dynamic pressure, \( \frac{1}{2} \rho \|v\|^2 \)
\( \alpha \) angle of attack, component axes B relative aerodynamic axes A
\( \beta \) sideslip angle, component axes B relative aerodynamic axes A
\( \ell_f \) ratio flap chord to airfoil chord, \( \epsilon_f/c \)
\( \delta_f \) flap deflection
\( F \) force
\( M \) moment
\( D, Y, L \) aerodynamic drag, side, lift forces (component aerodynamic axes A)
\( M_x, M_y, M_z \) aerodynamic roll, pitch, yaw moments (component aerodynamic axes A)
\( c_{d}, c_{l} \) section drag, lift coefficients
\( C_D, C_Y, C_L \) component drag, side, lift force coefficients
\( C_{c}, C_{M}, C_{N} \) component roll, pitch, yaw moment coefficients
\( D/q \) drag area, \( SC_D \) (\( S = \) reference area of component)

Aircraft
\( DL \) disk loading, \( W_D/A_{ref} \)
\( A_{ref} \) reference rotor area, \( \sum f AA \); typically projected area of lifting rotors
\( WL \) wing loading, \( W_D/S_{ref} \)
\( S_{ref} \) reference wing area, \( \sum S \); sum area all wings
\( c_{AC} \) aircraft control
\( T \) control matrix
\( c \) component control, \( c = STc_{AC} + c_0 \)
\( \alpha_{tilt} \) tilt control variable
\( M \) aircraft hover figure of merit, \( W \sqrt{W/2\rho A_{ref}}/P \)
\( D_e \) aircraft effective drag, \( P/V \)
\( L/D_e \) aircraft effective lift-to-drag ratio, \( WV/P \)
### Nomenclature

#### Rotor

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( W/A )</td>
<td>disk loading, ( W = f_W W_D )</td>
</tr>
<tr>
<td>( C_W/\sigma )</td>
<td>design blade loading, ( W/\rho A V_{tip}^2 \sigma ) (( V_{tip} ) = hover tip speed)</td>
</tr>
<tr>
<td>( R )</td>
<td>blade radius</td>
</tr>
<tr>
<td>( A )</td>
<td>disk area</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>solidity (ratio blade area to disk area)</td>
</tr>
<tr>
<td>( T_{des} )</td>
<td>design thrust of antitorque or auxiliary-thrust rotor</td>
</tr>
<tr>
<td>( r )</td>
<td>direction of rotation (1 for counter-clockwise, (-1) for clockwise)</td>
</tr>
<tr>
<td>( \rho )</td>
<td>blade span coordinate</td>
</tr>
<tr>
<td>( \psi )</td>
<td>blade azimuth coordinate</td>
</tr>
<tr>
<td>( \mu )</td>
<td>advance ratio</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>inflow ratio</td>
</tr>
<tr>
<td>( M_{at} )</td>
<td>advancing tip Mach number</td>
</tr>
<tr>
<td>( \nu )</td>
<td>blade flap frequency (per-rev)</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>blade Lock number</td>
</tr>
<tr>
<td>( C_T/\sigma )</td>
<td>thrust coefficient divided by solidity, ( T/\rho A (\Omega R)^2 \sigma )</td>
</tr>
<tr>
<td>( \beta_c, \beta_s )</td>
<td>longitudinal, lateral flapping (tip-path plane tilt relative shaft)</td>
</tr>
<tr>
<td>( \theta_{0.75} )</td>
<td>blade collective pitch angle (at 75% radius)</td>
</tr>
<tr>
<td>( \theta_s, \theta_c )</td>
<td>lateral, longitudinal blade pitch angle</td>
</tr>
<tr>
<td>( H, Y, T )</td>
<td>drag, side, thrust force on hub (shaft axes)</td>
</tr>
<tr>
<td>( M_x, M_y )</td>
<td>roll, pitch moment on hub</td>
</tr>
<tr>
<td>( Q )</td>
<td>shaft torque</td>
</tr>
<tr>
<td>( P_i, P_t, P_o, P_p )</td>
<td>induced, interference, profile, parasite power</td>
</tr>
<tr>
<td>( \kappa )</td>
<td>induced power factor, ( P_i = \kappa P_{ideal} )</td>
</tr>
<tr>
<td>( c_{dmean} )</td>
<td>profile power mean drag coefficient, ( C_{P_o} = (\sigma/8)c_{dmean} F_P )</td>
</tr>
<tr>
<td>( M )</td>
<td>rotor hover figure of merit, ( T f_D v/P )</td>
</tr>
<tr>
<td>( L/D_e )</td>
<td>rotor effective lift-to-drag ratio, ( V L/(P_i + P_o) )</td>
</tr>
<tr>
<td>( \eta )</td>
<td>propulsive efficiency, ( TV/P )</td>
</tr>
</tbody>
</table>

#### Wing

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( W/S )</td>
<td>wing loading, ( W = f_W W_D )</td>
</tr>
<tr>
<td>( S )</td>
<td>area</td>
</tr>
<tr>
<td>( b )</td>
<td>span</td>
</tr>
<tr>
<td>( c )</td>
<td>chord, ( S/b )</td>
</tr>
<tr>
<td>( AR )</td>
<td>aspect ratio, ( b^2/S )</td>
</tr>
</tbody>
</table>
The NDARC code performs design and analysis tasks. The design task involves sizing the rotorcraft to satisfy specified design conditions and missions. The analysis tasks can include mission performance analysis, flight performance calculation for point operating conditions, and generation of subsystem or component performance maps.

### 3–1 Size Aircraft for Design Conditions and Missions

#### 3-1.1 Sizing Method

The sizing task determines the dimensions, power, and weight of a rotorcraft that can perform a specified set of design conditions and missions. The aircraft size is characterized by parameters such as design gross weight \( W_D \) or weight empty \( W_E \), rotor radius \( R \), and engine power available \( P_{eng} \). The relationships between dimensions, power, and weight generally require an iterative solution. From the design flight conditions and missions, the task can determine the total engine power or the rotor radius (or both power and radius can be fixed), as well as the design gross weight, maximum takeoff weight, drive system torque limit, and fuel tank capacity. For each propulsion group, the engine power or the rotor radius can be sized:

a) **Engine power:** Determine \( P_{eng} \), for fixed \( R \). The engine power is the maximum of the power required for all designated sizing flight conditions and sizing missions (typically including vertical flight, forward flight, and one-engine inoperative). Hence the engine power is changed by the ratio \( \max(P_{reqPG}/P_{avPG}) \) (excluding flight states for which zero power margin is calculated, such as maximum gross weight or maximum effort). This approach is the one most commonly used for the sizing task.

b) **Rotor radius:** Determine \( R \) for input \( P_{eng} \). The maximum power required for all designated sizing flight conditions and sizing missions is calculated, and then the rotor radius determined such that the power required equals the input power available. The change in radius is estimated as \( R = R_{old} \sqrt{P_{reqPG}/P_{avPG}} \) (excluding flight states for which zero power margin is calculated, such as maximum gross weight or maximum effort). For multi-rotor aircraft, the radius can be fixed rather than sized for some rotors.

Alternatively, \( P_{eng} \) and \( R \) can be input rather than sized. For each jet group, the design thrust can be sized:

Determine \( T_{jet} \). The design thrust is the maximum of the thrust required for all designated sizing flight conditions and sizing missions. Hence the design thrust
is changed by the ratio $\max(T_{reqJG}/T_{avJG})$ (excluding flight states for which zero thrust margin is calculated).

For each charge group, the design power can be sized:

Determine $P_{\text{chrg}}$. The design power is the maximum of the power required for all designated sizing flight conditions and sizing missions. Hence the design power is changed by the ratio $\max(P_{reqCG}/P_{avCG})$ (excluding flight states for which zero power margin is calculated).

Aircraft parameters can be determined by a subset of the design conditions and missions:

a) Design gross weight $W_D$: maximum gross weight from designated conditions and missions (for which gross weight is not fixed).

b) Maximum takeoff gross weight $W_{MTO}$: maximum gross weight from designated conditions (for which gross weight is not fixed).

c) Drive system torque limit $P_{DS\text{limit}}$: maximum torque from designated conditions and missions (for each propulsion group; specified as power limit at reference rotor speed).

d) Fuel tank capacity: maximum fuel weight $W_{\text{fuel-cap}}$ or energy $E_{\text{fuel-cap}}$ from designated missions (without auxiliary tanks).

e) Antitorque or auxiliary thrust rotor design thrust $T_{\text{design}}$: maximum rotor thrust from designated conditions and missions.

Alternatively, these parameters can be fixed at input values. The design gross weight ($W_D$) can be fixed. The weight empty can be fixed, which is implemented by changing the contingency weight.

A successive substitution method is used for the sizing iteration, with an input tolerance $\epsilon$. Relaxation is applied to $P_{\text{eng}}$ or $R$, $T_{\text{jet}}$, $P_{\text{chrg}}$, $W_D$, $W_{MTO}$, $P_{DS\text{limit}}$, $W_{\text{fuel-cap}}$ or $E_{\text{fuel-cap}}$, and $T_{\text{design}}$. Convergence is tested in terms of these parameters, and the aircraft weight empty $W_E$. Two successive substitution loops are used. The outer loop is an iteration on performance: engine power or rotor radius, jet thrust, charger power. The inner loop is an iteration on parameters: $W_D$, $W_{MTO}$, $P_{DS\text{limit}}$, $W_{\text{fuel-cap}}$ or $E_{\text{fuel-cap}}$, and $T_{\text{design}}$. Either loop can be absent, depending on the definition of the size task.

For each flight condition and each mission, the gross weight and useful load are specified. The gross weight can be input, maximized, or fallout. For flight conditions, the payload or fuel weight can be specified, and the other calculated; or both payload and fuel weight specified, with gross weight fallout. For missions, the payload or fuel weight can be specified, the other fallout, and then time or distance of mission segments adjusted; or fuel weight calculated from mission, and payload fallout; or both payload and fuel weight specified (or payload specified and fuel weight calculated from mission), with gross weight fallout. For each flight condition and mission segment, the following checks are performed:

a) The power required does not exceed the power available: $P_{reqPG} \leq (1 + \epsilon)P_{avPG}$ (for each propulsion group).

b) The torque required does not exceed the drive system limit: $P_{reqPG}/\Omega \leq (1 + \epsilon)P_{DS\text{limit}}/\Omega_{\text{prin}}$ (for each propulsion group). Rotor shaft torque and engine shaft torque are also checked.

c) The jet thrust required does not exceed the thrust available: $T_{reqJG} \leq (1 + \epsilon)T_{avJG}$ (for each jet group).
d) The charger power required does not exceed the power available: \( P_{reqCG} \leq (1 + \epsilon)P_{avCG} \) (for each charge group).

e) The fuel weight does not exceed the fuel capacity: \( W_{fuel} \leq (1 + \epsilon)(W_{fuel-cap} + \sum N_{auxtank}w_{aux-cap}) \) (including auxiliary tanks).

These checks are performed using an input tolerance \( \epsilon \).

Sizing flight conditions typically include takeoff (hover or specified vertical rate of climb), one-engine inoperative, cruise or dash, perhaps transmission, and perhaps mission midpoint hover. Sizing missions typically include a design mission and a mission to determine fuel tank capacity.

### 3-1.2 Component Sizing

#### 3-1.2.1 Propulsion System

The engine size is described by the power \( P_{eng} \), which is the sea-level static power available per engine at a specified takeoff rating. The number of engines \( N_{eng} \) is specified for each engine group.

If the sizing task determines the engine power for a propulsion group, the power \( P_{eng} \) of at least one engine group is found (including the first engine group). The total power required is \( P_{PG} = r \sum N_{eng}P_{eng} \), where \( r = \max(P_{reqPG}/P_{avPG}) \). The sized power is \( P_{sized} = P_{PG} - \sum \text{fixed} N_{eng}P_{eng} \), where the sum is over the engine groups for which the power is fixed. Then the sized engine power is \( P_{eng} = f_{n}P_{sized}/N_{eng} \) for the \( n \)-th engine group (with \( f_{n} \) an input ratio and \( f_{1} = \sum_{n \neq 1, sized} f_{n} \) for the first group).

The jet size is described by the thrust \( T_{jet} \), which is the sea-level static thrust available per jet at a specified takeoff rating. The number of jets \( N_{jet} \) is specified for each jet group. If the sizing task determines the jet thrust for a jet group, the thrust \( T_{jet} \) is scaled by the factor \( r = \max(T_{reqJG}/T_{avJG}) \).

The charger size is described by the power \( P_{chrg} \), which is the sea-level static power available. The number of systems \( N_{chrg} \) is specified for each charge group. If the sizing task determines the charger power for a charge group, the power \( P_{chrg} \) is scaled by the factor \( r = \max(P_{reqCG}/P_{avCG}) \).

#### 3-1.2.2 Main Rotor

The main-rotor size is defined by the radius \( R \) or disk loading \( W/A \), thrust-weighted solidity \( \sigma \), hover tip speed \( V_{tip} \), and blade loading \( C_{W}/\sigma = W/\rho AV_{tip}^{2}/\sigma \). With more than one main-rotor, the disk loading and blade loading are obtained from an input fraction of design gross weight, \( W = f_{W}W_{D} \). The air density \( \rho \) for \( C_{W}/\sigma \) is obtained from a specified takeoff condition.

If the rotor radius is fixed for the sizing task, three of \( (R \text{ or } W/A), C_{W}/\sigma, V_{tip} \) and \( \sigma \) are input, and the other parameters are derived. Optionally the radius can be calculated from a specified ratio to the radius of another rotor.

If the sizing task determines the rotor radius \( (R \text{ and } W/A) \), then two of \( C_{W}/\sigma, V_{tip}, \text{ and } \sigma \) are input, and the other parameter is derived. The radius can be sized for just a subset of the rotors, with fixed radius for the others. The radii of all sized rotors are changed by the same factor.

#### 3-1.2.3 Antitorque or Auxiliary Thrust Rotor

For antitorque and auxiliary thrust rotors, three of \( (R \text{ or } W/A), C_{W}/\sigma, V_{tip}, \text{ and } \sigma \) are input, and the other parameters are derived. Optionally the radius can be calculated from a specified ratio to the radius of another rotor. Optionally the radius can be scaled with the main-rotor radius. The disk loading and
blade loading are based on \( f_T T_{\text{design}} \), where \( f_T \) is an input factor and \( T_{\text{design}} \) is the maximum thrust from designated design conditions and missions.

### 3-1.2.4 Wing

The wing size is defined by the wing area \( S \) or wing loading \( W/S \), span (perhaps calculated from other geometry), chord, and aspect ratio. With more than one wing, the wing loading is obtained from an input fraction of design gross weight, \( W = f_W W_D \).

Two of the following parameters are input: area (or wing loading), span, chord, and aspect ratio; the other parameters are derived. Optionally the span can be calculated from the rotor radius, fuselage width, and clearance (typically used for tiltrotors). Optionally the span can be calculated from a specified ratio to the span of another wing.

### 3-1.2.5 Fuel Tank

The fuel tank capacity \( W_{\text{fuel-cap}} \) (maximum usable fuel weight) or \( E_{\text{fuel-cap}} \) (maximum usable fuel energy) is determined from designated sizing missions. The maximum mission fuel required, \( W_{\text{fuel-miss}} \) or \( E_{\text{fuel-miss}} \) (excluding reserves and any fuel in auxiliary tanks), gives

\[
W_{\text{fuel-cap}} = \max(f_{\text{fuel-cap}} W_{\text{fuel-miss}}, W_{\text{fuel-miss}} + W_{\text{reserve}})
E_{\text{fuel-cap}} = \max(f_{\text{fuel-cap}} E_{\text{fuel-miss}}, E_{\text{fuel-miss}} + E_{\text{reserve}})
\]

or

\[
W_{\text{fuel-cap}} = d_{\text{fuel-cap}} + f_{\text{fuel-cap}} (W_{\text{fuel-miss}} + W_{\text{reserve}})
E_{\text{fuel-cap}} = d_{\text{fuel-cap}} + f_{\text{fuel-cap}} (E_{\text{fuel-miss}} + E_{\text{reserve}})
\]

where \( f_{\text{fuel-cap}} \geq 1 \) is an input factor. Alternatively, the fuel tank capacity \( W_{\text{fuel-cap}} \) or \( E_{\text{fuel-cap}} \) can be input. Optionally the maximum mission battery discharge power gives \( P_{\text{cap}} \), from which \( E_{\text{fuel-cap}} = \max(E_{\text{fuel-cap}}, (e_{\text{tank}}/\pi_{\text{tank}}) P_{\text{cap}}) \) (MJ from kW).

### 3-1.2.6 Weights

The structural design gross weight \( W_{\text{SD}} \) and maximum takeoff weight \( W_{\text{MTO}} \) can be input, or specified as an increment \( d \) plus a fraction \( f \) of a weight \( W \):

\[
W_{\text{SD}} = d_{\text{SDGW}} + f_{\text{SDGW}} W = \begin{cases} d_{\text{SDGW}} + f_{\text{SDGW}} W_D & \text{if } W_D - W_{\text{fuel}} + f_{\text{fuel}} W_{\text{fuel-cap}} > 0 \\ d_{\text{SDGW}} + f_{\text{SDGW}} W_{\text{MTO}} & \text{if } W_D - W_{\text{fuel}} + f_{\text{fuel}} W_{\text{fuel-cap}} < 0 \end{cases}
\]

\[
W_{\text{MTO}} = d_{\text{WMTOW}} + f_{\text{WMTOW}} W = \begin{cases} d_{\text{WMTOW}} + f_{\text{WMTOW}} W_D & \text{if } W_D - W_{\text{fuel}} + W_{\text{fuel-cap}} > 0 \\ d_{\text{WMTOW}} + f_{\text{WMTOW}} W_{\text{SD}} & \text{if } W_D - W_{\text{fuel}} + W_{\text{fuel-cap}} < 0 \end{cases}
\]

This convention allows the weights to be input directly \((f = 0)\), or scaled with a design weight. For \( W_{\text{SD}}, W \) is the design gross weight \( W_D \), or \( W_D \) adjusted for a specified fuel state (input fraction of fuel capacity), or the maximum takeoff weight \( W_{\text{MTO}} \). Alternatively, \( W_{\text{SD}} \) can be calculated as the gross weight at a designated sizing flight condition. For \( W_{\text{MTO}}, W \) is the design gross weight \( W_D \), or \( W_D \) adjusted for maximum fuel capacity, or the structural design gross weight \( W_{\text{SD}} \). Alternatively, \( W_{\text{MTO}} \) can be calculated as the maximum gross weight possible at a designated sizing flight condition.
3-1.2.7 Drive System Limit

The drive system limit is defined as a power limit, $P_{DS\text{limit}}$. The limit is properly a torque limit, $Q_{DS\text{limit}} = P_{DS\text{limit}}/\Omega_{\text{ref}}$, but is expressed as a power limit for clarity. The drive system limit can be specified as follows (with $f_{\text{limit}}$ an input factor):

a) Input $P_{DS\text{limit}}$.
b) From the engine takeoff power limit, $P_{DS\text{limit}} = f_{\text{limit}} \sum N_{\text{eng}} P_{\text{eng}}$ (summed over all engine groups).
c) From the power available at the transmission sizing conditions and missions, $P_{DS\text{limit}} = f_{\text{limit}} (\Omega_{\text{ref}}/\Omega_{\text{prim}}) \sum N_{\text{eng}} P_{\text{av}}$ (largest of all conditions and segments).
d) From the power required at the transmission sizing conditions and missions, $P_{DS\text{limit}} = f_{\text{limit}} (\Omega_{\text{ref}}/\Omega_{\text{prim}}) \sum N_{\text{eng}} P_{\text{req}}$ (largest of all conditions and segments).

The drive system limit is a limit on the entire propulsion system. To account for differences in the distribution of power through the drive system, limits are also used for the torque of each rotor shaft ($P_{RS\text{limit}}$) and of each engine group ($P_{ES\text{limit}}$). The engine shaft limit is calculated as for the drive system limit, without the sum over engine groups. The rotor shaft limit is either input or calculated from the rotor power required at the transmission sizing flight conditions. The power limit is associated with a reference rotational speed, and when applied the limit is scaled with the rotational speed of the flight state. The rotation speed for the drive system limit $P_{DS\text{limit}}$ is the hover speed of the primary rotor of the propulsion group (for the first drive state). The rotation speed for the engine shaft limit $P_{ES\text{limit}}$ is the corresponding engine turbine speed. The rotation speed for the rotor shaft limit $P_{RS\text{limit}}$ is the corresponding speed of that rotor.

The drive system limits can be specified for several levels, analogous to engine ratings. The limit $P_{DS\text{limit}}$ is associated with the maximum continuous rating (MCQ or MCP). An alternate rating changes the torque limit by the factor $x$. Typically $x > 1$ for ratings associated with short duration operation. The torque limit is calculated from $Q_{\text{limit}} = Q/x$ for the flight condition or mission segment. The torque limit is applied as $Q = xQ_{\text{limit}}$.

3–2 Mission Analysis

For the mission analysis, the fuel weight or payload weight is calculated. Power required, torque (drive system, engine shaft, and rotor shaft), and fuel weight are then verified to be within limits. Missions can be fixed or adjustable.

3–3 Flight Performance Analysis

For each performance flight condition, the power required is calculated or maximum gross weight is calculated. Power required, torque (drive system, engine shaft, and rotor shaft), and fuel weight are then verified to be within limits.

3–4 Maps

3-4.1 Engine Performance

The engine performance can be calculated for a specified range of power, altitude, and speed.
3-4.2 Airframe Aerodynamics

The airframe aerodynamic loads can be calculated for a specified range of angle of attack, sideslip angle, and control angles. The aerodynamic analysis evaluates the component lift, drag, and moments given the velocity. The aircraft velocity is $v_{AC}^F = C^{FA}(v 0 0)^T$; interference velocity from the rotors is not considered. From the angle of attack $\alpha$ and sideslip angle $\beta$, the transformation from wind axes to airframe axes is $C^{FA} = Y_\alpha Z_{-\beta}$ (optionally $C^{FA} = Z_{-\beta} Y_\alpha$ can be used, for better behavior in sideward flight). The loads are summed in the airframe axes (with and without tail loads), and then the wind axis loads are:

$$F^A = \begin{pmatrix} -D \\ Y \\ -L \end{pmatrix} = C^{AF} F^F$$
$$M^A = \begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix} = C^{AF} M^F$$

The center of action for the total loads is the fuselage location $z_{\text{fuse}}$. The ratio of the loads to dynamic pressure is required, so a nominal velocity $v = 100$ (ft/sec or m/sec) and sea level standard density are used.
Flight conditions are specified for the sizing task and for the flight performance analysis. For each condition, a flight state is also defined. For the sizing task, certain flight conditions are designated for engine sizing, design gross weight calculations, transmission sizing, maximum takeoff weight calculations, or rotor thrust sizing. The flight condition parameters include gross weight and useful load. The gross weight can be specified as follows, consistent with the sizing method:

a) Design gross weight, $W_D$ (calculated or input).
b) Structural design gross weight, $W_{SD}$, or maximum takeoff weight, $W_{MTO}$ (which may depend on $W_D$).
c) Function of $W_D$: $W = d + fW_D$ (with $d$ an input weight and $f$ an input factor).
d) Function of $W_{SD}$ ($W = d + fW_{SD}$); or function of $W_{MTO}$ ($W = d + fW_{MTO}$).
e) Input $W$.
f) Gross weight from specified mission segment or flight condition; or function of source ($W = d + fW_{source}$).
g) Gross weight maximized, such that power required equals specified power: $P_{reqPG} = fP_{avPG} + d$, with $d$ an input power and $f$ an input factor; in general, $\min((fP_{avPG} + d) - P_{reqPG}) = 0$, minimum over all propulsion groups; default $d = 0$ and $f = 1$ gives zero power margin, $\min(P_{avPG} - P_{reqPG}) = 0$.
h) Gross weight maximized, such that thrust required equals specified thrust: $T_{reqJG} = fT_{avJG} + d$, with $d$ an input thrust and $f$ an input factor; in general, $\min((fT_{avJG} + d) - T_{reqJG}) = 0$, minimum over all jet groups; default $d = 0$ and $f = 1$ gives zero thrust margin, $\min(T_{avJG} - T_{reqJG}) = 0$.
i) Gross weight maximized, such that transmission torque equals limit: zero torque margin, $\min(P_{limit} - P_{req}) = 0$ (minimum over all propulsion groups, engine groups, and rotors).
j) Gross weight maximized, such that power required equals specified power, or thrust required equals specified thrust, or transmission torque equals limit (most restrictive).
k) Gross weight fallout from input payload and fuel weights: $W_G = W_O + W_{pay} + W_{fuel}$.

Only the last five options are available for $W_D$ design conditions in the sizing task. The gross weight can be obtained from a mission segment only for the sizing task. Optionally the altitude can be obtained from the specified mission segment or flight condition. The secant method or the method of false position is used to solve for the maximum gross weight. A tolerance $\epsilon$ and a perturbation $\Delta$ are specified.
The useful load can be specified as follows, consistent with the sizing method and the gross weight specification.

1. Input payload weight $W_{\text{pay}}$, fuel weight fallout:
   \[ W_{\text{fuel}} = W_G - W_O - W_{\text{pay}}. \]
2. Input fuel weight $W_{\text{fuel}}$, payload weight fallout:
   \[ W_{\text{pay}} = W_G - W_O - W_{\text{fuel}}. \]
3. Input payload and fuel weights, gross weight fallout (must match gross weight option):
   \[ W_G = W_O + W_{\text{pay}} + W_{\text{fuel}}. \]

The input fuel weight is
\[ W_{\text{fuel}} = \min(d_{\text{fuel}} + f_{\text{fuel}} W_{\text{fuel}} - \text{cap}, W_{\text{fuel}} - \text{cap}) + \sum N_{\text{auxtank}} W_{\text{aux}} - \text{cap}. \]
For fallout fuel weight, $N_{\text{auxtank}}$ is changed (optionally only increased). If the auxiliary tank weight is greater than the increment in fuel weight needed, then the fallout fuel weight $W_{\text{fuel}} = W_G - W_O - W_{\text{pay}}$ can not be achieved; in such a case, the fuel weight is capped at the maximum fuel capacity and the payload weight changed instead. The fixed useful load can have increments, including crew weight increment; equipment weight increment; and installed folding, wing, wing extension, and other kits. These increments are reflected in the fallout weight. If the motive device burns energy not weight, then the fuel weight is zero and the input fuel energy is
\[ E_{\text{fuel}} = \min(d_{\text{fuel}} + f_{\text{fuel}} E_{\text{fuel}} - \text{cap}, E_{\text{fuel}} - \text{cap}) + \sum N_{\text{auxtank}} E_{\text{aux}} - \text{cap}. \]

4-2 Mission

Missions are defined for the sizing task and for the mission performance analysis. A mission consists of a specified number of mission segments. A flight state is defined for each mission segment. For the sizing task, certain missions are designated for engine sizing, design gross weight calculations, transmission sizing, or fuel tank sizing. The mission parameters include mission takeoff gross weight and useful load. The gross weight can be specified as follows, consistent with the sizing method:

1. Design gross weight, $W_D$ (calculated or input).
2. Structural design gross weight, $W_{SD}$, or maximum takeoff weight, $W_{MTO}$ (which may depend on $W_D$).
3. Function of $W_D$: $W = d + fW_D$ (with $d$ an input weight and $f$ an input factor).
4. Function of $W_{SD}$, $W = d + fW_{SD}$; or function of $W_{MTO}$, $W = d + fW_{MTO}$.
5. Input $W$.
6. Gross weight maximized at specified mission segments, such that power required equals specified power: $P_{\text{reqPG}} = fP_{avPG} + d$, with $d$ an input power and $f$ an input factor; in general, $\min((fP_{avPG} + d) - P_{\text{reqPG}}) = 0$, minimum over all propulsion groups; default $d = 0$ and $f = 1$ gives zero power margin, $\min(P_{avPG} - P_{\text{reqPG}}) = 0$.
7. Gross weight maximized at specified mission segments, such that thrust required equals specified thrust: $T_{\text{reqJG}} = fT_{avJG} + d$, with $d$ an input power and $f$ an input factor; in general, $\min((fT_{avJG} + d) - T_{\text{reqJG}}) = 0$, minimum over all jet groups; default $d = 0$ and $f = 1$ gives zero thrust margin, $\min(T_{avJG} - T_{\text{reqJG}}) = 0$.
8. Gross weight maximized at specified mission segments, such that transmission torque equals limit: zero torque margin, $\min(P_{\text{limit}} - P_{\text{req}}) = 0$ (minimum over all propulsion groups, engine groups, and rotors).
9. Gross weight maximized at specified mission segments, such that power required equals specified power, or thrust required equals specified thrust, or transmission torque equals limit (most restrictive).
10. Gross weight fallout from input initial payload and fuel weights: $W_G = W_O + \ldots$
\[ W_{\text{pay}} + W_{\text{fuel}}. \]

k) Gross weight fallout from input initial payload weight and calculated mission fuel weight: \( W_G = W_O + W_{\text{pay}} + W_{\text{fuel}}. \)

If maximum gross weight is specified for more than one mission segment, then the minimum takeoff gross weight increment is used; so the power or torque margin is zero for the critical segment and positive for other designated segments. Only the last six options are available for \( W_D \) design conditions in the sizing task. The secant method or the method of false position is used to solve for the maximum gross weight. A tolerance \( \epsilon \) and a perturbation \( \Delta \) are specified.

The useful load can be specified as follows, consistent with the sizing method and the gross weight specification:

a) Input initial payload weight \( W_{\text{pay}} \), fuel weight fallout: \( W_{\text{fuel}} = W_G - W_O - W_{\text{pay}}. \)

b) Input fuel weight \( W_{\text{fuel}} \), initial payload weight fallout: \( W_{\text{pay}} = W_G - W_O - W_{\text{fuel}}. \)

c) Calculated mission fuel weight, initial payload weight fallout: \( W_{\text{pay}} = W_G - W_O - W_{\text{fuel}}. \)

d) Input payload and fuel weights, takeoff gross weight fallout (must match gross weight option): \( W_G = W_O + W_{\text{pay}} + W_{\text{fuel}}. \)

The input fuel weight is \( W_{\text{fuel}} = \min(d_{\text{fuel}} + f_{\text{fuel}}W_{\text{fuel-cap}}, W_{\text{fuel-cap}}) + \sum N_{\text{auxtank}}W_{\text{aux-cap}}; \) if the fuel weight is fallout, then this is the initial value for the mission iteration. If the fuel weight is not calculated from the mission, then the mission is changed. The fixed useful load can have increments, including installed folding kits; other increments are specified for individual mission segments. If the motive device burns energy not weight, then the fuel weight is zero and the input fuel energy is \( E_{\text{fuel}} = \min(d_{\text{fuel}} + f_{\text{fuel}}E_{\text{fuel-cap}}, E_{\text{fuel-cap}}) + \sum N_{\text{auxtank}}E_{\text{aux-cap}}. \)

The takeoff gross weight is evaluated at the start of the mission, perhaps maximized for zero power margin at a specified mission segment (either takeoff conditions or midpoint). Then the aircraft is flown for all segments. For calculated mission fuel weight, the fuel weight at takeoff is set equal to the fuel required for the mission (burned plus reserve). For specified takeoff fuel weight with adjustable segments, the mission time or distance is adjusted so the fuel required for the mission (burned plus reserve) equals the takeoff fuel weight. The mission iteration is thus on mission fuel weight or energy. Range credit segments (defined below) can also require an iteration. A successive substitution method is used if an iteration is required, with a tolerance \( \epsilon \) specified. The iteration to maximize takeoff gross weight could be an outer loop around the mission iteration, but instead it is executed as part of the mission iteration. At the specified mission segment, the gross weight is maximized for zero power margin, and the resulting gross weight increment added to the takeoff gross weight for the next mission iteration. Thus takeoff gross weight is also a variable of the mission iteration.

Each mission consists of a specified number of mission segments. The following segment types can be specified:

a) Taxi or warm-up (fuel burned but no distance added to range).

b) Distance: fly segment for specified distance (calculate time).

c) Time: fly segment for specified time (calculate distance).

d) Hold: fly segment for specified time (loiter, so fuel burned but no distance added.
to range).
e) Climb: climb or descend from present altitude to next segment altitude (calculate
time and distance).
f) Spiral: climb or descend from present altitude to next segment altitude (fuel
burned but no distance added to range).
g) Fuel: use or replace specified fuel amount (calculate time and distance).
h) Burn: use or replace specified fuel amount (calculate time but no distance added
to range).

For each mission segment a payload weight can be specified; or a payload weight change can be specified,
as an increment from the initial payload or as a fraction of the initial payload. If the payload is calculated
from the number of passengers, then for each mission segment a change in the number of passengers
can be specified.

The number of auxiliary fuel tanks can change with each mission segment: \( N_{aux\text{tank}} \) is changed
based on the fuel weight (optionally only increased relative to the input number at takeoff, optionally fixed
during mission). For input fuel weight, \( N_{aux\text{tank}} \) is specified at takeoff. For fallout fuel weight, the takeoff
fuel weight is changed for the auxiliary fuel tank weight given \( N_{aux\text{tank}} \) (fixed \( W_G - W_{\text{pay}} = W_O + W_{\text{fuel}} \)).
If the auxiliary tank weight is greater than the increment in fuel weight needed, then the fallout fuel
weight \( W_{\text{fuel}} = W_G - W_O - W_{\text{pay}} \) can not be achieved; in such a case, the fuel weight is capped at the
maximum fuel capacity and the takeoff payload weight changed instead. For fuel tank design missions,
\( N_{aux\text{tank}} \) and fuel tank capacity is determined from \( W_{\text{fuel}} \). Optionally the aircraft can refuel (either on the
ground or in the air) at the start of a mission segment, by either filling all tanks to capacity or adding a
specified fuel weight. Optionally fuel can be dropped at the start of a mission segment. The fixed useful
load can have changes, including crew weight increment, equipment weight increment, and installed
wing extension and other kits.

For calculation of the time or distance in a mission segment, a headwind or tailwind can be specified.
The wind velocity is a linear function of altitude \( h \): \( V_w = \pm (\max(0, d_{\text{wind}} + f_{\text{wind}}h)) \), with the plus sign
for a headwind and the minus sign for a tailwind. For example, California-to-Hawaii 85th percentile
winter quartile headwind profile is \( V_w = 9.59 + 0.00149h \) (with altitude \( h \) in ft).

Mission fuel reserves can be specified in several ways for each mission. Fuel reserves can be
defined in terms of specific mission segments, for example 200 miles plus 20 minutes at speed for best
endurance. Fuel reserves can be an input fraction of the fuel burned by all (except reserve) mission
segments, so \( W_{\text{fuel}} = (1 + f_{\text{res}})W_{\text{burn}} \) or \( E_{\text{fuel}} = (1 + f_{\text{res}})E_{\text{burn}} \). Fuel reserves can be an input fraction
of the fuel capacity, so \( W_{\text{fuel}} = W_{\text{burn}} + f_{\text{res}}W_{\text{fuel-cap}} \) or \( E_{\text{fuel}} = E_{\text{burn}} + f_{\text{res}}E_{\text{fuel-cap}} \). If more than
one criterion for reserve fuel is specified, the maximum reserve is used. Time and distance in reserve
segments are not included in endurance and range.

To facilitate specification of range, range calculated for a group of segments (typically climb and
descent segments) can be credited to a designated distance segment. For mission analysis, missions can
be fixed or adjustable. In an adjustable mission, the fuel is input, so the time or distance in specified
segments is adjusted based on the calculated fuel burned. If more than one segment is adjusted, all must
be distance or all must be time or hold. Each segment can have only one special designation: reserve,
adjustable, or range credit.

A segment with a large distance, time, or altitude change can be split into several segments, for
more accurate calculation of the performance and fuel burned. The number of segments \( n \) can be input,
or calculated from an input increment $\Delta$: $n = [x/\Delta] + 1$, where the brackets indicate integer truncation, and $x$ is the total distance, time, or altitude change. Then the change for each split segment is $\Delta = x/n$.

Table 4-1 summarizes the time $T$, distance $D$, and range $dR$ calculations for each segment. The segment fuel burned is $dW_{\text{burn}} = T\dot{w}$, where $\dot{w}$ is the fuel flow; and $dE_{\text{burn}} = T\dot{E}$, where $\dot{E}$ is the energy flow. The horizontal velocity is $V_h$, and the vertical velocity (climb or descent) is $V_c$. The altitude at the start of the segment is $h_s$, and at the end of the segment (start of next segment) $h_{\text{end}}$. The wind speed is $V_w$, and the ground speed is $V_h - V_w$. The air distance is calculated from the time and speed ($D/V_h$), without the wind speed.

To use or replace fuel (weight or energy), the increment is specified in terms of the capacity ($dW_{\text{burn}} = d_{\text{tank}} + f_{\text{tank}}W_{\text{fuel-cap}}$) or the current fuel ($dW_{\text{burn}} = d_{\text{tank}} + f_{\text{tank}}W_{\text{fuel}}$). Alternatively, a target fuel is specified and the increment calculated from the current fuel. The tank is charged if the energy rate is negative. The segment time $T$ is the minimum of $dW_{\text{burn}}/\dot{w}$ or $dE_{\text{burn}}/\dot{E}$ for all fuel tank systems.

In an adjusted mission, the distances or times are changed at the end of the mission such that the sum of the fuel burned increments will equal the difference between takeoff fuel weight (plus any added fuel) and the calculated mission fuel: $\sum dW_{\text{burn}} = \sum \dot{w}dT = \sum \dot{w}dD/(V_h - V_w) = \Delta W_{\text{fuel}}$. The increments are apportioned among the adjusted segments by the factor $f$, determined from the ratio of the input distances or times: $dD = f\Delta D$ or $dT = f\Delta T$. Hence $\Delta D = \Delta W_{\text{fuel}}/(\sum f\dot{w}/(V_h - V_w))$ or $\Delta T = \Delta W_{\text{fuel}}/(\sum f\dot{w})$. The approach is similar if fuel energy is used, not weight. For a segment that is a source of range credit, the range increment is set to zero and the distance $D$ is added to $D_{\text{other}}$ of the destination segment. For the destination segment, the range contribution remains fixed at the input value, but the time and hence fuel burned are calculated from $(\text{dist} - D_{\text{other}})$. It is necessary to separately accumulate $D_{\text{other}}$ from earlier segments and $D_{\text{other}}$ from later segments; $D_{\text{other}}$ from later segments are estimated initially from the last iteration. At the end of the mission, the times and fuel burned are recalculated for all range credit destination segments.

<table>
<thead>
<tr>
<th>segment kind</th>
<th>time $T$</th>
<th>distance $D$</th>
<th>range $dR$</th>
</tr>
</thead>
<tbody>
<tr>
<td>taxi</td>
<td>time</td>
<td>0</td>
<td>$D$</td>
</tr>
<tr>
<td>distance</td>
<td>$D/(V_h - V_w)$</td>
<td>dist</td>
<td>$D$</td>
</tr>
<tr>
<td>time</td>
<td>time</td>
<td>$T(V_h - V_w)$</td>
<td>$D$</td>
</tr>
<tr>
<td>hold</td>
<td>time</td>
<td>0</td>
<td>$D$</td>
</tr>
<tr>
<td>climb</td>
<td>$(h - h_{\text{end}})/V_c$</td>
<td>$T(V_h - V_w)$</td>
<td>$D$</td>
</tr>
<tr>
<td>spiral</td>
<td>$(h - h_{\text{end}})/V_c$</td>
<td>0</td>
<td>$D$</td>
</tr>
<tr>
<td>fuel</td>
<td>$dW_{\text{burn}}/\dot{w}$ or $dE_{\text{burn}}/\dot{E}$</td>
<td>$T(V_h - V_w)$</td>
<td>$D$</td>
</tr>
<tr>
<td>burn</td>
<td>$dW_{\text{burn}}/\dot{w}$ or $dE_{\text{burn}}/\dot{E}$</td>
<td>0</td>
<td>$D$</td>
</tr>
</tbody>
</table>

| range credit | source $T$ | destination $D/(V_h - V_w)$ | range $dR$ | adjusted |
|--------------|------------|-----------------------------|------------|
| source       |            | $(\text{dist} - D_{\text{other}})$ | 0          | $D_{\text{new}}$ |
| destination  |            | $D/(V_h - V_w)$              | dist       | $D_{\text{new}}$ |

| adjusted | distance $T + dD/(V_h - V_w)$ | $D + dD = D + f\Delta D$ | $D_{\text{new}}$ |
|          | time $T + dT = T + f\Delta T$ | $D + dT(V_h - V_w)$      | $D_{\text{new}}$ |
|          | hold $T + dT = T + f\Delta T$ | 0                         | $D_{\text{new}}$ |
The segment time, distance, and fuel burned are evaluated by integrating over the segment duration. This integration can be performed by using the horizontal velocity, climb velocity, and fuel flow obtained for the flight state with the gross weight and altitude at the start of the segment; or at the middle of the segment; or the average of the segment start and segment end values (trapezoidal integration). The gross weight at the segment middle equals the gross weight at the segment start, less half the segment fuel burned (obtained from the previous mission iteration). The gross weight at the segment end equals the gross weight at the segment start, less the segment fuel burned. With trapezoidal integration, for the output the flight state is finally evaluated at the segment middle.

The mission endurance (block time), range, and fuel burned are:

\[ E = \sum T, \quad R = \sum dR, \quad W_{\text{burn}} = \sum dW_{\text{burn}} \] (sum over all non-reserve segments). The reserve fuel from mission segments is \( W_{\text{res}} = \sum dW_{\text{burn}} \) (sum over all reserve segments). Optionally the reserve fuel is the maximum of that from mission segments and the fraction \( f_{\text{res}} W_{\text{burn}} \), or the fraction \( f_{\text{res}} W_{\text{fuel-cap}} \). The calculated mission fuel is then \( W_{\text{fuel}} = W_{\text{burn}} + W_{\text{res}} \).

A fuel efficiency measure for the mission is the product of the payload and range, divided by the fuel weight:

\[ e = \frac{W_{\text{pay}} R}{W_{\text{burn}}} \] (ton-nm/lb or ton-nm/kg). A productivity measure for the mission is

\[ p = \frac{W_{\text{pay}} V}{W_{\text{burn}}} \] (ton-kt/lb or ton-kt/kg), where \( W_{\text{O}} \) is the operating weight and \( V \) the block speed; or

\[ p = \frac{W_{\text{pay}} V}{W_{\text{burn}}} \] (ton-kt/lb or ton-kt/kg). The Bréguet range equation \( R = RF \ln(W_0/W_1) \) is obtained by integrating \( dR = -RF(dW/W) \) for constant range factor

\[ RF = \frac{L/D_e}{sfc} = \frac{WV/P}{sfc} \]

The endurance \( E = EF 2(\sqrt{W_0/W_1} - 1) \) is obtained by integrating \( dE = -EF\sqrt{W_0}(dW/W^{3/2}) \) for constant endurance factor

\[ EF = \frac{L/D_e \sqrt{W/W_0}}{V} = \frac{W/P \sqrt{W/W_0}}{sfc} \]

Constant \( RF \) implies operation at constant \( L/D_e = WV/P \). Constant \( EF \) implies operation at constant \( (L/D_e)\sqrt{W/V} = W^{3/2}/P \) (or constant \( C_{L}^{3/2}/C_D \) for an airplane). It follows that overall range and endurance factors can be calculated from the mission performance:

\[ RF = \frac{R}{\ln W_0/W_1} \]

\[ EF = \frac{E}{2(\sqrt{W_0/W_1} - 1)} \]

where \( W_0 = W_{\text{to}} \) is the takeoff weight, and \( W_1 = W_{\text{to}} - W_{\text{burn}} \). If energy is burned, not weight, the efficiency metrics can be based on the equivalent fuel burned \( E_{\text{burn}}/e_{\text{ref}} \) and the range factor can be based on the equivalent specific fuel consumption. But since the weight does not change as energy is used, the equation \( dR = -RF(dW/W) \) integrates to \( R = RF(W_{\text{burn}}/W) \).

### 4–3 Takeoff Distance

The takeoff distance can be calculated, either as ground run plus climb to clear an obstacle or accelerate-stop distance in case of engine failure. The obstacle height \( h_o \) is typically 35 ft for commercial transport aircraft, or 50 ft for military aircraft and general aviation. This calculation allows determination of the balanced field length: engine failure at critical speed, such that the distance to clear the obstacle
equals the distance to stop. Landing and VTOL takeoff calculations are not implemented, as these are best solved as an optimal control problem.

The takeoff distance consists of a ground run, from zero ground speed to liftoff speed $V_{LO}$, perhaps including engine failure at speed $V_{EF}$; then rotation, transition, and climb; or decelerate to stop. Figure 4-1 describes the elements of the takeoff distance and the accelerate-stop distance, with the associated speeds. The ground is at angle $\gamma_G$ relative to the horizontal (inertial axes), with $\gamma_G$ positive for takeoff up hill. The takeoff profile is defined in terms of ground speed or climb speed, input as calibrated airspeed (CAS). The aircraft speed relative to the air is obtained from the ground speed, wind, and ground slope. The aircraft acceleration as a function of ground speed is integrated to obtain the ground distance, as well as the time, height, and fuel burned. Usually the speed increases from the start to liftoff (or engine failure), but the calculated acceleration depends on the flight state specification. The analysis checks for consistency of the input velocity and the calculated acceleration (on the ground), and for consistency of the input height and input or calculated climb angle (during climb).

The takeoff profile consists of a set of mission segments. The first segment is at the start of the takeoff, $V = 0$. Subsequent segments correspond to the ends of the integration intervals. The last segment has the aircraft at the required obstacle height, or stopped on the ground. The mission can consist of just one takeoff, more than one takeoff, or both takeoff and non-takeoff segments. Takeoff segments contribute to the mission fuel burned, but do not contribute to the mission time, distance, or range. The takeoff distance calculation is performed for a set of adjacent segments, the first segment specified as the takeoff start, and the last segment identified as before a non-takeoff segment or before
another takeoff start. The takeoff distance is calculated if a liftoff segment (with $V_{LO}$) is specified; otherwise the accelerate-stop distance is calculated. Table 4-2 summarizes the mission segments for takeoff calculations. There can be only one liftoff, engine failure, rotation, and transition segment (or none). The engine failure segment must occur before the liftoff segment. Rotation and transition segments must occur after liftoff. All ground run segments must be before liftoff, and all climb segments must be after liftoff. Takeoff segments (except start, rotation, and transition) can be split, in terms of height for climb and in terms of velocity for other segments. Splitting the takeoff or engine failure segment produces additional ground run segments. Separately defining multiple ground run, climb, or brake segments allows configuration variation during the takeoff.

Each takeoff segment requires that the flight state specify the appropriate configuration, trim option, and maximum effort. In particular, the number of inoperative engines for a segment is part of the flight state specification, regardless of whether or not an engine failure segment is defined. The engine failure segment (if present) serves to implement a delay in decision after failure: for a time $t_1$ after engine failure, the engine rating, power fraction, and friction of the engine failure segment are used (so the engine failure segment corresponds to conditions before failure). The number of inoperative engines specified must be consistent with the presence of the engine failure segment. The takeoff is assumed to occur at fixed altitude (so the maximum-effort variable can not be altitude). The flight state velocity specification is superseded by the ground or climb speed input for the takeoff segment. The flight state specification of height above ground level is superseded by the height input for the takeoff segment.

The ground distance, time, height, and fuel burned are calculated for each takeoff segment. The takeoff distance or accelerate-stop distance is the sum of the ground distance of all segments. Takeoff segments do not contribute to mission time, distance, or range.

<table>
<thead>
<tr>
<th>takeoff distance</th>
<th>accelerate-stop distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>start $V = 0$</td>
<td>start $V = 0$</td>
</tr>
<tr>
<td>ground run $V$</td>
<td>ground run $V$</td>
</tr>
<tr>
<td>engine failure $V_{EF}$</td>
<td>engine failure $V_{EF}$</td>
</tr>
<tr>
<td>ground run $V$</td>
<td>brake $V$</td>
</tr>
<tr>
<td>liftoff $V_{LO}$</td>
<td>brake $V = 0$</td>
</tr>
<tr>
<td>rotation $V_R$</td>
<td></td>
</tr>
<tr>
<td>transition $V_{TR}$</td>
<td></td>
</tr>
<tr>
<td>climb, to $h$ $V_{CL}$</td>
<td></td>
</tr>
<tr>
<td>climb, to $h_o$ $V_{CL}$</td>
<td></td>
</tr>
</tbody>
</table>

**4.3.1 Ground Run**

The takeoff starts at zero ground speed and accelerates to liftoff ground speed $V_{LO}$ (input as CAS). Possibly an engine failure speed $V_{EF} < V_{LO}$ is specified. Start, liftoff, and engine failure segments designate events, but otherwise are analyzed as ground run segments. The decision speed $V_1$ is $t_1$ seconds after engine failure (typically $t_1 = 1$ to $2$ sec). Up to $t_1$ after engine failure, conditions of the engine failure segment are used (so the engine failure segment corresponds to conditions before failure). The aircraft acceleration is obtained from the thrust minus drag ($T - D$ in airplane notation), plus a
friction force proportional to the weight on wheels \((W - L)\) in airplane notation:

\[
Ma = T - D - \mu(W - L) = \sum F_x - \mu \sum F_z
\]

from the force components in ground axes (rotated by the ground slope angle \(\gamma_G\) from inertial axes). Table 4-3 gives typical values of the friction coefficient \(\mu\). The velocity of the aircraft relative to the air is obtained from the ground velocity \(V\), wind velocity \(V_w\) (assumed parallel to the ground here), and the ground slope: \(V_h = (V + V_w) \cos \gamma_G\) and \(V_c = (V + V_w) \sin \gamma_G\). The takeoff configuration is specified, including atmosphere, in ground effect, gear down, power rating, nacelle tilt, flap setting, and number of inoperative engines. An appropriate trim option is specified, typically fixed attitude with longitudinal force trimmed using collective, for a given longitudinal acceleration. Perhaps the net aircraft yaw moment is trimmed with pedal. The maximum-effort condition is specified: maximum longitudinal acceleration (ground axes) for zero power margin. The aircraft acceleration as a function of ground speed is integrated to obtain the segment time, ground distance, height, and fuel burned:

\[
t_G = \int dt = \int \frac{dt}{dv} dv = \int \frac{dv}{a} = \frac{1}{2} \sum_{\text{seg}} \left( \frac{1}{a_2} + \frac{1}{a_1} \right) (v_2 - v_1) = \sum \Delta t
\]

\[
s_G = \int v dt = \int v \frac{dt}{dv} dv = \int \frac{dv}{a} = \frac{1}{2} \sum_{\text{seg}} \left( \frac{1}{2a_2} + \frac{1}{2a_1} \right) (v_2^2 - v_1^2) = \sum \left( \frac{v_2 + v_1}{2} \right) \Delta t
\]

\[
h_G = 0
\]

\[
w_G = \int \dot{w} dt = \frac{\sum (\ddot{w} f_2 + \dot{w} f_1)}{2} \Delta t
\]

Trapezoidal integration is used; each segment corresponds to the end of an integration integral.

<table>
<thead>
<tr>
<th>surface</th>
<th>rolling</th>
<th>braking</th>
</tr>
</thead>
<tbody>
<tr>
<td>dry and hard</td>
<td>0.03–0.05</td>
<td>0.30–0.50</td>
</tr>
<tr>
<td>grass</td>
<td>0.08</td>
<td>0.20</td>
</tr>
<tr>
<td>ice</td>
<td>0.02</td>
<td>0.06–0.10</td>
</tr>
</tbody>
</table>

4-3.2 Brake

After engine failure, the aircraft can decelerate to a stop. The operating engines are at idle. Reverse thrust is not permitted for the accelerate-stop distance calculation. The braking configuration is specified. Typically no trim option is executed; rather the aircraft has fixed attitude with controls for zero rotor thrust (such as zero collective and pedal). The aircraft acceleration as a function of ground speed is integrated, as for ground run.

4-3.3 Rotation

Rotation occurs at speed \(V_R\); usually \(V_R = V_{LO}\) is used. The duration \(t_R\) is specified, then \(s_R = V_R t_R\), \(h_R = 0\), and \(w_R = \dot{w} f R\) are the ground distance, height, and fuel burned. Typically \(t_R = 1\) to 3 sec.
4-3.4 Transition

Transition from liftoff to climb is modeled as a constant load factor pull-up to the specified climb angle $\gamma$, at speed $V_{TR}$. Usually $V_{TR} = V_{LO}$ is used, and typically $n_{TR} \approx 1.2$. From the load factor $n_{TR} = 1 + V_{TR}^2 / g R_{TR}$, the flight path radius is $R_{TR} = V_{TR}^2 / (g(n_{TR} - 1))$ and the pitch rate is $\dot{\theta} = V_{TR} / R_{TR}$.

$$
t_{TR} = \gamma / \dot{\theta} = \gamma \frac{R_{TR}}{V_{TR}} = \gamma \frac{V_{TR}}{g(n_{TR} - 1)}
$$

$s_{TR} = R_{TR} \sin \gamma$

$h_{TR} = R_{TR}(1 - \cos \gamma)$

$w_{TR} = \dot{w}_{ft_{TR}}$

are the time, ground distance, height, and fuel burned.

4-3.5 Climb

Climb occurs at an angle $\gamma$ relative to the ground and air speed $V_{CL}$, from the transition height $h_{TR}$ to the obstacle height $h_o$ (perhaps in several climb segments). The climb configuration is specified, including atmosphere, in ground effect, gear down or retracted, power rating, nacelle tilt, flap setting, and number of inoperative engines. An appropriate trim option is specified, typically aircraft force and moment trimmed with attitude and controls. The climb angle and air speed can be fixed or a maximum-effort condition can be specified. The maximum-effort options are fixed air speed and maximum rate of climb for zero power margin; or airspeed for best climb rate or best climb angle with maximum rate of climb for zero power margin. Not implemented is a maximum-effort calculation of maximum flight path acceleration for zero power margin, for specified climb angle; this calculation would require integration of the acceleration as a function of flight speed. For the climb segment, the input $V_{CL}$ is the magnitude of the aircraft velocity relative to the air, and the climb angle relative to the horizon is $\theta_V = \gamma + \gamma_G$. Hence from the maximum-effort calculation, the climb angle relative to the ground is $\gamma = \theta_V - \gamma_G$ and the ground speed is $V_{ground} = V_{CL} \cos \gamma - V_w$ (from the wind speed $V_w$).

$$
t_{CL} = s_{CL} / V_{ground}
$$

$s_{CL} = (h - h_{last}) / \tan \gamma$

$h_{CL} = h$

$w_{CL} = \dot{w}_{ft_{CL}}$

are the time, ground distance, height, and fuel burned.

4-4 Flight State

A flight state is defined for each flight condition (sizing task design conditions and flight performance analysis), and for each mission segment. The following parameters are required:

a) Speed: flight speed and vertical rate of climb, with the following options:

1) Specify horizontal speed (or forward speed or velocity magnitude), rate of climb (or climb angle), and sideslip angle.
2) Hover or vertical flight (input vertical rate of climb; climb angle 0 or $\pm 90$ deg).
3) Left or right sideward flight (input velocity and rate of climb; sideslip
angle ±90 deg).
4) Rearward flight (input velocity and rate of climb; sideslip angle 180 deg).

b) Aircraft motion:

1) Pitch and roll angles (Aircraft values or flight state input; initial values for trim variables, fixed otherwise).
2) Turn, pull-up, or linear acceleration.

c) Altitude: For mission segment, optionally input, or from last mission segment; climb segment end altitude from next segment.

d) Atmosphere:

1) Standard day, polar day, tropical day, or hot day at specified altitude.
2) Standard day, polar day, or tropical day plus temperature increment.
3) Standard day, polar day, or tropical day and specified temperature.
4) Input density and temperature.
5) Input density, speed of sound, and viscosity.

e) Height of landing gear above ground level. Landing gear state (extended or retracted).

f) Aircraft control state: input, or conversion schedule.

g) Aircraft control values (Aircraft values or flight state input; initial values for trim variables, fixed otherwise).

h) Aircraft center-of-gravity position (increment or input value).

For each propulsion group, the following parameters are required:

i) Drive system state.

j) Rotor tip speed for primary rotor:

1) Input.
2) Reference.
3) Conversion schedule or function speed.
4) Default for hover, cruise, maneuver, one engine inoperative (OEI), or transmission sizing condition.
5) From input $C_T/\sigma = t_0 - \mu t_1$, or $\mu$, or $M_{at}$ (where $\mu$ is the rotor advance ratio, and $M_{at}$ is the rotor advancing tip Mach number).

For each engine group (which is associated with a propulsion group):

k) Number of inoperative engines.

l) Infrared suppressor state: off (hot exhaust) or on (suppressed exhaust).

m) Engine rating, fraction of rated engine power available, and drive system rating.

For each jet group:

n) Number of inoperative jets.

o) Jet rating, fraction of rated thrust available.
For each charge group:

p) Number of inoperative chargers.
q) Charger rating, fraction of rated power available.

Aircraft and rotor performance parameters for each flight state:

r) Aircraft drag: forward flight drag increment, accounting for payload aerodynamics.
s) Rotor performance: induced power factor $\kappa$ and profile power mean $c_d$.

The aircraft trim state and trim targets are also specified.

The aircraft performance can be analyzed for the specified state, or a maximum-effort performance can be identified. For the maximum effort, a quantity and variable are specified. The available maximum-effort quantities include:

a) Best endurance: maximum $1/\dot{w}$.
b) Best range: 99% maximum $V/\dot{w}$ (high side), or low side, or 100%.
c) Best climb or descent rate: maximum $V_z$ or $1/P$.
d) Best climb or descent angle: maximum $V_z/V_h$ or $V/P$.
e) Ceiling: maximum altitude.
f) Power limit: zero power margin, $\min(P_{avPG} - P_{reqPG}) = 0$ (minimum over all propulsion groups).
g) Torque limit: zero torque margin, $\min(Q_{limit} - Q_{req}) = 0$ (minimum over all propulsion groups, engine groups, and rotors; $Q_{limit}$ expressed as power at reference rotation speed).
j) Jet thrust limit: zero thrust margin, $\min(T_{avJG} - T_{reqJG}) = 0$ (minimum over all jet groups).
i) Power limit or torque limit or thrust limit: most restrictive.
j) Battery limit: zero power margin, $\min(P_{max} - |\dot{E}_{batt}|) = 0$ (minimum over all fuel tanks).
k) Rotor stall: zero rotor thrust margin, $(C_T/\sigma)_{max} - C_T/\sigma = 0$ (for designated rotor, steady or transient limit).
l) Wing stall: zero wing lift margin, $C_{L,\max} - C_L = 0$ (for designated wing).

Here $\dot{w}$ is the aircraft fuel flow (or equivalent fuel flow $\dot{E}/e_{ref}$), and $P$ is the aircraft power. The available maximum-effort variables include:

a) Horizontal velocity $V_h$ or vertical rate of climb $V_z$ (times an input factor).
b) Aircraft sideslip angle.
c) Altitude.
d) Aircraft angular rate, $\dot{\theta}$ (pull-up) or $\dot{\psi}$ (turn).
e) Aircraft linear acceleration (airframe, inertial, or ground axes).
f) Aircraft controls.
g) Aircraft orientation, $\theta$ (pitch) or $\phi$ (roll).
h) Propulsion group tip speed or engine speed.

If the variable is velocity, first the velocity is found for the specified maximum effort; then the performance is evaluated at that velocity times an input factor. For endurance, range, or climb, the slope of the quantity
to be maximized must be zero; hence in all cases the target is zero. The slope of the quantity is evaluated by first-order backward difference. For the range, first the variable is found such that \( V/\dot{w} \) is maximized (slope zero), then the variable is found such that \( V/\dot{w} \) equals 99% of that maximum. Two maximum-effort quantity/variable pairs can be specified, and solved in nested iterations. The secant method or the method of false position is used to solve for the maximum effort. The task of finding maximum endurance, range, or climb is usually solved using the golden-section or curve-fit method. A tolerance \( \epsilon \) and a perturbation \( \Delta \) are specified.

Given the gross weight and useful load (from the flight condition or mission specification), the performance is calculated for this flight state. The calculated state information includes weight, speed and velocity orientation (climb and sideslip), aircraft Euler angles, rotor tip speeds, and aircraft controls.

A number of performance metrics are calculated for each flight state. The aircraft effective drag is \( D_e = P/V \), hence the effective lift-to-drag ratio is \( L/D_e = WV/P \). For these metrics, the aircraft power is the sum of the engine group power and jet group propulsive power: \( P = P_{\text{req}} + VT_{\text{jet}} \). The specific range is the ratio of the speed to the fuel flow: \( V/\dot{w} \) (nm/lb or nm/kg). From the Bréguet range equation, it follows that the range for which the fuel equals 1% of the gross weight is

\[
R_{1\%GW} = \frac{L/D_e}{\text{sfc}} \ln \left( \frac{1}{.99} \right)
\]

A fuel efficiency measure is the product of the payload and specific range: \( e = W_{\text{pay}}(V/\dot{w}) \) (ton-nm/lb or ton-nm/kg). A productivity measure is \( P = W_{\text{pay}}V/W_O \) (ton-kt/lb or ton-kt/kg), where \( W_O \) is the operating weight. If fuel energy is used, not weight, these performance metrics are based on the equivalent \( \dot{w} \) and equivalent sfc.

The aircraft weight statement defines the fixed useful load and operating weight for the design configuration. For each flight state, the fixed useful load may be different from the design configuration, because of changes in auxiliary fuel tank weight or kit weights or increments in crew or equipment weights. Thus the fixed useful load weight is calculated for the flight state; and from it the useful load weight and operating weight. The gross weight, payload weight, and usable fuel weight (in standard and auxiliary tanks) completes the weight information for the flight state.

### 4–5 Environment and Atmosphere

The aerodynamic environment is defined by the speed of sound \( c_s \), density \( \rho \), and kinematic viscosity \( \nu = \mu/\rho \) of the air (or other fluid). These quantities can be obtained from the standard day (International Standard Atmosphere), or input directly. Polar day, tropical day, and hot day atmospheres can also be used. The following options are implemented:

a) Input the altitude \( h_{\text{geom}} \) and a temperature increment \( \Delta T \). Calculate the temperature from altitude and pressure from the standard day (or polar day, tropical day, hot day), add \( \Delta T \), and then calculate the density from the equation of state for a perfect gas. Calculate the speed of sound and viscosity from the temperature.

b) Input the pressure altitude \( h_{\text{geom}} \) and the temperature \( \tau \) (°F or °C). Calculate the pressure (from temperature vs. altitude) for the standard day (or polar day, tropical day, hot day), and then calculate the density from the equation of state for a perfect gas. Calculate the speed of sound and viscosity from the temperature.
c) Input the density $\rho$ and the temperature $\tau$ (°F or °C). Calculate the speed of sound and viscosity from the temperature.

d) Input the density $\rho$, sound speed $c_s$, and viscosity $\mu$. Calculate the temperature from the sound speed.

e) For a fluid that is not air on earth, input the density $\rho$, sound speed $c_s$, and viscosity $\mu$.

Here $h_{\text{geom}}$ is the geometric altitude above mean sea level. The sources of the atmosphere descriptions are references 1–6.

The gravitational acceleration $g$ can have the standard value or an input value.

The International Standard Atmosphere (ISA) is a model for the variation with altitude of pressure, temperature, density, and viscosity, published as International Standard ISO 2533 by the International Organization for Standardization (ISO) (ref. 1). The ISA is intended for use in calculations and design of flying vehicles, to present the test results of flying vehicles and their components under identical conditions, and to allow unification in the field of development and calibration of instruments. The ISA is defined up to 80 km geopotential altitude and is identical to the ICAO Standard Atmosphere up to 32 km.

Dry air is modeled in the ISA as a perfect gas with a mean molecular weight, and hence a gas constant $R$, defined by adopted values for sea level pressure, temperature, and density ($p_0$, $T_0$, $\rho_0$). The speed of sound at sea level $c_{s0}$ is defined by an adopted value for the ratio of specific heats $\gamma$. The variation of temperature with geopotential altitude is defined by adopted values for vertical temperature gradients (lapse rates, $L_b$) and altitudes ($h_b$). The variation of pressure with geopotential altitude is further defined by an adopted value for the standard acceleration of free fall ($g$). The variation of dynamic viscosity $\mu$ with temperature is defined by adopted values for Sutherland’s empirical coefficients $\beta$ and $S$.

The required parameters are given in table 4-4, including the acceleration produced by gravity, $g$. The temperature $T$ is in °K, while $\tau$ is °C (perhaps input as °F); $T = T_{\text{zero}} + \tau$. The gas constant is $R = p_0/\rho_0 T_0$, and $\mu_0$ is actually obtained from $S$ and $\beta$. The standard atmosphere is defined in SI units. Although table 4-4 gives values in both SI and English units, all the calculations for the aerodynamic environment are performed in SI units. As required, the results are converted to English units using the exact conversion factors for length and force.

The ISA consists of a series of altitude ranges with constant lapse rate $L_b$ (linear temperature change with altitude). Thus at altitude $h_g$, the standard day temperature is

$$T_{\text{std}} = T_b + L_b(h_g - h_b)$$

for $h_g > h_b$. The altitude ranges and lapse rates are given in table 4-5. Note that $h_0$ is sea level, and $h_1$ is the boundary between the troposphere and the stratosphere. This altitude $h_g$ is the geopotential height, calculated assuming constant acceleration due to gravity. The geometric height $h$ is calculated using an inverse square law for gravity. Hence $h_g = rh/(r + h)$, where $r$ is the nominal radius of the Earth. The standard day pressure is obtained from hydrostatic equilibrium ($dp = -\rho g dh_g$) and the equation of state for a perfect gas ($p = \rho RT$, so $dp/p = -(g/RT)dh_g$). In isothermal regions ($L_b = 0$) the standard day pressure is

$$\frac{p_{\text{std}}}{p_b} = e^{-(g/RT)(h_g - h_b)}$$
and in gradient regions \((L_b \neq 0)\)

\[
\frac{p_{\text{std}}}{p_b} = \left(\frac{T}{T_b}\right)^{-g/RL_b}
\]

where \(p_b\) is the pressure at \(h_b\), obtained from these equations by working up from sea level. Let \(T_0, p_0, \rho_0, c_{s0}, \mu_0\) be the temperature, pressure, density, sound speed, and viscosity at sea level standard conditions. Then the density, sound speed, and viscosity are obtained from

\[
\rho = \rho_0 \left(\frac{p}{p_0}\right) \left(\frac{T}{T_0}\right)^{-1},
\]

\[
c_s = c_{s0} \left(\frac{T}{T_0}\right)^{1/2},
\]

\[
\mu = \mu_0 \left(\frac{(T/T_0)^{3/2}}{\alpha(T/T_0) + 1 - \alpha}\right)
\]

where \(\mu_0 = \beta T_0^{5/2}/(T_0 + S)\) and \(\alpha = T_0/(T_0 + S)\). For the cases using input temperature, \(T = T_{\text{zero}} + \tau\).

The density altitude and pressure altitude are calculated for reference. From the density and the standard day (troposphere only), the density altitude is:

\[
h_d = T_0 \left|L_0\right| \left(1 - \left(\frac{\rho}{\rho_0}\right)^{-1} \right)
\]

From the pressure \(p = \rho RT\) and the standard day,

\[
h_p = T_0 \left|L_0\right| \left(1 - \left(\frac{\rho}{\rho_0}\right)^{-1} \right) = T_0 \left|L_0\right| \left(1 - \left(\frac{\rho}{\rho_0}\right)^{-1} \right)
\]

is the pressure altitude.

The polar and tropical days are atmospheric models that describe realistic profiles of extremes of temperature and density, needed to calculate performance in near-worst-case conditions (ref. 6). These atmospheres are hydrodynamically balanced and can be used in calculations involving engine performance and aerodynamic characteristics, including calculations of true vertical velocity (ref. 4).

Tables of air properties for the polar and tropical days are given in MIL-STD-3013A, based on MIL-STD-210A. The break points in the lapse rates are evident in MIL-STD-210A, in terms of degrees C as a function of altitude in ft (geopotential altitude in MIL-C-5011B). The pressure ratio at sea level is \(\delta = 30.268/29.92 = 1.0116\) for the polar day, and \(\delta = 1\) for the tropical day. The altitude ranges and lapse rates are given in table 4-6 for the polar day and in table 4-7 for the tropical day. Figure 4-2 compares the temperature profiles. The air properties for the polar and tropical days are calculated from \(h_b\), \(L_b\), and \(T_b\) using the equations of hydrostatic equilibrium, as for the standard day. The primary data for the polar and tropical days are the altitude in ft and temperature in °C; \(h_b\) in km and \(L_b\) are derived using the exact conversion factor for length.

The conditions of the standard day, polar day, and tropical day are applicable to free air conditions. Temperatures close to the surface of the earth, even at high elevations, can be considerably higher than those for free air. The hot day is a model of ground-level atmospheric conditions, to be used for takeoff and other ground operations at elevations up to 15,000 ft (ref. 6). The hot day properties are statistically sampled and are not hydrodynamically balanced, hence should be used for approximately constant altitude conditions (ref. 4).
The origins of the hot day are in ref. 2, which has 103°F for sea level, and a lapse rate $-3.7^\circ F$ per 1000 ft (geometric) to 40,000 ft. The pressure in the table is from the equations for equilibrium, which gives the pressure altitude; these data are not used further. According to MIL-STD-210A (paragraph 3.1.1), the pressure (hence pressure altitude) as a function of altitude was obtained from statistics. Up to 15,000 ft, the ratio of geometric altitude to pressure altitude is 1.050. The $-3.7$ geometric lapse rate plus the mapping of geometric to pressure altitude gave °F vs. pressure altitude, rounded to 1 decimal place. Then °C was calculated from °F, rounded to 1 decimal place. The pressure ratio $\delta$ followed from the pressure altitude. The geometric altitude (to 35,000 ft) in MIL-STD-210A was calculated from the temperature and the $-3.7$ lapse rate, rounded to 100s. MIL-C-5011B has the same data as MIL-STD-210A, but does not give geometric altitude, and truncates the table at 15,000 ft. MIL-STD-3013A took °C (already rounded to 1 decimal place) vs. pressure altitude (ft) from MIL-STD-210A as the temperature profile up to 15,000 ft, and calculated °F (2 decimal places, so no more loss of information) from °C. The geopotential altitude was calculated from °C and $-3.7^\circ F$ lapse rate, rounded to 100s; but this altitude information is not meaningful, since the atmosphere is not in equilibrium; only the pressure altitude is used. Thus the hot day model has a sea level temperature of 39.4°C (102.92°F). Curve fitting the data to 15,000 ft gives a lapse rate of $\approx -7.065^\circ C$ per 1000 m = $-2.1534^\circ C$ per 1000 ft (table 4-8).

Given the pressure altitude $h$, the hot day temperature and pressure ratio are

$$T_{\text{hot}} = T_b + L_b(h - h_b)$$

$$\frac{p_{\text{hot}}}{p_0} = \left(1 + \frac{L_0}{T_0} \frac{R}{R} \right)^{-g/RL_b}$$

using the standard day $L_0 = -6.5$. A constant lapse rate fits the tabular data for the hot day atmosphere to only about 0.1°C. Thus the tabular data can be used directly instead (table 4-9, from MIL-C-5011B), with linear interpolation to the specified pressure altitude. MIL-STD-3013A has the same data, but only for altitudes a multiple of 1000 ft.

4–6 References


Figure 4-2. Temperature as a function of altitude.

Table 4-4. Constants adopted for calculation of the ISA.

<table>
<thead>
<tr>
<th>parameter</th>
<th>SI units</th>
<th>English units</th>
</tr>
</thead>
<tbody>
<tr>
<td>units $h$</td>
<td>m</td>
<td>ft</td>
</tr>
<tr>
<td>units $\tau$</td>
<td>°C</td>
<td>°F</td>
</tr>
<tr>
<td>m per ft</td>
<td>0.3048</td>
<td>0.3048</td>
</tr>
<tr>
<td>kg per lbm</td>
<td>0.45359237</td>
<td>0.45359237</td>
</tr>
<tr>
<td>$T_0$</td>
<td>288.15 °K</td>
<td>518.67 °R</td>
</tr>
<tr>
<td>$T_{zero}$</td>
<td>273.15 °K</td>
<td>459.67 °R</td>
</tr>
<tr>
<td>$p_0$</td>
<td>101325.0 N/m²</td>
<td>2116.22 lb/ft²</td>
</tr>
<tr>
<td>$\rho_0$</td>
<td>1.225 kg/m³</td>
<td>0.002377 slug/ft³</td>
</tr>
<tr>
<td>$c_{a0}$</td>
<td>340.294 m/sec</td>
<td>1116.45 ft/sec</td>
</tr>
<tr>
<td>$\mu_0$</td>
<td>1.7894E-5 kg/m·sec</td>
<td>3.7372E-7 slug/ft·sec</td>
</tr>
<tr>
<td>$S$</td>
<td>110.4 °K</td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>1.458E-6</td>
<td></td>
</tr>
<tr>
<td>$\gamma$</td>
<td>1.4</td>
<td></td>
</tr>
<tr>
<td>$g$</td>
<td>9.80665 m/sec²</td>
<td>32.17405 ft/sec²</td>
</tr>
<tr>
<td>$r$</td>
<td>6356766 m</td>
<td>20855531 ft</td>
</tr>
</tbody>
</table>
Table 4-5. Temperatures and vertical temperature gradients: standard day.

<table>
<thead>
<tr>
<th>level</th>
<th>base altitude ( h_b ) km</th>
<th>lapse rate ( L_b ) °K/km</th>
<th>temperature ( T_b ) °K</th>
<th>°C</th>
<th>°F</th>
</tr>
</thead>
<tbody>
<tr>
<td>troposphere</td>
<td>-2</td>
<td>-6.5</td>
<td>301.15</td>
<td>28</td>
<td>82.4</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>-6.5</td>
<td>288.15</td>
<td>15</td>
<td>59</td>
</tr>
<tr>
<td>1</td>
<td>11</td>
<td>0</td>
<td>216.65</td>
<td>-56.5</td>
<td>-69.7</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>+1.0</td>
<td>216.65</td>
<td>-56.5</td>
<td>-69.7</td>
</tr>
<tr>
<td>3</td>
<td>32</td>
<td>+2.8</td>
<td>228.65</td>
<td>-44.5</td>
<td>-48.1</td>
</tr>
<tr>
<td>4</td>
<td>47</td>
<td>0</td>
<td>270.65</td>
<td>-2.5</td>
<td>27.5</td>
</tr>
<tr>
<td>5</td>
<td>51</td>
<td>-2.8</td>
<td>270.65</td>
<td>-2.5</td>
<td>27.5</td>
</tr>
<tr>
<td>6</td>
<td>71</td>
<td>-2.0</td>
<td>214.65</td>
<td>-58.5</td>
<td>-73.3</td>
</tr>
<tr>
<td>7</td>
<td>80</td>
<td>0</td>
<td>196.65</td>
<td>-76.5</td>
<td>-105.7</td>
</tr>
</tbody>
</table>

* derived

Table 4-6. Temperatures and vertical temperature gradients: polar day.

<table>
<thead>
<tr>
<th>base altitude ( h_b ) ft</th>
<th>lapse rate ( L_b ) °K/km</th>
<th>temperature ( T_b ) °K</th>
<th>°C</th>
<th>°F *</th>
<th>( \delta = \frac{p}{p_0} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>6.326</td>
<td>246.15</td>
<td>-27</td>
<td>-16.6</td>
</tr>
<tr>
<td>3111.871</td>
<td>0.948</td>
<td>-1.083</td>
<td>252.15</td>
<td>-21</td>
<td>-5.8</td>
</tr>
<tr>
<td>9172.604</td>
<td>2.796</td>
<td>-5.511</td>
<td>250.15</td>
<td>-23</td>
<td>-9.4</td>
</tr>
<tr>
<td>28224.543</td>
<td>8.603</td>
<td>-0.476</td>
<td>218.15</td>
<td>-55</td>
<td>-67.0</td>
</tr>
<tr>
<td>83363.393</td>
<td>25.409</td>
<td>0</td>
<td>210.15</td>
<td>-63</td>
<td>-81.4</td>
</tr>
</tbody>
</table>

* derived

Table 4-7. Temperatures and vertical temperature gradients: tropical day.

<table>
<thead>
<tr>
<th>base altitude ( h_b ) ft</th>
<th>lapse rate ( L_b ) °K/km</th>
<th>temperature ( T_b ) °K</th>
<th>°C</th>
<th>°F *</th>
<th>( \delta = \frac{p}{p_0} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>-6.687</td>
<td>305.25</td>
<td>32.1</td>
<td>89.78</td>
</tr>
<tr>
<td>55000</td>
<td>16.764</td>
<td>4.374</td>
<td>193.15</td>
<td>-80</td>
<td>-112.00</td>
</tr>
<tr>
<td>70000</td>
<td>21.336</td>
<td>2.401</td>
<td>213.15</td>
<td>-60</td>
<td>-76.00</td>
</tr>
<tr>
<td>100745</td>
<td>30.707</td>
<td>305.25</td>
<td>235.65</td>
<td>-37.5</td>
<td>-35.50</td>
</tr>
</tbody>
</table>

* derived

Table 4-8. Temperatures and vertical temperature gradients: hot day.

<table>
<thead>
<tr>
<th>base altitude ( h_b ) ft</th>
<th>lapse rate ( L_b ) °K/km</th>
<th>temperature ( T_b ) °K</th>
<th>°C</th>
<th>°F *</th>
<th>( \delta = \frac{p}{p_0} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>-7.065</td>
<td>312.55</td>
<td>39.4</td>
<td>102.92</td>
</tr>
<tr>
<td>15000</td>
<td>4.572</td>
<td>280.25</td>
<td>280.25</td>
<td>7.10</td>
<td>44.78</td>
</tr>
</tbody>
</table>

* derived
Table 4-9. Temperature table for hot day (from MIL-C-5011B).

<table>
<thead>
<tr>
<th>pressure altitude</th>
<th>temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td>ft</td>
<td>°K</td>
</tr>
<tr>
<td>0</td>
<td>312.55</td>
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<tr>
<td>500</td>
<td>311.55</td>
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<tr>
<td>1000</td>
<td>310.45</td>
</tr>
<tr>
<td>1500</td>
<td>309.45</td>
</tr>
<tr>
<td>2000</td>
<td>308.35</td>
</tr>
<tr>
<td>2500</td>
<td>307.25</td>
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<td>3000</td>
<td>306.25</td>
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<td>3500</td>
<td>305.15</td>
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<tr>
<td>4000</td>
<td>304.05</td>
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<td>4500</td>
<td>302.95</td>
</tr>
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<td>5000</td>
<td>301.85</td>
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<td>5500</td>
<td>300.75</td>
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<td>6000</td>
<td>299.65</td>
</tr>
<tr>
<td>6500</td>
<td>298.55</td>
</tr>
<tr>
<td>7000</td>
<td>297.45</td>
</tr>
<tr>
<td>7500</td>
<td>296.35</td>
</tr>
<tr>
<td>8000</td>
<td>295.25</td>
</tr>
<tr>
<td>8500</td>
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<td>291.95</td>
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<td>10500</td>
<td>289.85</td>
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<tr>
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<td>281.35</td>
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<tr>
<td>15000</td>
<td>280.35</td>
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</tbody>
</table>

* derived
Chapter 5

Solution Procedures

The NDARC code performs design and analysis tasks. The design task involves sizing the rotorcraft to satisfy specified design conditions and missions. The analysis tasks can include off-design mission performance analysis, flight performance calculation for point operating conditions, and generation of subsystem or component performance maps. Figure 5-1 illustrates the tasks. The principal tasks (sizing, mission analysis, and flight performance analysis) are shown in the figure as boxes with dark borders. Dark black arrows show control of subordinate tasks.

The aircraft description (fig. 5-1) consists of all the information, input and derived, that defines the aircraft. The aircraft consists of a set of components, including fuselage, rotors, wings, tails, and propulsion. This information can be the result of the sizing task; can come entirely from input, for a fixed model; or can come from the sizing task in a previous case or previous job. The aircraft description information is available to all tasks and all solutions (indicated by light green arrows).

Missions are defined for the sizing task and for the mission performance analysis. A mission consists of a specified number of mission segments, for which time, distance, and fuel burn are evaluated. For specified takeoff fuel weight with adjustable segments, the mission time or distance is adjusted so the fuel required for the mission (burned plus reserve) equals the takeoff fuel weight. The mission iteration is on fuel weight or energy.

Flight conditions are specified for the sizing task and for the flight performance analysis.

For flight conditions and mission takeoff, the gross weight can be maximized such that the power required equals the power available.

A flight state is defined for each mission segment and each flight condition. The aircraft performance can be analyzed for the specified state, or a maximum-effort performance can be identified. The maximum effort is specified in terms of a quantity such as best endurance or best range, and a variable such as speed, rate of climb, or altitude. The aircraft must be trimmed, by solving for the controls and motion that produce equilibrium in the specified flight state. Different trim solution definitions are required for various flight states. Evaluating the rotor hub forces may require solution of the blade flap equations of motion.

The sizing task is described in more detail in chapter 3. The flight condition, mission, and flight state calculations are described in chapter 4. The solution of the blade flap equations of motion is described in chapter 11. The present chapter provides details of the solution procedures implemented for each iteration of the analysis.

The nested iteration loops involved in the solution process are indicated by the subtitles in the boxes of figure 5-1, and illustrated in more detail in figure 5-2. The flight state solution involves up
to three loops. The innermost loop is the solution of the blade flap equations of motion, needed for an accurate evaluation of the rotor hub forces. The next loop is the trim solution, which is required for most flight states. The flight state optionally has one or two maximum-effort iterations. The flight state solution is executed for each flight condition and for each mission segment. A flight condition solution or any mission segment solution can optionally maximize the aircraft gross weight. The mission usually requires an iterative solution, for fuel weight or for adjustable segment time or distance. Thus each flight condition solution involves up to four nested iterations: maximum gross weight (outer), maximum effort, trim, and blade motion (inner). Each mission solution involves up to five nested iterations: mission (outer), and then for each segment maximum gross weight, maximum effort, trim, and blade motion (inner). Finally, the design task introduces a sizing iteration, which is the outermost loop of the process.
Figure 5-2. Design and analysis tasks, with nested loops and solution methods.
5–1 Iterative Solution Tasks

5-1.1 Tolerance and Perturbation

For each solution procedure, a tolerance \( \varepsilon \) and a perturbation \( \Delta \) may be required. Single values are specified for the task and then scaled for each element tested or perturbed.

The scaling is based on a reference weight \( W \) (design gross weight, or derived from aircraft \( C_T/\sigma = 0.07 \)), a reference length \( L \) (fuselage length, rotor radius, or wing span), and a reference power \( P \) (aircraft installed power, or derived from \( P = W \sqrt{W/2 \rho A} \)). Then the force reference is \( F = W \), the moment reference is \( M = WL/10 \), and the angle reference is \( A = 1 \) deg. The velocity reference is \( V = 400 \) knots. The angular velocity reference is \( \Omega = V/L \) (in deg/sec). The coefficient reference is \( C = 0 \) for wings and \( C = 0.1 \) for rotors. Altitude scale is \( H = 10000 \) ft. Acceleration scale is \( G = g \) (acceleration due to gravity). The range scale is \( X = 100 \) nm. These scaling variables are referred to in the subsections that follow, and in tables 5-1 to 5-4.

5-1.2 Size Aircraft

The sizing task determines the dimensions, power, and weight of a rotorcraft that can perform a specified set of design conditions and missions. The aircraft size is characterized by parameters such as design gross weight, weight empty, rotor radius, and engine power available. The relationships between dimensions, power, and weight generally require an iterative solution. From the design flight conditions and missions, the task can determine the total engine power or the rotor radius (or both power and radius can be fixed), as well as the design gross weight, maximum takeoff weight, drive system torque limit, and fuel tank capacity. For each propulsion group, the engine power or the rotor radius can be sized.

A successive substitution method is used for the sizing iteration, with an input tolerance \( \varepsilon \). Relaxation is applied to \( P_{\text{eng}} \) or \( R, T_{\text{jet}}, P_{\text{chrg}}, W_D, W_{\text{MTO}}, P_{\text{DS\_limit}}, W_{\text{fuel\_cap}} \) or \( E_{\text{fuel\_cap}} \) and \( T_{\text{design}} \). Two successive substitution loops are used. The outer loop is an iteration on performance: engine power or rotor radius, jet thrust, charger power. The inner loop is an iteration on parameters: \( W_D, W_{\text{MTO}}, P_{\text{DS\_limit}}, W_{\text{fuel\_cap}} \) or \( E_{\text{fuel\_cap}} \), and \( T_{\text{design}} \). Either loop can be absent, depending on the definition of the size task. Convergence is tested in terms of these parameters, and the aircraft weight empty \( W_E \). The tolerance is \( 0.1P\varepsilon \) for engine power and drive system limit; \( 0.01W\varepsilon \) for gross weight, maximum takeoff weight, drive system torque limit, and fuel tank capacity. For each propulsion group, the engine power or the rotor radius can be sized.

5-1.3 Mission

Missions consist of a specified number of segments, for which time, distance, and fuel burn are evaluated. For calculated mission fuel weight, the fuel weight at takeoff is adjusted to equal the fuel required for the mission (burned plus reserve). For specified takeoff fuel weight with adjustable segments, the mission time or distance is adjusted so the fuel required for the mission (burned plus reserve) equals the takeoff fuel weight. The mission iteration is thus on fuel weight or energy. Range credit segments can also require an iteration.

A successive substitution method is used if an iteration is required, with a tolerance \( \varepsilon \) specified. The principal iteration variable is takeoff fuel weight, for which the tolerance is \( 0.01W\varepsilon \). For calculated mission fuel weight, the relaxation is applied to the mission fuel value used to update the takeoff fuel weight. For specified takeoff fuel weight, the relationship is applied to the fuel weight increment used to adjust the mission segments. The tolerance for the distance flown in range credit segments is \( X\varepsilon \). The relaxation is applied to the distance flown in the destination segments for range credit.
5-1.4 Maximum Gross Weight

Flight conditions are specified for the sizing task and for the flight performance analysis. Mission takeoff conditions are specified for the sizing task and for the mission analysis. Optionally for flight conditions and mission takeoff, the gross weight can be maximized, such that the power required equals the power available,

$$\min(P_{avPG} - P_{reqPG}) = 0 \quad \text{(zero power margin, minimum over all propulsion groups);}$$

or such that the power required equals an input power,

$$\min((d + fP_{avPG}) - P_{reqPG}) = 0 \quad \text{(minimum over all propulsion groups, with } d \text{ an input power and } f \text{ an input factor; this convention allows the power to be input directly, } f = 0, \text{ or scaled with power available).}$$

Similarly, the gross weight can be maximized for zero jet thrust margin, or zero torque margin.

The secant method or the method of false position is used to solve for the maximum gross weight. A tolerance $\epsilon$ and a perturbation $\Delta$ are specified. The variable is gross weight, with initial increment of $W\Delta$, and tolerance of $0.01W\epsilon$. Note that the convergence test is applied to the magnitude of the gross weight increment.

5-1.5 Maximum Effort

The aircraft performance can be analyzed for the specified state or a maximum-effort performance can be identified. The secant method or the method of false position is used to solve for the maximum effort. The task of finding maximum endurance, range, or climb is usually solved using the golden-section or curve-fit method. A tolerance $\epsilon$ and a perturbation $\Delta$ are specified.

A quantity and variable are specified for the maximum-effort calculation. Tables 5-1 and 5-2 summarize the available choices, with the tolerance and initial increment used for the variables. Note that the convergence test is applied to the magnitude of the variable increment. Optionally two quantity/variable pairs can be specified, solved in nested iterations. The two variables must be unique. The two variables can maximize the same quantity (endurance, range, or climb). If the variable is velocity, first the velocity is found for the specified maximum effort; the performance is then evaluated at that velocity times an input factor. For endurance, range, or climb, the slope of the quantity to be maximized must be zero; hence in all cases the target is zero. The slope of the quantity is evaluated by first-order backward difference. For the range, first the variable is found such that $V/\dot{w}$ is maximized (slope zero), and then the variable is found such that $V/\dot{w}$ equals 99% of that maximum; for the latter the variable perturbation is increased by a factor of 4 to ensure that the solution is found on the correct side of the maximum.

<table>
<thead>
<tr>
<th>Table 5-1. Maximum-effort solution.</th>
</tr>
</thead>
<tbody>
<tr>
<td>maximum-effort variable</td>
</tr>
<tr>
<td>--------------------------</td>
</tr>
<tr>
<td>horizontal velocity</td>
</tr>
<tr>
<td>vertical rate of climb</td>
</tr>
<tr>
<td>aircraft velocity</td>
</tr>
<tr>
<td>altitude</td>
</tr>
<tr>
<td>aircraft angular rate</td>
</tr>
<tr>
<td>aircraft linear acceleration</td>
</tr>
<tr>
<td>aircraft control</td>
</tr>
<tr>
<td>aircraft orientation</td>
</tr>
<tr>
<td>propulsion group tip speed</td>
</tr>
<tr>
<td>propulsion group engine speed</td>
</tr>
</tbody>
</table>
### Table 5-2. Maximum-effort solution.

<table>
<thead>
<tr>
<th>maximum-effort quantity</th>
<th>target</th>
</tr>
</thead>
<tbody>
<tr>
<td>best endurance</td>
<td>maximum $1/\dot{w}$</td>
</tr>
<tr>
<td>best range</td>
<td>$99%$ maximum $V/\dot{w}$</td>
</tr>
<tr>
<td>best climb or descent rate</td>
<td>maximum $V_z$ or $1/P$</td>
</tr>
<tr>
<td>best climb or descent angle ceiling</td>
<td>maximum altitude</td>
</tr>
<tr>
<td>power limit</td>
<td>power margin, ( \min(P_{avPG} - P_{reqPG}) = 0 ) over all propulsion groups</td>
</tr>
<tr>
<td>torque limit</td>
<td>torque margin, ( \min(Q_{limit} - Q_{req}) = 0 ) over all limits</td>
</tr>
<tr>
<td>thrust limit</td>
<td>thrust margin, ( \min(T_{avJG} - T_{reqJG}) = 0 ) over all jet groups</td>
</tr>
<tr>
<td>power, torque, thrust limit</td>
<td>power margin, power margin</td>
</tr>
<tr>
<td>battery limit</td>
<td>( P_{max} -</td>
</tr>
<tr>
<td>rotor stall</td>
<td>thrust margin, ( (C_T/\sigma)_{max} - C_T/\sigma = 0 ) for designated rotor</td>
</tr>
<tr>
<td>wing stall</td>
<td>lift margin, ( C_{L_{max}} - C_L = 0 ) for designated wing</td>
</tr>
</tbody>
</table>

### Table 5-3. Trim solution.

<table>
<thead>
<tr>
<th>trim quantity</th>
<th>target</th>
<th>tolerance</th>
</tr>
</thead>
<tbody>
<tr>
<td>aircraft total force</td>
<td>( x, y, z ) components</td>
<td>0</td>
</tr>
<tr>
<td>aircraft total moment</td>
<td>( x, y, z ) components</td>
<td>0</td>
</tr>
<tr>
<td>aircraft load factor</td>
<td>( x, y, z ) components</td>
<td>Flight State</td>
</tr>
<tr>
<td>propulsion group power</td>
<td></td>
<td>Flight State</td>
</tr>
<tr>
<td>power margin</td>
<td>( P_{avPG} - P_{reqPG} )</td>
<td>Flight State</td>
</tr>
<tr>
<td>torque margin</td>
<td>( P_{DSlimit} - P_{reqPG} )</td>
<td>Flight State</td>
</tr>
<tr>
<td>engine group power</td>
<td></td>
<td>Flight State</td>
</tr>
<tr>
<td>power margin</td>
<td>( P_{avEG} - P_{reqEG} )</td>
<td>Flight State</td>
</tr>
<tr>
<td>jet group thrust</td>
<td></td>
<td>Flight State</td>
</tr>
<tr>
<td>thrust margin</td>
<td>( T_{avJG} - T_{reqJG} )</td>
<td>Flight State</td>
</tr>
<tr>
<td>charge group power</td>
<td></td>
<td>Flight State</td>
</tr>
<tr>
<td>charge power margin</td>
<td>( P_{avCG} - P_{reqCG} )</td>
<td>Flight State</td>
</tr>
<tr>
<td>fuel tank energy flow</td>
<td>( \dot{E}_{batt} )</td>
<td>Flight State</td>
</tr>
<tr>
<td>battery power margin</td>
<td>( P_{max} -</td>
<td>\dot{E}_{batt}</td>
</tr>
<tr>
<td>rotor force</td>
<td>lift, vertical, propulsive</td>
<td>Flight State, component schedule</td>
</tr>
<tr>
<td>rotor thrust</td>
<td>( C_T/\sigma )</td>
<td>Flight State, component schedule</td>
</tr>
<tr>
<td>rotor thrust margin</td>
<td>( (C_T/\sigma)_{max} - C_T/\sigma )</td>
<td>Flight State</td>
</tr>
<tr>
<td>rotor flapping</td>
<td>( \beta_c, \beta_s )</td>
<td>Flight State</td>
</tr>
<tr>
<td>rotor hub moment</td>
<td>( x ) (roll), ( y ) (pitch)</td>
<td>Flight State</td>
</tr>
<tr>
<td>rotor torque</td>
<td></td>
<td>Flight State</td>
</tr>
<tr>
<td>wing force</td>
<td></td>
<td>Flight State, component schedule</td>
</tr>
<tr>
<td>wing lift coefficient</td>
<td>( C_L )</td>
<td>Flight State, component schedule</td>
</tr>
<tr>
<td>wing lift margin</td>
<td>( C_{L_{max}} - C_L )</td>
<td>Flight State</td>
</tr>
</tbody>
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Table 5-4. Trim solution.

<table>
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<tr>
<th>trim variable</th>
<th>perturbation</th>
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<tr>
<td>aircraft control</td>
<td>angle</td>
</tr>
<tr>
<td>aircraft orientation</td>
<td>$\theta$ (pitch), $\phi$ (roll)</td>
</tr>
<tr>
<td>aircraft velocity</td>
<td>$V_h$ (horizontal velocity)</td>
</tr>
<tr>
<td>aircraft velocity</td>
<td>$V_z$ (vertical velocity)</td>
</tr>
<tr>
<td>aircraft velocity</td>
<td>$\beta$ (sideslip)</td>
</tr>
<tr>
<td>aircraft angular rate</td>
<td>$\theta$ (pullup), $\psi$ (turn)</td>
</tr>
<tr>
<td>propulsion group tip speed</td>
<td>$V_{\text{tip}}$</td>
</tr>
<tr>
<td>propulsion group engine speed</td>
<td>$N_{\text{spec}}$</td>
</tr>
</tbody>
</table>

---

5-1.6 Trim

The aircraft trim operation solves for the controls and motion that produce equilibrium in the specified flight state. A Newton–Raphson method is used for trim. The derivative matrix is obtained by numerical perturbation. A tolerance $\epsilon$ and a perturbation $\Delta$ are specified.

Different trim solution definitions are required for various flight states. Therefore one or more trim states are defined for the analysis, and the appropriate trim state selected for each flight state of a performance condition or mission segment. For each trim state, the trim quantities, trim variables, and targets are specified. Tables 5-3 and 5-4 summarize the available choices, with the tolerances and perturbations used.

5-1.7 Rotor Flap Equations

Evaluating the rotor hub forces may require solution of the flap equations $E(v) = 0$. For tip-path plane command, the thrust and flapping are known, so $v = (\theta_{0.75}, \theta_c, \beta_s)^T$. For no-feathering plane command, the thrust and cyclic pitch are known, so $v = (\theta_{0.75}, \beta_c, \beta_s)^T$. A Newton–Raphson solution method is used: from $E(v_{n+1}) = E(v_n) + (dE/dv)(v_{n+1} - v_n) = 0$, the iterative solution is

$$v_{n+1} = v_n - C E(v_n)$$

where $C = f(dE/dv)^{-1}$, including the relaxation factor $f$. The derivative matrix for axial flow can be used. Alternatively, the derivative matrix $dE/dv$ can be obtained by numerical perturbation. Convergence of the Newton–Raphson iteration is tested in terms of $|E| < \epsilon$ for each equation, where $\epsilon$ is an input tolerance.

5-2 Theory

The analysis uses several methods to solve nonlinear algebraic equations. Such equations may be written in two forms: (a) fixed point $x = G(x)$, and (b) zero point $f(x) = 0$; where $x$, $G$, and $f$ are vectors. The analysis provides operations that implement the function $G$ or $f$. Solution procedures appropriate for the zero point form can be applied to equations in fixed point form, by defining $f(x) = x - G(x)$. In this context, $f$ can be considered the iteration error.

Efficient and convergent methods are required to find the solution $x = \alpha$ of these equations. Note that $f'(\alpha) = 0$ or $G'(\alpha) = 1$ means that $\alpha$ is a higher-order root. For nonlinear problems, the method will
successive substitution iteration
save: \( x_{\text{old}} = x \)
evaluate \( x \)
relax: \( x = \lambda x + (1 - \lambda)x_{\text{old}} \)
test convergence: error = \( \|x - x_{\text{old}}\| \leq \lambda \) tolerance \times \) weight

Figure 5-3. Outline of successive substitution method.

be iterative: \( x_{n + 1} = F(x_n) \). The operation \( F \) depends on the solution method. The solution error is:

\[
\epsilon_{n+1} = \alpha - x_{n+1} = F(\alpha) - F(x_n) = (\alpha - x_n)F'(\xi_n) \cong \epsilon_n F'(\alpha)
\]

Thus the iteration will converge if \( F \) is not too sensitive to errors in \( x \): \( |F'(\alpha)| < 1 \) for scalar \( x \). For \( x \) a vector, the criterion is that all the eigenvalues of the derivative matrix \( \partial F/\partial x \) have magnitude less than one. The equations in this section are generally written for scalar \( x \); the extension to vector \( x \) is straightforward. Convergence is linear for \( F' \) nonzero, quadratic for \( F' = 0 \). Iterative methods have a relaxation factor (and other parameters) to improve convergence, and a tolerance to measure convergence.

The following subsections describe the solution methods used for the various iterations, as shown in figure 5-2.

5-2.1 Successive Substitution Method

The successive substitution method (with relaxation) is an example of a fixed point solution. A direct iteration is simply \( x_{n+1} = G(x_n) \), but \( |G'| > 1 \) for many practical problems. A relaxed iteration uses \( F = (1 - \lambda)x + \lambda G \):

\[
x_{n+1} = (1 - \lambda)x_n + \lambda G(x_n) = x_n - \lambda f(x_n)
\]

with relaxation factor \( \lambda \). The convergence criterion is then

\[
|F'(\alpha)| = |1 - \lambda + \lambda G'| < 1
\]

so a value of \( \lambda \) can be found to ensure convergence for any finite \( G' \). Specifically, the iteration converges if the magnitude of \( \lambda \) is less than the magnitude of \( 2/(1 - G') = 2/f' \) (and \( \lambda \) has the same sign as \( 1 - G' = f' \)). Quadratic convergence \( (F' = 0) \) is obtained with \( \lambda = 1/(1 - G') = 1/f' \). Over-relaxation \( (\lambda > 1) \) can be used if \( |G'| < 1 \). Since the correct solution \( x = \alpha \) is not known, convergence must be tested by comparing the values of two successive iterations:

\[
\text{error} = \|x_{n+1} - x_n\| \leq \text{tolerance}
\]

where the error is some norm of the difference between iterations (typically absolute value for scalar \( x \)). Note that the effect of the relaxation factor is to reduce the difference between iterations:

\[
x_{n+1} - x_n = \lambda(G(x_n) - x_n)
\]

Hence the convergence test is applied to \( (x_{n+1} - x_n)/\lambda \), in order to maintain the definition of tolerance independent of relaxation. The process for the successive substitution method is shown in figure 5-3.
Solution Procedures

5.2.2 Newton–Raphson Method

The Newton–Raphson method (with relaxation and identification) is an example of a zero point solution. The Taylor series expansion of \( f(x) = 0 \) leads to the iteration operator \( F = x - \frac{f}{f'} \):

\[
x_{n+1} = x_n - \left[ f'(x_n) \right]^{-1} f(x_n)
\]

which gives quadratic convergence. The behavior of this iteration depends on the accuracy of the derivative \( f' \). Here it is assumed that the analysis can evaluate directly \( f \), but not \( f' \). It is necessary to evaluate \( f' \) by numerical perturbation of \( f \), and for efficiency the derivatives may not be evaluated for each \( x_n \). These approximations compromise the convergence of the method, so a relaxation factor \( \lambda \) is introduced to compensate. Hence a modified Newton–Raphson iteration is used, \( F = x - C f \):

\[
x_{n+1} = x_n - C f(x_n) = x_n - \lambda D^{-1} f(x_n)
\]

where the derivative matrix \( D \) is an estimate of \( f' \). The convergence criterion is then

\[
|F'(\alpha)| = |1 - C f'| = |1 - \lambda D^{-1} f'| < 1
\]

since \( f(\alpha) = 0 \). The iteration converges if the magnitude of \( \lambda \) is less than the magnitude of \( 2D / f' \) (and \( \lambda \) has the same sign as \( D / f' \)). Quadratic convergence is obtained with \( \lambda = D / f' \) (which would require \( \lambda \) to change during the iteration however). The Newton–Raphson method ideally uses the local derivative in the gain factor, \( C = 1 / f' \), so has quadratic convergence:

\[
F'(\alpha) = \frac{f f''}{f'^2} = 0
\]

since \( f(\alpha) = 0 \) (if \( f' \neq 0 \) and \( f'' \) is finite; if \( f' = 0 \), then there is a multiple root, \( F' = \lambda / 2 \), and the convergence is only linear). A relaxation factor is still useful, since the convergence is only quadratic sufficiently close to the solution. A Newton–Raphson method has good convergence when \( x \) is sufficiently close to the solution, but frequently has difficulty converging elsewhere. Hence the initial
initialize
evaluate $f_0$ at $x_0$, $f_1$ at $x_1 = x_0 + \Delta x$, $f_2$ at $x_2 = x_1 + \Delta x$

iteration
calculate derivative $f'$
secant: from $f_0$ and $f_1$
false position: from $f_0$, and $f_1$ or $f_2$ (opposite sign from $f_0$)
calculate gain: $C = \lambda / f$
increment solution: $\delta x = -C f$
shift: $f_2 = f_1$, $f_1 = f_0$
evaluate $f$
test convergence

Figure 5-5. Outline of secant method or method of false position.

estimate $x_0$ that starts the iteration is an important parameter affecting convergence. Convergence of the solution for $x$ may be tested in terms of the required value (zero) for $f$:

$$error = \|f\| \leq \text{tolerance}$$

where the error is some norm of $f$ (typically absolute value for scalar $f$).

The derivative matrix $D$ is obtained by an identification process. The perturbation identification can be performed at the beginning of the iteration, and optionally every $M_{PID}$ iterations thereafter. The derivative matrix is calculated from a one-step finite-difference expression (first order). Each element $x_i$ of the vector $x$ is perturbed, one at a time, giving the $i$-th column of $D$:

$$D = \begin{bmatrix} \cdots & \frac{\partial f}{\partial x_i} & \cdots \end{bmatrix} = \begin{bmatrix} \cdots & \frac{f(x_i + \delta x_i) - f(x_i)}{\delta x_i} & \cdots \end{bmatrix}$$

Alternatively, a two-step finite-difference expression (second order) can be used:

$$D = \begin{bmatrix} \cdots & \frac{\partial f}{\partial x_i} & \cdots \end{bmatrix} = \begin{bmatrix} \cdots & \frac{f(x_i + \delta x_i) - f(x_i - \delta x_i)}{2\delta x_i} & \cdots \end{bmatrix}$$

With this procedure, the accuracy of $D$ (hence convergence) can be affected by both the magnitude and sign of the perturbation (only the magnitude for a two-step difference).

The process for the Newton–Raphson method is shown in figure 5-4. A problem specified as $h(x) = h_{\text{target}}$ becomes a zero point problem with $f = h - h_{\text{target}}$. A successive substitution problem, $x = G(x)$, becomes a zero point problem with $f = x - G$. At the beginning of the solution, $x$ has an initial value. The perturbation identification can optionally never be performed (so an input matrix is required), be performed at the beginning of the iteration, or be performed at the beginning and every $M_{PID}$ iterations thereafter.

5-2.3 Secant Method

The secant method (with relaxation) is developed from the Newton–Raphson method. The modified Newton–Raphson iteration is:

$$x_{n+1} = x_n - Cf(x_n) = x_n - \lambda D^{-1} f(x_n)$$
Solution Procedures 53

initialize
   evaluate \( f_0 \) at \( x_0 \), \( f_1 \) at \( x_1 = x_0 + \Delta x \), \( f_2 \) at \( x_2 = x_1 + \Delta x \)

bracket maximum: while not \( f_1 \geq f_0, f_2 \)
   if \( f_2 > f_0 \), then \( x_3 = x_2 + (x_2 - x_1) \); 1,2,3 \( \to \) 0,1,2
   if \( f_0 > f_2 \), then \( x_3 = x_0 - (x_1 - x_0) \); 3,0,1 \( \to \) 0,1,2

iteration (search)
   if \( x_2 - x_1 > x_1 - x_0 \), then \( x_3 = x_1 + W(x_2 - x_1) \)
      if \( f_3 < f_1 \), then 0,1,3 \( \to \) 0,1,2
      if \( f_3 > f_1 \), then 1,3,2 \( \to \) 0,1,2
   if \( x_1 - x_0 > x_2 - x_1 \), then \( x_3 = x_1 - W(x_1 - x_0) \)
      if \( f_3 < f_1 \), then 3,1,2 \( \to \) 0,1,2
      if \( f_3 > f_1 \), then 0,3,1 \( \to \) 0,1,2

test convergence

Figure 5-6. Outline of golden-section search.

where the derivative matrix \( D \) is an estimate of \( f' \). In the secant method, the derivative of \( f \) is evaluated numerically at each step:

\[
f'(x_n) \approx \frac{f(x_n) - f(x_{n-1})}{x_n - x_{n-1}}
\]

It can be shown that then the error reduces during the iteration according to:

\[
|\epsilon_{n+1}| \approx |f''/2f'| |\epsilon_n| |\epsilon_{n-1}| \approx |f''/2f'|^{0.62} |\epsilon_n|^{1.62}
\]

which is slower than the quadratic convergence of the Newton–Raphson method \( (\epsilon^2_n) \), but still better than linear convergence. In practical problems, whether the iteration converges at all is often more important than the rate of convergence. Limiting the maximum amplitude of the derivative estimate may also be appropriate. Note that with \( f = x - G(x) \), the derivative \( f' \) is dimensionless, so a universal limit (say maximum \( |f'| = 0.3 \)) can be specified. A limit on the maximum increment of \( x \) (as a fraction of the \( x \) value) can also be imposed. The process for the secant method is shown in figure 5-5.

5-2.4 Method of False Position

The method of false position is a derivative of the secant method, based on calculating the derivative with values that bracket the solution. The iteration starts with values of \( x_0 \) and \( x_1 \) such that \( f(x_0) \) and \( f(x_1) \) have opposite signs. Then the derivative \( f' \) and new estimate \( x_{n+1} \) are

\[
f'(x_n) \approx \frac{f(x_n) - f(x_k)}{x_n - x_k}
\]

\[
x_{n+1} = x_n - \lambda D^{-1} f(x_n)
\]

using \( k = n - 1 \) or \( k = n - 2 \) such that \( f(x_n) \) and \( f(x_k) \) have opposite signs. The convergence is slower (roughly linear) than for the secant method, but by keeping the solution bracketed convergence is guaranteed. The process for the method of false position is shown in figure 5-5.

5-2.5 Golden-Section Search

The golden-section search method can be used to find the solution \( x \) that maximizes \( f(x) \). The problem of maximizing \( f(x) \) can be attacked by applying the secant method or method of false position to
the derivative $f'(x) = 0$, but that approach is often not satisfactory as it depends on numerical evaluation of the second derivative. The golden-section search method begins with a set of three values $x_0 < x_1 < x_2$ and the corresponding functions $f_0, f_1, f_2$. The $x$ value is incremented until the maximum is bracketed, $f_1 \geq f_0, f_2$. Then a new value $x_3$ is selected in the interval $x_0$ to $x_2$; $f_3$ evaluated; and the new set of $x_0 < x_1 < x_2$ determined such that the maximum is still bracketed. The new value $x_3$ is a fraction $W = (3 - \sqrt{5})/2 \approx 0.38197$ from $x_1$ into the largest segment. The process for the golden-section search is shown in figure 5-6.

5.2.6 Curve-Fit Method

The curve-fit method can be used to find the solution $x$ that maximizes $f(x)$, by fitting the solution to a polynomial. If the function $f$ is flat around the maximum and the inner loop tolerances are not tight enough, the golden-section search can become erratic, particularly for best range and best endurance calculations. Curve fitting the evaluated points and then solving the curve for the maximum has the potential to improve the behavior. The curve-fit method begins with a set of three values $x_0 < x_1 < x_2$ and the corresponding functions $f_0, f_1, f_2$. The $x$ value is incremented until the maximum is bracketed, $f_1 \geq f_0, f_2$, giving a course maximum $f_{max}$ at $x_{max}$. Next a set of $x$ and $f$ values are generated by incrementing $x$ above and below $x_{max}$, until $f < r_{fit}f_{max}$ is found (typically $r_{fit} = 0.98$ for best range).

This set of points is fit to the cubic polynomial $f = c_3 x^3 + c_2 x^2 + c_1 x + c_0$, $z = x/x_{max} - 1$ (or to a quadratic polynomial). Let $c^T = (c_0, c_1, c_2, c_3)$ and $\xi^T = (1, z, z^2, z^3)$. Then the least-squared-error solution for the coefficients is

$$ c = \left( \sum_i \xi_i \xi_i^T \right)^{-1} \left( \sum_i f_i \xi_i \right) $$

where the sums are over the set of points to be fit. For a quadratic polynomial fit, the solution is then

$$ x = x_{max} \left( 1 - c_1/2c_2 \pm \sqrt{1 - r (c_1/2c_2)^2 - c_0/c_2} \right) $$

where $r = 1$ for the maximum, or $r = 0.99$ for 99% best range. For a cubic polynomial fit, the maximum is at

$$ z = -\frac{c_2}{3c_3} \left( 1 - \sqrt{1 - \frac{3c_3 c_1}{c_2^2}} \right) \approx \frac{c_1}{2c_2} \left( 1 + \frac{1}{4} \frac{3c_3 c_1}{c_2^2} \right) $$

It is simplest to search the cubic for the peak ($z$ where $df/dz = 0$), and then if necessary search for the 99% range point ($f = 0.99 f_{peak}$). The process for the golden-section search is shown in figure 5-7.
Chapter 6

Cost

Costs are estimated using statistical models based on historical aircraft price and maintenance cost data, with appropriate factors to account for technology impact and inflation. The aircraft purchase price ($C_{AC}$, in dollars) covers airframe, mission equipment package (MEP), and flight control electronics (FCE) costs. The direct operating cost (DOC, in cents per available seat mile (ASM)) is the sum of maintenance cost ($C_{maint}$, in dollars per flight hour), flight crew salary and expenses, fuel and oil cost, depreciation, insurance cost, and finance cost.

Inflation factors can be input, or internal factors used. Table 6-1 gives the internal inflation factors for DoD (ref. 1) and CPI (ref. 2). For years beyond the data in the table, optionally the inflation factor is extrapolated based on the last yearly increase.

6–1  CTM Rotorcraft Cost Model

The CTM rotorcraft cost model (refs. 3–6) gives an estimate of aircraft purchase price, maintenance cost, and direct operating cost. The model was developed for shaft-driven helicopters and turboprop aircraft. If the aircraft shaft power is zero, $P/W_{AF} = 0.25$ is used in the equation for purchase price.

6-1.1 Aircraft Purchase Price

Aircraft purchase price is estimated from the statistical relationship of Harris and Scully (ref. 3, updated in 2001), based on a 1994 database of mostly civil aircraft (plus EH-101, UH-60L, CH-47D, CH-53E, and MV-22). The model starts with a function of aircraft weight and power; has several complexity factors; a factor for rotorcraft or turboprop aircraft; and a country or industry factor (specifically U.S. military). The model includes (as $$/lb) separate calculations of a composite construction increment (increase or decrease), mission equipment package cost, and flight control electronics cost. The model accounts for inflation and includes an overall technology factor. With these equations, the purchase price is predicted within 20% for 96% of 128 rotorcraft (figs. 6-1 and 6-2), implying a standard deviation of 10%. For the five military aircraft, price is predicted within $\pm$10% or less.

The basic statistical relationship for airframe purchase price is:

$$c_{AF} = 739.91 K_{ET} K_{EN} K_{LG} K_{R} W_{AF}^{1.0619} (P/W_{AF})^{0.5887} N_{blade}^{0.1465}$$

with $W_{AF} = W_{E} + \Delta W_{kit} - W_{MEP} - W_{FCE}$, including airframe kits $\Delta W_{kit}$ (the wing and wing extension kits, and optionally the folding kit). The configuration factor $K_{config} = K_{ET} K_{EN} K_{LG} K_{R}$ has the
Cost factors:

\[
\begin{align*}
K_{ET} &= 1.0 \text{ for turbine aircraft} \quad 0.557 \text{ for piston aircraft} \\
K_{EN} &= 1.0 \text{ for multi-engine aircraft} \quad 0.736 \text{ for single-engine aircraft} \\
K_{LG} &= 1.0 \text{ for retractable landing gear} \quad 0.884 \text{ for fixed landing gear} \\
K_R &= 1.0 \text{ for single main-rotor} \quad 1.057 \text{ for twin main-rotors, 1.117 for four main-rotors}
\end{align*}
\]

The number of blades and the configuration factor are essentially measures of complexity. In particular, retractable/fixed landing gear is a surrogate for general complexity. The term \(C_{\text{comp}} = r_{\text{comp}} W_{\text{comp}}\) accounts for additional costs for composite construction (negative for cost savings); \(W_{\text{comp}}\) is the composite structure weight, obtained as an input fraction of the component weight, with separate fractions for body, tail, pylon, and wing weight. The MEP and FCE costs are obtained from input cost-per-weight factors: \(C_{\text{MEP}} = r_{\text{MEP}} W_{\text{MEP}}\) and \(C_{\text{FCE}} = r_{\text{FCE}} W_{\text{FCE}}\).

The statistical cost equation for \(C_{\text{AC}}\) is based on 1994 dollars and current technology levels. Including an inflation factor \(F_i\) and technology factor \(\chi_{\text{AF}}\) gives the purchase price \(C_{\text{AC}}\):

\[
C_{\text{AC}} = \chi_{\text{AF}} (F_i C_{\text{AF}}) + C_{\text{comp}} + C_{\text{MEP}} + C_{\text{FCE}}
\]

In addition to technology, \(\chi\) accounts for calibration and industry factors; for example, \(\chi_{\text{AF}} = 0.87\) for U.S. Military (ref. 3). This equation also estimates turboprop airliner purchase price by setting \(N_{\text{rotor}} = N_{\text{blade}} = 1\) and using the additional factor 0.8754 (pressurized) or 0.7646 (unpressurized). The purchase price in \$/lb or \$/kg is

\[
\begin{align*}
r_{\text{AF}} &= (\chi_{\text{AF}} (F_i C_{\text{AF}}))/W_{\text{AF}} \\
r_{\text{AC}} &= C_{\text{AC}}/(W_E + \Delta W_{\text{kit}})
\end{align*}
\]

for the airframe and the aircraft. Parameters are defined in table 6-2, including units as used in these equations.

### 6-1.2 Maintenance Cost

Total maintenance cost per hour (dollars per flight hour) is estimated from the statistical relationship of Harris and Scully (ref. 3, updated in 2001). The maintenance cost per hour is:

\[
c_{\text{maint}} = 0.49885 W_E^{0.3746} P^{0.4635}
\]

This equation is based on 1994 dollars and current technology levels. Including an inflation factor \(F_i\) and technology factor \(\chi_{\text{maint}}\) gives the maintenance cost per flight hour \(C_{\text{maint}}\):

\[
C_{\text{maint}} = \chi_{\text{maint}} (F_i c_{\text{maint}})
\]

Parameters are defined in table 6-2, including units as used in these equations.

Alternatively, the maintenance cost \(c_{\text{maint}}\) can be calculated from separate estimates of labor, parts (airframe, engine, and avionics), engine overhaul, and major periodic maintenance costs. The equations for these maintenance cost components are from Harris (ref. 6), based on a 2011 civil database. The contributions to the dollars per flight hour are:

\[
\begin{align*}
C_{\text{labor}} &= r_{\text{labor}} (\text{MMH/FH}) \\
c_{\text{parts}} &= M_{\text{parts}} (C_{\text{AC}}/10^6)^{0.68} \\
c_{\text{engine}} &= M_{\text{engine}} P^{0.67} \\
c_{\text{major}} &= M_{\text{major}} (C_{\text{AC}}/10^6)
\end{align*}
\]
where $r_{\text{labor}}$ is the maintenance labor rate (dollars per hour). The maintenance-man-hours per flight-hour is estimated from

$$\text{MMH/FH} = M_{\text{labor}} W_E^{0.78}$$

or specified directly. These equations are based on 2011 dollars and current technology levels. For current best practice (bottom of data), the constants are $M_{\text{labor}} = 0.0017$, $M_{\text{parts}} = 34$, $M_{\text{engine}} = 1.45$, $M_{\text{major}} = 18$; while for current average practice $M_{\text{labor}} = 0.0027$, $M_{\text{parts}} = 56$, $M_{\text{engine}} = 1.74$, $M_{\text{major}} = 28$. Finally, the maintenance cost per flight hour is

$$C_{\text{maint}} = \chi_{\text{maint}} F_i (c_{\text{parts}} + c_{\text{engine}} + c_{\text{major}}) + C_{\text{labor}}$$

including an inflation factor $F_i$ and technology factor $\chi_{\text{maint}}$.

### 6.1.3 Direct Operating Cost

Given specifics of the operation (including available block hours, non-flight time, spares fraction, and financial numbers), the direct operating cost is calculated for each mission. The direct operating cost includes maintenance, fuel, crew, depreciation, insurance, and finance costs. The contributions to the yearly operating cost are:

$$C_{\text{fuel}} = (G_{\text{fuel}} (W_{\text{fuel}}/\rho_{\text{fuel}}) + G_{\text{energy}} E_{\text{fuel}}) N_{\text{dep}}$$

$$C_{\text{crew}} = 2.84 F_i K_{\text{crew}} W_{MTOG}^{0.4} B$$

$$C_{\text{dep}} = C_{\text{AC}} \frac{1 + S}{D} (1 - V)$$

$$C_{\text{ins}} = 0.0056 C_{\text{AC}}$$

$$C_{\text{fin}} = C_{\text{AC}} \frac{1 + S}{D} \frac{2L + 1}{4} \frac{i}{100}$$

There is a separate credit ($G_{\text{credit}}$, perhaps zero) for mission energy generation ($E_{\text{fuel}} < 0$). The crew factor $K_{\text{crew}} = 1$ corresponds to low-cost, domestic airlines (1994 dollars). The fuel burn $W_{\text{fuel}}$, block time $T_{\text{miss}}$, and block range $R_{\text{miss}}$ are obtained for a designated mission. The number of departures per year is $N_{\text{dep}} = B/T_{\text{miss}}$. The flight time per trip is $T_{\text{trip}} = T_{\text{miss}} - T_{\text{NF}}$. The flight hours per year are $T_F = T_{\text{trip}} N_{\text{dep}}$. Alternatively, the sum of the crew, insurance, and depreciation costs can be estimated from the purchase price:

$$C_{\text{crew}} + C_{\text{dep}} + C_{\text{ins}} = K_{\text{cdi}} (175000 + 29.8 (C_{\text{AC}}/1000))$$

where $K_{\text{cdi}}$ is a calibration factor (ref. 6). The yearly operating cost $C_{\text{OP}}$ and DOC (cents per available seat mile) are then:

$$C_{\text{OP}} = T_F C_{\text{maint}} + C_{\text{fuel}} + C_{\text{crew}} + C_{\text{dep}} + C_{\text{ins}} + C_{\text{fin}}$$

$$\text{DOC} = 100 C_{\text{OP}} / \text{ASM}$$

where the available seat miles per year are \(A S M = 1.1508 N_{\text{pass}} R_{\text{miss}} N_{\text{dep}} \) (range in nm).

### 6.2 References


Table 6-1. DoD and CPI inflation factors.

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<td>1945</td>
<td>12.15</td>
<td>76.86</td>
<td>72.60</td>
<td>1946</td>
<td>13.16</td>
<td>79.19</td>
<td>73.95</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>1947</td>
<td>15.05</td>
<td>81.87</td>
<td>76.65</td>
<td>1948</td>
<td>16.26</td>
<td>85.01</td>
<td>79.82</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1949</td>
<td>16.06</td>
<td>88.19</td>
<td>83.67</td>
<td>1950</td>
<td>15.37</td>
<td>91.30</td>
<td>88.19</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

* projected
Table 6-2. Cost model parameters.

<table>
<thead>
<tr>
<th>parameter</th>
<th>definition</th>
<th>units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_E$</td>
<td>weight empty</td>
<td>lb</td>
</tr>
<tr>
<td>$W_{MTO}$</td>
<td>maximum takeoff weight</td>
<td>lb</td>
</tr>
<tr>
<td>$N_{blade}$</td>
<td>number of blades per rotor</td>
<td>lb</td>
</tr>
<tr>
<td>$P$</td>
<td>rated takeoff power (all engines)</td>
<td>hp</td>
</tr>
<tr>
<td>$W_{MEP}$</td>
<td>fixed useful load weight, mission equipment package</td>
<td>lb or kg</td>
</tr>
<tr>
<td>$W_{FCE}$</td>
<td>fixed useful load weight, flight control electronics</td>
<td>lb or kg</td>
</tr>
<tr>
<td>$r_{MEP}$</td>
<td>cost factor, mission equipment package</td>
<td>$/lb or $/kg</td>
</tr>
<tr>
<td>$r_{FCE}$</td>
<td>cost factor, flight control electronics</td>
<td>$/lb or $/kg</td>
</tr>
<tr>
<td>$r_{comp}$</td>
<td>additional cost for composite construction</td>
<td>$/lb or $/kg</td>
</tr>
<tr>
<td>$F_i$</td>
<td>inflation factor, relative 1994</td>
<td></td>
</tr>
<tr>
<td>$W_{fuel}$</td>
<td>mission fuel burned</td>
<td>lb or kg</td>
</tr>
<tr>
<td>$T_{miss}$</td>
<td>mission time</td>
<td>hr</td>
</tr>
<tr>
<td>$R_{miss}$</td>
<td>mission range</td>
<td>nm</td>
</tr>
<tr>
<td>$G_{fuel}$</td>
<td>fuel cost</td>
<td>$/gallon or $/liter</td>
</tr>
<tr>
<td>$G_{energy}$</td>
<td>energy cost</td>
<td>$/MJ</td>
</tr>
<tr>
<td>$G_{credit}$</td>
<td>energy credit</td>
<td>$/MJ</td>
</tr>
<tr>
<td>$B$</td>
<td>available block hours</td>
<td>hr</td>
</tr>
<tr>
<td>$S$</td>
<td>spares per aircraft (fraction purchase price)</td>
<td></td>
</tr>
<tr>
<td>$D$</td>
<td>depreciation period</td>
<td>yr</td>
</tr>
<tr>
<td>$V$</td>
<td>residual value (fraction)</td>
<td></td>
</tr>
<tr>
<td>$L$</td>
<td>loan period</td>
<td>yr</td>
</tr>
<tr>
<td>$i$</td>
<td>interest rate</td>
<td>%</td>
</tr>
<tr>
<td>$T_{NF}$</td>
<td>non-flight time per trip</td>
<td>hr</td>
</tr>
<tr>
<td>$N_{pass}$</td>
<td>number of passengers</td>
<td></td>
</tr>
<tr>
<td>$\rho_{fuel}$</td>
<td>fuel density (weight per volume)</td>
<td>lb/gal or kg/liter</td>
</tr>
<tr>
<td>$N_{dep}$</td>
<td>number of departures per year</td>
<td></td>
</tr>
<tr>
<td>$K_{crew}$</td>
<td>crew factor</td>
<td></td>
</tr>
<tr>
<td>$T_F$</td>
<td>flight hours per year</td>
<td></td>
</tr>
</tbody>
</table>
Figure 6-1. Statistical estimation of rotorcraft purchase price ($/lb).
Figure 6-2. Statistical estimation of rotorcraft purchase price ($M).
Chapter 7

Emissions

Evaluating the environmental impact of an aircraft begins with calculating the engine emissions for the missions flown. It is best to use metrics that account for all relevant aircraft emissions, although metrics based on a single species can be useful. From the engine emissions, the atmospheric concentrations are evaluated, then radiative forcing ($RF$). Radiative forcing is a measure of the amount of heat trapped in the atmosphere by a particular pollutant, and is expressed in terms of trapped energy per unit area ($W/m^2$). Finally, climate changes and the resulting impacts and damages can be determined. The steps in this causal chain become increasingly important and increasingly uncertain. Any metric that is used to evaluate new aircraft concepts must balance uncertainty with relevance. The present environmental impact models were developed by Russell (ref. 1), based on the work of Dallara, Kroo, and Waitz (ref. 2).

The important aircraft emission species are the direct greenhouse gas carbon dioxide ($CO_2$), nitrogen oxides ($NO_x$), water vapor ($H_2O$), aerosols of soot and sulphate ($SO_4$), and aviation-induced cloudiness (AIC). Carbon dioxide has a long lifetime relative to chemical processes in the atmosphere, hence aviation $CO_2$ impacts do not vary with altitude and can be calculated as for other sources. $NO_x$ emissions cause changes in $RF$ indirectly through chemical processes in the atmosphere. Increases in $NO_x$ lead to increases in atmospheric ozone by a short lifetime process ($O_3$), a warming effect, and over long time-scales reductions in the greenhouse gas methane ($CH_4$), a cooling effect. The methane reduction has a secondary effect of reducing ozone ($O_3$), a cooling effect. Water vapor, short-lived ozone, soot, and sulfate aerosols have lifetimes much shorter than one year, hence have impacts only for a short time after emission. Aviation-induced cloudiness (AIC), including the effects of linear contrails and altered cirrus cloudiness, have a short-lived warming effect. Total radiative forcing from aircraft can be several times greater than forcing due to $CO_2$ alone, depending on altitude and AIC conditions. The impact of $NO_x$ emissions is similar in magnitude to that of $CO_2$, but there is greater uncertainty in the $NO_x$ $RF$ values. Estimates of the $RF$ due to induced cirrus are particularly uncertain. The uncertainty of the models and parameters for the various emission species is discussed in reference 2.

The environmental impact of the aircraft operation is estimated in terms of two metrics: emissions trading scheme (ETS) credits, and average temperature response (ATR). The impact of energy used is also modeled.

7–1 Emissions Trading Scheme Credits

The emissions trading scheme (ETS) is the European Union system to curb the effects of greenhouse gas emissions on global climate change, by limiting the amount of carbon dioxide that can be produced by large polluters such as energy and industrial installations. The ETS was extended to the aviation sector in early 2012. Under the ETS, each member nation has an emission cap that is used to allocate allowable carbon emissions to their industrial operators. Operators that do not use their entire allocation
can sell their unused “carbon credits” on the open market, while operators exceeding their allocation must purchase credits on the market. The price of carbon credits has varied widely, ranging from below 10 to above 30 Euro/ton CO$_2$. The metric used here is the weight of CO$_2$ produced:

$$W_{CO_2} = K_{fuel}W_{burn} + K_{energy}E_{burn}$$

per mission. Typically $K_{fuel} = EI_{CO_2} \approx 3.75$ (lb/lb or kg/kg) and $K_{energy} \approx 0.14$ (kg/MJ). Different aircraft designs or operations can be compared in terms of $W_{CO_2}$. Accounting for the influence on operating cost requires the price of carbon credits.

$K_{fuel}$ depends on the combustion process and the chemical composition of the fuel (ref. 3). For combustion of conventional jet fuel, $K_{fuel} = 3.16$ kg/kg = 74 g/MJ = 266 g/kWh (using 42.8 MJ/kg). Accounting for emissions from production of the fuel (principally carbon dioxide, methane, and nitrogen oxides), gives an equivalent CO$_2$ value perhaps 20% larger: $K_{fuel} = 3.75$ kg/kg = 88 g/MJ = 315 g/kWh. For combustion of alternative jet fuels, typically $K_{fuel} = 3.01$ kg/kg = 70 g/MJ = 263 g/kWh. Accounting for biomass credit (CO$_2$ absorbed during biomass growth), the equivalent CO$_2$ value might be 50% lower than for conventional fuel (ref. 3).

$K_{energy}$ depends on the energy source. Table 7-1 (from ref. 4) gives typical values of equivalent CO$_2$ for electricity from various sources. Worldwide average values for equivalent CO$_2$ range from 6 to 1100 g/kWh. The current average for the United States is about $K_{energy} = 500$ g/kWh = 139 g/MJ. Optionally there is no emissions credit for energy generation ($E_{burn} < 0$).

<table>
<thead>
<tr>
<th>source</th>
<th>median</th>
<th>range</th>
</tr>
</thead>
<tbody>
<tr>
<td>coal</td>
<td>860</td>
<td>780-1100</td>
</tr>
<tr>
<td>gas</td>
<td>500</td>
<td>400-600</td>
</tr>
<tr>
<td>small wind turbine</td>
<td>35</td>
<td>20-50</td>
</tr>
<tr>
<td>nuclear</td>
<td>22</td>
<td>10-30</td>
</tr>
<tr>
<td>large wind turbine</td>
<td>20</td>
<td>15-25</td>
</tr>
<tr>
<td>hydroelectric</td>
<td>18</td>
<td>10-20</td>
</tr>
</tbody>
</table>

7–2 Average Temperature Response

The average temperature response (ATR) quantifies the lifetime global mean temperature change caused by operation of an aircraft, as a measure of climate change (ref. 2). ATR can be used with a number of different climate models, but simple linear climate models are appropriate for the conceptual design of rotorcraft. While atmospheric processes are nonlinear, the globally averaged responses to small perturbations (in magnitude similar to commercial aircraft emissions) of CO$_2$, NO$_x$, and AIC perturbations are nearly linear (ref. 2). The ATR metric is based on the radiative forcing ($RF$) generated by each emission species. Radiative forcing measures the net imbalance of incoming and outgoing energies in the Earth-atmosphere system caused by a perturbation. $RF$ is an instantaneous measure that quantifies the change in energy that produces changes in climate properties, including temperature. Many climate change metrics, such as Global Warming Potentials, rely on $RF$, but do not specifically target emissions due to aviation. The total $RF$ for all emitted pollutants is used to calculate the global temperature response. The use of an altitude-sensitive climate model captures the effects of aircraft operating conditions. In addition, ATR includes parameters such as usage rates and operating lifetime of the aircraft to determine the total climate impact.
The average temperature response is calculated by integrating the temperature change during and after \( H \) years of sustained operation of an aircraft:

\[
\text{ATR} = \frac{1}{H} \int_0^\infty \Delta T_H(t) \, w(t) \, dt
\]

ATR has units of temperature. The metric can also be used in relative terms, by dividing by the value for a baseline design. \( \Delta T_H(t) \) is the global mean temperature change resulting from the operation of the given design, where emissions are assumed to be constant for the first \( H \) years and zero thereafter. This temperature change is a function of the radiative forcing caused by the emission of a number of different pollutant species. There are multiple models available with varying levels of fidelity for calculating \( \Delta T_H \). Here linear climate models and functions are used, based on reference 2. One limitation of this model is that the RF of NO\(_x\) and AIC is based on data that only goes down to 17500 ft. Below this altitude the effects of NO\(_x\) and AIC are assumed constant.

There are climate effects from perturbations remaining in the Earth-atmosphere system after the aircraft operating lifetime has ended. Hence \( \Delta T_H \) increases for time up to \( H \), and thereafter decays, but is not zero. The weighting function \( w(t) \) allows discounting of temperature change effects in the years following \( H \), so that long-term effects such as CO\(_2\) warming do not dominate ATR. The function

\[
w(t) = \begin{cases} 
1 & \text{if } t \leq H \\
\frac{1}{(1 + r)^{t-H}} & \text{if } H < t \leq t_{\text{max}} \\
0 & \text{if } t > t_{\text{max}} 
\end{cases}
\]

has unit weighting during the operating lifetime, and exponential devaluation thereafter, in terms of the discount rate \( r \) (typically \( r = 2 \) to 5\%). A rate of zero means that post-operation impacts are equally important compared with impacts during operating years; a rate of infinity means that post-operation impacts have no importance; and a positive, finite rate means that the temperature change each post-operation year is less important than the temperature change experienced the previous year (by the factor \( 1/(1 + r) \)).

Also contributing to the ATR metric is aviation induced cloudiness (AIC), which includes both contrails and aviation-induced cirrus clouds. Following the methodology of reference 2, the impact of AIC is assumed to be a function of cruise altitude and distance, and does not account for changes in water vapor emissions or exhaust temperature or atmospheric conditions. The consequence of these assumptions is that for a given rotorcraft mission, the radiative forcing due to AIC may be greatly overestimated. Also, strategies are currently being developed to reduce the effects of AIC by rerouting around areas in the atmosphere prone to aircraft contrails, so future aircraft may have significantly reduced climate impacts due to AIC. Since there is a high degree of uncertainty in radiative forcing due to AIC, calculations of ATR should be made both with and without its effects.

Design for minimum ATR depends on the time horizon, discount rate, operating altitude and speed, and the engine technology. If there is no discounting of long-term effects (\( r = 0 \)), radiative forcing due to CO\(_2\) dominates the ATR, favoring a higher cruise altitude. When long term effects are included and engine technology for low NO\(_x\) is assumed, ATR can be generally minimized by designing for low fuel burn. If only short-term impacts are considered (\( r = \infty \)), then NO\(_x\) emissions have a greater impact, and a lower cruise altitude results in decreased climate impact, despite increased fuel burn and CO\(_2\) emissions. The inclusion of AIC in the metric calculations also has a large impact on the design, and favors a low cruise altitude.
7-2.1 Aircraft Mission

ATR is calculated assuming constant emissions during the first $H$ years of operation and zero emissions thereafter. For a single operating mission, $\Delta T_H$ is the sum over all mission segments, times a utilization rate of $U$ missions per year. The information for each mission segment includes: fuel burned $W_{\text{burn}}$, energy used $E_{\text{burn}}$, altitude $h$, air distance flown $d$, engine fuel flow $\dot{w}$, and operating condition $(M, \theta, \delta)$.

7-2.2 Engine Emissions

Engine emissions can be quantified in terms of an emissions index ($EI$, lb/lb or kg/kg) that relates the amount of a pollutant species to the amount of fuel burned. For some emission species, such as CO$_2$ and water vapor, the $EI$ is nearly constant, assuming ideal or near-ideal fuel combustion. For others, particularly NO$_x$, the $EI$ varies widely throughout the operating range of the engine. The emission of species $i$ is thus obtained from the fuel consumption summed over all mission segments:

$$e_i = \sum_{\text{seg}} EI_i W_{\text{burn}}$$

The time-varying annual emissions $E$ is found by summing over all operations conducted in a year. Assuming a single operating mission, $E_i = U e_i$, where $U$ is the aircraft utilization rate (number of missions per year).

For each fuel, $EI$ is required for CO$_2$, H$_2$O, SO$_4$, soot, and NO$_x$. Optionally there is no emissions credit for energy generation ($E_{\text{burn}} < 0$).

7-2.3 Turboshaft Engine Emissions

For carbon dioxide, water vapor, sulfates, and soot the emissions indices are assumed to be constant. The $EI$ of CO$_2$, H$_2$O, or SO$_4$ depends solely on the composition of the fuel. The $EI$ for soot can vary with engine operating condition, but because soot comprises a small fraction of total climate impacts (typically less than 5%), this index can also be assumed constant. The values used for the constant $EI$ emissions are given in table 7-2 (from ref. 2).

<table>
<thead>
<tr>
<th>species</th>
<th>emission index $EI$ (lb/lb or kg/kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CO$_2$</td>
<td>3.16</td>
</tr>
<tr>
<td>H$_2$O</td>
<td>1.26</td>
</tr>
<tr>
<td>SO$_4$</td>
<td>$2.0 \times 10^{-4}$</td>
</tr>
<tr>
<td>soot</td>
<td>$4.0 \times 10^{-5}$</td>
</tr>
</tbody>
</table>

The $EI$ for oxides of nitrogen (including NO and NO$_2$, collectively called NO$_x$) depends on operating conditions, including engine throttle setting, Mach number, and altitude (temperature and pressure). Hence $EI_{\text{NO}_x}$ must be evaluated for each mission segment. Engines are required to comply with NO$_x$ emissions regulations during landing and takeoff. The International Civil Aviation Organization (ICAO) measures and publishes certification data relating fuel flow, thrust, and $EI_{\text{NO}_x}$ at four sea-level static throttle settings that simulate taxi, takeoff, climb, and approach operating conditions. However, ICAO does not measure NO$_x$ at cruise conditions, because these emissions are not currently regulated. Therefore, a model is needed to compute $EI_{\text{NO}_x}$ at specific flight conditions including cruise. Fuel flow
Emission methods have been developed for this purpose. While there is a large amount of published turbofan NO\textsubscript{x} emissions data and established methods for estimating variation with altitude, there is little public information for turboshaft engines. Russell (ref. 1) developed a method to estimate EI\textsubscript{NO\textsubscript{x}} based on engine performance information from the NDARC engine models, ICAO turbofan emissions data, and the German Aerospace Center (DLR) fuel flow method described in reference 5. The DLR semi-empirical method models the relationship between engine throttle setting and EI\textsubscript{NO\textsubscript{x}} at varying ambient temperature and pressure conditions. The DLR method has been shown to predict EI\textsubscript{NO\textsubscript{x}} to within approximately 10% of measurements at typical cruise conditions.

The emission index for NO\textsubscript{x} is calculated from the fuel flow \dot{\omega} as follows. The fuel flow is corrected to referred conditions at the inlet: \dot{\omega}_{corr} = \dot{\omega}/(\theta_t \delta_t \sqrt{\theta_t}) where the temperature and pressure ratios are

\[ \theta_t = \left(1 + \frac{\gamma - 1}{2} M^2\right) \theta = (1 + 0.2 M^2) \theta \]
\[ \delta_t = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma - 1}{\gamma}} \delta = (1 + 0.2 M^2)^{3.5} \delta \]

(for ratio of specific heats \(\gamma = 1.4\)). The engine model gives the fuel flow \dot{\omega} as a function of \(P_{req} = f_P P_{to}\) at SLS static conditions, which can be interpolated to find \(f_P\) such that \(\dot{\omega} = \dot{\omega}_{corr}\). The corrected emission index is scaled to SLS takeoff power, hence

\[ EI_{NO_x,corr} = K_{EI0} + K_{EI1} f_P \]

The constants depend on the engine design and technology. Typically \(1000 K_{EI0} = 3.6739\) and \(1000 K_{EI1} = 7.48\) for a low-emissions engine (based on CF34); \(1000 K_{EI0} = 2.4392\) and \(1000 K_{EI1} = 23.26\) for a high-emissions engine (based on HTF7000). Finally, the emission index is

\[ EI_{NO_x} = EI_{NO_x,corr} \theta_t^{0.5677} \delta_t^{0.4} F_H \]

with the humidity correction factor \(F_H = \exp \left(\frac{\left(6.29 - e^{-0.000143(h-12900)}}{53.2}\right)}{53.2}\), altitude \(h\) in ft.

There are two assumptions in this method. The first assumption is that the variation of \(EI_{NO_x}\) for a turboshaft is the same as that of a turbofan engine. Since both are gas turbine engines, employing the same thermal cycle, this assumption should be valid. The second assumption is that the RPTEM engine model is producing variations in fuel flow with speed and altitude in a manner consistent with the DLR fuel flow method. Since both the fuel flow method and the engine model in NDARC are based on real engine data, this assumption should be valid as well. Two different engines are used as references. The GE CF34-3B and the Honeywell HTF7000 have fuel flows similar to what is calculated for a turboshaft engine. Both engines are relatively modern, high bypass ratio, small turbofans. The HTF7000 and CF34 also represent the upper and lower bounds on NO\textsubscript{x} emissions for this category of engine, so they should bracket the expected quantity of NO\textsubscript{x} emissions for a turboshaft engine. Compared to the differences between the \(EI_{NO_x}\) values of the two reference engines, the speed and altitude corrections are relatively small.

An alternative approach is to calculate the emission index for NO\textsubscript{x} from the engine power required. Reference 6 gives the parametric relation

\[ EI_{NO_x} = K_{EI1} \left(\frac{P_{req}}{1000}\right)^{0.5677} \]
with $P_{\text{req}}$ in hp, and $1000K_{EI_s} = 4$ to 14, depending on the engine design and technology.

With more than one turboshaft engine group using fuel from a tank, the emission index for a segment is weighted by the fuel flow, hence by the fuel burned: $EI_{NO_x} = \sum EI \dot{w} / \sum \dot{w}$ (sum over engine groups).

### 7-2.4 Energy Emissions

The environmental impact of energy used is quantified in terms of constants ($K$, kg/MJ) that relate the amount of a pollutant species to the amount of energy burned. These constants depend on the source of the energy, and are required for CO$_2$, H$_2$O, SO$_4$, soot, and NO$_x$. The emission of species $i$ is thus obtained from the energy consumption summed over all mission segments:

$$e_i = \sum_{\text{seg}} K_i E_{\text{burn}}$$

The time-varying annual emissions are $E_i = U e_i$, which is added to the engine emissions. In the climate model, the altitude corresponding to these emissions is zero.

### 7-2.5 Climate Model and Radiative Forcing

A climate model is required to determine the temperature change resulting from the emissions produced by an aircraft operation. The simplest approach (ref. 2) is to use a linear climate model, so the time-varying radiative forcing of each species is obtained from the emissions and a climate impulse response function. Although derived from the globally averaged results of some complex, three-dimensional, global climate model, a linear model simplifies the physics and chemistry of aircraft-induced climate change, and captures first-order effects. The climate model used, developed in reference 2 specifically for aircraft operations, includes the altitude variation for NO$_x$ and AIC impacts. Different models are required for long-lived gases, short-lived pollutants, and AIC. Models for $RF$ from all emissions except CO$_2$, which has a very long atmospheric lifetime, depend on the assumed geographical distribution of emissions. The models used are based on average impacts from fleetwide routing in a single year within the last decade, quantifying the average forcing caused by emissions spatially distributed according to routing similar to current traffic.

For long-lived emission species $i$ (CO$_2$, CH$_4$, O$_{3L}$), the radiative forcing $RF_i$ is obtained by integrating the product of the $E_i$ (kg/yr) and the impulse response $G_i$ (W/m$^2$/kg):

$$RF_i(t) = s_i(h) \int_0^t G_i(t - \tau) E_i(\tau) \, d\tau$$

$$= U \sum_{\text{seg}} s_i(h) \left( EI_i W_{\text{burn}} + K_i E_{\text{burn}} \right) A_i \int_0^t \left( G_i(t - \tau)/A_i \right) u(\tau) \, d\tau$$

$$= U \sum_{\text{seg}} s_i(h) a_i A_i X_i(t)$$

where $a_i = (EI_i W_{\text{burn}} + K_i E_{\text{burn}})$, $u(t)$ is the emission time history (unit amplitude), and for $t > 0$

$$G_{CO_2}(t) = A_{CO_2} \left( 1 + \sum_{j=1}^{3} \alpha c_j \left( e^{-t/\tau_{c_j}} - 1 \right) \right)$$

$$G_{CH_4}(t) = A_{CH_4} e^{-t/\tau_n}$$

$$G_{O_{3L}}(t) = A_{O_{3L}} e^{-t/\tau_n}$$
Global mean temperature change is evaluated: $EI_{NOx}$ is used for CH$_4$, O$_{3L}$. The parameters are given in table 7-3. The forcing factors $s_i(h)$ are given as a function of altitude in table 7-4; $s = 1$ for carbon dioxide.

For short-lived emission species (H$_2$O, O$_{3S}$, soot, SO$_4$), $RF$ is assumed to be proportional to the emissions:

$$RF_i(t) = s_i(h) \left( \frac{RF_{ref}}{E_{ref}} \right)_i E_i(t)$$

$$= U \sum_{seg} s_i(h) \left( EI_{Wburn} + K_i E_{burn} \right) \left( \frac{RF_{ref}}{E_{ref}} \right)_i u(t)$$

$$= U \sum_{seg} s_i(h)a_i A_i X_i(t)$$

where $A_i = \frac{RF_{ref}}{E_{ref}}$ and $X_i(t) = u(t)$. $EI_{NOx}$ is used for O$_{3S}$. The parameters ($\frac{RF_{ref}}{E_{ref}}$) are given in table 7-3. The forcing factor $s = 1$ except for O$_{3S}$, which is given in table 7-4.

Aviation-induced cloudiness (AIC) effects are assumed to be proportional to the distance traveled:

$$RF_{AIC}(t) = s_{AIC}(h) f_{AIC} \left( \frac{RF_{ref}}{L_{ref}} \right)_{AIC} L(t)$$

$$= U \sum_{seg} s_{AIC}(h) f_{AIC} \left( \frac{RF_{ref}}{L_{ref}} \right)_{AIC} u(t)$$

$$= U \sum_{seg} s_{AIC}(h) a_{AIC} A_{AIC} X_{AIC}(t)$$

where $L = U \sum_{seg} d$ is the stage length flown per year, $a_{AIC} = d$; $A_{AIC} = f_{AIC} \frac{RF_{ref}}{L_{ref}}$ and $X_{AIC}(t) = u(t)$. The parameter ($\frac{RF_{ref}}{L_{ref}}$) is given in table 7-3, and the forcing factor $s_{AIC}$ in table 7-4; $f_{AIC}$ is an input correction factor.

These forcing factors (table 7-4) characterize globally and annually averaged RFs based on current flight routes. Forcing factors, particularly for AIC impacts, are expected to vary with time of day, season, and earth latitude, effects which are averaged in this model.

The normalized RF for each species is based on the RF that would result from a doubling of carbon dioxide:

$$RF_i^*(t) = f_i \frac{RF_i(t)}{RF_{2xCO_2}} = U \sum_{seg} s_i(h) a_i A_i \frac{f_i}{RF_{2xCO_2}} X_i(t)$$

for CO$_2$, CH$_4$, O$_{3L}$, O$_{3S}$, H$_2$O, soot, SO$_4$, and AIC. The efficacy factors $f_i$ are given in table 7-3; $f_{CO_2} = 1$. From the total normalized RF and a climate impulse response function, the time-varying global mean temperature change is evaluated:

$$\Delta T(t) = \int_0^t G_T(t - \tau) \sum_i RF_i^*(\tau) d\tau$$

$$= \sum_i U \sum_{seg} s_i(h) a_i A_i \frac{f_i}{RF_{2xCO_2}} S \int_0^t (G_T(t - \tau)/S) X_i(\tau) d\tau$$

$$= \sum_i U \sum_{seg} s_i(h) a_i A_i \frac{f_i}{RF_{2xCO_2}} SY_i(t)$$

where

$$G_T(t) = S \left(\frac{\alpha_t}{\tau_1} e^{-t/\tau_1} + \frac{1 - \alpha_t}{\tau_2} e^{-t/\tau_2}\right)$$

The time constants in $G_T$ describe the thermal response of the earth system to an energy perturbation, and $S$ is the steady-state temperature change produced by a constant annual forcing of $RF_{2xCO_2}$. The parameters are given in table 7-3.
The corresponding weight emitted for each species is $W_i = \sum_{\text{seg}} (E_i W_{\text{burn}} + K_i E_{\text{burn}}) = \sum_{\text{seg}} a_i$ per mission ($W_i = HU \sum_{\text{seg}} a_i$ over the operating duration $H$).

### 7-2.6 ATR Evaluation

The average temperature response (ATR) is calculated from $\Delta T = \Delta T_H$, assuming constant emissions during the first $H$ years of operation and zero emissions thereafter. Hence the shape function $u(t) = 1$ for $0 < t < H$, and zero at other times. For a single operating mission, $\Delta T_H$ is the sum over all mission segments (each giving fuel and energy burned and distance traveled), with a utilization rate of $U$ missions per year. The radiative forcing factors are:

$$X_{\text{CO}_2}(t) = \int_0^t \left( 1 + \sum_j \alpha_{cj} \left( e^{-(t-\tau)/\tau_{cj}} - 1 \right) \right) u(\tau) \, d\tau$$

$$= \begin{cases} t(1 - \sum_j \alpha_{cj}) + \sum_j \alpha_{cj} \tau_{cj} (1 - e^{-t/\tau_{cj}}) & t < H \\
H(1 - \sum_j \alpha_{cj}) + \sum_j \alpha_{cj} \tau_{cj} (e^{-(t-H)/\tau_{cj}} - e^{-t/\tau_{cj}}) & t > H 
\end{cases}$$

$$X_{\text{CH}_4}(t) = X_{\text{O}_3\text{H}_2}(t) = \int_0^t e^{-(t-\tau)/\tau_n} u(\tau) \, d\tau$$

$$= \begin{cases} \tau_n (1 - e^{-t/\tau_n}) & t < H \\
\tau_n (e^{-(t-H)/\tau_n} - e^{-t/\tau_n}) & t > H 
\end{cases}$$

and $X_i(t) = u(t)$ for the other species. The temperature change factors $Y_i(t) = \int_0^t (G_T (t-\tau)/S) X_i(\tau) \, d\tau$ are:

$$Y_{\text{CO}_2}(t) = \begin{cases} \alpha_{t} \left[ \left( t - \tau_{t1} (1 - e^{-t/\tau_{t1}}) \right) (1 - \sum_j \alpha_{cj}) \\
+ \sum_j \alpha_{cj} \tau_{cj} \left\{ 1 + \frac{\tau_{t1}}{\tau_{cj} - \tau_{t1}} e^{-t/\tau_{t1}} - \frac{\tau_{cj}}{\tau_{cj} - \tau_{t1}} e^{-t/\tau_{cj}} \right\} \right] & t < H \\
\alpha_{t} \left[ \left( H - \tau_{t1} (e^{-1/\tau_{t1}}) \right) (1 - \sum_j \alpha_{cj}) \\
+ \sum_j \alpha_{cj} \tau_{cj} \left\{ -\frac{\tau_{t1}}{\tau_{cj} - \tau_{t1}} \left( e^{-H/\tau_{t1}} - e^{-t/\tau_{t1}} \right) \\
\quad + \frac{\tau_{cj}}{\tau_{cj} - \tau_{t1}} \left( e^{-(t-H)/\tau_{t1}} - e^{-t/\tau_{cj}} \right) \right\} \right] & t > H 
\end{cases}$$

$$Y_{\text{CH}_4}(t) = Y_{\text{O}_3\text{H}_2}(t) = \int_0^t e^{-(t-\tau)/\tau_n} u(\tau) \, d\tau$$

$$= \begin{cases} \tau_n (1 - e^{-t/\tau_n}) & t < H \\
\tau_n (e^{-(t-H)/\tau_n} - e^{-t/\tau_n}) & t > H 
\end{cases}$$

$$Y_i(t) = u(t)$$ for the other species.
Emissions \( Y \) \( CH_4 \) \( (t) = Y_{O_{3L}} \) \( (t) = \) \( \frac{\alpha t}{n} \left[ \frac{\tau_1}{n - \tau_1} \left( e^{-t/\tau_1} - e^{-t/\tau_2} \right) \right. \\
\quad + \left( 1 - \alpha t \right) \tau_n \left[ 1 + \frac{\tau_2}{n - \tau_2} e^{-t/\tau_2} - \frac{\tau_n}{n - \tau_2} e^{-t/\tau_n} \right] \] 
\quad \frac{t < H}{t > H}

\[ Y_{i} \] \( (t) = \) \[ \alpha t \left[ 1 - e^{-t/\tau_1} \right] + \left( 1 - \alpha t \right) \left[ 1 - e^{-t/\tau_2} \right] \] 
\[ \alpha t \left[ e^{-\left( t-H \right)/\tau_1} - e^{-t/\tau_1} \right] + \left( 1 - \alpha t \right) \left[ e^{-\left( t-H \right)/\tau_2} - e^{-t/\tau_2} \right] \] 
\[ t < H \] 
\[ t > H \]

Then the average temperature response is
\[ ATR = \frac{1}{H} \int_{0}^{t_{\text{max}}} \Delta T_H (t) w(t) \, dt \]
\[ = \sum_{i} U \sum_{\text{seg}} s_i (h) a_i A_i \frac{f_i}{RF_{2xCO_2}} S \frac{1}{H} \int_{0}^{t_{\text{max}}} Y_i (t) w(t) \, dt \]
\[ = \sum_{i} U \sum_{\text{seg}} s_i (h) a_i A_i \frac{f_i}{RF_{2xCO_2}} S Z_i \]
\[ = \sum_{i} ATR_i \]

Given \( H \) and \( r \), \( Z_i \) is evaluated by numerical integration. For the special cases of \( r = 0 \) (no discounting) and \( r = \infty \) (no effects beyond \( H \)):
and for the other species:

\[ Z_i = \begin{cases} 
1 + \alpha_t \left[ \frac{\tau_{t1}}{H} \left( 1 - e^{-H/\tau_{t1}} \right) \right] + (1 - \alpha_t) \left[ \frac{\tau_{t2}}{H} \left( 1 - e^{-H/\tau_{t2}} \right) \right] & r = \infty \\
1 + \alpha_t \left[ e^{-\left( t_{\text{max}} - H \right)/\tau_{t1}} - e^{-t_{\text{max}}/\tau_{t1}} \right] + (1 - \alpha_t) \left[ e^{-\left( t_{\text{max}} - H \right)/\tau_{t2}} - e^{-t_{\text{max}}/\tau_{t2}} \right] & r = 0 
\end{cases} \]
Since $\Sigma_j \alpha_{cj} \neq 1$, the radiative forcing for CO$_2$ is finite at long times ($X_{CO_2} \rightarrow H(1 - \Sigma_j \alpha_{cj})$), and CO$_2$ dominates the temperature response. It is necessary to introduce $t_{max}$ so that $Z_{CO_2}$ (and ATR) is finite in the absence of discounting ($r = 0$).

### 7–3 References


<table>
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Table 7-4. Radiative forcing models.

| CO₂ parameters | | |
|----------------|-----------------|
| $A_{CO₂}$      | $1.80 \times 10^{-15}$ (W/m²)/(kg CO₂) |
| $\alpha_{c1}$  | 0.259            |
| $\alpha_{c2}$  | 0.338            |
| $\alpha_{c3}$  | 0.186            |
| $\tau_{c1}$    | 172.9 yr         |
| $\tau_{c2}$    | 18.51 yr         |
| $\tau_{c3}$    | 1.186 yr         |

| NOₓ parameters | | |
|----------------|-----------------|
| $A_{CH₄}$      | $-5.16 \times 10^{-13}$ (W/m²)/(kg NOₓ) |
| $A_{O₃L}$      | $-1.21 \times 10^{-13}$ (W/m²)/(kg NOₓ) |
| $\tau_n$       | 12.0 yr         |
| $f_{CH₄}$      | 1.18            |
| $f_{O₃}$       | 1.37            |
| $(RF_{ref}/E_{ref})_{O₃L}$ | $1.01 \times 10^{-11}$ (W/m²)/(kg NOₓ) |

| H₂O, SO₄, soot, and AIC parameters | | |
|-----------------------------------|-----------------|
| $f_{H₂O}$                         | 1.14            |
| $f_{SO₄}$                         | 0.9             |
| $f_{soot}$                        | 0.7             |
| $f_{AIC}$                         | 0.59            |
| $(RF_{ref}/E_{ref})_{H₂O}$        | $7.43 \times 10^{-15}$ (W/m²)/(kg H₂O) |
| $(RF_{ref}/E_{ref})_{SO₄}$        | $-1.00 \times 10^{-10}$ (W/m²)/(kg SO₄) |
| $(RF_{ref}/E_{ref})_{soot}$       | $5.00 \times 10^{-10}$ (W/m²)/(kg soot) |
| $(RF_{ref}/L_{ref})_{AIC}$        | $2.21 \times 10^{-12}$ (W/m²)/nm |

| Temperature change model parameters | | |
|-----------------------------------|-----------------|
| $RF_{2xCO₂}$                      | 3.70 W/m²       |
| $S$                                | 3.0 K           |
| $\alpha_t$                        | 0.595           |
| $\tau_{t1}$                       | 8.4 yr          |
| $\tau_{t2}$                       | 409.5 yr        |
Chapter 8

Aircraft

The aircraft consists of a set of components, including rotors, wings, tails, fuselage, and propulsion. For each component, attributes such as performance, drag, and weight can be calculated. The aircraft attributes are obtained from the sum of the component attributes. Description and analysis of conventional rotorcraft configurations is facilitated, while retaining the capability to model novel and advanced concepts. Specific rotorcraft configurations considered include: single-main-rotor and tail-rotor helicopter, tandem helicopter, coaxial helicopter, tiltrotor, compound helicopter, multicopter, and airplane.

The following components form the aircraft:

a) Systems: The systems component contains weight information (fixed useful load, vibration, contingency, and systems and equipment) for the aircraft.

b) Fuselage: There is one fuselage for the aircraft.

c) Landing Gear: There is one landing gear for the aircraft.

d) Rotors: The aircraft can have one or more rotors, or no rotors. In addition to main-rotors, the component can model tail-rotors, propellers, proprotors, and ducted fans.

e) Wings: The aircraft can have one or more wings, or no wings.

f) Tails: The aircraft can have one or more horizontal or vertical tail surfaces, or no tails.

g) Fuel Tanks: There are one or more fuel tank systems for the aircraft. Fuel tank systems are associated with the engine groups, jet groups, and charge groups. Fuel quantity is measured as either weight or energy. There can be one or more sizes of auxiliary fuel tanks.

h) Propulsion Groups: The aircraft can have one or more propulsion groups, or none. Each propulsion group is a set of components (rotors) and engine groups, connected by a drive system. The components define the power required. The engine groups define the power available.

i) Engine Groups: An engine group consists of one or more engines of a specific type. An engine group transfers power by shaft torque, so it is associated with a propulsion group. For each engine type an engine model is defined. The engine model describes a particular engine, used in one or more engine groups.

j) Jet Groups: The aircraft can have one or more jet groups, or none. A jet group produces a force on the aircraft. A jet model describes a particular jet, used in one or more jet groups.

k) Charge Groups: The aircraft can have one or more charge groups, or none. A charge group generates energy for the aircraft. A charge model describes a particular charger, used in one or more charge groups.
8–1 Loading

The aircraft disk loading is the ratio of the design gross weight and a reference rotor area: $DL = \frac{W_D}{A_{ref}}$. The reference area is a sum of specified fractions of the rotor areas, $A_{ref} = \sum f_A A$ (typically the projected area of the lifting rotors). The disk loading of a rotor is the ratio of a specified fraction of the design gross weight and the rotor area:

$$(DL)_{rotor} = \frac{T}{A} = \frac{f_W W_D}{A} = \frac{f_W W_D}{A_{ref} A_{ref}}$$

where probably $\sum f_A = 1$, and the lifting rotors are all rotors not designated antitorque or auxiliary-thrust. If there are $N$ lifting rotors of the same area, with no overlap, then $f_A = 1$, $A_{ref} = N A$, $f_W = A_{ref} = 1/N$, and $(DL)_{rotor} = DL$. For rotors designated antitorque or auxiliary-thrust, the disk loading is calculated from the design rotor thrust: $(DL)_{rotor} = \frac{T_{design}}{A}$. For coaxial rotors, the default reference area is the area of one rotor: $f_A = \frac{1}{2}$, $A_{ref} = A$, $f_W = \frac{1}{2}$, and $(DL)_{rotor} = \frac{1}{2} DL$. For tandem rotors, the default reference area is the projected area: $A_{ref} = (2 - m) A$, where $mA$ is the overlap area ($m = 0$ for no overlap, $m = 1$ for coaxial). Then $f_A = \frac{2 - m}{2}$, $f_W = \frac{1}{2}$, and $(DL)_{rotor} = \frac{2 - m}{2} DL$. Optionally, the reference area for tandem rotors can be total rotor area instead: $A_{ref} = 2 A$.

The aircraft wing loading is the ratio of the design gross weight and a reference wing area: $WL = \frac{W_D}{S_{ref}}$. The reference area is a sum of the wing areas, $S_{ref} = \sum S$. The wing loading of an individual wing is the ratio of a specified fraction of the design gross weight and the wing area:

$$(WL)_{wing} = \frac{W}{S} = \frac{f_W W_D}{S} = \frac{f_W}{S_{ref}} \frac{W_D}{S_{ref}}$$

where probably $\sum f_W = 1$. If there are $N$ wings of the same area, then $f_W = S/S_{ref} = 1/N$, and $(WL)_{wing} = WL$.

The aircraft power loading is the ratio of the design gross weight and the total installed takeoff power: $W/P = \frac{W_D}{\sum N_{eng} P_{eng}}$, where the sum is over all engine groups.

8–2 Controls

A set of aircraft controls $c_{AC}$ are defined, and these aircraft controls are connected to the component controls. The connection to the component control $c$ is typically of the form $c = ST c_{AC} + c_0$, where $T$ is an input matrix and $c_0$ the component control for zero aircraft control. The connection (matrix $T$) is defined for a specified number of control system states (allowing change of control configuration with flight state). The factor $S$ is available for internal scaling of the matrix. The control state and initial control values are specified for each flight state. Figure 8-1 illustrates the control relationships.

Typical (default) aircraft controls are the pilot’s controls: collective stick, lateral and longitudinal cyclic sticks, pedal, and tilt. Units and sign convention of the pilot’s controls are contained in the matrix $T$. For the single-main-rotor and tail-rotor configuration, it is often convenient for the collective and cyclic stick motion to equal the collective and cyclic pitch input of the main-rotor, and the pedal motion to equal the collective pitch input of the tail-rotor. The aircraft controls should be scaled to approximately the same amplitude, by appropriate definition of the matrix $T$ and scale factor $S$.

These aircraft controls are available for trim of the aircraft. Any aircraft controls not selected for trim will remain fixed at the values specified for the flight state. Thus by defining additional aircraft controls, component controls can be specified as required for a flight state.
Each aircraft control variable $c_{AC}$ can be zero, constant, or a function of flight speed (piecewise linear input). The flight state input can override this value of the aircraft control. The input value is an initial value if the control is a trim variable.

Each component control variable $c_0$ (value for zero aircraft control) can be zero, constant, or a function of flight speed (piecewise linear input). Optionally the use of $c_0$ can be suppressed for a flight state. The component control from aircraft control ($Tc_{AC}$) is a fixed value, or a function of speed, or a linear function of another control (perhaps a trim variable).
The tilt control variable $\alpha_{\text{tilt}}$ is intended for nacelle tilt angle or conversion control, particularly for tiltrotors. The convention is $\alpha_{\text{tilt}} = 0$ for cruise, and $\alpha_{\text{tilt}} = 90$ deg for helicopter mode. If $\alpha_{\text{tilt}}$ exists as a control, it can be zero, constant, or a function of flight speed (piecewise linear input).

An optional control conversion schedule is defined in terms of conversion speeds: hover and helicopter mode for speeds below $V_{\text{Chover}}$, cruise mode for speeds above $V_{\text{Cr Cruise}}$, and conversion mode between. The nacelle angle is $\alpha_{\text{tilt}} = 90$ in helicopter mode, $\alpha_{\text{tilt}} = 0$ in airplane mode, and it varies linearly with speed in conversion mode. The tip speed is $V_{\text{tip-hover}}$ in helicopter and conversion mode, and $V_{\text{tip-cruise}}$ in airplane mode. Control states and drive system states are defined for helicopter, cruise, and conversion mode flight. The flight state specifies the nacelle tilt angle, tip speeds, control state, and drive system state, including the option to obtain any or all of these quantities from the conversion schedule.

The flight speed used for control scheduling is usually the calibrated airspeed (CAS), hence variation with dynamic pressure. Velocity schedules are used for conversion, controls and motion, rotor tip speed, landing gear retraction, and trim targets. Optionally these velocity schedules use either calibrated airspeed $V_{\text{cal}}$ or the true airspeed $V$.

The control matrices $T$ can be defined based on the configuration. Let $c_{\text{AC0}}, c_{\text{ACc}}, c_{\text{ACs}}, c_{\text{ACP}}$ be the pilot’s controls (collective, lateral cyclic, longitudinal cyclic, and pedal). For the helicopter, the first rotor is the main-rotor and the second rotor is the tail-rotor; then

$$
\begin{bmatrix}
T_{M\text{coll}} \\
T_{M\text{lat}} \\
T_{M\text{lng}} \\
T_{T\text{coll}}
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & -r & 0 & 0 \\
0 & 0 & -1 & 0 \\
0 & 0 & 0 & -r
\end{bmatrix}
\begin{bmatrix}
c_{\text{AC0}} \\
c_{\text{ACc}} \\
c_{\text{ACs}} \\
c_{\text{ACP}}
\end{bmatrix}
$$

where $r$ is the main-rotor direction of rotation ($r = 1$ for counter-clockwise rotation, $r = -1$ for clockwise rotation). For the tandem configuration, the first rotor is the front rotor and the second rotor is the rear rotor; then

$$
\begin{bmatrix}
T_{F\text{coll}} \\
T_{F\text{lat}} \\
T_{R\text{coll}} \\
T_{R\text{lat}}
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & -1 & 0 \\
0 & -r_F & 0 & -r_F \\
1 & 0 & 1 & 0 \\
0 & -r_R & 0 & r_R
\end{bmatrix}
\begin{bmatrix}
c_{\text{AC0}} \\
c_{\text{ACc}} \\
c_{\text{ACs}} \\
c_{\text{ACP}}
\end{bmatrix}
$$

For the coaxial configuration:

$$
\begin{bmatrix}
T_{1\text{coll}} \\
T_{1\text{lat}} \\
T_{1\text{lng}} \\
T_{2\text{coll}} \\
T_{2\text{lat}} \\
T_{2\text{lng}}
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 & r_1 \\
0 & -r_1 & 0 & 0 \\
0 & 0 & -1 & 0 \\
1 & 0 & 0 & r_2 \\
0 & -r_2 & 0 & 0 \\
0 & 0 & -1 & 0
\end{bmatrix}
\begin{bmatrix}
c_{\text{AC0}} \\
c_{\text{ACc}} \\
c_{\text{ACs}} \\
c_{\text{ACP}}
\end{bmatrix}
$$

For the tiltrotor, the first rotor is the right rotor and the second rotor is the left rotor; then

$$
\begin{bmatrix}
T_{R\text{coll}} \\
T_{R\text{lng}} \\
T_{L\text{coll}} \\
T_{L\text{lng}} \\
T_{\text{ail}} \\
T_{\text{elev}} \\
T_{\text{rud}}
\end{bmatrix} =
\begin{bmatrix}
1 & -1 & 0 & 0 \\
0 & 0 & -1 & 1 \\
1 & 1 & 0 & 0 \\
0 & 0 & -1 & -1 \\
0 & -1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
c_{\text{AC0}} \\
c_{\text{ACc}} \\
c_{\text{ACs}} \\
c_{\text{ACP}}
\end{bmatrix}
$$
with cyclic stick and pedal connected to rotor controls only for helicopter mode. For the multicopter configuration:

\[ T_{\text{coll}} = c_{AC0} - c_{ACc} \sin \psi_i + c_{ACc} \cos \psi_i + c_{ACP} r_i \]

where \( \psi_i \) is the angle of the \( i \)-th rotor, measured clockwise from the forward longitudinal axis. Alternative multicopter control strategies use rotor rotational speed instead of collective, or cyclic for yaw control.

The sign conventions for the pilot’s controls are collective stick positive up, lateral cyclic stick positive right, longitudinal cyclic stick positive forward, and pedal positive nose right. The rotor controls are a positive Fourier series, with azimuth measured in the direction of rotation.

### 8–3 Trim

The aircraft trim operation solves for the controls and motion that produce equilibrium in the specified flight state. In steady flight (including hover, level flight, climb and descent, and turns), equilibrium implies zero net force and moment on the aircraft. In general, there can be additional quantities that at equilibrium must equal target values. In practice, the trim solution can deal with a subset of these quantities. Usually it is at least necessary to achieve equilibrium in the aircraft lift and drag forces, as well as in yaw moment for torque balance. The basic purpose of the trim solution is to determine the component states, including aircraft drag and rotor thrust, sufficient to evaluate the aircraft performance.

Different trim solution definitions are required for various flight states. Therefore one or more trim states are defined for the analysis, and the appropriate trim state selected for each flight state of a performance condition or mission segment. For each trim state, the trim quantities, trim variables, and targets are specified. The available trim quantities include:

- aircraft total force and moment; aircraft load factor;
- propulsion group power;
- power margin \( P_{avPG} - P_{reqPG} \); torque margin \( P_{DS\text{limit}} - P_{reqPG} \);
- engine group power; power margin \( P_{avEG} - P_{reqEG} \);
- jet group thrust; thrust margin \( T_{avJG} - T_{reqJG} \);
- charge group power; charge power margin \( P_{avCG} - P_{reqCG} \);
- fuel tank energy flow \( \dot{E}_{\text{batt}} \); battery power margin, \( P_{\text{max}} - |\dot{E}_{\text{batt}}| \);
- rotor force (lift, vertical, or propulsive);
- rotor thrust \( C_T/\sigma \); rotor thrust margin \( (C_T/\sigma)_{\text{max}} - C_T/\sigma \);
- rotor flapping \( \beta_c, \beta_s \); rotor hub moment, roll and pitch; rotor torque;
- wing force; wing lift coefficient \( C_L \); wing lift margin \( C_{L\text{max}} - C_L \);
- tail force.

Targets for aircraft total force and total moment (including inertial loads in turns) are always zero. The available trim variables include:

- aircraft controls;
- aircraft orientation, \( \theta \) (pitch), \( \phi \) (roll);
- aircraft horizontal velocity \( V_h \);
- aircraft vertical rate of climb \( V_c \); aircraft sideslip angle;
- aircraft angular rate, \( \dot{\theta} \) (pullup), \( \dot{\psi} \) (turn);
- propulsion group tip speed or engine speed.
The aircraft orientation variables are the Euler angles of the body axes relative to inertial axes. The aircraft controls (appropriately scaled) are connected to the component controls.

A Newton–Raphson method is used for trim. The derivative matrix is obtained by numerical perturbation. A tolerance $\epsilon$ and a perturbation $\Delta$ are specified.

### 8–4 Geometry

The aircraft coordinate system has the $x$-axis forward, $y$-axis to the right, and $z$-axis down, measured from the center of gravity (fig. 8-2). These aircraft axes are body axes ($x$ is not aligned with the wind), the orientation determined by the convention used for the input geometry. The center of gravity is the appropriate origin for describing the motion of the aircraft, and summing the forces and moments acting on the aircraft.

Layout of the geometry is typically in terms of station line (SL, positive aft), buttline (BL, positive right), and waterline (WL, positive up), measured relative to some arbitrary origin (fig. 8-2). The $x$-$y$-$z$ axes are parallel to the SL-BL-WL directions. One or more locations are defined for each component of the aircraft. Each component will at least have a location that is the point where component forces and moments act on the aircraft. Each location is input in fixed or scaled form. The fixed form input is SL/BL/WL (dimensional). The scaled form input is $x/L$ (positive aft), $y/L$ (positive right), and $z/L$ (positive up), based on a reference length $L$, from a reference point. The reference length is the rotor radius or wing span of a designated component, or the fuselage length. The reference point can optionally be input, or the location (hub) of a designated rotor, or the location (center of action) of a designated wing component, or the location (center of action) of the fuselage, or the location of the center of gravity. Fixed input can be used for the entire aircraft, or just for certain components.

From this fixed or scaled input and the current aircraft size, the actual geometry $(x, y, z)$ can be...
calculated for each location. There are also options to calculate geometry from other parameters (such as tiltrotor span from rotor radius and clearance). This calculated geometry has the sign convention of the aircraft axes \((x\) positive forward, \(y\) positive right, \(z\) positive down), but has the origin at the reference point (which may or may not be the center of gravity). All input uses the same sign convention; all internal calculations use the same sign conventions. Table 8-1 summarizes the conventions.

The location of the aircraft center of gravity is specified for a baseline configuration. With tilting rotors, this location is in helicopter mode. For each flight state the aircraft center of gravity is calculated, from the baseline location plus any shift due to nacelle tilt, plus an input center-of-gravity increment. Alternatively, the aircraft center-of-gravity location for the flight state can be input. Any change of the center-of-gravity position with fuel burn during a mission is not automatically calculated, but could be accounted for using the flight state input.

The aircraft operating length and width are calculated from the component positions and dimensions:
\[
\ell_{\text{total}} = x_{\text{max}} - x_{\text{min}} \quad \text{and} \quad w_{\text{total}} = y_{\text{max}} - y_{\text{min}},
\]
where the maximum and minimum dimensions are for the fuselage and all rotors, wings, and tails. The corresponding footprint area is then \(S_{\text{total}} = \ell_{\text{total}}w_{\text{total}}\).

Table 8-1. Geometry conventions.

<table>
<thead>
<tr>
<th>layout</th>
<th>scaled input</th>
<th>calculated</th>
<th>motion and loads</th>
</tr>
</thead>
<tbody>
<tr>
<td>origin</td>
<td>arbitrary</td>
<td>reference point</td>
<td>reference point</td>
</tr>
<tr>
<td>(x)</td>
<td>SL (+ aft)</td>
<td>(x/L) (+ aft)</td>
<td>(x) (+ forward)</td>
</tr>
<tr>
<td>(y)</td>
<td>BL (+ right)</td>
<td>(y/L) (+ right)</td>
<td>(y) (+ right)</td>
</tr>
<tr>
<td>(z)</td>
<td>WL (+ up)</td>
<td>(z/L) (+ up)</td>
<td>(z) (+ down)</td>
</tr>
</tbody>
</table>

8–5 Aircraft Motion

The aircraft velocity and orientation are defined by the following parameters: flight speed \(V\); turn rate; orientation of the body frame relative to inertial axes (Euler angles); and orientation of the velocity frame relative to inertial axes (flight path angles). Aircraft conventions are followed for the direction and orientation of axes: the \(z\)-axis is down, the \(x\)-axis forward, and the \(y\)-axis to the right; and a yaw-pitch-roll sequence is used for the Euler angles. However, the airframe axes are body axes (fixed to the airframe, regardless of the flight direction) rather than wind axes (which have the \(x\)-axis in the direction of the flight speed). The orientation of the body frame \(F\) relative to inertial axes \(I\) is defined by yaw, pitch, and roll Euler angles, which are rotations about the \(z\), \(y\), and \(x\) axes, respectively:

\[
C^F_I = X_{\phi_F}Y_{\theta_F}Z_{\psi_F}
\]

So yaw is positive to the right, pitch is positive nose up, and roll is positive to the right. The flight path is specified by the velocity \(V\), in the positive \(x\)-axis direction of the velocity axes. The orientation of the velocity axes \(V\) relative to inertial axes \(I\) is defined by yaw (sideslip) and pitch (climb) angles:

\[
C^V_I = Y_{\theta_V}Z_{\psi_V}Z_{\phi_V}
\]

Sideslip is positive for the aircraft moving to the right, and climb is positive for the aircraft moving up. Then

\[
C^{FV} = C^F_IC^IV = X_{\phi_F}Y_{\theta_F}Z_{-\psi_V}Y_{-\theta_V}
\]
In straight flight, all these angles and matrices are constant. In turning flight at a constant yaw rate, the 
yaw angle is $\psi_F = \psi_{\text{fly}}$; the turn radius is $R_T = V_h/\psi_F$; and the nominal bank angle and load factor are 
$\tan \phi_F = \sqrt{n^2 - 1} = \psi_F V_h/g$. Then the forward, sideward, and climb velocities are:

$$
V_f = V \cos \theta_V \cos \psi_V = V_h \cos \psi_V \\
V_s = V \cos \theta_V \sin \psi_V = V_h \sin \psi_V \\
V_c = V \sin \theta_V = V_h \tan \theta_V
$$

where $V_h = V \cos \theta_V$ is the horizontal velocity component. The velocity components in airframe axes are 
$v_{AC}^F = v_{F1/F} = C_{FV}^F \{V \ 0 \ 0 \}^T$ (aircraft velocity relative to the air). The calibrated airspeed is calculated 
from the true airspeed $V$:

$$
V_{\text{cal}} = V \sqrt{\delta \left( 1 + 0.2 M^2 \right)^{7/2} - 1} + 1^{2/7} - 1 \approx V \sqrt{\delta} \left[ 1 + \frac{1}{8} (1 - \delta) M^2 + \frac{3}{640} (1 - 10 \delta + 9 \delta^2) M^4 \right]
$$

where $\sigma = p/p_0$ is the density ratio, $\delta = p/p_0$ is the pressure ratio, and $M$ is the Mach number. The 
aircraft angular velocity is

$$
\omega_{AC}^F = \omega_{F1/F}^F = R \begin{bmatrix} \phi_F \\ \theta_F \\ \psi_F \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\sin \theta_F \\ 0 & \cos \phi_F & \sin \phi_F \cos \theta_F \\ 0 & -\sin \phi_F & \cos \phi_F \cos \theta_F \end{bmatrix} \begin{bmatrix} \dot{\phi}_F \\ \dot{\theta}_F \\ \dot{\psi}_F \end{bmatrix}
$$

For steady state flight, $\dot{\phi}_F = \dot{\theta}_F = \dot{\psi}_F = 0$; $\dot{\psi}_F$ is nonzero in a turn.

Accelerated flight is also considered, in terms of linear acceleration $a_{AC}^F = a_{F1/F}^F$ and pitch 
rate $\dot{\theta}_F$. The nominal pullup load factor is $n = 1 + \dot{\theta}_F V_h/g$. For accelerated flight, the instantaneous 
equilibrium of the forces and moments on the aircraft is evaluated, for specified acceleration and angular 
velocity; the equations of motion are not integrated to define a maneuver. Note that the fuselage and 
wing aerodynamic models do not include all roll and yaw moment terms needed for general unsteady 
flight (notably derivatives $L_u, L_p, L_r, N_u, N_p, N_r$).

The aircraft pitch and roll angles are available for trim of the aircraft. Any motion not selected for 
trim will remain fixed at the values specified for the flight state. The pitch and roll angles each can be 
zero, constant, or a function of flight speed (piecewise linear input). The flight state input can override 
this value of the aircraft motion. The input value is an initial value if the motion is a trim variable.

### 8–6 Loads and Performance

For each component, the power required and the net forces and moments acting on the aircraft can 
be calculated. The aerodynamic forces $F$ and moments $M$ are typically calculated in wind axes and then 
resolved into body axes $(x, y, z)$, relative to the origin of the body axes (the aircraft center of gravity). The 
power and loads of all components are summed to obtain the aircraft power and loads. Typically 
the trim solution drives the net forces and moments on the aircraft to zero.

The aircraft equations of motion, in body axes $F$ with origin at the aircraft center of gravity, are the 
equations of force and moment equilibrium:

$$
m \left( \ddot{v}_{F1/F} + \dot{\omega}_{F1/F} \dot{v}_{F1/F} \right) = \dot{F}_F + \dot{F}_{\text{grav}} \\
I_F \dot{\omega}_{F1/F} + \dot{\omega}_{F1/F} I_F \omega_{F1/F} = \dot{M}_F
$$
where \( m = W/g \) is the aircraft mass; the gravitational force is \( F_{grav}^F = mC^{FI}g^I = mC^{FI}(00g)^T \); and the moment of inertia matrix is

\[
I^F = \begin{bmatrix}
I_{xx} & -I_{xy} & -I_{xz} \\
-I_{yx} & I_{yy} & -I_{yz} \\
-I_{zx} & -I_{zy} & I_{zz}
\end{bmatrix}
\]

For steady flight, \( \dot{\omega}^{FI/F} = \dot{\omega}^{F1/F} = 0 \), and \( \omega^{FI/F} = R(00\dot{\psi})^T \) is nonzero only in turns. For accelerated flight, \( \dot{\psi}^{FI/F} \) can be nonzero, and \( \omega^{FI/F} = R(0\dot{\theta} F \dot{\psi})^T \). The equations of motion are thus:

\[
m(a_{AC}^F + \omega_{AC}^F v_{AC}^F) = F^F + F_{grav}
\]

\[
\omega_{AC}^F I^F \omega_{AC}^F F = M^F
\]

The body axis load factor is \( n = (C^{FI}g^I - (a_{AC}^F + \omega_{AC}^F v_{AC}^F))/g \). The \( a_{AC}^F \) term is absent for steady flight. The forces and moments are the sum of loads from all components of the aircraft:

\[
F^F = F_{fus}^F + \sum F_{rotor}^F + \sum F_{wing}^F + \sum F_{tail}^F + F_{tank}^F + \sum F_{engine}^F + \sum F_{jet}^F + \sum F_{charge}^F
\]

\[
M^F = M_{fus}^F + \sum M_{rotor}^F + \sum M_{wing}^F + \sum M_{tail}^F + M_{tank}^F + \sum M_{engine}^F + \sum M_{jet}^F + \sum M_{charge}^F
\]

Forces and moments in inertial axes are also of interest (\( F^I = C^{IF} F^F \) and \( M^I = C^{IF} M^F \)). A particular component can have more than one source of loads; for example, the rotor component produces hub forces and moments, but also includes hub and pylons drag. The equations of motion are \( E_f = F^F + F_{grav}^F - F_{inertial}^F = 0 \) and \( E_m = M^F - M_{inertial}^F = 0 \).

The component power required \( P_{comp} \) is evaluated for all components (rotors, motors, and compressors) of the propulsion group. The total power required for the propulsion group \( P_{reqPG} \) is obtained by adding the transmission losses and accessory power. The power required for the propulsion group must be distributed to the engine groups. The fuel flow is calculated from the engine power, jet thrust, charger power, and equipment power required. The total fuel flow is the sum from all components of the aircraft: \( \dot{w} = \sum \dot{w}_{reqEG} + \sum \dot{w}_{reqIG} + \sum \dot{w}_{reqCG} + \dot{w}_{eq} \).

### 8–7 Aerodynamics

Each component has a position \( z^F \) in aircraft axes F, relative to the reference point; and orientation of component axes B relative to aircraft axes given by the rotation matrix \( C^{BF} \). It is expected that the component axes are (roughly) \( x \) forward and \( z \) down (or in negative lift direction). The aerodynamic model must be consistent with the convention for component orientation. Acting at the component are interference velocities \( v_{int}^F \) (velocity of air, in F axes), from all other components. Then the total component velocity relative to the air is

\[
v^F = v_{AC}^F + \omega_{AC}^F \Delta z^F - \sum v_{int}^F
\]

where \( \Delta z^F = z^F - z_{cg}^F \). Then \( v^B = C^{BF} v^F \) is the velocity in component axes. The aerodynamic environment is defined in the component axes: velocity magnitude \( v = |v^B| \), dynamic pressure \( q = \frac{1}{2} \rho v^2 \), angle of attack \( \alpha \), and sideslip angle \( \beta \). The angle of attack and sideslip angle provide the transformation between airframe axes and velocity axes:

\[
C^{BA} = Y_\alpha Z_\beta
\]

This is the conventional aircraft definition, corresponding to yaw-then-pitch of the airframe axes relative to the velocity vector. By definition, the velocity is along the \( x \)-axis in the A axes, \( v^B = C^{BA}(v 0 0)^T \).
from which the angle of attack and sideslip in terms of the components of \( v^B \) are obtained:
\[
\alpha = \tan^{-1} \frac{v^B_3}{v^B_1} \\
\beta = \sin^{-1} \frac{v^B_2}{|v^B|}
\]
This definition is not well behaved for \( v^B_1 = 0 \) (it gives \( \alpha = 90 \frac{\text{sign}(v^B_3)}{v^B_1} \)), so for sideward flight a pitch-then-yaw definition can be useful: \( C^{BA} = Z_{-\beta} Y_{\alpha} \). Then
\[
\alpha = \sin^{-1} \frac{v^B_3}{|v^B|} \\
\beta = \tan^{-1} \frac{v^B_2}{v^B_1}
\]
which gives \( \beta = 90 \frac{\text{sign}(v^B_2)}{v^B_1} \) for \( v^B_1 = 0 \).

The component aerodynamic model may include coefficient values for sideward flight, but not have equations for a continuous variation of the coefficients with sideslip angle. For such cases, sideward flight is defined as \( |\beta| = 80 \) to 100 degrees.

From \( v, q, \alpha, \) and \( \beta \), the aerodynamic model calculates the component force and moment, in wind axes acting at \( z^F \):
\[
F^A = \begin{pmatrix} -D \\ Y \\ -L \end{pmatrix} \\
M^A = \begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix}
\]
where \( D, Y, \) and \( L \) are the drag, side force, and lift; \( M_x, M_y, \) and \( M_z \) are the roll, pitch, and yaw moments.

The aerodynamic loads in aircraft axes acting at the center of gravity are then:
\[
F^F = C^{FB} C^{BA} F^A \\
M^F = C^{FB} C^{BA} M^A + \Delta z^F F^F
\]
where \( \Delta z^F = z^F - z_{cg} \). In hover and low speed, the download is calculated: \( F^I_z = k^T (C^{IF} F^F) \), the downward component of the aerodynamic force in inertial axes. Download can be expressed as a fraction of the total rotor vertical force or as a fraction of gross weight. The aerodynamic model also calculates the interference velocities caused by this component at all other components: \( \dot{v}^F_{\text{int}} = C^{FB} v^B_{\text{int}} \).

Equations for the aerodynamics models are defined for all angles in radians. Input values of angles are, however, in degrees.

The aircraft neutral point is calculated from the airframe aerodynamics with all controls set to zero. The neutral point is here defined as the longitudinal position about which the derivative of the pitch moment with lift is zero. Hence \( SL_{na} = SL_{cg} - \Delta M/\Delta L, \) with the change in lift and moment calculated from the loads at angles of attack of 0 and 5 deg.

**8–8 Compressible Aerodynamics**

The aerodynamic coefficients of lifting surfaces (wing and empennage) can be corrected for compressibility effects. Based on Prandtl-Glauert, the incompressible lift-curve slope is multiplied by the factor
\[
F_c = \begin{cases} 
1 & M < M_{\text{div}} \\
\frac{1 - M}{(1 - M_{\text{div}})\sqrt{1 - M_{\text{div}}^2}} & M_{\text{div}} < M < M_{\text{div}} + .1 \\
\frac{1 - M}{(1 - M_{\text{div}})\sqrt{1 - M_{\text{div}}^2}} + \frac{M - M_{\text{div}} - .1}{1 - M_{\text{div}} - .1} & M_{\text{div}} + .1 < M < 1 \\
1 & M > 1 \end{cases}
\]
Typically the lift-divergence Mach number $M_{\text{div}} \approx 0.75$, about 0.1 less than the drag-divergence Mach number. A compressible drag increment is modeled following ref. 1 (fig. 6):

$$C_{Dc} = C_{Dcc} \left( 6.5x^2 + 28x^4 + 2870x^8 \right)$$

where $x = (M/M_{cc} - 0.7)$ (with $M/M_{cc}$ restricted to the range 0.7 to 1.1) and $C_{Dcc} \approx 0.0011$ is the drag increment at $M = M_{cc}$. The critical Mach number depends on the lift coefficient:

$$M_{cc} = M_{cc0} - M_{cc1} \left( 0.03C_L + 0.19C_L^2 + 0.78C_L^3 \right)$$

with typically $M_{cc0} \approx 0.74$, $M_{cc1} \approx 0.31$, and $C_L$ restricted to the range 0 to 1.

### 8–9 Trailing-Edge Flaps

The lifting surfaces have controls in the form of trailing-edge flaps: flap, flaperon, and aileron for wings; elevator or rudder for tails. The aerodynamic loads generated by flap deflection $\delta_f$ (radians) are estimated based on two-dimensional aerodynamic data (as summarized in refs. 2 and 3). Let $\ell_f = c_f/c$ be the ratio of the flap chord to the wing chord. The lift coefficient is

$$c_{\ell f} = \frac{c_{\ell f} \alpha + \tau \eta \delta_f}{\pi} \approx \left( \sin \left( \frac{\pi}{2} \ell_f \right) \right)^n$$

with $\theta_f = \cos^{-1}(2\ell_f - 1)$ (ref. 2, eq. 3.56 and fig. 3.35; ref. 3, eq. 5.40). The last expression is an approximation that is a good fit to the thin airfoil theory result for $n = 1/2$, and a good approximation including the effects of real flow for $n = 2/3$ (ref. 3, fig. 5.18); the last expression with $n = 2/3$ is used here. The increase of maximum lift coefficient caused by flap deflection is less than the increase in lift coefficient, so the stall angle of attack is decreased. Approximately

$$\frac{\Delta c_{\ell f}}{\Delta c_{\ell f} \max} \approx (1 - \ell_f) (1 + \ell_f - 5\ell_f^2 + 3\ell_f^3)$$

(ref. 2, fig. 3.37). Thin airfoil theory gives the moment coefficient increment about the quarter chord:

$$\Delta c_m = -0.85 \left( (1 - \ell_f) \sin \theta_f \right) \delta_f = -0.85 \left( (1 - \ell_f) \sqrt{(1 - \ell_f) \ell_f} \right) \delta_f$$

(ref. 2, eq. 3.57; ref. 3, eq. 5.41); with the factor of 0.85 accounting for real flow effects (ref. 3, fig. 5.19). The drag increment is estimated using

$$\Delta C_D = 0.9 \ell_f^{3.8} \frac{S_f}{S} \sin^2 \delta_f$$

for slotted flaps (ref. 2, eq. 3.51). In summary, the section load increments are:

$$\Delta c_{\ell f} = \frac{c_{\ell f} \alpha c_f}{c} L_f \eta_f \delta_f$$

$$\Delta c_{\ell f \max} = X_f \Delta c_{\ell f}$$

$$\Delta c_m = \frac{c_f}{c} M_f \delta_f$$

The decrease in angle of attack for maximum lift is

$$\Delta \alpha_{\text{max}} = -\frac{\Delta c_{\ell f} - \Delta c_{\ell f \max}}{c_{\ell f} \alpha} = -(1 - X_f) \frac{\Delta c_{\ell f}}{c_{\ell f} \alpha}$$
The coefficients

\[ \eta_f = 0.85 - 0.43 \delta_f = \eta_0 - \eta_1 \delta_f \]
\[ L_f = \frac{1}{\ell_f} \left( \sin \left( \frac{\pi}{2} \ell_f \right) \right)^{2/3} \]
\[ X_f = (1 - \ell_f) \left( 1 + \ell_f - 5 \ell_f^2 + 3 \ell_f^3 \right) \]
\[ M_f = -0.85 \frac{1}{\ell_f} \left( (1 - \ell_f) 2 \sqrt{(1 - \ell_f) \ell_f} \right) \]
\[ D_f = 0.9 \ell_f^{1.38} \]

follow from these equations.

For three-dimensional aerodynamic loads, these two-dimensional coefficients are corrected by using the three-dimensional lift-curve slope, and multiplying by the ratio of flap span to wing span \( b_f/b \).

Then the wing load increments caused by flap deflection, in terms of coefficients based on the wing area, are:

\[ \Delta C_L = \frac{S_f}{S} C_{L \alpha} L_f \eta_f \delta_f \]
\[ \Delta C_M = \frac{S_f}{S} M_f \delta_f \]
\[ \Delta C_D = \frac{S_f}{S} D_f \sin^2 \delta_f \]

where \( S_f/S \) is the ratio of flap area to wing area.

### 8–10 Drag

Each component can contribute drag to the aircraft. A fixed drag can be specified as a drag area \( D/q \); or the drag can be scaled, specified as a drag coefficient \( C_D \) based on an appropriate area \( S \). There may also be other ways to define a scaled drag value. For fixed drag, the coefficient is \( C_D = (D/q)/S \) (the aerodynamic model is formulated in terms of drag coefficient). For scaled drag, the drag area is \( D/q = S C_D \). For all components, the drag \( (D/q)_{\text{comp}} \) or \( C_{D_{\text{comp}}} \) is defined for forward flight or cruise; typically this is the minimum drag value. For some components, the vertical drag \( (D/q)_{V_{\text{comp}}} \) or sideward drag \( (D/q)_{S_{\text{comp}}} \) or \( C_{D_{V_{\text{comp}}}} \) or \( C_{D_{S_{\text{comp}}}} \) is defined. For some components, the aerodynamic model includes drag due to lift, angle of attack, or stall.

Table 8-2 summarizes the component contributions to drag, and the corresponding reference areas. If no reference area is indicated, then the input is only drag area \( D/q \). An appropriate drag reference area is defined for each component, and either input or calculated. Wetted area is calculated for each component, even if it is not the reference area. The component wetted areas are summed to obtain the aircraft wetted area. Some of the weight models also require the wetted area. The component drag contributions must be consistent. In particular, a rotor with a spinner (such as on a tiltrotor aircraft) would likely not have hub drag. The pylon is the rotor support and the nacelle is the engine support. The drag model for a tiltrotor aircraft with tilting engines would use the pylon drag (and no nacelle drag), since the pylon is connected to the rotor shaft axes; with non-tilting engines it would use the nacelle drag as well.

Optionally the aircraft drag can be fixed. The quantity specified is the sum (over all components) of the drag area \( D/q \) (minimum drag, excluding drag due to lift and angle of attack), without accounting for interference effects on dynamic pressure. The input parameter can be \( D/q \); or the drag can be scaled, specified as a drag coefficient based on the rotor disk area, so \( D/q = A_{\text{ref}} C_D \) (\( A_{\text{ref}} \) is the reference rotor
disk area); or the drag can be estimated based on the gross weight, \( D/q = k(W_{MTO}/1000)^{2/3} \) (\( W_{MTO} \) is the maximum takeoff gross weight; units of \( k \) are \( \text{ft}^2/\text{k-lb}^{2/3} \) or \( \text{m}^2/\text{Mg}^{2/3} \)). Based on historical data, the drag coefficient \( C_D = 0.02 \) for old helicopters, \( C_D = 0.008 \) for current low-drag helicopters. Based on historical data, \( k = 9 \) for old helicopters, \( k = 2.5 \) for current low-drag helicopters, \( k = 1.6 \) for current tiltrotors, and \( k = 1.4 \) for turboprop aircraft (English units). If the aircraft drag is input, then the fuselage contingency drag is adjusted so the total aircraft \( D/q \) equals the input value.

<table>
<thead>
<tr>
<th>component</th>
<th>drag contribution</th>
<th>reference area</th>
</tr>
</thead>
<tbody>
<tr>
<td>fuselage</td>
<td>fuselage</td>
<td>fuselage wetted area</td>
</tr>
<tr>
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<td>fuselage vertical</td>
<td>fuselage projected area</td>
</tr>
<tr>
<td></td>
<td>fittings and fixtures</td>
<td>fuselage wetted area</td>
</tr>
<tr>
<td></td>
<td>rotor-body interference</td>
<td>fuselage wetted area</td>
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<tr>
<td></td>
<td>contingency (aircraft)</td>
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<td>payload increment (flight state)</td>
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<td>duct, duct vertical</td>
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<td>wing-body interference</td>
<td>wing planform area</td>
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</tr>
<tr>
<td></td>
<td>momentum drag</td>
<td></td>
</tr>
</tbody>
</table>

Optionally the aircraft vertical drag (download fraction) can be fixed. The quantity specified is the sum over all components of the vertical drag area \( (D/q)_V \). The input parameter can be \( (D/q)_V \), or \( k = (D/q)_V A_{ref} \) (\( A_{ref} \) is reference rotor disk area). Approximating the dynamic pressure in the wake as \( q = \frac{1}{2} \rho (2v_w)^2 = T/A_{ref} \), the download fraction is \( DL/T = q(D/q)_V/T = k \). If the aircraft vertical drag is input, then the fuselage contingency vertical drag is adjusted so the total aircraft \( (D/q)_V \) equals the input value.

The nominal drag areas of the components and the aircraft are part of the aircraft description and are used when the aircraft drag is fixed. The nominal drag area is calculated for low-speed helicopter flight, for high-speed cruise flight, and for vertical flight. An incidence angle \( i \) is specified for the rotors, wings, and nacelles, to be used solely to calculate the nominal helicopter and vertical drag areas. The convention is that \( i = 0 \) if the component does not tilt. Table 8-3 summarizes the contributions to the nominal drag areas, with \( D \) for the drag in normal flow and \( D_V \) for the drag in vertical flow.
While vertical drag parameters are part of the aerodynamic model for the hub, duct, pylon, and nacelle, aerodynamic interference at the rotor and at the propulsion components is not considered, so these terms do not contribute to download. In the context of download, only the fuselage, wing, tail, and contingency contribute to the nominal vertical drag.

From the input and the current aircraft size, the drag areas $D/q$ and coefficients $C_D$ are calculated. The aerodynamic analysis is usually in terms of coefficients. If the aircraft drag is fixed for the aircraft model, then the fuselage contingency drag is set:

$$(D/q)_{cont} = (D/q)_{fixed} - \sum (D/q)_{comp}$$

and similarly for fixed vertical drag. Note that this adjustment ignores changes caused by interference in the dynamic pressure and the velocity direction, which will affect the actual component drag.

The component aerodynamic model calculates the drag, typically from a drag coefficient $C_D$, a reference area, and the air velocity of the component. The drag force is then $D = q_{comp} S_{ref} C_D$, where the dynamic pressure $q_{comp}$ includes interference. From the aerodynamic forces and moments in wind axes, the total force and moment in body axes ($F_F$ and $M_F$) are calculated. For reference, the aircraft total drag and total drag area are

$$D_{AC} = \sum c_d^T F_{aero}^F$$
$$\frac{(D/q)_{AC}}{q} = D_{AC}/q$$

where the aircraft velocity (without interference) gives the direction $c_d = -v_{AC}^F/|v_{AC}^F|$ and dynamic pressure $q = \frac{1}{2}\rho |v_{AC}^F|^2$; and $F_{aero}^F$ is the component aerodynamic force. An overall skin friction drag coefficient is then $C_{D,AC} = (D/q)_{ACwet}/S_{AC}$, based on the aircraft wetted area $S_{AC} = \sum S_{wet}$ and excluding drag terms not associated with skin friction (specifically landing gear, rotor hub, and contingency drag).

Table 8-3. Component contributions to nominal drag area.

<table>
<thead>
<tr>
<th>component</th>
<th>drag contribution</th>
<th>cruise</th>
<th>helicopter</th>
<th>vertical</th>
</tr>
</thead>
<tbody>
<tr>
<td>fuselage</td>
<td>fuselage</td>
<td>$D$</td>
<td>$D$</td>
<td>$D_V$</td>
</tr>
<tr>
<td></td>
<td>fittings and fixtures</td>
<td>$D$</td>
<td>$D$</td>
<td>$D$</td>
</tr>
<tr>
<td></td>
<td>rotor-body int</td>
<td>$D$</td>
<td>$D$</td>
<td>$D$</td>
</tr>
<tr>
<td>landing gear</td>
<td>landing gear</td>
<td>$D$</td>
<td>$D$</td>
<td>$0$</td>
</tr>
<tr>
<td></td>
<td>retractable</td>
<td>$0$</td>
<td>$D$</td>
<td>$0$</td>
</tr>
<tr>
<td>rotor</td>
<td>hub</td>
<td>$D$</td>
<td>$D cos^2 i + D_V sin^2 i$</td>
<td>$0$</td>
</tr>
<tr>
<td></td>
<td>duct</td>
<td>$D$</td>
<td>$D cos^2 i + D_V sin^2 i$</td>
<td>$0$</td>
</tr>
<tr>
<td></td>
<td>pylon</td>
<td>$D$</td>
<td>$D cos^2 i + D_V sin^2 i$</td>
<td>$0$</td>
</tr>
<tr>
<td></td>
<td>spinner</td>
<td>$D$</td>
<td>$D$</td>
<td>$0$</td>
</tr>
<tr>
<td>wing</td>
<td>wing</td>
<td>$D$</td>
<td>$D cos^2 i + D_V sin^2 i$</td>
<td>$D sin^2 i + D_V cos^2 i$</td>
</tr>
<tr>
<td></td>
<td>wing-body int</td>
<td>$D$</td>
<td>$D cos^2 i + D_V sin^2 i$</td>
<td>$D$</td>
</tr>
<tr>
<td>tail</td>
<td>horizontal tail</td>
<td>$D$</td>
<td>$D$</td>
<td>$D_V cos^2 \phi$</td>
</tr>
<tr>
<td></td>
<td>vertical tail</td>
<td>$D$</td>
<td>$D$</td>
<td>$D_V sin^2 \phi$</td>
</tr>
<tr>
<td>engine</td>
<td>nacelle</td>
<td>$D$</td>
<td>$D cos^2 i + D_V sin^2 i$</td>
<td>$0$</td>
</tr>
<tr>
<td>jet</td>
<td>nacelle</td>
<td>$D$</td>
<td>$D cos^2 i + D_V sin^2 i$</td>
<td>$0$</td>
</tr>
<tr>
<td>charger</td>
<td>nacelle</td>
<td>$D$</td>
<td>$D cos^2 i + D_V sin^2 i$</td>
<td>$0$</td>
</tr>
<tr>
<td>contingency</td>
<td></td>
<td>$D$</td>
<td>$D$</td>
<td>$D_V$</td>
</tr>
</tbody>
</table>
8–11 Performance Metrics

The following performance metrics are calculated for the aircraft. The aircraft hover figure of merit is \( M = W \sqrt{\frac{W}{2\rho A_{ref}}} / P \). The aircraft effective drag is \( D_e = P/V \), hence the effective lift-to-drag ratio is \( L/D_e = WV/P \). For these metrics, the aircraft power is the sum of the engine group power, jet group propulsive power, and charge group power: \( P = P_{req} + VT_{jet} + P_{chrg} \). The aircraft power loading is \( W/P \) (lb/hp or kg/kW). Isolated rotor performance metrics are described in Chapter 12.

8–12 Weights

The design gross weight \( W_D \) is a principal parameter defining the aircraft, either input or determined by the sizing task from designated design conditions and missions. Typically \( W_D \) is the takeoff weight for the primary design mission. The design gross weight is used by the analysis to calculate the rotor disk loading and blade loading, wing loading, power loading, and thrust loading; to obtain aircraft moments of inertia from the radii of gyration; for tolerance and perturbation scales of the solution procedures; optionally to define structural design gross weight and maximum takeoff weight; and optionally to specify the gross weight for missions and flight conditions.

The aircraft weight statement defines the empty weight, fixed useful load, and operating weight for the design configuration. The aircraft weight statement is the sum of the weight statements for all the aircraft components, the component weight determined by input or by parametric calculations with technology factors. The definitions of the weight terms are as follows:

- **gross weight** \( W_G = W_E + W_{UL} = W_O + W_{pay} + W_{fuel} \)
- **operating weight** \( W_O = W_E + W_{FUL} \)
- **useful load** \( W_{UL} = W_{FUL} + W_{pay} + W_{fuel} \)

where \( W_E \) is the weight empty; \( W_{FUL} \) the fixed useful load; \( W_{pay} \) the payload weight; and \( W_{fuel} \) the usable fuel weight. Aircraft weight definitions are given in SAWE RP7D (ref. 4), including:

- Payload is any item which is being transported and is directly related to the purpose of the flight as opposed to items that are necessary for the flight operation. Payload can include, but is not limited to, passengers, cargo, passenger baggage, ammo, internal and external stores, and fuel which is to be delivered to another aircraft or site. Payload may or may not be expended in flight.

- Operating weight is the sum of aircraft weight empty and operating items. Operating weight is equivalent to takeoff gross weight less usable fuel, payload, and any item to be expended in flight.

- Weight empty is an engineering term which is defined as the weight of the complete aircraft as defined in the aircraft specifications, dry, clean, and empty except for fluids in closed systems such as a hydraulic system.

The weight empty consists of structure, propulsion group, systems and equipment, vibration, and contingency weights. If the weight empty is input, then the contingency weight is adjusted so \( W_E \) equals the required value. If the design gross weight is input, then the payload or fuel weight must be fallout.
The structural design gross weight $W_{SD}$ and maximum takeoff weight $W_{MTO}$ can be input, or specified as an increment $d$ plus a fraction $f$ of a weight $W$:

$$W_{SD} = d_{SDGW} + f_{SDGW}W = \left\{ \begin{array}{l} d_{SDGW} + f_{SDGW}W_D \\ d_{SDGW} + f_{SDGW}(W_D - W_{fuel} + f_{fuel}W_{fuel-cap}) \\ d_{SDGW} + f_{SDGW}W_{MTO} \end{array} \right.$$  

$$W_{MTO} = d_{W_MTO} + f_{W_MTO}W = \left\{ \begin{array}{l} d_{W_MTO} + f_{W_MTO}W_D \\ d_{W_MTO} + f_{W_MTO}(W_D - W_{fuel} + W_{fuel-cap}) \\ d_{W_MTO} + f_{W_MTO}W_{SD} \end{array} \right.$$  

This convention allows the weights to be input directly ($f = 0$), or scaled with a design weight. For $W_{SD}, W$ is the design gross weight $W_D$, or $W_D$ adjusted for a specified fuel state (input fraction of fuel capacity), or the maximum takeoff weight $W_{MTO}$. Alternatively, $W_{SD}$ can be calculated as the gross weight at a designated sizing flight condition. The structural design gross weight is used in the weight estimation. For $W_{MTO}, W$ is the design gross weight $W_D$, or $W_D$ adjusted for maximum fuel capacity, or the structural design gross weight $W_{SD}$. Alternatively, $W_{MTO}$ can be calculated as the maximum gross weight possible at a designated sizing flight condition. The maximum takeoff weight is used in the cost model, in the scaled aircraft and hub drag, and in the weight estimation.

The design ultimate load factor $n_{ult}$ at the structural design gross weight $W_{SD}$ is specified, in particular for use in the component weight estimates. The structural design gross weight $W_{SD}$ and design ultimate load factor $n_{ult}$ are used for the fuselage, rotor, and wing weight estimations. The maximum takeoff weight $W_{MTO}$ is used for the cost and drag (scaled aircraft and hub), and for the weights (system, fuselage, landing gear, and engine group).

The gross weight $W_G$ is specified for each flight condition and mission, perhaps in terms of the design gross weight $W_D$. For each flight state, the fixed useful load may be different from the design configuration because of changes in auxiliary fuel tank weight, or kit weights, or increments in crew or equipment weights. Thus the fixed useful load weight is calculated for the flight state; and from it the useful load weight and operating weight are calculated. The gross weight, payload weight, and usable fuel weight (in standard and auxiliary tanks) complete the weight information for the flight state.

For each weight group, fixed (input) weights can be specified; or weight increments $dW$ added to the results of the parametric weight model. The parametric weight model includes technology factors $\chi$. Thus typically a component or element weight is obtained from $W = \chi W_{model} + dW$. Weight of individual elements in a group can be fixed by using $dW$ and setting the corresponding technology factor $\chi = 0$. With $\chi \neq 0$, the increment $dW$ can account for something not included in the parametric model.

For scaled weights of all components, the AFDD weight models are implemented. The user can incorporate custom weight models as well.

The operating weight is composed of scaled and fixed weights, so the design gross weight can be written $W_D = W_O + W_{pay} + W_{fuel} = W_{O\text{fixed}} + W_{O\text{scaled}} + W_{pay} + W_{fuel}$. The growth factor is the change in gross weight due to a change in payload:

$$\frac{\partial W_D}{\partial W_{pay}} = 1 + \frac{\partial W_{O\text{scaled}}}{\partial W_{pay}} + \frac{\partial W_{fuel}}{\partial W_{pay}} = 1 + \left( \frac{\partial W_{O\text{scaled}}}{\partial W_D} + \frac{\partial W_{fuel}}{\partial W_D} \right) \frac{\partial W_D}{\partial W_{pay}} \cong 1 + \left( \frac{W_{O\text{scaled}}}{W_D} + \frac{W_{fuel}}{W_D} \right) \frac{\partial W_D}{\partial W_{pay}} = \frac{1}{1 - \phi_{O\text{scaled}} - \phi_{fuel}}$$

in terms of the weight fractions $\phi = W/W_D$. 


The aircraft operating weight can be divided into core vehicle weight and military load. The core vehicle weight is the weight in minimum airworthy state, with the aircraft capable of normal flight throughout the envelope, but not mission capable. Military load is the sum of fixed useful load and military features in weight empty. Thus

\[
\text{weight empty} = \text{core vehicle weight} + \text{military features} \\
\text{military load} = \text{fixed useful load} + \text{military features in weight empty} \\
\text{operating weight} = W_E + W_{FUL} = \text{core vehicle weight} + \text{military load}
\]

In terms of the weight breakdown used here, military features in weight empty consist of folding weight (wing, rotor, tail, fuselage terms), crashworthiness weight (fuselage, landing gear terms), marinization weight (fuselage), rotor brake (drive system), avionics group (mission equipment), armament group, furnishings and equipment group, anti-icing group (including electrical group term), and load and handling group.

8–13 Weight Statement

Aircraft weight information is stored in a data structure that follows SAWE RP8A Group Weight Statement format (ref. 5), as outlined in figure 8-3. The asterisks designate extensions of RP8A for the purposes of this analysis. Typically only the lowest elements of the hierarchy are specified; higher elements are obtained by summation. Fixed (input) weight elements are identified in the data structure. A weight statement data structure exists for each component. The aircraft weight statement is the sum of the structures from all components.

8–14 References


5) “Weight and Balance Data Reporting Forms for Aircraft (including Rotorcraft), Revision A.” Society of Allied Weight Engineers, Recommended Practice Number 8, June 1997.
WEIGHT EMPTY
STRUCTURE
wing group
  basic structure
  secondary structure
    fairings (*), fittings (*), fold/tilt (*)
  control surfaces
rotor group
  blade assembly
  hub & hinge
    basic (*), fairing/spinner (*), blade fold (*), shaft (*)
  rotor support structure (*), duct (*)
empennage group
  horizontal tail (*)
    basic (*), fold (*)
  vertical tail (*)
    basic (*), fold (*)
  tail rotor (*)
    blades, hub & hinge, rotor supports, rotor/fan duct
fuselage group
  basic (*)
  wing & rotor fold/retraction (*)
  tail fold/tilt (*)
  marinization (*)
  pressurization (*)
  crashworthiness (*)
alighting gear group
  basic (*), retraction (*), crashworthiness (*)
engine section or nacelle group
  engine support (*), engine cowling (*), pylon support (*)
air induction group
PROPULSION GROUP
  engine system
    engine
    exhaust system
    accessories (*)
propeller/fan installation
  blades (*), hub & hinge (*), rotor supports (*), rotor/fan duct (*)
fuel system
  tanks and support
  plumbing
drive system
  gear boxes
  transmission drive
  rotor shaft
  rotor brake (*)
  clutch (*)
  gas drive

Figure 8-3a. Weight statement (* indicates extension of RP8A).
SYSTES AND EQUIPMENT
  flight controls group
    cockpit controls
    automatic flight control system
  system controls
    fixed wing systems
      non-boosted (*), boost mechanisms (*)
    rotary wing systems
      non-boosted (*), boost mechanisms (*), boosted (*)
  conversion systems
    non-boosted (*), boost mechanisms (*)
 auxiliary power group
 instruments group
 hydraulic group
  fixed wing (*), rotary wing (*), conversion (*)
  equipment (*)
 pneumatic group
 electrical group
  aircraft (*), anti-icing (*)
 avionics group (mission equipment)
 armament group
  armament provisions (*), armor (*)
 furnishings & equipment group
 environmental control group
 anti-icing group
 load & handling group
 VIBRATION (*)
 CONTINGENCY

FIXED USEFUL LOAD
 crew
 fluids (oil, unusable fuel) (*)
 auxiliary fuel tanks
 other fixed useful load (*)
 equipment increment (*)
 folding kit (*)
 wing extension kit (*)
 wing kit (*)
 other kit (*)

PAYLOAD
 USABLE FUEL
  standard tanks (*)
  auxiliary tanks (*)

OPERATING WEIGHT = weight empty + fixed useful load
USEFUL LOAD = fixed useful load + payload + usable fuel

GROSS WEIGHT = weight empty + useful load
GROSS WEIGHT = operating weight + payload + usable fuel

Figure 8-3b. Weight statement (* indicates extension of RP8A).
The systems component contains weight information (fixed useful load, vibration, contingency, and systems and equipment).

9–1 Weights

The weight empty consists of structure, propulsion group, systems and equipment, vibration, and contingency weights. The vibration control weight can be input, or specified as a fraction of weight empty:

\[ W_{vib} = f_{vib} W_E. \]

The contingency weight can be input, or specified as a fraction of weight empty:

\[ W_{cont} = f_{cont} W_E. \]

However, if the weight empty is input, then the contingency weight is adjusted so \( W_E \) equals the required value. The weights of all components are evaluated and summed, producing the aircraft weight empty less vibration and contingency weight, \( W_X \). Then:

a) Fixed weight empty: \( W_{vib} \) input or \( W_{vib} = f_{vib} W_E, W_{cont} = W_E - (W_X + W_{vib}). \)

b) Both fractional: \( W_E = W_X/(1 - f_{vib} - f_{cont}), W_{vib} = f_{vib} W_E, W_{cont} = f_{cont} W_E. \)

c) Only vibration weight fractional: \( W_{cont} \) input, \( W_E = (W_X + W_{cont})/(1 - f_{vib}), W_{vib} = f_{vib} W_E. \)

d) Only contingency weight fractional: \( W_{vib} \) input, \( W_E = (W_X + W_{vib})/(1 - f_{cont}), W_{cont} = f_{cont} W_E. \)

e) Both input: \( W_{vib} \) and \( W_{cont} \) input, \( W_E = W_X + W_{vib} + W_{cont}. \)

Finally, the operating weight \( W_O = W_E + W_{FUL} \) is recalculated.

Systems and equipment includes the following fixed (input) weights: auxiliary power group, instruments group, pneumatic group, electrical group (aircraft), avionics group (mission equipment), armament group (armor and armament provisions), furnishings and equipment group, environmental control group, and load and handling group. Systems and equipment includes the following scaled weights: flight controls group, hydraulic group, electrical group (anti-icing), and anti-icing group.

Flight controls group includes the following fixed (input) weights: cockpit controls and automatic flight control system. Flight controls group includes the following scaled weights: fixed wing systems, rotary wing systems, and conversion or thrust vectoring systems. Rotary wing flight control weights can be calculated for the entire aircraft (using rotor parameters such as chord and tip speed for a designated rotor), an approach that is consistent with parametric weight equations developed for conventional two-rotor configurations. Alternatively, rotary wing flight control weights can be calculated separately for each rotor and then summed. The fixed wing flight controls and the conversion controls can be absent.

The fixed useful load \( W_{FUL} \) consists of crew (\( W_{crew} \)), trapped fluids (oil and unusable fuel, \( W_{trap} \)), auxiliary fuel tanks (\( W_{auxtank} \)), equipment increment, kits (folding, wing, wing extension, other), and other fixed useful load (\( W_{FULOther} \)). \( W_{crew}, W_{trap} \), and \( W_{FULOther} \) are input. For each flight state, the
fixed useful load may be different from the design configuration because of changes in auxiliary fuel
tank weight, kit weight, and crew or equipment weight increments.

Folding weights can be calculated in several weight groups, including wing, rotor, empennage, and
fuselage. These weights are the total weights for folding and the impact of folding on the group. A
fraction \( f_{\text{foldkit}} \) of these weights can be in a kit, hence optionally removable. Thus, of the total folding
weight, the fraction \( f_{\text{foldkit}} \) is a kit weight in the fixed useful load of the weight statement, while the
remainder is kept in the component group weight.

### 9–2 Detailed Weight Definition

Figure 9-1 presents a more detailed description of the weights for the systems and equipment, and
the useful load. The input information for a group can be just the total weight (\( \Delta W \) in the following
equations), or can include all the terms.

The electrical group consists of power supply, power conversion, power distribution and controls,
lights and signal devices, and equipment supports; plus a term associated with the anti-icing group. The
total weight is

\[
W_{\text{electrical}} = W_{\text{elect aircraft}} + W_{\text{DI elec}}
\]

\[
W_{\text{elect aircraft}} = W_{\text{supply}} + W_{\text{conv}} + W_{\text{distrib}} + W_{\text{lights}} + W_{\text{support}} + \Delta W_{\text{electrical}}
\]

The avionics group consists of equipment and installation; here the equipment weights include installa-
tion. The equipment items are communications, navigation, identification, control and display, aircraft
survivability, and mission system equipment; plus armament electronics. The total weight is

\[
W_{\text{MEQ}} = W_{\text{com}} + W_{\text{nav}} + W_{\text{ident}} + W_{\text{display}} + W_{\text{survive}} + W_{\text{mission}} + \Delta W_{\text{MEQ}} + W_{\text{arm elect}}
\]

The armament group consists of armament provisions (gun provisions, turret systems, expendable
weapons provisions) and armor. Armament electronics weight \( W_{\text{arm elect}} \) (such as targeting, sights,
radar) is part of the avionics group.

\[
W_{\text{arm prov}} = W_{\text{gun}} + W_{\text{turret}} + W_{\text{expend}} + \Delta W_{\text{arm prov}}
\]

\[
W_{\text{armor}} = U_{\text{armor floor}} S_{\text{cabin floor}} + U_{\text{armor wall}} S_{\text{cabin wall}} + U_{\text{armor crew}} N_{\text{crew seat}} + \Delta W_{\text{armor}}
\]

Here \( U_{\text{armor floor}} \) and \( U_{\text{armor wall}} \) are the armor weights per surface area; \( S_{\text{cabin floor}} \) and \( S_{\text{cabin wall}} \) are the
the cabin floor and wall areas; \( U_{\text{armor crew}} \) is the armor weight per crew; and \( N_{\text{crew seat}} \) is number of crew
seats.

The furnishings and equipment group consists of accommodation for personnel, miscellaneous
equipment, furnishings, and emergency equipment. Accommodation for personnel consists of seats,
miscellaneous accommodation (including galleys, toilets), and the oxygen system. Miscellaneous
equipment includes cockpit displays. Furnishings includes floor covering, trim, partitions, crash padding,
and acoustic and thermal insulation; but excludes vibration absorbers. Emergency equipment consists of fire
detection and extinguishing, and other emergency equipment (including first aid, survival kit, and life
raft). The total weight is

\[
W_{\text{furnish}} = W_{\text{accom}} + W_{\text{misc}} + W_{\text{furn}} + W_{\text{emerg}} + \Delta W_{\text{furnish}}
\]

\[
W_{\text{accom}} = (U_{\text{seat crew}} + U_{\text{accom crew}} + U_{\text{ox crew}}) N_{\text{crew seat}} + (U_{\text{seat pass}} + U_{\text{accom pass}} + U_{\text{ox pass}}) N_{\text{pass seat}}
\]

\[
W_{\text{furn}} = W_{\text{trim}} + U_{\text{insulation}} S_{\text{cabin}}
\]

\[
W_{\text{emerg}} = W_{\text{fire}} + W_{\text{other}}
\]
Here $U_{zz\text{ crew}}$, $U_{zz\text{ pass}}$ are the weights per crew and passenger; $N_{\text{crew seat}}$, $N_{\text{pass seat}}$ are the number of crew and passenger seats; $U_{\text{insulation}}$ is the acoustic and thermal insulation weight per area; and $S_{\text{cabin}}$ is the total cabin surface area. The load and handling group consists of aircraft handling and load handling (cargo handling, hoist, external load provisions). The total weight is

\[ W_{\text{load}} = W_{\text{aircraft}} + W_{\text{cargo}} + \Delta W_{\text{load}} \]

\[ W_{\text{cargo}} = U_{\text{handling}} S_{\text{cabin floor}} + W_{\text{hoist}} + W_{\text{ext prov}} \]

Here $U_{\text{handling}}$ is the cargo handling weight per cabin floor area, and $S_{\text{cabin floor}}$ is the cabin floor area.

The crew weight is

\[ W_{\text{crew}} = U_{\text{crew}} N_{\text{crew}} + \Delta W_{\text{crew}} \]

Here $U_{\text{crew}}$ is the weight per crew, and $N_{\text{crew}}$ is the number of crew. A crew weight increment for a flight condition or mission is given by $\delta N_{\text{crew}}$ and $\delta W_{\text{crew}}$. Other fixed useful load consists of various categories, such as baggage, gun installations, weapons provisions, aircraft survivability equipment (chaff, flares), survival kits, life rafts, and oxygen.

An equipment increment can be defined for a flight condition or mission, in terms of $\delta N_{\text{crew seat}}$, $\delta N_{\text{pass seat}}$, and $\delta W_{\text{equip}}$:

\[ W_{\text{equip inc}} = U_{\text{crew seat inc}} \delta N_{\text{crew seat}} + U_{\text{pass seat inc}} \delta N_{\text{pass seat}} + \delta W_{\text{equip}} \]

The default weights per crew and passenger seats are $U_{\text{crew seat inc}} = U_{\text{seat crew}} + U_{\text{accom crew}} + U_{\text{ox crew}} + U_{\text{armor crew}}$ and $U_{\text{pass seat inc}} = U_{\text{seat pass}} + U_{\text{accom pass}} + U_{\text{ox pass}}$.

The payload consists of passengers or troops, cargo (internal and external), ammunition, and weapons:

\[ W_{\text{payload}} = U_{\text{pass}} N_{\text{pass}} + W_{\text{cargo}} + W_{\text{ext load}} + W_{\text{ammo}} + W_{\text{weapons}} + \Delta W_{\text{payload}} \]

Here $U_{\text{pass}}$ is the weight per passenger, and $N_{\text{pass}}$ is the number of passengers. A payload increment is defined by $\Delta W_{\text{payload}}$. For fallout payload, the value of $\Delta W_{\text{payload}}$ is adjusted.
WEIGHT EMPTY
SYSTEMS AND EQUIPMENT
  electrical group
    aircraft
      power supply
      power conversion
      power distribution and controls
      lights and signal devices
      equipment supports
      anti-icing
  avionics group (mission equipment)
    equipment
    installation
  armament group
    armament provisions
    armor
  furnishings & equipment group
    accommodation for personnel
      seats
      miscellaneous accommodation
      oxygen system
    miscellaneous equipment
    furnishings
    emergency equipment
      fire detection and extinguishing
      other emergency equipment
  load & handling group
    aircraft handling
    load handling

USEFUL LOAD
FIXED USEFUL LOAD
  crew
  other fixed useful load
    various categories
    equipment increment

PAYLOAD
  passengers/troops
  cargo
  ammunition
  weapons

Figure 9-1. Details of weight descriptions (based on RP8A).
Chapter 10

Fuselage

There is one fuselage component for the aircraft.

10–1 Geometry

The fuselage length $\ell_{\text{fus}}$ can be input or calculated. The calculated length depends on the longitudinal positions of all components. Let $x_{\text{max}}$ and $x_{\text{min}}$ be the maximum (forward) and minimum (aft) position of all rotors, wings, and tails. Then the calculated fuselage length is

$$\ell_{\text{fus}} = \ell_{\text{nose}} + (x_{\text{max}} - x_{\text{min}}) + \ell_{\text{aft}}$$

The nose length $\ell_{\text{nose}}$ (distance forward of hub) and aft length $\ell_{\text{aft}}$ (distance aft of hub) are input, or calculated as $\ell_{\text{nose}} = f_{\text{nose}} L$ and $\ell_{\text{aft}} = f_{\text{aft}} L$. Typically $f_{\text{aft}} = 0$ or negative for the main-rotor and tail-rotor configuration, and $f_{\text{aft}} = 0.75$ for the coaxial configuration. The fuselage width $w_{\text{fus}}$ is input. The reference length $L$ is the rotor radius or wing span of a designated component, or the input fuselage length.

The fuselage wetted area $S_{\text{wet}}$ (reference area for drag coefficients) and projected area $S_{\text{proj}}$ (reference area for vertical drag) are input (excluding or including the tail boom terms); or calculated from the nose length:

$$S_{\text{wet}} = f_{\text{wet}} (2\ell_{\text{nose}} h_{\text{fus}} + 2\ell_{\text{nose}} w_{\text{fus}} + 2h_{\text{fus}} w_{\text{fus}}) + C_{\text{boom}} L$$
$$S_{\text{proj}} = f_{\text{proj}} (\ell_{\text{nose}} w_{\text{fus}}) + w_{\text{boom}} L$$

using input fuselage height $h_{\text{fus}}$, and factors $f_{\text{wet}}$ and $f_{\text{proj}}$; or calculated from the fuselage length:

$$S_{\text{wet}} = f_{\text{wet}} (2\ell_{\text{fus}} h_{\text{fus}} + 2\ell_{\text{fus}} w_{\text{fus}} + 2h_{\text{fus}} w_{\text{fus}}) + C_{\text{boom}} L$$
$$S_{\text{proj}} = f_{\text{proj}} (\ell_{\text{fus}} w_{\text{fus}}) + w_{\text{boom}} L$$

Using the nose length and the tail boom area is probably best for a single-main-rotor and tail-rotor helicopter. Here $C_{\text{boom}}$ is the effective tail boom circumference (boom wetted area divided by reference length), and $w_{\text{boom}}$ is the effective tail boom width (boom vertical area divided by reference length).

Cabin areas are required for weight estimates: total cabin surface area $S_{\text{cabin}}$ for acoustic and thermal insulation weight (furnishings and equipment group); cabin floor area $S_{\text{cabin floor}}$ for armor and cargo handling weights; and cabin wall area $S_{\text{cabin wall}}$ for armor weight. These areas are input, or calculated from the fuselage dimensions:

$$S_{\text{cabin}} = f_{\text{cabin}} (2\ell_{\text{fus}} h_{\text{fus}} + 2\ell_{\text{fus}} w_{\text{fus}})$$
$$S_{\text{cabin floor}} = f_{\text{floor}} (\ell_{\text{fus}} w_{\text{fus}})$$
$$S_{\text{cabin wall}} = f_{\text{wall}} (2\ell_{\text{fus}} h_{\text{fus}})$$

with typically $f \cong 0.6$. 

The fuselage contribution to the aircraft operating length is \( x_{\text{fus}} + f_{\text{ref}} \ell_{\text{fus}} \) (forward) and \( x_{\text{fus}} - (1 - f_{\text{ref}}) \ell_{\text{fus}} \) (aft). Here \( f_{\text{ref}} \) is the position of the fuselage aerodynamic reference location aft of the nose, as a fraction of the fuselage length. If the fuselage length is input, then \( f_{\text{ref}} \) is input; if the fuselage length is calculated, then \( f_{\text{ref}} = (x_{\text{max}} + \ell_{\text{nose}} - x_{\text{fus}})/\ell_{\text{fus}} \).

### 10–2 Control and Loads

The fuselage has a position \( z^F \), where the aerodynamic forces act; and the component axes are aligned with the aircraft axes, \( C_{BF} = I \). The fuselage has no control variables.

### 10–3 Aerodynamics

The aerodynamic velocity of the fuselage relative to the air, including interference, is calculated in component axes, \( v^B \). The angle of attack \( \alpha_{\text{fus}} \), sideslip angle \( \beta_{\text{fus}} \) (hence \( C_{BA} \)), and dynamic pressure \( q \) are calculated from \( v^B \). The reference area for the fuselage forward flight drag is the fuselage wetted area \( S_{\text{wet}} \), which is input or calculated as described previously. The reference area for the fuselage vertical drag is the fuselage projected area \( S_{\text{proj}} \), which is input or calculated as described previously.

#### 10-3.1 Drag

The drag area or drag coefficient is defined for forward flight, vertical flight, and sideward flight. In addition, the forward flight drag area or drag coefficient is defined for fixtures and fittings, and for rotor-body interference. The effective angle of attack is \( \alpha_e = \alpha_{\text{fus}} - \alpha_{\text{Dmin}} \), where \( \alpha_{\text{Dmin}} \) is the angle of minimum drag; in reverse flow (\( |\alpha_e| > 90 \)), \( \alpha_e \leftarrow \alpha_e - 180 \, \text{sign} \alpha_e \). For angles of attack less than a transition angle \( \alpha_t \), the drag coefficient equals the forward flight (minimum) drag \( C_{D0} \), plus an angle of attack term. Thus if \( |\alpha_e| \leq \alpha_t \)

\[
C_D = C_{D0} (1 + K_d |\alpha_e| X_d)
\]

and otherwise

\[
C_{Dt} = C_{D0} (1 + K_d |\alpha_t| X_d)
\]

\[
C_D = C_{Dt} + \left( \frac{S_{\text{proj}}}{S_{\text{wet}}} C_{DV} - C_{Dt} \right) \sin \left( \frac{\pi}{2} \frac{|\alpha_e| - \alpha_t}{\pi/2 - \alpha_t} \right)
\]

and similarly for the transition of payload drag \( (D/q)_{\text{pay}} \) and contingency drag \( (D/q)_{\text{cont}} \). Optionally there might be no angle-of-attack variation at low angles \( (K_d = 0) \), or quadratic variation \( (X_d = 2) \). With an input transition angle, there will be a jump in the slope of the drag coefficient at \( \alpha_t \). For a smooth transition, the transition angle that matches slopes as well as coefficients is found by solving

\[
\left( \frac{2X_d}{\pi} - 1 \right) \alpha_t X_d - X_d \alpha_t^{-1} + \left( \frac{S_{\text{proj}}/S_{\text{wet}}}{C_{DV} - C_{D0}} \right) K_d C_{D0} = 0
\]

This calculation of the transition angle is only implemented with quadratic variation, for which

\[
\alpha_t = \frac{1}{a} \left( 1 + \sqrt{1 - a \left( \frac{S_{\text{proj}}/S_{\text{wet}}}{C_{DV} - C_{D0}} \right)} \right)
\]

with \( a = (4/\pi) - 1 \); \( \alpha_t \) is however required to be between 15 and 45 deg. For sideward flight \( (v^B_x = 0) \) the drag is obtained using \( \phi_v = \tan^{-1}(-v^B_z/v^B_y) \) to interpolate between sideward and vertical coefficients:

\[
C_D = C_{DS} \cos^2 \phi_v + \frac{S_{\text{proj}}}{S_{\text{wet}}} C_{DV} \sin^2 \phi_v
\]
Then the drag force is

\[ D = qS_{\text{wet}} (C_D + C_{D_{\text{fit}}} + \sum C_{D_{\text{rb}}}) + q\left( (D/q)_{\text{pay}} + (D/q)_{\text{cont}} \right) \]

including drag coefficient for fixtures and fittings \( C_{D_{\text{fit}}} \) and rotor-body interference \( C_{D_{\text{rb}}} \) (summed over all rotors); drag area of the payload (specified for flight state); and contingency drag area.

### 10-3.2 Lift and Pitch Moment

The fuselage lift and pitch moment are defined in fixed form \((L/q, M/q)\), or scaled form \((C_L, C_M)\), based on the fuselage wetted area and fuselage length). The effective angle of attack is \( \alpha_e = \alpha_{\text{fus}} - \alpha_{zl} \), where \( \alpha_{zl} \) is the angle of zero lift; in reverse flow \((|\alpha_e| > 90)\), \( \alpha_e \leftarrow \alpha_e - 180 \text{ sign}\alpha_e \). Let \( \alpha_{\text{max}} \) be the angle-of-attack increment (above or below zero lift angle) for maximum lift. If \(|\alpha_e| \leq \alpha_{\text{max}}\)

\[ C_L = C_{L0}\alpha_e \]
\[ C_M = C_{M0} + C_{M\alpha}\alpha_e \]

and otherwise

\[ C_L = C_{L0}\alpha_{\text{max}}\text{sign}\alpha_e \left( \frac{\pi/2 - |\alpha_e|}{\pi/2 - |\alpha_{\text{max}}|} \right) \]
\[ C_M = (C_{M0} + C_{M\alpha}\alpha_{\text{max}}\text{sign}\alpha_e) \left( \frac{\pi/2 - |\alpha_e|}{\pi/2 - |\alpha_{\text{max}}|} \right) \]

for zero lift and moment at 90 deg angle of attack. In sideward flight, these coefficients are zero. Then \( L = qS_{\text{wet}}C_L \) and \( M = qS_{\text{wet}}\ell_{\text{fus}}C_M \) are the lift and pitch moment.

### 10-3.3 Side Force and Yaw Moment

The fuselage side force and yaw moment are defined in fixed form \((Y/q, N/q)\), or scaled form \((C_Y, C_N)\), based on the fuselage wetted area and fuselage length). The effective sideslip angle is \( \beta_e = \beta_{\text{fus}} - \beta_{zy} \), where \( \beta_{zy} \) is the angle of zero side force; in reverse flow \((|\beta_e| > 90)\), \( \beta_e \leftarrow \beta_e - 180 \text{ sign}\beta_e \). Let \( \beta_{\text{max}} \) be the sideslip angle increment (above or below zero side force angle) for maximum side force. If \(|\beta_e| \leq \beta_{\text{max}}\)

\[ C_Y = C_{Y\beta}\beta_e \]
\[ C_N = C_{N0} + C_{N\beta}\beta_e \]

and otherwise

\[ C_Y = C_{Y\beta}\beta_{\text{max}}\text{sign}\beta_e \left( \frac{\pi/2 - |\beta_e|}{\pi/2 - |\beta_{\text{max}}|} \right) \]
\[ C_N = (C_{N0} + C_{N\beta}\beta_{\text{max}}\text{sign}\beta_e) \left( \frac{\pi/2 - |\beta_e|}{\pi/2 - |\beta_{\text{max}}|} \right) \]

for zero side force and yaw moment at 90 deg sideslip angle. Then \( Y = qS_{\text{wet}}C_Y \) and \( N = qS_{\text{wet}}\ell_{\text{fus}}C_N \) are the side force and yaw moment. The roll moment is zero.

### 10-4 Weights

The fuselage group consists of the basic structure; wing and rotor fold/retraction; tail fold/tilt; and marinization, pressurization, and crashworthiness structure.
Chapter 11

Landing Gear

There is one landing gear component for the aircraft. The landing gear can be located on the body or on the wing. The landing gear can be fixed or retractable; a gear retraction speed is specified (CAS), or the landing gear state can be specified in the flight state.

11–1 Geometry

The landing gear has a position $z^F$, where the aerodynamic forces act. The component axes are aligned with the aircraft axes, $C^{BF} = I$. The landing gear has no control variables. The height of the bottom of the landing gear above ground level, $h_{LG}$, is specified in the flight state. The landing gear position $z^F$ is a distance $d_{LG}$ above the bottom of the gear.

11-1.1 Drag

The drag area is specified for landing gear extended, $(D/q)_{LG}$. The velocity relative to the air at $z^F$ gives the drag direction $e_d = -v^F/|v^F|$ and dynamic pressure $q = \frac{1}{2}\rho|v^F|^2$ (no interference). Then

$$F^F = e_d q (D/q)_{LG}$$

is the total drag force.

11–2 Weights

The alighting gear group consists of basic structure, retraction, and crashworthiness structure.
The aircraft can have one or more rotors, or no rotors. In addition to main-rotors, the rotor component can model tail-rotors, propellers, proprotors, ducted fans, thrust vectoring rotors, and auxiliary-thrust rotors. The principal configuration designation (main-rotor, tail-rotor, or propeller) is identified for each rotor component, and in particular determines where the weights are put in the weight statement (summarized in table 12-1). Each configuration can possibly have a separate performance or weight model, which is separately specified. Antitorque rotors and auxiliary-thrust rotors can be identified, for special sizing options. Other configuration features are variable diameter and ducted fan, and reaction drive.

Multi-rotor systems (such as coaxial or tandem configuration) are modeled as a set of separate rotors, in order to accommodate the description of the position, orientation, controls, and loads. Optionally the location of the center of the rotor system can be specified and the rotor locations calculated based on input separation parameters. The performance calculation for twin and multiple rotor systems can include the mutual influence of the induced velocity on the power.

The main-rotor size is defined by the radius $R$ or disk loading $W/A$, thrust-weighted solidity $\sigma$, hover tip speed $V_{tip}$, and blade loading $C_W/\sigma = W/\rho A V_{tip}^2/\sigma$. With more than one main-rotor, the disk loading and blade loading are obtained from an input fraction of design gross weight, $W = f_W W_D$. The air density $\rho$ for $C_W/\sigma$ is obtained from a specified takeoff condition. If the rotor radius is fixed for the sizing task, three of ($R$ or $W/A$), $C_W/\sigma$, $V_{tip}$, and $\sigma$ are input, and the other parameters are derived. Optionally the radius can be calculated from a specified ratio to the radius of another rotor. If the sizing task determines the rotor radius ($R$ and $W/A$), then two of $C_W/\sigma$, $V_{tip}$, and $\sigma$ are input, and the other parameter is derived. The radius can be sized for just a subset of the rotors, with fixed radius for the others.

For antitorque and auxiliary-thrust rotors, three of ($R$ or $W/A$), $C_W/\sigma$, $V_{tip}$, and $\sigma$ are input, and the other parameters are derived. Optionally the radius can be calculated from a specified ratio to the radius of another rotor. The disk loading and blade loading are based on $fT$, where $f$ is an input factor and $T$ is the maximum thrust from designated design conditions. Optionally the tail-rotor radius can be scaled with the main-rotor radius: $R = f R_{mr}(0.1348 + 0.0071 W/A)$, where $f$ is an input factor and the units of disk loading $W/A$ are lb/ft$^2$. Figure 12-1 shows the basis for this scaling.

<table>
<thead>
<tr>
<th>configuration</th>
<th>weight statement</th>
<th>weight model</th>
<th>performance model</th>
</tr>
</thead>
<tbody>
<tr>
<td>main-rotor</td>
<td>rotor group</td>
<td>rotor</td>
<td>rotor</td>
</tr>
<tr>
<td>tail-rotor</td>
<td>empennage group</td>
<td>tail-rotor</td>
<td>rotor</td>
</tr>
<tr>
<td>propeller</td>
<td>propulsion group</td>
<td>rotor, aux thrust</td>
<td>rotor</td>
</tr>
</tbody>
</table>
12–1 Drive System

The drive system defines gear ratios for all the components it connects. The gear ratio is the ratio of the component rotational speed to that of the primary rotor. There is one primary rotor per propulsion group (for which the reference tip speed is specified); other components are dependent (for which a gear ratio is specified). There can be more than one drive system state, in order to model a multiple-speed or variable-speed transmission. Each drive system state corresponds to a set of gear ratios.

For the primary rotor, a reference tip speed $V_{\text{tip-ref}}$ is defined for each drive system state. By convention, the “hover tip speed” refers to the reference tip speed for drive state #1. If the sizing task changes the hover tip speed, then the ratios of the reference tip speeds at different engine states are kept constant. By convention, the gear ratio of the primary rotor is $r = 1$. For dependent rotors, either the gear ratio is specified (for each drive system state) or a tip speed is specified and the gear ratio is calculated ($r = \Omega_{\text{dep}}/\Omega_{\text{prim}}$, $\Omega = V_{\text{tip-ref}}/R$). For the engine group, either the gear ratio is specified (for each drive system state) or the gear ratio is calculated from the specification engine turbine speed $\Omega_{\text{spec}} = (2\pi/60)N_{\text{spec}}$ and the reference tip speed of the primary rotor ($r = \Omega_{\text{spec}}/\Omega_{\text{prim}}$, $\Omega_{\text{prim}} = V_{\text{tip-ref}}/R$). The latter option means the specification engine turbine speed $N_{\text{spec}}$ corresponds to $V_{\text{tip-ref}}$ for all drive system states. To determine the gear ratios, the reference tip speed and radius are used, corresponding to hover.
The flight state specifies the tip speed of the primary rotor and the drive system state, for each propulsion group. The drive system state defines the gear ratio for dependent rotors and the engine groups. From the rotor radius, the rotational speed of the primary rotor is obtained \(\Omega_{\text{prim}} = \frac{V_{\text{tip}}}{R}\); from the gear ratios, the rotational speed of dependent rotors \(\Omega_{\text{dep}} = r\Omega_{\text{prim}}\) and the engine groups \(N = \frac{(60/2\pi)}{r_{\text{eng}}\Omega_{\text{prim}}}\) are obtained; and from the rotor radius, the tip speed of the dependent rotor \(V_{\text{tip}} = \Omega_{\text{dep}}R\) is obtained. The flight state specification of the tip speed can be an input value, the reference tip speed, a function of flight speed or a conversion schedule, or one of several default values. These relationships between tip speed and rotational speed use the actual radius of the rotors in the flight state, which for a variable-diameter rotor may not be the same as the reference, hover radius.

A designated drive system state can have a variable speed (variable gear ratio) transmission, by introducing a factor \(f_{\text{gear}}\) on the gear ratio when the speeds of the dependent rotors and engines are evaluated. The factor \(f_{\text{gear}}\) is a component control, which can be connected to an aircraft control and thus set for each flight state.

An optional conversion schedule is defined in terms of two speeds: hover and helicopter mode for speeds below \(V_{\text{C, hover}}\), cruise mode for speeds above \(V_{\text{C, cruise}}\), and conversion mode for speeds between \(V_{\text{C, hover}}\) and \(V_{\text{C, cruise}}\). The tip speed is \(V_{\text{tip, hover}}\) in helicopter and conversion mode, and \(V_{\text{tip, cruise}}\) in airplane mode. Drive system states are defined for helicopter, cruise, and conversion mode flight. The flight state specifies the nacelle tilt angle, tip speeds, control state, and drive system state, including the option to obtain any or all of these quantities from the conversion schedule.

Several default values of the tip speed are defined for use by the flight state, including cruise, maneuver, one-engine inoperative, drive system limit conditions, and a function of flight speed (piecewise linear input). Optionally these default values can be input as a fraction of the hover tip speed. Optionally the tip speed can be calculated from \(\mu = \frac{V}{V_{\text{tip}}}, \text{so } V_{\text{tip}} = \frac{V}{\mu}\); or from \(M_{\text{at}} = M_{\text{tip}}\sqrt{(1 + \mu^2) + \mu^2}, \text{so } V_{\text{tip}} = \sqrt{\left(c_sM_{\text{at}}\right)^2 - V^2} - V\). Optionally the tip speed can be the minimum of the input value or that for \(M_{\text{at}}\).

The sizing task might change the hover tip speed (reference tip speed for drive system state #1), the reference tip speed of a dependent rotor, a rotor radius, or the specification engine turbine speed \(N_{\text{spec}}\). In such cases the gear ratios and other parameters are recalculated. Note that it is not consistent to change the reference tip speed of a dependent rotor if the gear ratio is a fixed input.

An increment on the primary rotor rotational speed (or primary engine group, if there are no rotors) is a control variable of the propulsion group.

12–2 Geometry

The rotor rotation direction is described by the parameter \(r\): \(r = 1\) for counter-clockwise rotation and \(r = -1\) for clockwise rotation (as viewed from the positive thrust side of the rotor).

The rotor solidity and blade mean chord are related by \(\sigma = \frac{Nc}{\pi R}\); usually thrust-weighted values are used, but geometric values are also required by the analysis. The mean chord is the average of the chord over the rotor blade span, from root cutout to tip. The thrust-weighted chord is the average of the chord over the rotor blade span \(r\), from root cutout to tip, weighted by \(r^2\). A general blade chord distribution is specified as \(c(r) = c_{\text{ref}}\tilde{c}(r)\), where \(c_{\text{ref}}\) is the thrust-weighted chord. Linear taper is specified in terms of a taper ratio \(t = c_{\text{tip}}/c_{\text{root}}\), or in terms of the ratio of thrust-weighted and geometric chords, \(f = \sigma_t/\sigma_g = c.75R/c.50R\).
The rotor hub is at position \( z_{hub}^F \). Optionally, a component of the position can be calculated, superseding the location input. The calculated geometry depends on the configuration. For a coaxial rotor, the rotor separation is \( s = |k^T C^{SF} (z_{hub1}^F - z_{hub2}^F)/(2R)| \) (fraction rotor diameter), or the hub locations are calculated from the input separation \( s \), and the input location midway between the hubs:

\[
\begin{align*}
z_{hub}^F &= z_{center}^F \pm C^{FS} \begin{pmatrix} 0 \\ 0 \\ sR \end{pmatrix}
\end{align*}
\]

For a tandem rotor, the rotor longitudinal overlap is \( o = \Delta \ell/(2R) = 1 - \ell/(2R) \) (fraction rotor diameter), or the hub locations are calculated from the input overlap \( o \), and the input location midway between the hubs:

\[
x_{hub} = x_{center} \pm R(1 - o)
\]

For a tail-rotor, the longitudinal position can be calculated from the main-rotor radius \( R \), tail-rotor radius \( R_{tr} \), and tail-rotor/main-rotor clearance \( d_{tr} \):

\[
x_{hub_{tr}} = x_{hub_{mr}} - (R_{mr} + d_{tr} + R_{tr})
\]

For a multicopter, the longitudinal and lateral position can be calculated from the rotor radius \( R \), the scaled arm length \( \ell_i \) of the \( i \)-th rotor, and the angle \( \psi_i \) (measured clockwise from the forward longitudinal axis):

\[
\begin{align*}
x_{hub} &= x_{center} + R\ell_i \cos \psi_i \\
y_{hub} &= y_{center} + R\ell_i \sin \psi_i
\end{align*}
\]

For a tiltrotor, the lateral position can be calculated from the rotor radius \( R \) (cruise value for variable-diameter rotor), fuselage/rotor clearance \( d_{fus} \), and fuselage width \( w_{fus} \):

\[
y_{hub} = \pm (fR + d_{fus} + \frac{1}{2}w_{fus})
\]

with the pivot, pylon, and nacelle center-of-gravity lateral positions adjusted to keep the same relative position to the hub. The calculated clearance between the rotor and fuselage is \( d_{fus} = |y_{hub}| - (R + \frac{1}{2}w_{fus}) \).

Alternatively, for a tiltrotor the lateral position can be calculated from the wing span, \( y_{hub} = \pm b/2 \), so the rotors are at the wing tips, or from a designated wing panel edge, \( y_{hub} = \pm \eta p(b/2) \).

For twin rotors (tandem, side-by-side, or coaxial), the overlap is \( o = \Delta \ell/(2R) = 1 - \ell/(2R) \) (fraction of diameter; 0 for no overlap and 1 for coaxial), where the hub-to-hub separation is \( \ell = [(x_{hub1} - x_{hub2})^2 + (y_{hub1} - y_{hub2})^2]^{1/2} \) \( (\ell = 2R \) for no overlap and \( \ell = 0 \) for coaxial). The overlap area is \( mA \), with \( A \) the area of one rotor disk and

\[
m = \frac{2}{\pi} \left[ \cos^{-1}(\ell/2R) - (\ell/2R)\sqrt{1 - (\ell/2R)^2} \right]
\]

The vertical separation is \( s = |z_{hub1} - z_{hub2}|/(2R) \).

The reference areas for the component drag coefficients are the rotor disk area \( A = \pi R^2 \) (for hub drag), pylon wetted area \( S_{pylon} \), duct wetted area \( 2S_{duct} \), and spinner wetted area \( S_{spin} \). The pylon wetted area is input, or calculated from the drive system (gear box and rotor shaft) weight, or from the drive system plus engine system (engine, exhaust, and accessories) weight:

\[
S_{pylon} = k\left(\frac{w}{N_{rotor}}\right)^{2/3}
\]
where \( w = W_{gbrs} \) or \( w = W_{gbrs} + \sum W_{ES} \) and the units of \( k \) are \( \text{ft}^2/\text{lb}^{2/3} \) or \( \text{m}^2/\text{kg}^{2/3} \). The pylon area is included in the aircraft wetted area if the pylon drag coefficient is nonzero. The duct wetted area is twice the duct area \( S_{duct} \). The duct area is input, or calculated from the rotor circumference and the duct length \( \ell_{duct} = kR \):

\[
S_{duct} = (2\pi R)\ell_{duct}
\]

The spinner wetted area is input, or calculated from the spinner frontal area:

\[
S_{spin} = k(\pi R^2_{spin})
\]

where \( R_{spin} \) is the spinner radius, which is specified as a fraction of the rotor radius.

The rotor contribution to the aircraft operating length and width is calculated from the locus of the rotor disk: \( z_{disk} = z_{hub} + RC_{FS}(\cos \psi \sin \psi)T \). The longitudinal distance from the hub position is \( \Delta x = R(a \cos \psi + b \sin \psi) \), so the maximum distance is \( \Delta x = \pm R\sqrt{a^2 + b^2} \). The lateral distance from the hub position is \( \Delta y = R(c \cos \psi + d \sin \psi) \), so the maximum distance is \( \Delta y = \pm R\sqrt{c^2 + d^2} \).

12–3 Control and Loads

The rotor controls consist of collective, lateral cyclic, longitudinal cyclic, and perhaps shaft incidence (tilt) and cant angles. Rotor cyclic control can be defined in terms of tip-path plane or no-feathering plane command. The collective control variable is the rotor thrust amplitude or the collective pitch angle.

The relationship between tip-path plane tilt and hub moment is

\[
M = N^2 I_b \Omega^2 (\nu^2 - 1) \beta = K_{hub} \beta,
\]

where \( N \) is the number of blades, \( \Omega \) the rotor speed, and \( \nu \) the dimensionless fundamental flap frequency. The flap moment of inertia \( I_b \) is obtained from the Lock number: \( \gamma = \rho acR^4/I_b \), for seal level standard (SLS) density \( \rho \), lift-curve slope \( a = 5.7 \), and thrust-weighted chord (or from the blade weight, or from an autorotation index). The flap frequency and Lock number are specified for hover radius and rotational speed. The flap frequency and hub stiffness are required for the radius and rotational speed of the flight state. For a hingeless rotor, the blade flap spring is \( K_{flap} = I_b \Omega^2 (\nu^2 - 1) \), obtained from the hover quantities; then \( K_{hub} = \frac{N}{2} K_{flap} \) and

\[
\nu^2 = 1 + \frac{K_{flap}}{I_b \Omega^2}
\]

For an articulated rotor, the hinge offset is \( e = Rx/(1 + x) \), \( x = \frac{3}{5} (\nu^2 - 1) \) from the hover quantities; then

\[
\nu^2 = 1 + \frac{3}{2} \frac{e/R}{1 - e/R}
\]

and \( K_{hub} = \frac{N}{2} I_b \Omega^2 (\nu^2 - 1) \), using \( I_b \) from \( \gamma \) (and scaled with \( R \) for a variable diameter rotor) and \( \Omega \) for the flight state.

Optionally the rotor can have a variable diameter. The rotor diameter is treated as a control, allowing it to be connected to an aircraft control and thus set for each flight state. The basic variation can be specified based on the conversion schedule, or input as a function of flight speed (piecewise linear input). For the conversion schedule, the rotor radius is \( R_{\text{hover}} \) for speeds below \( V_{\text{hover}} \), \( R_{\text{cruise}} = f R_{\text{hover}} \) for speeds above \( V_{\text{cruise}} \), and linear with flight speed in conversion mode. During the diameter change, the chord, chord radial distribution, and blade weight are assumed fixed; hence solidity scales as \( \sigma \sim 1/R \), blade flap moment of inertia as \( I_b \sim R^2 \), and Lock number as \( \gamma \sim R^2 \).
12-3.1 Control Variables

The collective control variable is direct command of rotor thrust magnitude $T$ or $C_T/\sigma$ (in shaft axes), from which the collective pitch angle can be calculated; or rotor collective pitch angle $\theta_{0.75}$, from which the thrust and inflow can be calculated.

Shaft tilt control variables are incidence (tilt) and cant angles, acting at a pivot location.

Tip-path plane command is direct control of the tip-path plane tilt, hence tilt of the thrust vector. This control mode requires calculation of rotor cyclic pitch angles from the flapping. The control variables are longitudinal tilt $\beta_c$ (positive forward) and lateral tilt $\beta_s$ (positive toward retreating side). Alternatively, the cyclic control can be specified in terms of hub moment or lift offset, if the blade flap frequency is greater than 1/rev. The relationship between tip-path plane tilt and hub moment is $M = K_{\text{hub}}\beta$, and between moment and lift offset is $M = \rho V_{\text{tip}}^2 A_{\text{blade}}(a/6)$. Thus the flapping is

$$\begin{bmatrix} \beta_s \\ \beta_c \end{bmatrix} = \frac{1}{K_{\text{hub}}} \begin{bmatrix} rM_x \\ -M_y \end{bmatrix} = \frac{TR}{K_{\text{hub}}} \begin{bmatrix} \alpha_x \\ -\alpha_y \end{bmatrix}$$

for hub moment command or lift offset command, respectively.

No-feathering plane command is control of rotor cyclic pitch angles, usually producing tilt of the thrust vector. This control mode requires calculation of rotor tip-path plane tilt from the cyclic control, including the influence of inflow. The control variables are longitudinal cyclic pitch angle $\theta_s$ (positive aft) and lateral cyclic pitch angle $\theta_c$ (positive toward retreating side).

12-3.2 Aircraft Controls

Each control can be connected to the aircraft controls $c_{AC}$: $c = c_0 + STc_{AC}$, with $c_0$ zero, constant, or a function of flight speed (piecewise linear input). The factor $S$ can be introduced to automatically scale the collective matrix: $S = \rho V_{\text{tip}}^2 A_{\text{blade}}(a/6)$ if the collective control variable is rotor thrust $T$; $S = 1$ if the collective control variable is pitch angle $\theta_{0.75}$. For cyclic matrices, $S = 1$ with no-feathering plane command, and $S = -1$ for tip-path plane command.

12-3.3 Rotor Axes and Shaft Tilt

The rotor hub is at position $z_h^F$, where the rotor forces and moments act; the orientation of the rotor shaft axes relative to the aircraft axes is given by the rotation matrix $C_{SF}$. The pivot is at position $z_{\text{pivot}}^F$. The hub or shaft axes $S$ have origin at the hub node; the $z$-axis is the shaft, positive in the positive thrust direction; and the $x$-axis downstream or up. The rotor orientation is specified by selecting a nominal direction in body axes (positive or negative $x$-, $y$-, or $z$-axis) for the positive thrust direction; the other two axes are then the axes of control. For a main-rotor, the nominal direction would be the negative $z$-axis; for a tail-rotor, it would be the lateral axis ($y$-axis, depending on the direction of rotation of the main-rotor); and for a propeller, the nominal direction would be the positive $x$-axis. This selection defines a rotation matrix $W$ from $F$ to $S$ axes. The hub and pivot axes have a fixed orientation relative to the body axes:

hub incidence and cant: $C_{HF} = U_{\theta_h} V_{\phi_h}$
pivot dihedral, pitch, and sweep: $C_{PF} = X_{\phi_p} Y_{\theta_p} Z_{\psi_p}$

where $U$ and $V$ depend on the nominal direction, as described in table 12-2. The shaft control consists of incidence and cant about the pivot axes, from reference angles $i_{\text{ref}}$ and $c_{\text{ref}}$:

$C_{\text{cont}} = U_{1-i_{\text{ref}}} V_{c-c_{\text{ref}}}$
For a tiltrotor aircraft, one of the aircraft controls is the nacelle angle, with the convention $\alpha_{\text{tilt}} = 0$ for cruise, and $\alpha_{\text{tilt}} = 90$ deg for helicopter mode. The rotor shaft incidence angle is then connected to $\alpha_{\text{tilt}}$ by defining the matrix $T_i$ appropriately. For the locations and orientation input in helicopter mode, $i_{\text{ref}} = 90$. Thus the orientation of the shaft axes relative to the body axes is:

$$C^{SF} = WC^{HF}C^{FP}C_{\text{cont}}C^{PF}$$

or just $C^{SF} = WC^{HF}$ with no shaft control. From the pivot location $z_{\text{pivot}}^F$ and the hub location for the reference shaft control $z_{\text{hub0}}^F$, the hub location in general is

$$z_{\text{hub}}^F = z_{\text{pivot}}^F + (C^{FP}C_{\text{cont}}C^{PF})^T(z_{\text{hub0}}^F - z_{\text{pivot}}^F)$$

Similarly, the pylon location and nacelle center-of-gravity location can be calculated for given shaft control. The shift in the aircraft center of gravity produced by nacelle tilt is

$$W(z_{\text{cg}}^F - z_{\text{cg0}}^F) = W_{\text{move}}(z_{\text{nac}}^F - z_{\text{nac0}}^F) = W_{\text{move}}(C^{FP}C_{\text{cont}}C^{PF} - I)(z_{\text{nac0}}^F - z_{\text{pivot}}^F)$$

where $W$ is the gross weight and $W_{\text{move}}$ the weight moved. Table 12-2 summarizes the geometry options.

### Table 12-2. Rotor shaft axes.

<table>
<thead>
<tr>
<th>nominal thrust</th>
<th>$z^S$-axis</th>
<th>$x^S$-axis</th>
<th>$W$</th>
<th>incidence + for $T$</th>
<th>cant + for $T$</th>
<th>$U_{\theta h}V_{\phi h}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>main-rotor</td>
<td>$-z^F$</td>
<td>up</td>
<td>$-z^F$ aft</td>
<td>$Y_{180}$ aft</td>
<td>right</td>
<td>$Y_{\theta}X_{\phi}$</td>
</tr>
<tr>
<td>other</td>
<td>$z^F$</td>
<td>down</td>
<td>$-z^F$ aft</td>
<td>$Z_{180}$ aft</td>
<td>right</td>
<td>$Y_{-\theta}X_{-\phi}$</td>
</tr>
<tr>
<td>propeller</td>
<td>$x^F$</td>
<td>forward</td>
<td>$-z^F$ up</td>
<td>$Y_{90}$ up</td>
<td>right</td>
<td>$Y_{\theta}Z_{\phi}$</td>
</tr>
<tr>
<td>other</td>
<td>$-x^F$ aft</td>
<td>$-z^F$ up</td>
<td>$Z_{180}Y_{-90}$ up</td>
<td>right</td>
<td>$Y_{-\theta}Z_{-\phi}$</td>
<td></td>
</tr>
<tr>
<td>tail-rotor ($r = 1$)</td>
<td>$y^F$ right</td>
<td>$-x^F$ aft</td>
<td>$Z_{180}X_{-90}$ aft</td>
<td>up</td>
<td>$Z_{\theta}X_{-\phi}$</td>
<td></td>
</tr>
<tr>
<td>tail-rotor ($r = -1$)</td>
<td>$-y^F$ left</td>
<td>$-x^F$ aft</td>
<td>$Z_{180}X_{90}$ aft</td>
<td>up</td>
<td>$Z_{-\theta}X_{\phi}$</td>
<td></td>
</tr>
</tbody>
</table>

$$Y_{180} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \quad Z_{180}Y_{-90} = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

$$Z_{180} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad Z_{180}X_{-90} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$Y_{90} = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \quad Z_{180}X_{90} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

#### 12-3.4 Hub Loads

The rotor controls give the thrust magnitude and the tip-path plane tilt angles $\beta_c$ and $\beta_s$, either directly or from the collective and cyclic pitch. The forces acting on the hub are the thrust $T$, drag $H$, and side force $Y$ (positive in $z$, $x$, $y$-axis directions, respectively). The hub pitch and roll moments are
proportional to the flap angles. The hub torque is obtained from the shaft power $P_{\text{shaft}}$ and rotor speed $\Omega$. The force and moment acting on the hub, in shaft axes, are then:

$$F^S = \begin{pmatrix} H \\ Y \\ T \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -f_B T \end{pmatrix}$$

$$M^S = \begin{pmatrix} M_x \\ M_y \\ -rQ \end{pmatrix} = \begin{pmatrix} K_{\text{hub}} (r \beta_s) \\ K_{\text{hub}} (-\beta_c) \\ -rP_{\text{shaft}}/\Omega \end{pmatrix}$$

The force includes a term proportional to the rotor thrust and an input blockage factor $f_B = \Delta T/T \geq 0$. This term accounts for blockage or download, as an alternative to including the drag of the fuselage or a lifting surface in the aircraft trim. For example, $f_B$ can model the tail-rotor blockage caused by operation near the vertical tail. The rotor loads in aircraft axes acting at the center of gravity are then:

$$F^F = C^{FS} F^S$$

$$M^F = C^{FS} M^S + \Delta z^F F^F$$

where $\Delta z^F = z^F_{\text{hub}} - z^F_{\text{cg}}$.

The wind axis lift $L$ and drag $X$ are calculated from the net rotor hub force $F^F$ and the rotor velocity $v^F$. The velocity relative to the air gives the propulsive force direction $e_p = v^F/|v^F|$ (no interference) and the velocity magnitude $V = |v^F|$. The drag and lift components of the force are $X = -e_p^T F^F$ and $L = |F^F|^2 - |X|^2$. The rotor contribution to vertical force is the $z$-axis component of the force in inertial axes, $F_V = -k^T C^{IF} F^F$.

### 12-4 Aerodynamics

The rotor velocity relative to the air is $v^F = v^F_{AC} + \omega^F_{AC} \Delta z^F$ in aircraft axes. The velocities in shaft axes are

$$v^S = C^{SF} v^F = \Omega R \begin{pmatrix} -\mu_x \\ r \mu_y \\ \mu_z \end{pmatrix}$$

$$\omega^S = C^{SF} \omega^F_{AC} = \Omega \begin{pmatrix} r \dot{\alpha}_x \\ \dot{\alpha}_y \\ r \dot{\alpha}_z \end{pmatrix}$$

where $\Omega R$ is the rotor tip speed. The advance ratio $\mu$, inflow ratio $\lambda$, and shaft angle of attack $\alpha$ are defined as

$$\mu = \sqrt{\mu_x^2 + \mu_y^2}$$

$$\lambda = \lambda_i + \mu_z$$

$$\alpha = \tan^{-1}(\mu_z/\mu)$$

The blade velocity relative to the air has the maximum amplitude (advancing tip velocity) of $\mu_{at} = \sqrt{(1+\mu)^2 + \mu_z^2}$, from which the advancing tip Mach number is $M_{at} = M_{tip} \mu_{at}$, using the tip Mach number $M_{tip} = (\Omega R)/c_s$. The rotor thrust coefficient is defined as $C_T = T/\rho A (\Omega R)^2$. The dimensionless ideal induced velocity $\lambda_i$ is calculated from $\mu$, $\mu_z$, and $C_T$; then the dimensional velocity is $v_i = \Omega R \lambda_i$. The ideal induced power is then $P_{\text{ideal}} = T v_i$. Note that for these inflow velocities, the subscript “i” denotes “ideal.”

### 12-4.1 Ideal Inflow

The ideal wake-induced velocity is obtained from Glaubert’s momentum theory:

$$\lambda_i = \frac{C_T}{2 \sqrt{\lambda^2 + \mu^2}} = \frac{s \lambda_i^2}{\sqrt{\lambda^2 + \mu^2}}$$
where \( \lambda = \lambda_i + \mu_z, \lambda_h^2 = |C_T|/2 \) (\( \lambda_h \) is always positive), and \( s = \text{sign} \ C_T \). This expression is generalized to
\[
\lambda_i = \lambda_h \ s \ F(\mu/\lambda_h, s\mu_z/\lambda_h)
\]
If \( \mu \) is zero, the equation for \( \lambda_i \) can be solved analytically. Otherwise, for non-axial flow, the equation is written as follows:
\[
\lambda = \frac{s\lambda_h^2}{\sqrt{\lambda_h^2 + \mu_z^2}} + \mu_z
\]
Using \( \lambda \) instead of \( \lambda_i \) as the independent variable simplifies implementation of the ducted fan model. A Newton–Raphson solution for \( \lambda \) gives:
\[
\hat{\lambda}_{in} = \frac{s\lambda_h^2}{\sqrt{\lambda_h^2 + \mu_z^2}}
\]

\[
\lambda_{n+1} = \lambda_n - \frac{\lambda_n - \mu_z - \hat{\lambda}_{in}}{1 + \lambda_n \lambda_n/(\lambda_h^2 + \mu_z^2)} f
\]

A relaxation factor of \( f = 0.5 \) is used to improve convergence. Three or four iterations are usually sufficient, using
\[
\lambda \approx \frac{s\lambda_h^2}{\sqrt{(s\lambda_h + \mu_z)^2 + \mu_z^2}} + \mu_z
\]
to start the solution. To eliminate the singularity of the momentum theory result at ideal autorotation, the expression
\[
\lambda = \mu_z \left[ 0.373\mu_z^2 + 0.598\mu_z^2 - 0.991 \right]
\]
is used when
\[
1.5\mu_z^2 + (2s\mu_z + 3\lambda_h)^2 < \lambda_h^2
\]
The equation \( \lambda = \mu_z(\alpha\mu_z^2 - b\lambda_h^2 + c\mu^2)/\lambda_h^2 \) is an approximation for the induced power measured in the turbulent-wake and vortex-ring states. Matching this equation to the axial-flow momentum theory result at \( \mu_z = -2\lambda_h \) and \( \mu_z = -\lambda_h \) gives \( \alpha = \sqrt{5}/6 \) \( \approx 0.3726780 \) and \( b = (4\sqrt{5} - 3)/6 \approx 0.9907120 \). Then matching to the forward-flight momentum theory result at \( (\mu = \lambda_h, \mu_z = -1.5\lambda_h) \) gives \( c = 0.5980197 \).

For axial flow \( (\mu = 0) \) the solution is:
\[
\lambda = \begin{cases} 
\frac{\mu_z}{2} + s\sqrt{\left(\frac{\mu_z}{2}\right)^2 + \lambda_h^2} & -\lambda_h < s\mu_z \\
\mu_z \left[ 0.373\mu_z^2 - 0.991 \right] & -2\lambda_h < s\mu_z < -\lambda_h \\
\frac{\mu_z}{2} - s\sqrt{\left(\frac{\mu_z}{2}\right)^2 - \lambda_h^2} & s\mu_z < -2\lambda_h
\end{cases}
\]

Note that \( \lambda_i \) and \( v_i \) are the ideal induced velocities; additional factors are required for the wake-induced velocity or induced power calculations.

12.4.1 Ducted Fan

Rotor momentum theory can be extended to the case of a ducted fan. Consider a rotor system with disk area \( A \), operating at speed \( V \), with an angle \( \alpha \) between \( V \) and the disk plane. The induced velocity at the rotor disk is \( v \), and in the far wake \( w = f_w v \). The far wake area is \( A_\infty = A/f_A \). The axial velocity
at the fan is \( f_{Vz} V_z \), with \( f_{Vz} \) accounting for acceleration or deceleration through the duct. The edgewise velocity at the fan is \( f_{Vz} V_z \), with \( f_{Vz} = 1.0 \) for wing-like behavior, or \( f_{Vz} = 0 \) for tube-like behavior of the flow. The total thrust (rotor plus duct) is \( T \), and the rotor thrust is \( T_{rotor} = f_T T \). For this model, the duct aerodynamics are defined by the thrust ratio \( f_T \) or far wake area ratio \( f_A \), plus the fan velocity ratio \( f_{V} \). The mass flux through the rotor disk is \( \dot{m} = \rho AU = \rho A_{\infty} U_{\infty} \), where \( U \) and \( U_{\infty} \) are respectively the total velocity magnitudes at the fan and in the far wake:

\[
U^2 = (f_{Vz} V \cos \alpha)^2 + (f_{Vz} V \sin \alpha + v)^2 \\
U_{\infty}^2 = (V \cos \alpha)^2 + (V \sin \alpha + w)^2
\]

Mass conservation (\( f_A = A/A_{\infty} = U_{\infty}/U \)) relates \( f_A \) and \( f_W \). Momentum and energy conservation give

\[
T = \dot{m} w = \rho A U_{\infty} w / f_A = \rho A U f_{W} v \\
P = \frac{1}{2} \dot{m} w (2V \sin \alpha + w) = T \left( V \sin \alpha + \frac{w}{2} \right)
\]

With these expressions, the span of the lifting system in forward flight is assumed equal to the rotor diameter \( 2R \). Next it is required that the power equals the rotor induced and parasite loss:

\[
P = T_{rotor}(f_{Vz} V \sin \alpha + v) = T f_T (f_{Vz} V \sin \alpha + v)
\]

In axial flow, this result can be derived from Bernoulli’s equation for the pressure in the wake. In forward flight, any induced drag on the duct is being neglected. From these two expressions for power, \( V_z + f_{W} w/2 = f_T (f_{Vz} V_z + v) \) is obtained, relating \( f_T \) and \( f_{W} \). With no duct (\( f_T = f_{Vz} = f_{Vz} = 1 \)), the far wake velocity is always \( w = 2v \), hence \( f_{W} = 2 \). With an ideal duct (\( f_A = f_{Vx} = f_{Vz} = 1 \)), the far wake velocity is \( f_{W} = 1 \). In hover (with or without a duct), \( f_{W} = f_A = 2f_T \), and \( v = \sqrt{2/f_{W} v_{h}} \). The rotor ideal induced power is \( P_{\text{ideal}} = Tw/2 = f_D T w \), introducing the duct factor \( f_D = f_{W}/2 \).

For a ducted fan, the thrust \( C_T \) is calculated from the total load (rotor plus duct). To define the duct effectiveness, either the thrust ratio \( f_T = T_{rotor}/T \) or the far wake area ratio \( f_A = A/A_{\infty} \) is specified (and the fan velocity ratio \( f_{V} \)). The wake-induced velocity is obtained from the momentum theory result for a ducted fan: \( \lambda_{h}^2 = (f_{W} \lambda_{i}/2)\sqrt{(f_{Vz} \mu)^2 + (f_{Vz} \mu_{z} + \lambda_{i})^2} \). If the thrust ratio \( f_T \) is specified, this can be written

\[
f_{Vz} \mu_{z} + \lambda_{i} = \frac{s\lambda_{h}^2 / f_T}{\sqrt{(f_{Vz} \mu_{z} + \lambda_{i})^2 + (f_{Vz} \mu)^2}} + \frac{\mu_{z}}{f_T}
\]

In this form, \( \lambda_{i} \) can be determined using the free-rotor expressions given previously: replacing \( \lambda_{h}^2 \), \( \mu_{z} \), \( \mu \), and \( \lambda \) with \( \lambda_{h}^2 / f_T \), \( \mu_{z} / f_T \), \( f_{Vz} \mu \), and \( f_{Vz} \mu_{z} + \lambda_{i} \), respectively. Then from \( \lambda_{i} \) the velocity and area ratios are obtained:

\[
f_{W} = 2 \left( f_T - (1 - f_T f_{Vz}) \frac{\mu_{z}}{\lambda_{i}} \right) \\
f_{A} = \sqrt{\frac{\mu_{z}^2 + (\mu_{z} + f_{W} \lambda_{i})^2}{(f_{Vz} \mu_{z} + \lambda_{i})^2}}
\]

If instead the area ratio \( f_A \) is specified, it is simplest to first solve for the far wake velocity \( f_{W} \lambda_{i} \):

\[
\mu_{z} + f_{W} \lambda_{i} = \frac{s\lambda_{h}^2 f_{A}}{\sqrt{(\mu_{z} + f_{W} \lambda_{i})^2 + \mu^2}} + \mu_{z}
\]

In this form, \( f_{W} \lambda_{i} \) can be determined using the free-rotor expressions given previously: replacing \( \lambda_{h}^2 \) and \( \lambda \) with \( \lambda_{h}^2 f_{A} \) and \( \mu_{z} + f_{W} \lambda_{i} \), respectively. The induced velocity is

\[
(f_{Vz} \mu_{z} + \lambda_{i})^2 = \frac{1}{f_{A}} \left[ \mu^2 + (\mu_{z} + f_{W} \lambda_{i})^2 \right] - (f_{Vz} \mu)^2
\]
The velocity ratio is \( f_W = (f_W \lambda_i)/\lambda_i \), and

\[
f_T = \frac{\mu_z + f_W \lambda_i/2}{f_V \mu_z + \lambda_i}
\]
is the thrust ratio. However, physical problems and convergence difficulties are encountered with this approach in descent, if an arbitrary value of \( f_T \) is permitted. From the expression for \( f_T \), \( f_T \) should approach \( 1/f_V \) at high rates of climb or descent. To avoid problems with an arbitrary value of \( f_T \), it is assumed that the input value of \( f_T \) defines the velocity ratio \( f_W = 2f_T \) in descent. So in descent \( \mu_z \) is not replaced by \( \mu_z/f_T \).

12-4.1.2 Ground Effect

The wake-induced velocity is reduced when the rotor disk is in the proximity of the ground plane. Ground effect in hover can be described in terms of the figure of merit

\[
M = \left( \frac{T^3}{2\sqrt{2\rho A}} \right) / P
\]
as a function of scaled rotor height above the ground, \( z_g/D = z_g/2R \). Usually the test data are given as the ratio of the thrust to OGE thrust, for constant power: \( T/T_{\infty} = (M/M_{\infty})^{2/3} = \kappa_g \geq 1 \). The effect on power at constant thrust is then \( P = P_{\infty} f_g \), where \( f_g = \kappa_g^{-3/2} \leq 1 \). Ground effect is generally negligible at heights above \( z_g/D = 1.5 \) and at forward speeds above \( \mu = 3\lambda_h \).

The ground plane is assumed to be perpendicular to the inertial frame \( z \)-axis. The ground normal (directed downward) is \( k_g^F = C^F k \) in airframe axes, or \( k_g^S = C^SF k_g^F \) in rotor shaft axes. The height of the landing gear above ground level, \( h_{LG} \), is specified in the flight state. The height of the rotor hub above ground level is then

\[
z_g = h_{LG} - (k_g^F)^T (z_{hub}^F - z_{LG}^F) + d_{LG}
\]
where \( z_{LG}^F \) is the position of the landing gear in the airframe, and \( d_{LG} \) is the distance from the bottom of the gear to the location \( z_{LG} \). From the velocity

\[
v^S = \begin{pmatrix} \mu_x \\
-\mu_y \\ -\lambda \end{pmatrix}
\]
the angle \( \epsilon \) between the ground normal and the rotor wake is evaluated: \( \cos \epsilon = (k_g^S)^T v^S / |v^S| \) (\( \epsilon = 0 \) for hover, \( \epsilon = 90 \) deg in forward flight). Note that if the rotor shaft is vertical, then \( \cos \epsilon = \lambda / \sqrt{\mu_x^2 + \lambda^2} \) (see ref. 1). The expressions for ground effect in hover are generalized to forward flight by using \( (z_g/\cos \epsilon) \) in place of \( z_g \). No ground effect correction is applied if the wake is directed upward (\( \cos \epsilon \leq 0 \)), or if \( z_g/\cos \epsilon > 1.5D \). From \( z_g/D \cos \epsilon \), the ground effect factor \( f_g = \kappa_g^{-3/2} \) is calculated. Then

\[
(\lambda_i)_{IGE} = f_g (\lambda_i)_{OGE}
\]
is the effective ideal induced velocity.

Several empirical ground effect models are implemented: from Cheeseman and Bennett (ref. 1, basic model and using blade-element (BE) theory to incorporate influence of thrust), from Law (ref. 2), from Hayden (ref. 3), and a curve fit of the interpolation from Zbrozek (ref. 4):
These equations break down at small height above the ground, and so are restricted to \( z_g/D \geq 0.15 \); however, the database for ground effect extends only to about \( z/D = 0.3 \). Also, \( f_g \leq 1 \) is required. Figure 12-2 shows \( T/T_\infty = \kappa_g = f_g^{2/3} \) as a function of \( z/R \) for these models (\( C_T/\sigma = 0.05, 0.10, 0.15 \)), compared with test data from several sources.

The influence of the ground on tiltrotor power required is stronger than for an isolated rotor. This further reduction of power is probably due to a reduction of wing and fuselage download when operating near the ground. For quad tiltrotors, an upload on the airframe has been measured at low heights. These effects can be modeled by using an effective distance above the ground: \( z_e = C_g z_g \), with typically \( C_g = 0.5 \) for a tiltrotor.

### Inflow Gradient

As a simple approximation to nonuniform induced velocity distribution, a linear variation over the disk is used: \( \Delta \lambda = \lambda_x r \cos \psi + \lambda_y r \sin \psi \). There are contributions to \( \Delta \lambda \) from forward flight and from hub moments, which influence the relationship between flapping and cyclic. The linear inflow variation caused by forward flight is \( \Delta \lambda_f = \lambda_i (\kappa_x r \cos \psi + \kappa_y r \sin \psi) \), where \( \lambda_i \) is the mean inflow. Typically \( \kappa_x \) is positive, and roughly 1 at high speed, and \( \kappa_y \) is smaller in magnitude and negative. Both \( \kappa_x \) and \( \kappa_y \) must be zero in hover. Based on references 5–8, the following models are considered:

- **Coleman and Feingold:**
  \[
  \kappa_x = f_x \frac{15\pi}{32} \tan \chi / 2 = f_x \frac{15\pi}{32} \frac{\mu}{\sqrt{\mu^2 + \lambda^2 + |\lambda|}} \\
  \kappa_y = -f_y 2\mu
  \]

- **White and Blake:**
  \[
  \kappa_x = f_x \sqrt{2} \sin \chi = f_x \sqrt{2} \frac{\mu}{\sqrt{\mu^2 + \lambda^2}} \\
  \kappa_y = -2f_y \mu
  \]

where \( \tan \chi = |\lambda|/\mu \) is the wake angle. Extending these results to include sideward velocity gives \( \kappa_x = (\kappa_{x0} \mu_x + \kappa_{y0} \mu_y) / \mu \) and \( \kappa_y = (-\kappa_{x0} \mu_y + \kappa_{y0} \mu_x) / \mu \). For flexibility, the empirical factors \( f_x \) and \( f_y \) have been introduced (values of 1.0 give the baseline model). There is also an inflow variation produced by any net aerodynamic moment on the rotor disk, which can be evaluated using a differential form of momentum theory:

\[
\Delta \lambda_m = \frac{f_m}{\sqrt{\mu^2 + \lambda^2}} (-2C_{M_y} r \cos \psi + 2C_{M_x} r \sin \psi) = \lambda_{xm} r \cos \psi + \lambda_{ym} r \sin \psi
\]
including empirical factor $f_m$. Note that the denominator of the hub moment term is zero for a hovering rotor at zero thrust; so this inflow contribution should not be used for cases of low speed and low thrust.

### 12-4.2 Rotor Forces

When direct control of the rotor thrust magnitude is used, the rotor collective pitch angle $\theta_{0.75}$ must be calculated from the thrust $C_T/\sigma$. If the commanded variable is the collective pitch angle, then it is necessary to calculate the rotor thrust, resulting in more computation, particularly since all quantities depending on the thrust (inflow, induced power factor, and mean drag coefficient) are also unknown. There may be flight states where the commanded thrust can not be produced by the rotor, even with stall neglected in the section aerodynamics. This condition manifests as an inability to solve for the collective pitch given the thrust. In this circumstance the trim method can be changed so the required or specified thrust is an achievable value, or commanded collective pitch control can be used.

Cyclic control consists of tip-path plane command, requiring calculation of the rotor cyclic pitch angles from the flapping; or no-feathering plane command, requiring calculation of the tip-path plane tilt from the cyclic control angles. The longitudinal tip-path plane tilt is $\beta_c$ (positive forward) and the lateral tilt is $\beta_s$ (positive toward retreating side). The longitudinal cyclic pitch angle is $\theta_s$ (positive aft), and the lateral cyclic pitch angle is $\theta_c$ (positive toward retreating side). Tip-path plane command is appropriate.
for main-rotors. For rotors with no cyclic pitch, no-feathering plane command must be used.

The forces acting on the hub are the thrust $T$, drag $H$, and side force $Y$ (positive in $z$-, $x$-, $y$-axis directions, respectively). The aerodynamic analysis is conducted for a clockwise rotating rotor, with appropriate sign changes for lateral velocity, flapping, and force. The analysis is conducted in dimensionless form, based on the actual radius and rotational speed of the flight state. The inplane hub forces are produced by tilt of the thrust vector with the tip-path plane, plus forces in the tip-path plane, and profile terms (produced by the blade drag coefficient). The orientation of the tip-path axes relative to the shaft axes is then $C_{PS} = X_r\beta_s Y - \beta_c$. Then

$$
\begin{pmatrix}
C_H \\
C_Y \\
C_T
\end{pmatrix} = C^{SP} \begin{pmatrix}
0 \\
0 \\
C_T/C_{33}^{SP}
\end{pmatrix} + \left( C_{H\text{tip}} r C_{Y\text{tip}}/C_{33}^{SP} \right)
$$

The inplane forces relative to the tip-path plane can be neglected, or calculated by blade element theory. Note that with thrust and tip-path plane command and $C_{H\text{tip}}$ and $C_{Y\text{tip}}$ neglected, it is not necessary to solve for the rotor collective and cyclic pitch angles. In general the inplane forces relative to the tip-path plane are not zero, and may be significant, as for a rotor with large flap stiffness. Figures 12-3a and b show, respectively, the tip-path plane tilt and thrust vector tilt with cyclic pitch control (no-feathering plane tilt), as functions of flap stiffness (frequency), for several rotor thrust values. The difference between tip-path plane tilt (fig. 12-3a) and thrust vector tilt (fig. 12-3b) is caused by tilt of the thrust vector relative to the tip-path plane.

The profile inplane forces can be obtained from simplified equations, or calculated by blade element theory. The simplified method uses:

$$
\begin{pmatrix}
C_{H_s} \\
C_{Y_s}
\end{pmatrix} = \frac{\sigma}{8} c_{d\text{mean}} F_H \begin{pmatrix}
\mu_x/\mu \\
\mu_y/\mu
\end{pmatrix}
$$

where the mean drag coefficient $c_{d\text{mean}}$ is from the profile power calculation. The function $F_H$ accounts for the increase of the blade section velocity with rotor edgewise and axial speed: $C_{H_s} = \int \frac{1}{2} \sigma c_d u^2 (r \sin \psi + \mu) dr = \int \frac{1}{2} \sigma c_d (u_T^2 + u_R^2 + u_P^2)^{1/2} (r \sin \psi + \mu) dr$; so (from ref. 9)

$$
F_H = 4 \frac{1}{2\pi} \int_0^{2\pi} \int_0^1 \left( (r + \mu \sin \psi)^2 + (\mu \cos \psi)^2 + \mu^2 \right)^{1/2} (r \sin \psi + \mu) dr d\psi
$$

$$
\approx \sqrt{1+V^2} \left( 3\mu + \frac{3}{4} \mu^3 \frac{V^2 - 1}{(1+V^2)^2} \right) + \left( \mu \mu^2 + \frac{3}{4} \mu^3 \right) \ln \left[ \sqrt{1+V^2} + 1 \right]
$$

with $V^2 = \mu^2 + \mu_z^2$.

### 12-4.3 Blade Element Theory

Blade element theory is the basis for the solution for the collective and cyclic pitch angles (or thrust and flap angles) and evaluation of the rotor inplane hub forces. The section aerodynamics are described by lift varying linearly with angle of attack, $c_\ell = c_{\ell\alpha\alpha}$ (no stall), and a constant mean drag coefficient $c_{d\text{mean}}$ (from the profile power calculation). The analysis is conducted in dimensionless form (based on density $\rho$, rotor rotational speed $\Omega$, and blade radius $R$ of the flight state). So in the following $\sigma$, $\nu$, and $\gamma$ are for the actual $\rho$, $\Omega$, and $R$; and $a = 5.7$ is the lift-curve slope used in the Lock number $\gamma$. The blade
section aerodynamic environment is described by the three components of velocity, from which the yaw and inflow angles are obtained, and then the angle of attack:

\[
\begin{align*}
    u_T &= r + \mu_x \sin \psi + \mu_y \cos \psi \\
    u_R &= \mu_x \cos \psi - \mu_y \sin \psi \\
    u_P &= \lambda + r(\beta + \dot{\alpha}_x \sin \psi - \dot{\alpha}_y \cos \psi) + u_R \beta
\end{align*}
\]

\[
U^2 = u_T^2 + u_P^2
\]

\[
\cos \Lambda = \frac{U}{\sqrt{u_T^2 + u_P^2 + u_R^2}}
\]

\[
\phi = \tan^{-1} \frac{u_P}{u_T}
\]

\[
\alpha = \theta - \phi
\]

In reverse flow (|\alpha| > 90), \( \alpha \leftarrow \alpha - 180 \text{sign} \alpha \), and then \( c_t = c_t \alpha \) still (airfoil tables are not used). The blade pitch consists of collective, cyclic, twist, and pitch-flap coupling terms. The flap motion is rigid rotation about a hinge with no offset, and only coning and once-per-revolution terms are considered:

\[
\begin{align*}
    \theta &= \theta_{0.75} + \theta_{tw} + \theta_c \cos \psi + \theta_s \sin \psi - K_P \beta \\
    \beta &= \beta_0 + \beta_c \cos \psi + \beta_s \sin \psi
\end{align*}
\]

where \( K_P = \tan \delta_3 \). The twist is measured relative to 0.75R; \( \theta_{tw} = \theta_L(r - 0.75) \) for linear twist. The mean inflow is \( \lambda_0 = \kappa \lambda_t \), using the induced velocity factor \( \kappa \) from the induced power model. The inflow includes gradients caused by edgewise flight and hub moments:

\[
\begin{align*}
    \lambda &= \mu_z + \lambda_0(1 + \kappa_x r \cos \psi + \kappa_y r \sin \psi) + \Delta \lambda_m \\
    &= \mu_z + \lambda_0(1 + \kappa_x r \cos \psi + \kappa_y r \sin \psi) + \frac{f_m}{\sqrt{\mu^2 + \lambda^2}} (-2C_{My} r \cos \psi + 2C_{Mx} r \sin \psi)
\end{align*}
\]

From the hub moments

\[
\begin{pmatrix}
    -C_{My} \\
    C_{Mx}
\end{pmatrix} = \frac{\sigma a}{2} \frac{\nu^2 - 1}{\gamma} \begin{pmatrix}
    \beta_c \\
    \beta_s
\end{pmatrix}
\]
the inflow gradient is

$$\Delta \lambda_m = \left( \frac{f_m}{\sqrt{\mu^2 + \lambda^2}} \right) \frac{\sigma a}{8} \frac{\nu^2 - 1}{\gamma/8} (r\beta_c \cos \psi + r\beta_s \sin \psi) = K_m \nu^2 - 1 \frac{1}{\gamma/8} (r\beta_c \cos \psi + r\beta_s \sin \psi)$$

The constant $K_m$ is associated with a lift-deficiency function:

$$C = \frac{1}{1 + K_m} = \frac{1}{1 + f_m \sigma a / (8 \sqrt{\mu^2 + \lambda^2})}$$

The blade chord is $c(r) = c_{ref} \hat{c}(r)$, where $c_{ref}$ is the thrust-weighted chord (chord at 0.75R for linear taper). Yawed flow effects increase the section drag coefficient, hence $c_d = c_{dmean}/\cos \Lambda$. The section forces in velocity axes and shaft axes are

$$L = \frac{1}{2} \rho U^2 cc_{\ell} \quad F_z = L \cos \phi - D \sin \phi = \frac{1}{2} \rho U c (c_{\ell} u_T - c_d u_P)$$

$$D = \frac{1}{2} \rho U^2 cc_{d} \quad F_x = L \sin \phi + D \cos \phi = \frac{1}{2} \rho U c (c_{\ell} u_P + c_d u_T)$$

$$R = \frac{1}{2} \rho U^2 cc_{r} = D \tan \Lambda \quad F_r = -\beta F_z + R = -\beta F_z + \frac{1}{2} \rho U c c_{d} u_R$$

These equations for the section environment and section forces are applicable to high inflow (large $\mu_z$), sideward flight ($\mu_y$), and reverse flow ($u_T < 0$). The total forces on the rotor hub are

$$T = N \int F_z \, dr$$

$$H = N \int F_x \sin \psi + F_r \cos \psi \, dr$$

$$Y = N \int -F_x \cos \psi + F_r \sin \psi \, dr$$

Figure 12-3b. Thrust vector tilt with cyclic pitch.
with an average over the rotor azimuth implied, along with the integration over the radius. Lift forces are integrated from the root cutout \( r_{root} \) to the tip loss factor \( B \). Drag forces are integrated from the root cutout to the tip.

In coefficient form (forces divided by \( \rho AV_{tip}^2 \)) the rotor thrust and inplane forces are:

\[
\begin{align*}
C_T &= \sigma \int \bar{F}_z \, dr \\
C_H &= \sigma \int \bar{F}_x \sin \psi + \bar{F}_r \cos \psi \, dr \\
C_Y &= \sigma \int -\bar{F}_x \cos \psi + \bar{F}_r \sin \psi \, dr
\end{align*}
\]

\( \bar{F}_z = \frac{1}{2} \bar{c}U (c_L u_T - c_d u_P) \)

\( \bar{F}_x = \frac{1}{2} \bar{c}U (c_L u_P + c_d u_T) \)

\( \bar{F}_r = -\beta \bar{F}_z + \frac{1}{2} \bar{c}U c_d u_R \)

(and the sign of \( C_Y \) is changed for a clockwise rotating rotor). The terms \( \Delta \bar{F}_z \equiv \bar{F}_z \beta \) and \( \Delta \bar{F}_r = -\bar{F}_z \beta \) produce tilt of the thrust vector with the tip-path plane \( (C_H = -C_T \beta_c \) and \( C_Y = -C_T \beta_s \)), which are accounted for directly. The section drag coefficient \( c_d \) produces the profile inplane forces. The approximation \( u_P \equiv \mu_2 \) is consistent with the simplified method (using the function \( F_H \)), hence

\[
\begin{align*}
\bar{F}_{zo} &= \frac{1}{2} \bar{c}U_0 c_d u_T \\
\bar{F}_{ro} &= \frac{1}{2} \bar{c}U_0 c_d u_R
\end{align*}
\]

\( C_{Ho} = \sigma \int \bar{F}_{zo} \sin \psi + \bar{F}_{ro} \cos \psi \, dr = \frac{\sigma}{2} \int \bar{c}U_0 c_d (r \sin \psi + \mu_2) \, dr \\
C_{Yo} = \sigma \int -\bar{F}_{xo} \cos \psi + \bar{F}_{ro} \sin \psi \, dr = -\frac{\sigma}{2} \int \bar{c}U_0 c_d (r \cos \psi + \mu_2) \, dr
\]

where \( U_0^2 = u_T^2 + \mu_2^2 \), and \( c_d = c_{dmean} / \cos \Lambda \). Using blade element theory to evaluate \( C_{Ho} \) and \( C_{Yo} \) accounts for the planform \( \beta \) and root cutout. Using the function \( F_H \) implies a rectangular blade and no root cutout (plus at most a 1% error approximating the exact integration). The remaining terms in the section forces produce the inplane loads relative to the tip-path plane:

\[
\begin{align*}
\bar{F}_{xi} &= \bar{F}_x - \bar{F}_z \beta - \bar{F}_{zo} = \frac{1}{2} \bar{c}U c_d (u_P - u_T \beta) + \frac{1}{2} \bar{c}U c_d ((1 - U_0 / U) u_T + u_P \beta) \\
\bar{F}_{ri} &= \bar{F}_r + \bar{F}_z \beta - \bar{F}_{ro} = \frac{1}{2} \bar{c}U c_d (1 - U_0 / U) u_R \\
C_{H_{tip}} &= \sigma \int \bar{F}_{xi} \sin \psi + \bar{F}_{ri} \cos \psi \, dr \\
C_{Y_{tip}} &= \sigma \int -\bar{F}_{xi} \cos \psi + \bar{F}_{ri} \sin \psi \, dr
\end{align*}
\]

(including small profile terms from \( U_0 \neq U \)).

Evaluating these inplane forces requires the collective and cyclic pitch angles and the flapping motion. The thrust equation must be solved for the rotor collective pitch or the rotor thrust. The relationship between cyclic pitch and flapping is defined by the rotor flap dynamics. The flap motion is rigid rotation about a central hinge, with a flap frequency \( \nu > 1 \) for articulated or hingeless rotors. The flapping equation of motion is

\[
\ddot{\beta} + \nu^2 \beta + 2 \dot{\alpha}_y \sin \psi + 2 \dot{\alpha}_x \cos \psi = \frac{\gamma}{a} \int \bar{F}_z \, dr + (\nu^2 - 1) \beta_p
\]

including precone angle \( \beta_p \); the Lock number \( \gamma = \rho ac_{root} R^4 / I_b \). This equation is solved for the mean (coning) and 1/rev (tip-path plane tilt) flap motion:

\[
\nu^2 \beta_0 = \frac{\gamma}{a} \int \bar{F}_z \, dr + (\nu^2 - 1) \beta_p \\
(\nu^2 - 1) \left( \frac{\beta_c}{\beta_s} \right) = \frac{\gamma}{a} \int \bar{F}_z \, dr \left( \frac{2 \cos \psi}{2 \sin \psi} \right) + \left( \frac{2 \dot{\alpha}_x}{2 \dot{\alpha}_y} \right)
\]
with an average over the rotor azimuth implied. The solution for the coning is largely decoupled by introducing the thrust:

$$v_0^2 \beta_0 = \frac{\gamma}{8} \frac{6 C_T}{\sigma a} + (v_0^2 - 1) \beta_p + \frac{\gamma}{a} \int \hat{F}_z (r - 3/4) \, dr$$

A separate flap frequency $v_0$ is used for coning, in order to model teetering and gimballed rotors. For an articulated rotor, $\beta_p = 0$ should be used.

The thrust and flapping equations of motion that must be solved are:

$$E_t = \frac{6}{a} \int \hat{F}_z \, dr - \frac{6 C_T}{\sigma a}$$

$$\left( \begin{array}{c} E_c \\ E_s \end{array} \right) = \frac{8}{a} \int \hat{F}_z r \, dr \left( \begin{array}{c} 2 \cos \psi \\ 2 \sin \psi \end{array} \right) - \frac{\nu^2 - 1}{\gamma/8} \left( \beta_c \beta_s \right) + \frac{16}{\gamma} \left( \begin{array}{c} \dot{\alpha}_x \\ \dot{\alpha}_y \end{array} \right)$$

The solution $v$ such that $E(v) = 0$ is required. The variables are $v = (\theta_{0.75} \theta_c \theta_s)^T$ for thrust and tip-path plane command; $v = (\theta_0 \beta_c \beta_s)^T$ for thrust and no-feathering plane command; $v = (C_T/\sigma \theta_c \theta_s)$ for collective pitch and tip-path plane command; $v = (C_T/\sigma \beta_c \beta_s)$ for collective pitch and no-feathering plane command. Note that since $\alpha = \epsilon_\alpha a$ is used (no stall), these equations are linear in $\theta$. However, if $\partial T/\partial \theta_{0.75}$ is small, the solution may not produce a reasonable collective for commanded thrust. A Newton–Raphson solution method is used: from $E(v_{n+1}) \approx E(v_n) + (dE/dv)(v_{n+1} - v_n) = 0$, the iterative solution is

$$v_{n+1} = v_n - C \frac{dE}{dv}(v_n)$$

where $C = f(dE/dv)^{-1}$, including the relaxation factor $f$. The derivative matrix $dE/dv$ is obtained by numerical perturbation. Convergence of the Newton–Raphson iteration is tested in terms of $|E| < \epsilon$ for each equation, where $\epsilon$ is an input tolerance.

### 12–5 Power

The rotor power consists of induced, profile, interference, and parasite terms: $P = P_i + P_o + P_t + P_p$. The parasite power (including climb/descent power for the aircraft) is obtained from the wind axis drag force: $P_p = -XV = (v^F)^T F^F$.

The interference power can be produced by interactions from the wing. The component of the wing interference velocity $v^F_{\text{ind}}$ parallel to the rotor force vector $F^F$ (velocity roughly normal to the rotor disk) produces a power change $P_i \approx v^T_{\text{ind}} F$. The component of the interference velocity perpendicular to the rotor force vector (velocity roughly in the plane of the rotor disk) produces interference through the swirl, hence $P_i \propto (V/\Omega R)|v_{\text{ind}}||F|$. Thus the interference power is calculated from

$$P_i = -K_{\text{intn}} v^T_{\text{ind}} F + K_{\text{intp}} \frac{V}{\Omega R} \sqrt{|v_{\text{ind}}||F|) \, V - (v^T_{\text{ind}} F)^2}$$

Separate interference factors $K_{\text{int}}$ are used for the two terms. $K_{\text{intp}}$ is negative for favorable interference.

The induced power is calculated from the ideal power: $P_i = \kappa P_{\text{ideal}} = \kappa f_D T v_{\text{ideal}}$. The empirical factor $\kappa$ accounts for the effects of nonuniform inflow, non-ideal span loading, tip losses, swirl, blockage, and other phenomena that increase the induced power losses ($\kappa > 1$). For a ducted fan, $f_D = f_W/2$ is introduced. The induced power at zero thrust is zero in this model (or accounted for as a profile power increment). If $\kappa$ is deduced from an independent calculation of induced power, nonzero $P_i$ at low thrust will be reflected in large $\kappa$ values.
The profile power is calculated from a mean blade drag coefficient: \( P_\rho = \rho A(\Omega R)^3 C_{\rho \alpha} \), where \( C_{\rho \alpha} = (\sigma/8) c_{d\text{mean}} F_P \). The function \( F_P(\mu, \mu_z) \) accounts for the increase of the blade section velocity with rotor edgewise and axial speed: \( C_{\rho \alpha} = \int \frac{1}{2} \sigma c_d U^3 \, dr = \int \frac{1}{2} \sigma c_d (u_x^2 + u_y^2 + u_z^2)^{3/2} \, dr \); so (from ref. 9)

\[
F_P = 4 \frac{1}{2\pi} \int_0^{2\pi} \int_0^1 ((r + \mu \sin \psi)^2 + (\mu \cos \psi)^2 + \mu_z^2)^{3/2} \, dr \, d\psi
\]

\[
\cong \sqrt{1 + V^2} \left( 1 + \frac{5}{8} V^2 + \frac{3}{8} \mu_z^4 \right) \frac{9}{16} V^4 \right) \\
+ \left( 3 \frac{\mu_z^4}{2} + \frac{3}{2} \frac{\mu_z^2}{2} \mu^2 + \frac{9}{16} \mu^4 \right) \ln \left[ \frac{\sqrt{1 + V^2} + 1}{V} \right]
\]

with \( V^2 = \mu^2 + \mu_z^2 \). This expression is exact when \( \mu = 0 \), and \( f_P \approx 4V^3 \) for large \( V \).

Two performance methods are implemented, the energy method and the table method. The induced power factor and mean blade drag coefficient are obtained from equations with the energy method, or from tables with the table method. Optionally \( \kappa \) and \( c_{d\text{mean}} \) can be specified for each flight state, superseding the performance method values.

### 12-5.1 Energy Performance Method

#### 12-5.1.1 Induced Power

The induced power is calculated from the ideal power: \( P_i = \kappa P_{\text{ideal}} = \kappa f_P T v_{\text{ideal}} \). Reference values of \( \kappa \) are specified for hover, axial cruise (propeller), and edgewise cruise (helicopter): \( \kappa_{\text{hover}}, \kappa_{\text{prop}}, \) and \( \kappa_{\text{edge}} \). Two models are implemented: constant model and standard model. The constant model uses \( \kappa = \kappa_{\text{hover}} \) if \( \mu = \mu_z = 0 \); or \( \kappa = \kappa_{\text{prop}} \) if \( |\mu| < 0.1 |\mu_z| \); or \( \kappa = \kappa_{\text{edge}} \) otherwise.

The standard model calculates an axial flow factor \( \kappa_{\text{axial}} \) from \( \kappa_{\text{hover}}, \kappa_{\text{climb}}, \) and \( \kappa_{\text{prop}} \). Let \( \Delta = C_T/\sigma - (C_T/\sigma)_{\text{ind}} \). For hover and low-speed axial climb, including a variation with thrust, the inflow factor is

\[
\kappa_h = \kappa_{\text{hover}} + k_h \Delta_h + k_h \Delta_h |\mu_z|^{X_{\mu_z}} + (\kappa_{\text{climb}} - \kappa_{\text{hover}}) \frac{2}{\pi} \tan^{-1} \left[ \left( (\mu_z/\lambda_h) / M_{\text{axial}} \right)^X_{\mu_z} \right]
\]

where \( |\mu_z|/\lambda_h = M_{\text{axial}} \) is the midpoint of the transition between hover and climb, and \( X_{\mu_z} \) is large for a fast transition. Figure 12-4 illustrates \( \kappa \) in hover (with a minimum value). Figure 12-5 shows the behavior of this function for a helicopter in climb \( (X_{\mu_z} = 0.65) \). A polynomial describes the variation with axial velocity, scaled so \( \kappa = \kappa_h \) at \( \mu_z = 0 \) and \( \kappa = \kappa_p \) at \( \mu_z = \mu_{\text{prop}} \). Including variations with thrust and shaft angle:

\[
\kappa_p = \kappa_{\text{prop}} + k_p \Delta_p + k_p \Delta_p |\mu_z|^{X_{\mu_z}} + k_{p,\mu} \mu_{\mu_z}^{X_{\mu_z}}
\]

\[
\kappa_{\text{axial}} = \kappa_h + k_{a1} |\mu_z| + S (k_{a2} \mu_z^2 + k_{a3} \mu_z ^{X_{\mu_z}})
\]

where \( S = (\kappa_p - (\kappa_h + k_{a1} \mu_{\text{prop}})) / (k_{a2} \mu_{\text{prop}} ^2 + k_{a3} \mu_{\text{prop}} ^{X_{\mu_z}}) \); \( S = 0 \) if \( k_{a2} = k_{a3} = 0 \) (not scaled); and \( k_{\text{axial}} = \kappa_h \) if \( \mu_{\text{prop}} = 0 \). A polynomial describes the variation with edgewise advance ratio, scaled so \( \kappa = \kappa_{\text{axial}} \) at \( \mu = 0 \) and \( \kappa = f_\alpha f_\text{off} \kappa_{\text{edge}} \) at \( \mu = \mu_{\text{edge}} \). Thus the induced power factor is

\[
\kappa = \kappa_{\text{axial}} + k_{e1} \mu + S (k_{e2} \mu_z^2 + k_{e3} \mu_z ^{X_{\mu_z}})
\]

where \( S = (f_\alpha f_\text{off} \kappa_{\text{edge}} - (\kappa_{\text{axial}} + k_{e1} \mu_{\text{edge}})) / (k_{e2} \mu_{\text{edge}} ^2 + k_{e3} \mu_{\text{edge}} ^{X_{\mu_z}}) \); \( S = 0 \) if \( k_{e2} = k_{e3} = 0 \) (not scaled); and \( \kappa = \kappa_{\text{axial}} \) if \( \mu_{\text{edge}} = 0 \). The function \( f_\alpha = 1 - k_{e0} \mu_z \) accounts for the influence of angle of
Figure 12-4. Induced power factor for rotor in hover.

Figure 12-5. Induced power factor for rotor in axial flight.

attack ($\mu_z/\mu$) or rotor drag ($C_X$). The function $f_{off} = 1 - k_{o1}(1 - e^{-k_2\sigma})$ accounts for the influence of lift offset, $o_x = rM_x/TR = (K_{hub}/TR)\beta_s$. Figure 12-6 illustrates $\kappa$ in edgewise flight. Minimum and maximum values of the induced power factor, $\kappa_{min}$ and $\kappa_{max}$, are also specified.

12-5.1.2 Profile Power

The profile power is calculated from a mean blade drag coefficient: $C_{P_o} = (\sigma/8)c_{dmean}F_P$. Since the blade mean lift coefficient is $c_L \cong 6C_T/\sigma$, the drag coefficient is estimated as a function of blade loading $C_T/\sigma$ (using thrust-weighted solidity). With separate estimates of the basic, stall, and compressibility drag, the mean drag coefficient is:

$$c_{dmean} = \chi S (c_{dbasic} + c_{dSTALL} + c_{dCOMP})$$
where $\chi$ is a technology factor. The factor $S = (R_{\text{ref}}/Re)^{X_{\text{ref}}}$ accounts for Reynolds number effects on the drag coefficient; typically $X_{\text{ref}} = 0.2$ for a turbulent boundary layer. $Re$ is based on the thrust-weighted chord, $0.75V_{\text{tip}}$, and the flight state; $R_{\text{ref}}$ corresponds to the input $c_d$ information. The following models are implemented for the basic drag:

a) Array model: The basic drag $c_d^{\text{basic}}$ is input as a function of $C_T/\sigma$; the array is linearly interpolated.

b) Equation model: The basic drag $c_d^{\text{basic}}$ is a quadratic function of $C_T/\sigma$, plus an additional term allowing faster growth at high (sub-stall) angles of attack. Let $\Delta = |C_T/\sigma| - (C_T/\sigma)^{D_{\text{min}}}$, where $(C_T/\sigma)^{D_{\text{min}}}$ corresponds to the minimum drag and $\Delta_{\text{sep}} = |C_T/\sigma| - (C_T/\sigma)^{\text{sep}}$. Values of the basic drag equation are specified for helicopter (hover and edgewise) and propeller (axial climb and cruise) operation:

\[
\begin{align*}
c_{dh} &= d_{0\text{hel}} + d_{1\text{hel}}\Delta + d_{2\text{hel}}\Delta^2 + d_{\text{sep}}X_{\text{sep}} \\
c_{dp} &= d_{0\text{prop}} + d_{1\text{prop}}\Delta + d_{2\text{prop}}\Delta^2 + d_{\text{sep}}X_{\text{sep}} + d_{\alpha\mu}\mu^2|\mu|^X_{\alpha}
\end{align*}
\]

The separation term is present only if $\Delta_{\text{sep}} > 0$. The helicopter and propeller values are interpolated as a function of $\mu_z$:

\[
c_{d^{\text{basic}}} = c_{dh} + (c_{dp} - c_{dh})\frac{2}{\pi}\tan^{-1}(|\mu_z|/\lambda_h) + d_{f1}\mu + d_{f2}\mu^{X_f}
\]

so $|\mu_z|/\lambda_h = 1$ is the midpoint of the transition. The last terms are the effect of edgewise flow.

The stall drag increment represents the rise of profile power caused by the occurrence of significant stall on the rotor disk. Let $\Delta_s = |C_T/\sigma| - (f_s/f_0,f_{\text{off}})(C_T/\sigma)$, ($f_s$ is an input factor). The function $f_\alpha = 1 - d_{s\alpha}\mu$ accounts for the influence of angle of attack ($\mu_z/\mu$) or rotor drag ($C_X$). The function $f_{\text{off}} = 1 - d_{a}(1 - e^{-d_{a}\mu})$ accounts for the influence of lift offset, $a = rM_{\text{z}}/TR = (K_{\text{hub}}/TR)\beta$. Then $c_{d\text{stall}} = d_{s1}\Delta_{X_1} + d_{s2}\Delta_{X_2}$ (zero if $\Delta_s \leq 0$). The blade loading at which the stall affects the entire rotor power, $(C_T/\sigma)_{s}$, is an input function of the velocity ratio $V = \sqrt{\mu^2 + \mu_z^2}$.

The compressibility drag increment depends on the advancing tip Mach number $M_{\text{off}}$, and the tip airfoil thickness-to-chord ratio $\tau$. The following models are implemented:
a) Drag divergence model: Let \( \Delta M = M_{at} - M_{dd} \), where \( M_{dd} \) is the drag divergence Mach number of the tip section. Then the compressibility increment in the mean drag coefficient is

\[
c_{d\text{comp}} = d_{m1} \Delta M + d_{m2} \Delta M X_m
\]
(ref. 10). \( M_{dd} \) is a function of the advancing tip lift coefficient, \( c_{\ell(1,90)} \). The advancing tip lift is estimated from \( \alpha_{(1,90)} = (\theta, 0.25\theta_L + \theta_s - (\lambda - \beta_s)/(1 + \mu)) \approx 1.6(1 - 2.97\mu + 2.21\mu^3)(6C_T/\sigma a) + 0.25\theta_t \) (zero above \( \mu = 0.6 \)). Then the Korn expression (ref. 11) gives \( M_{dd} \) for small lift coefficient:

\[
M_{dd} = \kappa_A - \kappa|c| - \tau = M_{dd0} - \kappa|c|
\]
where \( M_{dd0} \) is the drag divergence Mach number at zero lift, and typically \( \kappa = 0.16 \).

b) Similarity model: From transonic small-disturbance theory (refs. 12–13), the scaled wave drag must be a function only of \( K_1 = (M_{at}^2 - 1)/[M_{at}^2(1 + \gamma)]^{1/3} \). An approximation for the wave drag increment is

\[
\Delta c_d = \frac{\tau^{5/3}}{[M_{at}^2(1 + \gamma)]^{1/3}} D(K_1) = \frac{\tau^{5/3}}{[M_{at}^2(1 + \gamma)]^{1/3}} 1.774(K_1 + 1.674)^{5/2}
\]
(constant for \( K_1 > -0.2 \)). Integration of \( \Delta c_d \) over the rotor disk gives the compressibility increment in the profile power. Following Harris, the resulting compressibility increment in the mean drag coefficient is approximately:

\[
c_{d\text{comp}} = 1.52 f(K_1 + 1)^2[(1 + \mu)\tau]^{5/2}(1 + \gamma)^{1/2}
\]
including the input correction factor \( f \); \( c_{d\text{comp}} \) is zero for \( K_1 < -1 \), and constant for \( K_1 > -0.2 \).

Figure 12-7 shows typical stall functions \( (C_T/\sigma)_s \) for two rotors with different stall characteristics, designated high-stall and low-stall, resulting from design features such as different airfoils. Figure 12-7 also shows, for reference, typical helicopter rotor steady and transient load limits. Figure 12-8 illustrates the mean drag coefficient in hover, showing \( c_{dh} \) with and without the separation term, and the total for the high-stall and low-stall cases. Figure 12-9 illustrates the mean drag coefficient in forward flight, showing the compressibility term \( c_{d\text{comp}} \), and the growth in profile power with \( C_T/\sigma \) and \( \mu \) as the stall drag increment increases.

### 12.5.1.3 Twin and Multiple Rotors

For twin rotors, the induced power is determined by the induced velocity of the rotor system, not the individual rotors. The induced power is still obtained using \( P_i = \kappa P_{i\text{ideal}} = \kappa f_D TV_{i\text{ideal}} \) for each rotor, but the ideal induced velocity is calculated for an equivalent thrust \( C_{Te} \) based on the thrust and geometry of both rotors. The profile power calculation is not changed for twin rotors.

In hover, the twin rotor induced velocity is \( v_i = \kappa_{\text{twin}} \sqrt{T/2\rho A_p} \), from the total thrust \( T \) and the projected disk area \( A_p = (2 - m)A \). The overlap fraction \( m \) is calculated from the rotor hub separation \( \ell \). A correction factor for the twin rotor ideal power is also included. For a coaxial rotor, typically \( \kappa_{\text{twin}} \approx 0.90 \). So the ideal inflow is calculated for \( C_{Te} = (C_{T1} + C_{T2})/(2 - m) \).

In forward flight, the induced velocity of a coaxial rotor is \( v_i = \kappa_{\text{twin}} T/(2\rho A_v) \), from the total thrust \( T \) and a span of \( 2R \). The correction factor for ideal induced power (biplane effect) is \( \kappa_{\text{twin}} \approx 0.88 \) to 0.81 for rotor separations of 0.06\( D \) to 0.12\( D \). The ideal inflow is thus calculated for \( C_{Te} = C_{T1} + C_{T2} \). The induced velocity of side-by-side rotors is \( v_i = \kappa_{\text{twin}} T/(2\rho A_v) \), from the total thrust \( T \) and a span of \( 2R + \ell \), hence \( A_v = A(1+\ell/2R)^2 \). The ideal inflow is thus calculated for \( C_{Te} = (C_{T1} + C_{T2})/(1+\ell/2R)^2 \).
The induced velocity of tandem rotors is \( v_F = \kappa_{twin} \left( \frac{T_F}{2\rho AV} + x_R T_R/(2\rho AV) \right) \) for the front rotor and \( v_R = \kappa_{twin} \left( \frac{T_R}{2\rho AV} + x_F T_F/(2\rho AV) \right) \) for the rear rotor. For large separation, \( x_R \approx 0 \) and \( x_F \approx 2 \); for the coaxial limit \( x_R = x_F = 1 \) is appropriate. Here \( x_R = m \) and \( x_F = 2 - m \) is used.

To summarize, the model for twin rotor ideal induced velocity uses \( C_{T_x} = x_1 C_{T_1} + x_2 C_{T_2} \) and the correction factor \( \kappa_{twin} \). In hover, \( x_h = 1/(2 - m) \); in forward flight of coaxial and tandem rotors, \( x_f = 1 \) for this rotor and \( x_f = m \) or \( x_f = 2 - m \) for the other rotor; in forward flight of side-by-side rotors,
\[ x_f = \frac{1}{1 + \ell/2R}^2 \quad (x = 1/2 \text{ if there is no overlap, } \ell/2R > 1) \]. The transition between hover and forward flight is accomplished using

\[ x = \frac{x_f \mu^2 + x_h \lambda \mu^2}{\mu^2 + C \lambda^2} \]

with typically \( C = 1 \) to \( 4 \). This transition is applied to \( x \) for both rotors, and to \( \kappa_{\text{twin}} \).
With a coaxial rotor in hover, the lower rotor acts in the contracted wake of the upper rotor. Momentum theory gives the ideal induced power for coaxial rotors with large vertical separation (ref. 14): 
\[ P_u = T_u v_u, \quad \tau \, v_u^2 = T_u/2\rho A \text{ for the upper rotor; and } P_l = (\tilde{\alpha}s/\sqrt{\tau}) T_l v_l, \quad \tau \, v_l^2 = T_l/2\rho A \text{ for the lower rotor.} \]
Here \( \tau = T_l/T_u \); \( \tilde{\alpha} \) is the average of the disk loading weighted by the induced velocity, hence a measure of nonuniform loading on the lower rotor (\( \tilde{\alpha} = 1.05 \) to \( 1.10 \) typically); and the momentum theory solution is
\[
\frac{\tilde{\alpha}s}{\sqrt{\tau}} = \frac{1}{2\tau^{3/2}} \left( \sqrt{1 + 4(1 + \tau)^2 \tilde{\alpha}^2} - 1 \right)
\]
The optimum solution for equal power of the upper and lower rotors is \( \tilde{\alpha}s\tau = 1 \), giving \( \tau = T_l/T_u \cong 2/3 \). Hence for the coaxial rotor in hover the ideal induced velocity is calculated from \( C_{Te} = C_{Tu} \) for the upper rotor and from \( C_{Te} = (\tilde{\alpha}s/\sqrt{\tau})^2 C_{Te} \) for the lower rotor, with \( \kappa_{\text{twin}} = 1 \). Thus \( \tau_h = 1/(2-\mu) = 1/2 \) and the input hover \( \kappa_{\text{twin}} \) is not used, unless the coaxial rotor is modeled as a tandem rotor with zero longitudinal separation.

For multiple rotors, the model for ideal induced velocity is extended to use \( C_{Te} = \sum x_n C_{Te} \) (sum over all rotors, scaled with square of tip speed), and the correction factor \( \kappa_{\text{twin}} \). The thrust factors \( x_n \) are input for hover and forward flight.

### 12-5.2 Table Performance Method

The induced power is calculated from the ideal power: 
\[ P_i = \kappa P_{\text{ideal}} = \kappa f_D T v_{\text{ideal}}. \]
The induced power factor \( \kappa \) is obtained from an input table (linearly interpolated) that can be a function of up to three independent variables. Optionally \( \kappa \) from the equations can be retained as an increment to the table value.

The profile power is calculated from a mean blade drag coefficient: 
\[ P_o = \rho A(\Omega R)^3 C_{Po} = \rho A(\Omega R)^3 \frac{2}{\pi} c_{d\text{mean}} F_P. \]
The mean drag coefficient \( c_{d\text{mean}} \), or alternatively \( c_{d\text{mean}} F_P = 8C_{Po}/\sigma \), is obtained from an input table (linearly interpolated) that can be a function of up to three independent variables. Optionally \( c_{d\text{mean}} \) from the equations can be retained as an increment to the table value.

The independent variables can be edgewise advance ratio \( \mu \) (hub plane or tip-path plane), axial velocity ratio \( \mu_z \) (hub plane or tip-path plane), shaft angle of attack \( \alpha = \tan^{-1}(\mu_z/\mu) \) (hub plane or tip-path plane), blade loading \( C_T/\sigma \), lift offset \( M_z/TR \), or advancing tip Mach number \( M_{at} \).

### 12-6 Power Required and Reaction Drive

The total rotor power required is \( P_{\text{req}} = P_i + P_o + P_l + P_p \). In most helicopter designs the power is delivered to the rotor by a mechanical drive, through the rotor shaft torque. Such designs require a transmission and a means for balancing the main-rotor torque. The shaft power is \( P_{\text{shaft}} = P_{\text{req}} \), which contributes to the propulsion group power required, \( P_{\text{reqPG}} \), and produces a torque on the aircraft.

An alternative is to supply the power by a jet reaction drive of the rotor, using cold or hot air ejected out of the blade tips or trailing edges. Helicopters have also been designed with ram jets on the blade tips, or with jet flaps on the blade trailing edges that use compressed air generated in the fuselage. Since there is no torque reaction between the helicopter and rotor (except for the small bearing friction), no transmission or antitorque device is required, resulting in a considerable weight saving. With a jet reaction drive, the propulsion system is potentially lighter and simpler, although the aerodynamic and thermal efficiency are lower. The helicopter must still have a mechanism for yaw control. With reaction drive the shaft power is \( P_{\text{shaft}} = P_{\text{req}} - P_{\text{react}} \), where the reaction power \( P_{\text{react}} \) contributes to the engine
group or jet group power required. The reaction drive produces a force \( F_{\text{react}} \) on the rotor blades at effective radial station \( r_{\text{react}} \), so \( P_{\text{react}} = \Omega r_{\text{react}} F_{\text{react}} \). Momentum balance gives the total force

\[
F = \sum \left[ \dot{m}(V_{\text{react}} - \Omega r_{\text{react}}) e_{\psi} - \dot{m}V e_{x} \right]
\]

where \( e_{\psi} \) and \( e_{x} \) are unit vectors perpendicular to the blade and in the free stream direction; and the sum is over all blades. The average force in the nonrotating frame is the drag of the inlet momentum (\( \dot{m}_{\text{react}} V \)), which is accounted for in the engine group or jet group model. The mean in the rotating frame gives the total jet force required

\[
F_{\text{react}} = \dot{m}_{\text{react}} (V_{\text{react}} - \Omega r_{\text{react}})
\]

The engine group or jet group performance includes the blade duct and nozzle, perhaps even with tip burning. Optionally the reaction power can be set equal to the rotor power required \( (P_{\text{react}} = P_{\text{req}}) \), so \( P_{\text{shaft}} = 0 \). If the reaction drive is turned off \( (P_{\text{react}} = 0 \) and \( F_{\text{react}} = 0 \)), then the rotor must be trimmed such that \( P_{\text{req}} = 0 \).

### 12–7 Performance Metrics

Several performance metrics are calculated for each rotor. The induced power factor is \( \kappa = P_{i}/P_{\text{ideal}} \). The rotor mean drag coefficient is \( c_{d} = (8C_{p}/\sigma)/F_{P} \), using the function \( F(\mu, \mu_{z}) \) given previously. The rotor effective lift-to-drag ratio is a measure of the induced and profile power: \( L/D = V/(P_{i} + P_{a}) \). The hover figure of merit is \( M = T f_{D} v/P \). The propeller propulsive efficiency is \( \eta = TV/P \). These two metrics can be combined as a momentum efficiency: \( \eta_{\text{mom}} = T(V+w/2)/P \), where \( w/2 = f_{W} v/2 = f_{D} v \).

### 12–8 Interference

The rotor can produce aerodynamic interference velocities at the other components (fuselage, wings, and tails). The induced velocity at the rotor disk is \( \kappa v_{1} \), acting opposite the thrust \( (z\text{-axis of tip-path plane axes}) \). So \( v_{\text{ind}}^{P} = -k^{P}\kappa v_{1} \), and \( v_{\text{ind}}^{F} = C^{FP} v_{\text{ind}}^{P} \). The total velocity of the rotor disk relative to the air consists of the aircraft velocity and the induced velocity from this rotor: \( v_{\text{total}}^{P} = v^{P} - v_{\text{ind}}^{P} \). The direction of the wake axis is thus \( e_{\text{w}}^{P} = -C^{PF} v_{\text{total}}^{P}/v_{\text{total}}^{P} \). The angle of the wake axis from the thrust axis is \( \chi = \cos^{-1} \left( (k^{P})^{T} e_{\text{w}}^{P} \right) \).

The interference velocity \( v_{\text{ind}}^{F} \) at each component is proportional to the induced velocity \( v_{\text{ind}}^{F} \) (hence is in the same direction), with factors accounting for the stage of wake development and the position of the component relative to the rotor wake. The far wake velocity is \( w = f_{W} v_{1} \), and the contracted wake area is \( A_{c} = \pi R_{c}^{2} = A/f_{A} \). The solution for the ideal inflow gives \( f_{W} \) and \( f_{A} \). For an open rotor, \( f_{W} = 2 \). For a ducted rotor, the inflow and wake depend on the wake area ratio \( f_{A} \), or on the ratio of the rotor thrust to total thrust: \( f_{T} = T_{\text{rotor}}/T \). The corresponding velocity and area ratios at an arbitrary point on the wake axis are \( f_{w} \) and \( f_{a} \), related by

\[
f_{a} = \sqrt{\frac{\mu^{2} + (\mu z + f_{w} \lambda_{z})^{2}}{(f_{V} \mu)^{2} + (f_{V} \mu z + \lambda_{z})^{2}}}
\]

Vortex theory for hover gives the variation of the induced velocity with distance \( z \) below the rotor disk:

\[
v = v(0) \left( 1 + \frac{z/R}{\sqrt{1 + (z/R)^{2}}} \right)
\]
With this equation the velocity varies from zero far above the disk to \( v = 2v(0) \) far below the disk. To use this expression in edgewise flow and for ducted rotors, the distance \( z/R \) is replaced by \( \zeta w/tR \), where \( \zeta w \) is the distance along the wake axis, and the parameter \( t \) is introduced to adjust the rate of change (\( t \) small for faster transition to far wake limit). Hence the velocity inside the wake is \( f_wv_i \), where

\[
 f_w = f_Wf_z = \begin{cases} 
 1 + \frac{\zeta w/tR}{\sqrt{1+(\zeta w/tR)^2}} & \zeta w < 0 \\
 1 + (f_W - 1) \frac{\zeta w/tR}{\sqrt{1+(\zeta w/tR)^2}} & \zeta w > 0 
\end{cases}
\]

and the contracted radius is \( R_c = R/\sqrt{T_a} \).

The wake is a skewed cylinder, starting at the rotor disk and with the axis oriented by \( e^P_w \). The interference velocity is required at the position \( z^P_B \) on a component. Whether this point is inside or outside the wake cylinder is determined by finding its distance from the wake axis, in a plane parallel to the rotor disk. The position relative to the rotor hub is \( \xi^P_B = C^{PF}(z^P_B - z^P_{hub}) \); the corresponding point on the wake axis is \( \zeta_A = e^P_w\zeta_w \). Requiring \( \zeta^P_B \) and \( \zeta_A \) have the same \( z \) value in the tip-path plane axes gives

\[
 \zeta_w = \frac{(k^P)^T C^{PF}(z^P_B - z^P_{hub})}{(k^P)^T e^P_w}
\]

from which \( f_z, f_w, f_a, \) and \( R_c \) are evaluated. The distance \( r \) from the wake axis is then

\[
 r^2 = \left((i^P)^T (\xi^P_B - \xi^P_A)\right)^2 + \left((j^P)^T (\xi^P_B - \xi^P_A)\right)^2
\]

The transition from full velocity inside the wake to zero velocity outside the wake is accomplished in the distance \( sR_c \), using

\[
 f_r = \begin{cases} 
 1 & r \leq R_c \\
 1 - (r - R_c)/(sR_c) & r \geq (1 + s)R_c 
\end{cases}
\]

(\( s = 0 \) for an abrupt transition, \( s \) large for always in wake).

The interference velocity at the component (at \( z^P_B \)) is calculated from the induced velocity \( v^F_{\text{ind}} \), the factors \( f_Wf_z \) accounting for axial development of the wake velocity, the factor \( f_r \) accounting for immersion in the wake, and an input empirical factor \( K_{\text{int}} \):

\[
 v^F_{\text{int}} = K_{\text{int}} f_Wf_zf_rf_i v^F_{\text{ind}}
\]

An additional factor \( f_i \) for twin rotors is included. Optionally the development along the wake axis can be a step function (\( f_Wf_z = 0, 1, f_W \) above the rotor, on the rotor disk, and below the rotor disk, respectively); nominal (\( t = 1 \)); or use an input rate parameter \( t \). Optionally the wake immersion can use the contracted radius \( R_c \) or the uncontracted radius \( R \); can be a step function (\( s = 0 \), so \( f_r = 1 \) and 0 inside and outside the wake boundary); can be always immersed (\( s = \infty \) so \( f_r = 1 \) always); or can use an input transition distance \( s \). Optionally the interference factor \( K_{\text{int}} \) can be reduced from an input value at low speed to zero at high speed, with linear variation over a specified speed range.

To account for the extent of the wing or tail area immersed in the rotor wake, the interference velocity is calculated at several points along the span and averaged. The increment in position is \( \Delta z^P_B = C^{FB}(0 \Delta y)^T \), \( \Delta y = (b/2)(1 + (2t - 1)/N) \) for \( i = 1 \) to \( N \); where \( b \) is the wing span. The
average interference is calculated separately for each wing panel (left and right), by interpolating the interference velocity at \( N \) points along the wing to \( N/2 \) points along the panel span.

For twin main-rotors (tandem, side-by-side, or coaxial), the performance may be calculated for the rotor system, but the interference velocity is still calculated separately for each rotor, based on its disk loading. At the component, the velocities from all rotors are summed, and the total used to calculate the angle of attack and dynamic pressure. This sum must give the interference velocity of the twin rotor system, which requires the correction factor \( f_i \). Consider differential momentum theory to estimate the induced velocity of twin rotors in hover. For the first rotor, the thrust and area in the non-overlap region are \((1 - m)T_1\) and \((1 - m)A\), hence the induced velocity is \( v_1 = \kappa\sqrt{T_1/(2\rho A)}\); similarly \( v_2 = \sqrt{T_2/(2\rho A)}\). In the overlap region the thrust and area are \(mT_1 + mT_2\) and \(mA\), hence the induced velocity is \( v_m = \kappa\sqrt{(T_1 + T_2)/(2\rho A)}\). So for equal thrust, the velocity in the overlap region (everywhere for the coaxial configuration) is \(\sqrt{2}\) larger. The factor \(K_T\) is introduced to adjust the overlap velocity: \(v_m = \kappa(K_T/\sqrt{2})\sqrt{(T_1 + T_2)/(2\rho A)}\). The interference velocities are calculated separately for the two rotors, with the correction factor \(f_i\): \(v_{int1} = f_i\kappa\sqrt{T_1/(2\rho A)}\) and \(v_{int2} = f_i\kappa\sqrt{T_2/(2\rho A)}\). The sum \(v_{int1} + v_{int2}\) must take the required value. Below the non-overlap region, the component is in the wake of only one of the rotors, so the interference velocity from the other rotor is zero, and thus \(f_i = 1\). Below the overlap region, the component is in the wake of both rotors, and the sum of the interference velocities equals \(v_m\) if

\[
f_{th} = \frac{K_T/\sqrt{2}}{\sqrt{T_1} + \sqrt{T_2}}
\]

where \(\tau_n = T_n/(T_1 + T_2)\) is the thrust ratio. For equal thrusts, \(f_{th} = K_T/2\); or \(f_{th} = 1/\sqrt{2}\) for the nominal velocity. The expression \(f_i = f_{th}\cos^2\chi + \sin^2\chi\) gives the required correction factor, with \(f_i = 1\) in edgewise flight. Optionally the correction for twin rotors can be omitted \((f_i = 1)\); nominal \((K_T = \sqrt{2})\); or use an input velocity factor in the overlap region \((K_T)\).

### 12–9 Drag

The rotor component includes drag forces acting on the hub and spinner (at \(z_{hub}^F\)) and on the pylon (at \(z_{pylon}^F\)). The component drag contributions must be consistent. In particular, a rotor with a spinner (such as on a tiltrotor aircraft) would likely not have hub drag. The pylon is the rotor support and the nacelle is the engine support. The drag model for a tiltrotor aircraft with tilting engines would use the pylon drag (and no nacelle drag), since the pylon is connected to the rotor shaft axes; with non-tilting engines it would use the nacelle drag as well.

The body axes for the drag analysis are rotated about the \(y\)-axis relative to the rotor shaft axes: \(C_{BF}^{BS} = C_{BS}^{SF}\), where \(C_{BS} = Y_{\theta_{ref}}\). The pitch angle \(\theta_{ref}\) can be input, or the rotation appropriate for a helicopter rotor or a propeller can be specified.

a) Consider a helicopter rotor, with the shaft axes oriented \(z\)-axis up and \(x\)-axis downstream. It is appropriate that the angle of attack is \(\alpha = 0\) for forward flight and \(\alpha = -90\) deg for hover, meaning that the body axes are oriented \(z\)-axis down and \(x\)-axis forward. Hence \(\theta_{ref} = 180\) deg.

b) Consider a propeller or tiltrotor, with the shaft axes oriented \(z\)-axis forward and \(x\)-axis up. It is appropriate that the angle of attack is \(\alpha = 0\) in cruise and \(\alpha = 90\) deg for helicopter mode (with a tilting pylon), meaning that the body axes are oriented \(z\)-axis down and \(x\)-axis forward. Hence \(\theta_{ref} = 90\) deg.
The aerodynamic velocity relative to the air is calculated in component axes, \(v^B\). The angle of attack \(\alpha\) and dynamic pressure \(q\) are calculated from \(v^B\). The reference areas for the drag coefficients are the rotor disk area \(A = \pi R^2\) (for hub drag), pylon wetted area \(S_{\text{pylon}}\), ducted wetted area \(2S_{\text{duct}}\), and spinner wetted area \(S_{\text{spin}}\); these areas are input or calculated as described previously.

The hub drag can be fixed, specified as a drag area \(D/q\); or the drag can be scaled, specified as a drag coefficient \(C_D\) based on the rotor disk area \(A = \pi R^2\); or the drag can be estimated based on the gross weight, using a squared-cubed relationship or a square-root relationship. Based on historical data, the maximum takeoff gross weight per lifting rotor; units of \(k\) are ft

\[
\Delta = \frac{F}{D}\text{ is the maximum takeoff gross weight per lifting rotor; units of \(k\) are ft-lb or m\(^2\)N.}
\]

For the squared-cubed relationship: \((D/q)_{\text{hub}} = k((W_{\text{MTO}}/N_{\text{rotor}})/1000)^{2/3} (W_{\text{MTO}}/N_{\text{rotor}})\) is the maximum takeoff gross weight per lifting rotor; units of \(k\) are \(\text{ft}^2/\text{lb}^{2/3}\) or \(\text{m}^2/\text{N}^{2/3}\). Based on historical data, \(k = 1.4\) for typical hubs, \(k = 0.8\) for current low-drag hubs, and \(k = 0.5\) for faired hubs (English units). For the square-root relationship: \((D/q)_{\text{hub}} = k\sqrt{W_{\text{MTO}}/N_{\text{rotor}}} (W_{\text{MTO}}/N_{\text{rotor}})\) is the maximum takeoff gross weight per lifting rotor; units of \(k\) are \(\text{ft}^2/\text{lb}^{1/2}\) or \(\text{m}^2/\text{N}^{1/2}\); based on historical data (ref. 15), \(k = 0.074\) for single rotor helicopters, \(k = 0.049\) for tandem rotor helicopters (probably a blade number effect), \(k = 0.038\) for hingeless rotors, and \(k = 0.027\) for faired hubs (English units). To handle multi-rotor aircraft, the scaling weight \(w = W_{\text{MTO}}/N_{\text{rotor}}\) is calculated as for disk loading: \(w = f_W W_{\text{MTO}}\) for main-rotors or \(w = fT\) for antitorque and auxiliary-thrust rotors.

The hub vertical drag can be fixed, specified as a drag area \(D/q\); or the drag can be scaled, specified as a drag coefficient \(C_D\) based on the rotor disk area \(A = \pi R^2\).

The pylon forward flight drag and vertical drag are specified as drag area or drag coefficient, based on the pylon wetted area. The duct forward flight drag and vertical drag are specified as drag area or drag coefficient, based on the duct wetted area. The spinner drag is specified as drag area or drag coefficient, based on the spinner wetted area.

The drag coefficient for the hub or pylon or duct at angle of attack \(\alpha\) is

\[
C_D = C_{D0} + (C_{DV} - C_{D0})\sin|\alpha| X_d
\]

Optionally the variation can be quadratic \((X_d = 2)\). For sideward flight, \(C_{D\text{hub}} = C_{D0}\) for the hub, \(C_{D\text{pylon}} = C_{DV}\) for the pylon, and \(C_{D\text{duct}} = C_{DV}\) for the duct. Then the total component drag force is

\[
D = qAC_{D\text{hub}} + qS_{\text{pylon}}C_{D\text{pylon}} + qS_{\text{duct}}C_{D\text{duct}} + qS_{\text{spin}}C_{D\text{spin}}
\]

The dynamic pressure and position of the hub are used for the duct and spinner. The force and moment produced by the drag are

\[
F^F = \sum e_d D
\]

\[
M^F = \sum \Delta z^F F^F
\]

where \(\Delta z^F = z^F - z_{cg}^F\) (separate locations are defined for the rotor hub and for the pylon), and \(e_d\) is the drag direction. The velocity relative to the air gives \(e_d = -v^F/|v^F|\) (no interference).

12–10 Weights

The rotor configuration determines where the weights occur in the weight statement, as summarized in table 12-3. The rotor group consists of blade assembly, hub and hinge, fairing/spinner, blade fold structure, inter-rotor shaft, rotor support, and duct. The tail-rotor (in empennage group) or the propeller/fan installation (in propulsion group) consists of blade assembly, hub and hinge, rotor support, and duct.
There are separate weight models for main-rotors, tail-rotors, and auxiliary-thrust systems (propellers). The tail-rotor model requires a torque calculated from the drive system rated power and main-rotor rotational speed: $Q = P_{DS\text{limit}}/\Omega_{mr}$. The auxiliary-thrust model requires the design maximum thrust of the propeller. The engine section or nacelle group includes the engine support weight and pylon support weight; these must be consistent with the use of the rotor support structural weight.

<table>
<thead>
<tr>
<th>configuration</th>
<th>weight statement</th>
<th>weight model</th>
<th>performance model</th>
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</thead>
<tbody>
<tr>
<td>main-rotor</td>
<td>rotor group</td>
<td>rotor</td>
<td>rotor</td>
</tr>
<tr>
<td>tail-rotor</td>
<td>empennage group</td>
<td>tail-rotor</td>
<td>rotor</td>
</tr>
<tr>
<td>propeller</td>
<td>propulsion group</td>
<td>rotor, aux thrust</td>
<td>rotor</td>
</tr>
</tbody>
</table>

The flap moment of inertia $I_b$ and the Lock number $\gamma = \rho ac R^4/I_b$ are required for the blade motion solution. Several options are implemented to calculate $I_b$. The Lock number can be specified, and then $I_b = \rho ac R^4/\gamma$ used, independent of the blade weight; this is the only option for the tail–rotor and auxiliary-thrust weight models, which do not give separate blade and hub weight estimates. The moment of inertia $I_b$ can be calculated from the blade weight and the weight distribution. The Lock number can be specified, hence $I_b = \rho ac R^4/\gamma$, and then mass added to the blade to achieve this value. An autorotation index $AI = KE/P = \frac{1}{2}NI_b\Omega^2/P$ can be specified, hence the required $I_b$, and then mass added to the blade to achieve this value. Reference 16 describes this and other autorotation indices; $AI = KE/P \geq 3$ sec gives good autorotation characteristics for small helicopters.

In order to increase the moment of inertia, a tip weight $W_t$ can be added to each blade at radial station $r_1$. Thus the total blade weight is $W_b = \chi w_b + dW_b + (1 + f)W_t N$ (lb or kg); where $w_b$ is the blade weight estimate, $\chi$ the technology factor, $dW_b$ a specified weight increment; and the factor $f$ accounts for the blade weight increase required by the centrifugal force due to $W_t$. The mass per blade is $M_b = W_b/N$ (slug or kg), or $M_{b0}$ without the tip weight. The blade moment of inertia is

$$I_b = R^2(r_2^2(M_{b0} + fM_t) + r_1^2M_t) = I_{b0} + R^2(r_1^2 + fr_2^2)M_t$$

where $r_2$ is the radius of gyration of the distributed mass. Typically $r_2 \approx 0.6$; $r_2 = 1/\sqrt{3} = 0.577$ for uniform mass distribution. If the required moment of inertia $I_b$ is greater than $I_{b0}$, the tip mass $M_t$ is needed. Additional mass is required inboard to react the centrifugal force increase due to $M_t$. This additional mass is less effective than $M_t$ at increasing $I_b$. Assume a fraction $a$ of the blade mass reacts the centrifugal force $F$, so $\Delta M/M_{b0} = a \Delta F/F_0$. The reference values are $M_{b0} = R \int m \, dr$ and $F_0 = \Omega^2 R^2 \int rm \, dr = \Omega^2 R r_1 M_{b0}$, where $r_1 \approx \frac{1}{2}$. Then

$$\Delta M = \frac{a M_{b0}}{F_0} \Delta F = a \frac{1}{\Omega^2 R r_1} (\Omega^2 R r_1 \Delta M + \Omega^2 R r_1 M_t) = a \left( \Delta M + \frac{r_1}{r_1} M_t \right) = \frac{ar_1}{r_1} M_t = f M_t$$

With $a \approx \frac{1}{4}$, $f \approx 1$. The tip mass required to produce $\Delta I_b = I_b - I_{b0}$ is $M_t = \Delta I_b/(R^2(r_1^2 + fr_2^2))$, and the total blade weight increment is $\Delta W_b = (1 + f)M_t N$ (lb or kg).

### 12–11 References


The aircraft can have one or more wings, or no wings.

13–1 Geometry

The wing is described by planform area $S$, span $b$, mean chord $c = S/b$, and aspect ratio $AR = b^2/S$. These parameters are for the entire wing. The geometry is specified in terms of two of the following parameters: $S$ or wing loading $W/S$, $b$ (perhaps calculated from other geometry), $c$, $AR = b^2/S$. With more than one wing, the wing loading is obtained from an input fraction of design gross weight, $W = f_W W_D$. Optionally the span can be calculated from a specified ratio to the span of another wing; or the span can be calculated from a specified ratio to the radius of a designated rotor, $b = 2fR$. Optionally the wing span can be calculated from an appropriate specification of all wing panel widths.

Optionally for the tiltrotor configuration, the wing span can be calculated from the fuselage and rotor geometry:

$$b = 2\left(f_R + d_{fus}\right) + w_{fus},$$

where $R$ is the rotor radius (cruise value for variable-diameter rotor), $d_{fus}$ the rotor-fuselage clearance, and $w_{fus}$ the fuselage width. Note that the corresponding option for the rotor hub position is $y_{hub} = \pm(fR + d_{fus} + 1/2w_{fus})$. Optionally the wing span can be calculated from the rotor hub position: $b = 2|y_{hub}|$ (regardless of how the rotor position is determined). As implemented, symmetry is not assumed; rather the radius or hub position of the outermost designated rotors is used.

The wing is at position $z_F$, where the aerodynamic forces act. The component axes are the aircraft body axes, $C_{BF} = I$.

The wing planform is defined in terms of one or more wing panels (fig. 13-1). Symmetry of the wing is assumed. The number of panels is $P$, with the panel index $p = 1$ to $P$. The wing span station $\eta$ is scaled with the semi-span: $y = \eta(b/2)$, $\eta = 0$ to 1. Each panel is a trapezoid, with a straight aerodynamic center and linear taper. The aerodynamic center locus (in wing axes) is defined by sweep $\Lambda_p$; dihedral $\delta_p$; and offsets $(x_{ip}, z_{ip})$ at the inboard edge relative to the aerodynamic center of the previous panel. The wing position $z^F$ is the mean aerodynamic center. The offset $(\bar{x}_A, \bar{z}_A)$ of the mean aerodynamic center from the root chord aerodynamic center is calculated (so the wing planform can be drawn; typically the aerodynamic center is drawn as the quarter-chord). Outboard panel edges are at $\eta_{Ep}$ (input or calculated). A panel is characterized by span $b_p$ (each side), mean chord $c_p$, and area $S_p = 2b_pc_p$ (both sides). The taper is defined by inboard and outboard chord ratios, $\lambda = c/c_{ref}$ (where $c_{ref}$ is a panel or wing reference chord, depending on the options for describing the geometry).

The span for each panel (if there are more than two panels) can be a fixed input; a fixed ratio of the wing span, $b_p = f_{bp}(b/2)$; or free. The panel outboard edge (except at the wing tip) can be at a fixed input position $y_{Ep}$; at a fixed station $\eta_{Ep}$, $y_p = \eta_{Ep}(b/2)$; calculated from the rotor radius, $y_p = fR$; calculated from the fuselage and rotor geometry, $y_p = fR + d_{fus} + 1/2w_{fus}$ (for a designated rotor); calculated from
the hub position, \( y_p = |y_{hub}| \) (for a designated rotor); or adjusted. An adjusted station is obtained from the last station and the span of this panel, \( y_p = y_{p-1} + b_p \) or \( y_p = y_{p-1} + f_{bp}(b/2) \); or from the next station and the span of the next panel, \( y_p = y_{p+1} - b_{p+1} \) or \( y_p = y_{p+1} - f_{b(p+1)}(b/2) \). The specification of panel spans and panel edges must be consistent, and sufficient to determine the wing geometry. Determining the panel edges requires the following steps:

a) Calculate the panel edges that are either at fixed values (input, or from width, or from hub position) or at fixed stations; root and tip edges are known.

b) Working from root to tip, calculate the adjusted panel edge \( y_p \) if panel span \( b_p \) or ratio \( f_{bp} \) is fixed, and if previous edge \( y_{p-1} \) is known.

c) Working from tip to root, calculate the adjusted panel edge \( y_p \) (if not yet known) if panel span \( b_{p+1} \) or ratio \( f_{b(p+1)} \) is fixed, and if next edge \( y_{p+1} \) is known.

At the end of this process, all edges must be known and the positions \( y_p \) must be unique and sequential. If this geometry is being determined for a known span, then there must not be a fixed panel span or span
ratio that has not been used. Alternatively, if the wing span is being calculated from the specification of all panel widths, then the process must leave one and only one fixed panel span or span ratio that has not been used. Since the wing span is to be calculated, each panel edge is known in the form \( y_p = c_0 + c_1 b / 2 \). Then the unused fixed panel span gives the equation \((c_0 + c_1 b / 2)_O - (c_0 + c_1 b / 2)_I = b_p \) (subscript \( O \) denotes outboard edge, subscript \( I \) denotes inboard edge), or the unused fixed panel span ratio gives the equation \((c_0 + c_1 b / 2)_O - (c_0 + c_1 b / 2)_I = f_p b / 2 \), which can be solved for the semispan \( b / 2 \).

To complete the definition of the geometry, one of the following quantities is specified for each panel: panel area \( S_p \); ratio of panel area to wing area, \( f_s = S_p / S \); panel mean chord \( c_p \); ratio of panel mean chord to wing mean chord, \( f_c = c_p / c \); chord ratios \( \lambda_I = c_I / c_{ref} \) and \( \lambda_O = c_O / c_{ref} \) (taper); or free. The total wing area equals the sum of all panel areas:

\[
S = \sum S_p + S \sum f_s + 2 \sum b_p c_p + 2c \sum b_p f_c + 2c_{ref} \sum b_p \frac{1}{2} (\lambda_I + \lambda_O)
\]

If there is one or more taper specification (and no free), then \( c_{ref} \) is calculated from this equation for \( S \), and the mean chord is \( c_p = \frac{1}{2} (c_I + c_O) = c_{ref} \frac{1}{2} (\lambda_I + \lambda_O) \), \( S_p = 2b_p c_p \). If there is one (and only one) free specification, then \( S_p \) is calculated from this equation for \( S \), and the mean chord is \( c_p = S_p / (2b_p) \), with \( c_I = 2c_p / (1 + \lambda_O / \lambda_I) \), \( c_O = 2c_p - c_I \).

Since the panels have linear taper \((c = c_{ref} \lambda)\), the mean aerodynamic chord is

\[
S_{\lambda} = \int_{-b/2}^{b/2} c^2 dy = b \int_{0}^{1} c_{ref}^2 \lambda^2 d\eta
\]

\[
= b \sum c_{ref}^2 \frac{1}{3} (\lambda_I^2 + \lambda_I \lambda_O + \lambda_O^2) \Delta \eta_p = \sum \frac{1}{3} (c_I^2 + c_I c_O + c_O^2) 2b_p
\]

\[
S = \int_{-b/2}^{b/2} c dy = b \int_{0}^{1} c_{ref} \lambda d\eta
\]

\[
= b \sum c_{ref} \frac{1}{2} (\lambda_I + \lambda_O) \Delta \eta_p = \sum \frac{1}{2} (c_I + c_O) 2b_p
\]

These expressions are evaluated from panel \( c_I \) and \( c_O \), as calculated using \( \lambda_I \) and \( \lambda_O \), or using the ratio \( \lambda_O / \lambda_I \) (\( c_{ref} \) may not be the same for all panels).

The mean aerodynamic center is the point where there is zero moment due to lift: \( \bar{x}_A C_L S = \bar{x}_A \int c_{\ell} c \, dy = \int \bar{x} c_{\ell} c \, dy \), with \( c_{\ell} = \ell(y) \) the spanwise lift distribution. Thus

\[
\int_{0}^{1} \ell(\eta)(\bar{x}_A - x_{AC}(\eta)) \, d\eta = 0
\]

The locus of section aerodynamic centers \( x_{AC} \) is described by the panel sweep \( \Lambda_p \) and the offset \( x_{Ip} \) at the inboard end of the panel. These offsets can be a fixed input, a fraction of the root chord, or a fraction of the panel inboard chord. Assuming elliptical loading \((\ell = \sqrt{1 - \eta^2})\) gives

\[
\frac{\pi}{4} \bar{x}_A = \int_{0}^{1} \ell(\eta) x_{AC} \, d\eta = \sum \int \sqrt{1 - \eta^2} \left( \bar{x}_{Ip} + \frac{b_p}{2} \tan \Lambda_p \eta \right) \, d\eta
\]

\[
= \sum \left[ \bar{x}_{Ip} \frac{1}{2} \left( \eta \sqrt{1 - \eta^2} + \sin^{-1} \eta \right) - \frac{b_p}{2} \tan \Lambda_p \frac{1}{3} \left(1 - \eta^2 \right)^{3/2} \right]_{\eta_{ip}}^{\eta_{o}}
\]

where \( \bar{x}_{Ip} = \sum_{q=2}^{p} \left(x_{IQ} + (b/2) \tan \Lambda_{q-1} (\eta_{O(q-1)} - \eta_{1(q-1)}) - (b/2) \tan \Lambda_p \eta_{Ip} \right) \). The vertical position of the mean aerodynamic center is obtained in a similar fashion, from panel dihedral \( \delta_p \) and offset \( z_{Ip} \) at the inboard edge of the panel. Assuming uniform loading \((\ell = 1)\) gives

\[
\bar{z}_A = \int_{0}^{1} z_{AC} \, d\eta = \sum \int \left( \bar{z}_{Ip} + \frac{b_p}{2} \tan \delta_p \eta \right) \, d\eta = \sum \left[ \bar{z}_{Ip} \eta + \frac{b_p}{2} \tan \delta_p \frac{1}{2} \eta^2 \right]_{\eta_{ip}}^{\eta_{o}}
\]
Then \((\bar{x}_A, \bar{z}_A)\) is the offset of the mean aerodynamic center from the root chord aerodynamic center. Finally,

\[
\Lambda = \tan^{-1} \left( \sum b_p \frac{b}{b/2} \tan \Lambda_p \right)
\]

\[
\delta = \tan^{-1} \left( \sum b_p \frac{b}{b/2} \tan \delta_p \right)
\]

\[
\lambda = \frac{2c}{c_{\text{root}}} - 1
\]

are the wing overall sweep, dihedral, and taper.

The wing contribution to the aircraft operating length is \(x_{\text{wing}} + (0.25c) \cos i\) (forward), \(x_{\text{wing}} - (0.75c) \cos i\) (aft), and \(y_{\text{wing}} \pm b/2\) (lateral).

### 13–2 Control and Loads

The control variables are flap \(\delta_F\), flaperon \(\delta_f\), aileron \(\delta_a\), and incidence \(i\). The flaperon deflection can be specified as a fraction of flap deflection, or as an increment relative to the flap deflection, or the flaperon can be independent of the flap. The flaperon and aileron are the same surface, generating symmetric and antisymmetric loads, respectively, hence with different connections to pilot controls.

With more than one wing panel, each panel can have control variables: flap \(\delta_{Fp}\), flaperon \(\delta_{fp}\), aileron \(\delta_{ap}\), and incidence \(i_p\). The outboard panel \((p \geq 2)\) control or incidence can be specified independently, or in terms of the root panel \((p = 1)\) control or incidence (either fraction or increment).

Each control is described by the ratio of the control surface chord to the wing panel chord, \(\ell_f = c_f/c_p\); and by the ratio of the control surface span to wing panel span, \(f_b = b_f/b_p\), such that the control surface area is obtained from the panel area by \(S_f = \ell_f f_b S_p\).

### 13–3 Aerodynamics

The aerodynamic velocity of the wing relative to the air, including interference, is calculated in component axes, \(v^B\). The angle of attack \(\alpha_{\text{wing}}\) (hence \(C^{BA}\)) and dynamic pressure \(q\) are calculated from \(v^B\). The reference area for the wing aerodynamic coefficients is the planform area, \(S\). The wetted area contribution is twice the exposed area: \(S_{\text{wet}} = 2(S - cw_{\text{fus}})\), where \(w_{\text{fus}}\) is the fuselage width.

The wing vertical drag can be fixed, specified as a drag area \((D/q)_V\); or the drag can be scaled, specified as a drag coefficient \(C_{DV}\) based on the wing area; or calculated from an airfoil section drag coefficient (for \(-90\) deg angle of attack) and the wing area immersed in the rotor wake:

\[
C_{DV} = \frac{1}{S} \frac{c_{\text{drag}}}{c} \left( S - S_{\text{center}} - f_{\text{drag}} b_F c_F (1 - \cos \delta_F) - f_{\text{drag}} b_F c_F (1 - \cos \delta_F) \right)
\]

The term \(S_{\text{center}} = c(w_{\text{fus}} + 2d_{\text{fus}})\) (where \(w_{\text{fus}}\) is the fuselage width and \(d_{\text{fus}}\) the rotor-fuselage clearance) is the area not immersed in the rotor wake, and is used only for tiltrotors. The last two terms account for the change in wing area due to flap and flaperon deflection, with an effectiveness factor \(f_{\text{drag}}\).

From the control surface deflection and geometry, the lift coefficient, maximum lift angle, moment coefficient, and drag coefficient increments are evaluated: \(\Delta C_{L_f}\), \(\Delta \alpha_{\text{max}}\), \(\Delta C_{M_f}\), and \(\Delta C_{D_f}\). These increments are the sum of contributions from flap and flaperon deflection, hence weighted by the control surface area. The drag coefficient increment includes the contribution from aileron deflection.
13-3.1 Lift

The wing lift is defined in terms of lift-curve slope $C_{La}$ and maximum lift coefficient $C_{Lmax}$ (based on wing planform area). The three-dimensional lift-curve slope is input directly or calculated from the two-dimensional lift-curve slope:

$$C_{La} = \frac{c_{f\alpha}}{1 + c_{f\alpha}(1 + \tau)/(\pi AR)}$$

where $\tau$ accounts for non-elliptical loading. Optionally the input incompressible lift-curve slope is multiplied by the compressibility factor $F_c$. The effective angle of attack is

$$\alpha_e = \alpha_{\text{wing}} + \alpha_{dl} - \alpha_{zl},$$

where $\alpha_{zl}$ is the angle of zero lift; in reverse flow ($|\alpha_e| > 90$), $\alpha_e \rightarrow \alpha_e - 180 \text{ sign} \alpha_e$. Let $\alpha_{\text{max}} = C_{Lmax}/C_{La}$ be the angle-of-attack increment (above or below zero lift angle) for maximum lift. Including the change of maximum lift angle caused by control deflection, $A_{\text{max}} = \alpha_{\text{max}} + \Delta \alpha_{\text{max}}$ and $A_{\text{min}} = -\alpha_{\text{max}} + \Delta \alpha_{\text{max}}$. Then

$$C_{L} = \begin{cases} C_{La\alpha_e} + \Delta C_{Lf} & A_{\text{min}} \leq \alpha_e \leq A_{\text{max}} \\ (C_{La} A_{\text{max}} + \Delta C_{Lf}) \max \left(0, \frac{f\pi/2 - |\alpha_e|}{f\pi/2 - |A_{\text{max}}|} \right) & \alpha_e > A_{\text{max}} \\ (C_{La} A_{\text{min}} + \Delta C_{Lf}) \max \left(0, \frac{f\pi/2 - |\alpha_e|}{f\pi/2 - |A_{\text{min}}|} \right) & \alpha_e < A_{\text{min}} \end{cases}$$

(for zero lift at 90 deg angle of attack, $f = .9$). Note that $C_{La} A_{\text{max}} + \Delta C_{Lf} = C_{La\alpha_{\text{max}}} + \Delta C_{Lmax}$. In sideward flight, $C_L = 0$. Finally, $L = qSC_L$ is the lift force.

13-3.2 Pitch Moment

The wing pitch moment coefficient is $C_M = C_{Mac} + \Delta C_{Mf}$. Then $M = qScC_M$ is the pitch moment.

13-3.3 Roll Moment

The only wing roll moment considered is that produced by aileron control. Typically the flaperon and aileron are the same surface, but they are treated separately in this model. The aileron geometry is specified as for the flaperon and flap, hence includes both sides of the wing. The lift coefficient increment $\Delta C_{La}$ is evaluated as for the flaperon, so one-half of this lift acts up (on the right side) and one-half acts down. The roll moment is then $M_x = 2(\Delta L_a/2)y$, where $y$ is the lateral position of the aileron aerodynamic center, measured from the wing centerline (defined as a fraction of the wing semi-span).

The roll moment coefficient is $C_{\ell} = -\frac{y}{b^2} \frac{1}{2} \Delta C_{La}$. Then $M_x = qSbC_{\ell}$ is the roll moment.

13-3.4 Drag

The drag area or drag coefficient is defined for forward flight and vertical flight. The effective angle of attack is $\alpha_e = \alpha_{\text{wing}} + \alpha_{dl} - \alpha_{Dmin}$, where $\alpha_{Dmin}$ is the angle of minimum drag; in reverse flow ($|\alpha_e| > 90$), $\alpha_e \rightarrow \alpha_e - 180 \text{ sign} \alpha_e$. For angles of attack less than a transition angle $\alpha_t$, the drag coefficient equals the forward flight (minimum) drag $C_{D0}$, plus an angle of attack term and the control increment. If the angle of attack is greater than a separation angle $\alpha_s < \alpha_t$, there is an additional drag increase. Thus if $|\alpha_e| \leq \alpha_t$, the profile drag is

$$C_{Dp} = C_{D0} (1 + K_d|\alpha_e|^{Xd} + K_s(|\alpha_e| - \alpha_s)^{Xs}) + \Delta C_{Dc} + \Delta C_{Df}$$
where the separation \((K_s)\) term is present only for \(|\alpha_e| > \alpha_s\); and otherwise

\[ C_{Di} = C_{D0} (1 + K_d |\alpha_e| X_d + K_s (|\alpha_e| - \alpha_s)^X_s) + \Delta C_{Dc} + \Delta C_{Df} \]

\[ C_{Dp} = C_{Dt} + (C_{DV} - C_{Dt}) \sin \left( \frac{\pi}{2} \frac{|\alpha_e| - \alpha_t}{\pi/2 - \alpha_t} \right) \]

\(\Delta C_{Dc}\) is the compressibility drag increment. Optionally there might be no angle of attack variation at low angles \((K_d = 0 \text{ and/or } K_s = 0)\), or quadratic variation \((X_d = 2)\), or cubic variation for the separation term \((X_s = 3)\). For sideward flight \((v_x^B = 0)\) the drag is obtained using \(\phi_v = \tan^{-1}(-v_z^B/v_y^B)\) to interpolate the vertical coefficient: 

\[ C_D = C_{D0} \cos^2 \phi_v + C_{DV} \sin^2 \phi_v. \]

The induced drag is obtained from the lift coefficient, aspect ratio, and Oswald efficiency \(e\):

\[ C_{Di} = \frac{(C_L - C_{L0})^2}{\pi e A R} \]

Conventionally the Oswald efficiency \(e\) represents the wing parasite drag variation with lift, as well as the induced drag (hence the use of \(C_{L0}\)). If \(C_{Dp}\) varies with angle of attack, then \(e\) is just the span efficiency factor for the induced power (and \(C_{L0}\) should be zero). The wing-body interference is specified as a drag area, or a drag coefficient based on the wing area. Then

\[ D = qSC_D = qS \left( C_{Dp} + C_{Di} + C_{Dwb} \right) \]

is the drag force. The other forces and moments are zero.

### 13-3.5 Wing Panels

The wing panels can have separate controls, different incidence angles, and different interference from the rotors. Thus the lift, drag, and moment coefficients are evaluated separately for each panel, based on the panel area \(S_p\) and mean chord \(c_p\). The coefficient increments due to control surface deflection are calculated using the ratio of the control surface area to panel area, \(S_f/S_p = \ell_f/b\). The lateral position of the aileron aerodynamic center is \(\eta_a/b_p\) from the panel inboard edge, so \(y/(b/2) = \eta_E(p-1) + \eta_a/b_p/(b/2)\) from the wing centerline. Then the total wing loads are:

\[ L = \sum q_p S_p C_{Lp} \]
\[ M = \sum q_p S_p c_p C_{Mp} \]
\[ M_x = \sum q_p S_p b C_{lp} \]
\[ D = \sum q_p S_p C_{Dpp} + \langle qS \rangle (C_{Di} + C_{Dwb}) \]

The sums are over all panels (left and right). The reference area is \(S = \sum S_p\), accounting for possible absence of wing extensions. The total wing coefficients are based on \(\langle qS \rangle = \sum q_p S_p\). The three-dimensional lift-curve slope \(C_{L\alpha}\) is calculated for the entire wing and used for each panel. The induced drag is calculated for the entire wing, from the total \(C_L\). Since \(C_{BF} = I\),

\[ F^F = C^{FB} C^{BA} F^A = \sum C_p^{BA} q_p S_p \begin{pmatrix} -C_{Dpp} - C_{Di} - C_{Dwb} \\ 0 \\ -C_L \end{pmatrix} \]

is the wing aerodynamic force.
13-3.6 Interference

With more than one wing, the interference velocity at other wings is proportional to the induced velocity of the wing producing the interference: \( v_{\text{ind}}^F = K_{\text{int}} v_{\text{ind}}^F \). The induced velocity is obtained from the induced drag, assumed to act in the \( k^B \) direction: \( \alpha_{\text{ind}} = v_{\text{ind}}^F / |v_B| = C_{D_i}/C_L = C_L/(\pi eAR) \), \( v_{\text{ind}}^F = CFBk^B |v_B| \alpha_{\text{ind}} \). For tandem wings, typically \( K_{\text{int}} = 2 \) for the interference of the front wing on the aft wing, and \( K_{\text{int}} = 0 \) for the interference of the aft wing on the front wing. For biplane wings, the mutual interference is typically \( K_{\text{int}} = 0.7 \) (upper on lower, and lower on upper). The induced drag is then

\[
C_{D_i} = \left( \frac{C_L - C_{L0}}{\pi eAR} \right)^2 + C_L \sum_{\text{other wings}} \alpha_{\text{int}}
\]

\[
\alpha_{\text{int}} = K_{\text{int}} \alpha_{\text{ind}} = K_{\text{int}} \left( \frac{C_L}{\pi eAR} \right)_{\text{other wing}}
\]

The induced velocity from the rotors is included in the angle of attack of the wing. The rotor interference must also be accounted for in the wing induced power:

\[
C_{D_i} = \left( \frac{C_L - C_{L0}}{\pi eAR} \right)^2 + C_L \left[ \sum_{\text{other wings}} K_{\text{int}} \alpha_{\text{ind}} + \sum_{\text{rotors}} C_{\text{int}} \alpha_{\text{ind}} \right]
\]

The angle \( \alpha_{\text{ind}} = v_{\text{ind}}^F / V \) is obtained from the rotor induced velocity \( \lambda_i \). If the interference is wing-like, \( v_{\text{ind}} = \Omega R \kappa \lambda_i \) (so \( v_{\text{ind}} \propto L/pb^2 V \propto T/2\rho AV \)). If the interference is propeller-like, \( v_{\text{ind}} = V \kappa \lambda_i \) (so \( v_{\text{ind}} \propto \Gamma / b \propto T / 2 \rho A \Omega R \)). The interference factor can be evaluated from wing induced drag calculations:

\[
C_{\text{int}} = \Delta C_{D_i} / (C_L \kappa \lambda_i / \mu) \text{ or } C_{\text{int}} = \Delta C_{D_i} / (C_L \kappa \lambda_i)
\]

Typically for tiltrotors the interference is wing-like, with \( C_{\text{int}} \approx -0.06 \).

The wing interference at the tail produces an angle-of-attack change \( \epsilon = E(C_L/C_{La}) \), where \( E = d\epsilon / d\alpha \) is an input factor determined by the aircraft geometry. Then from the velocity \( v^B \) of the wing,

\[
v_{\text{int}}^F = CFB \begin{pmatrix} -e v_z^B \\ 0 \\ e v_x^B \end{pmatrix}
\]

is the interference velocity at the tail.

The wing interference at the rotor can produce interference power. The induced velocity at the rotor disk is \( v_{\text{int}}^F = K_{\text{int}} v_{\text{ind}}^F \), with \( v_{\text{ind}}^F = CFBk^B |v_B| \alpha_{\text{ind}} \) again. Separate interference factors \( K_{\text{int}} \) are used for the components of the interference velocity parallel to and perpendicular to the rotor force vector (roughly normal to and in the plane of the rotor disk).

13-4 Wing Extensions

The wing can have extensions, defined as wing portions of span \( b_X \) at each wing tip. For the tiltrotor configuration in particular, the wing weight depends on the distribution of wing area outboard (the extension) and inboard of the rotor and nacelle location. Wing extensions are defined as a set of wing panels at the tip. The extension span and area are the sum of the panel quantities, \( b_X = \sum_{\text{ext}} b_p \) and \( S_X = \sum_{\text{ext}} S_p \). The inboard span and area are then \( b_I = b - \sum b_X \), \( S_I = S - S_X \). Optionally the wing extensions can be considered a kit, hence the extensions can be absent for designated flight conditions or missions. As a kit, the wing extension weight is considered fixed useful load. With wing extensions removed, the aerodynamic analysis considers only the remaining wing panels. The total wing
coefficients are then based on the area without the extensions. For the induced drag and interference, the effective aspect ratio is then reduced by the factor \((b_I/b)^2\), since the lift and drag coefficients are still based on total wing area \(S\).

**13–5 Wing Kit**

The wing can be a kit, the kit weight an input fraction of the total wing weight. The wing kit weight can be part of the wing group, or considered fixed useful load. With the kit removed, there are no aerodynamic loads or aerodynamic interference generated by the wing, and the wing kit weight is omitted.

**13–6 Weights**

The wing group consists of: basic structure (primary structure, consisting of torque box and spars, plus extensions); fairings (leading edge and trailing edge); fittings (non-structural); fold/tilt structure; and control surfaces (flaps, ailerons, flaperons, and spoilers). There are separate models for a tiltrotor or tiltwing configuration and for other configurations (including a compound helicopter).

The AFDD wing weight models are based on parameters for the basic wing plus the wing tip extensions (not the total wing and extensions). The tiltrotor wing model requires the weight on the wing tips (both sides), consisting of: rotor group, engine system, drive system (except drive shaft), engine section or nacelle group, air induction group, rotary wing and conversion flight controls, hydraulic group, trapped fluids, and wing extensions. An adjustment of this calculated weight can be used; a negative increment is required when the engine and transmission are not at the tip location with the rotor.
Chapter 14

Empennage

The aircraft can have one or more tail surfaces, or no tail surface. Each tail is designated as horizontal or vertical, affecting some parameter definitions.

14–1 Geometry

The tail is described by planform area $S$, span $b$, chord $c = S/b$, and aspect ratio $AR = b^2/S$. The tail volume can be referenced to rotor radius and disk area, $V = Sl/RA$; to wing area and chord for horizontal tails, $V = Sl/S_wc_w$; or to wing area and span for vertical tails, $V = Sl/S_wb_w$. Here the tail length is $\ell = |x_{ht} - x_{cg}|$ or $\ell = |x_{vt} - x_{cg}|$ for horizontal tail or vertical tail, respectively. The geometry is specified in terms of $S$ or $V$; and $b$, or $AR$, or $c$. The elevator or rudder is described by the ratio of control surface chord to tail chord, $c_f/c$; and the ratio of control surface span to tail span, $b_f/b$.

The tail contribution to the aircraft operating length is $x_{tail} + 0.25c$ (forward), $x_{tail} - 0.75c$ (aft), and $y_{tail} \pm (b/2)C$ (lateral), where $C = \cos \phi$ for a horizontal tail and $C = \cos(\phi - 90)$ for a vertical tail.

14–2 Control and Loads

The tail is at position $z^F$, where the aerodynamic forces act. The scaled input for tail position can be referenced to the fuselage length, or to the rotor radius.

The horizontal tail can have a cant angle $\phi$ (positive tilt to left, becomes vertical tail for $\phi = 90$ deg). Thus the component axes are given by $C^{BF} = X_{-\phi}$. The control variables are elevator $\delta_e$ and incidence $i$.

The convention for nominal orientation of the vertical tail is positive lift to the left, so aircraft sideslip (positive to right) generates positive tail angle of attack and positive tail lift. The vertical tail can have a cant angle $\phi$ (positive tilt to right, becomes horizontal tail for $\phi = 90$), so the component axes are given by $C^{BF} = X_{-90+\phi}$. The control variables are rudder $\delta_r$ and incidence $i$.

14–3 Aerodynamics

The aerodynamic velocity of the tail relative to the air, including interference, is calculated in component axes, $v^B$. The angle of attack $\alpha_{tail}$ (hence $C^{B,A}$) and dynamic pressure $q$ are calculated from $v^B$. The reference area for the tail aerodynamic coefficients is the planform area, $S$. The wetted area contribution is $S_{wet} = 2S$. From the elevator or rudder deflection and geometry, the lift coefficient, maximum lift angle, and drag coefficient increments are evaluated: $\Delta C_{Lf}$, $\Delta \alpha_{max,f}$, and $\Delta C_{Df}$. 
14-3.1 Lift

The tail lift is defined in terms of lift-curve slope \( C_{L_{\text{tail}}} \) and maximum lift coefficient \( C_{L_{\text{max}}} \) (based on tail planform area). The three-dimensional lift-curve slope is input directly or calculated from the two-dimensional lift-curve slope:

\[
C_{L_t} = \frac{C_{L_{\text{tail}}} \alpha - \Delta C_L f}{1 + C_{L_{\text{tail}}} \left( \alpha - \Delta C_L f \right) / (\pi AR)}
\]

where \( \tau \) accounts for non-elliptical loading. Optionally the input incompressible lift-curve slope is multiplied by the compressibility factor \( F_c \). The effective angle of attack is \( \alpha_e = \alpha_{\text{tail}} + \beta - \alpha_{z_{l}} \), where \( \alpha_{z_{l}} \) is the angle of zero lift; in reverse flow \( (|\alpha_e| > 90) \), \( \alpha_e \leftarrow \alpha_e - 180 \text{ sign} \alpha_e \). Let \( \alpha_{\text{max}} = C_{L_{\text{max}}} / C_{L_{\text{tail}}} \) be the angle-of-attack increment (above or below zero lift angle) for maximum lift. Including the change of maximum lift angle caused by control deflection, \( A_{\text{max}} = \alpha_{\text{max}} + \Delta \alpha_{\text{max}} f \) and \( A_{\text{min}} = -\alpha_{\text{max}} + \Delta \alpha_{\text{max}} f \).

Then

\[
C_L = \begin{cases} 
C_{L_{\text{tail}}} \alpha_e + \Delta C_L f & A_{\text{min}} \leq \alpha_e \leq A_{\text{max}} \\
(C_{L_{\text{tail}}} A_{\text{max}} + \Delta C_L f) \max \left( 0, \frac{f \pi/2 - |\alpha_e|}{f \pi/2 - A_{\text{max}}} \right) & \alpha_e > A_{\text{max}} \\
(C_{L_{\text{tail}}} A_{\text{min}} + \Delta C_L f) \max \left( 0, \frac{f \pi/2 - |\alpha_e|}{f \pi/2 - A_{\text{min}}} \right) & \alpha_e < A_{\text{min}}
\end{cases}
\]

(for zero lift at \( 90 \) deg angle of attack, \( f = .9 \)). Note that \( C_{L_{\text{tail}}} A_{\text{max}} + \Delta C_L f = C_{L_{\text{tail}}} \alpha_{\text{max}} + \Delta C_{L_{\text{max}}} f \). In sideward flight (defined by \( (v_x^B)^2 + (v_z^B)^2 < (0.05|v_x^B|)^2 \)), \( C_L = 0 \). Finally, \( L = qS C_L \) is the lift force.

14-3.2 Drag

The drag area or drag coefficient is defined for forward flight and vertical flight. The effective angle of attack is \( \alpha_e = \alpha_{\text{tail}} + \beta - \alpha_{D_{\text{min}}} \), where \( \alpha_{D_{\text{min}}} \) is the angle of minimum drag; in reverse flow \( (|\alpha_e| > 90) \), \( \alpha_e \leftarrow \alpha_e - 180 \text{ sign} \alpha_e \). For angles of attack less than a transition angle \( \alpha_t \), the drag coefficient equals the forward flight (minimum) drag \( C_{D_{0}} \), plus an angle-of-attack term and the control increment. Thus if \( |\alpha_e| \leq \alpha_t \), the profile drag is

\[
C_{D_p} = C_{D_0} \left( 1 + K_d |\alpha_e| X_d \right) + \Delta C_{D_e} + \Delta C_{D_f}
\]

and otherwise

\[
C_{D_t} = C_{D_0} \left( 1 + K_d |\alpha_t| X_d \right) + \Delta C_{D_e} + \Delta C_{D_f} + \left( C_{D_V} - C_{D_t} \right) \sin \left( \frac{\pi |\alpha_e|}{2 \pi/2 - \alpha_t} \right).
\]

\( \Delta C_{D_e} \) is the compressibility drag increment. Optionally there might be no angle-of-attack variation at low angles \( (K_d = 0) \), or quadratic variation \( (X_d = 2) \). In sideward flight (defined by \( (v_x^B)^2 + (v_z^B)^2 < (0.05|v_x^B|)^2 \)), the drag is obtained using \( \phi_e = \tan^{-1}(-v_z^B/v_x^B) \) to interpolate the vertical coefficient: \( C_{D_p} = C_{D_0} \cos^2 \phi_e + C_{D_V} \sin^2 \phi_e \). The induced drag is obtained from the lift coefficient, aspect ratio, and Oswald efficiency \( e \):

\[
C_{D_i} = \frac{(C_L - C_{L_0})^2}{\pi e AR}
\]

Conventionally the Oswald efficiency \( e \) can represent the tail parasite drag variation with lift, as well as the induced drag (hence the use of \( C_{L_0} \)). Then

\[
D = qS C_D = qS \left( C_{D_p} + C_{D_i} \right)
\]
Empennage is the drag force. The other forces and moments are zero.

14–4 V-Tail

A V-tail is modeled as a pair of horizontal and vertical tails, each sized by area or tail volume. The aerodynamic loads are calculated separately for each tail. The weight is calculated only for the second tail, using the V-tail area and aspect ratio. The V-tail area and span are the sum of the two tails:

\[
S_V = S_{ht} + S_{vt}
\]
\[
b_V = b_{ht} + b_{vt}
\]

and the aspect ratio is \(AR = \frac{b_V^2}{S_V}\). The V-tail dihedral angle is \(\delta = \tan^{-1}\sqrt{S_{vt}/S_{ht}}\), and the cant angle \(\phi = 0\) for both. The location (which should be the same for the two tails) is the midpoint of the V-tail, vertically and laterally. An upward V-tail and a downward V-tail are identical in this model.

14–5 Weights

The empennage group consists of the horizontal tail, vertical tail, and tail-rotor. The tail plane weight consists of the basic structure and fold structure. The tail weight (empennage group) model depends on the configuration: helicopters and compounds, or tiltrotors and tiltwings. Separate weight models are available for horizontal and vertical tails.

The AFDD tail weight model depends on the design dive speed at sea level (input or calculated). The calculated dive speed is \(V_{\text{dive}} = 1.25V_{\text{max}}\), from the maximum speed at the design gross weight and sea level standard conditions.
Propulsion System

The aircraft propulsion system can be constructed from a number of components: propulsion groups, engine groups, jet groups, charge groups, and fuel tank systems. Figure 15-1 illustrates the configuration possibilities.

The aircraft can have one or more propulsion groups, or none. Each propulsion group is a set of components (rotors) and engine groups, connected by a drive system. The components define the power required, and the engine groups define the power available. There are one or more drive states, with a set of gear ratios for each state. The power required equals the sum of component power, transmission losses, and accessory losses.

An engine group consists of one or more engines of a specific type. An engine group transfers power by shaft torque, so it is associated with a propulsion group. For each engine type an engine model is defined. The engine model describes a particular engine, used in one or more engine groups. The models include turboshaft engines (perhaps convertible, for turbojet operation or reaction drive), reciprocating engines, compressors, electric motors (perhaps with fuel cells), electric generators, and generator-motors.

The aircraft can have one or more jet groups, or none. A jet group produces a force on the aircraft. A jet model describes a particular jet, used in one or more jet groups. The models include turbojet and turbofan engines (perhaps convertible, for reaction drive), reaction drive, and a simple force. A reaction drive supplies a blade force that provides the rotor power required.

The aircraft can have one or more charge groups, or none. A charge group generates energy for the aircraft. A charge model describes a particular charger, used in one or more charge groups. The models include fuel cells and solar cells.

There are one or more fuel tank systems for the aircraft. Fuel tank systems are associated with the engine groups, jet groups, and charge groups. Fuel quantity is measured as either weight or energy. Fuels considered include jet fuel, gasoline, diesel, and hydrogen. Storage systems considered include batteries, capacitors, and flywheels.

15-1 Referred Performance

Referred performance parameters are used for propulsion system components that operate with air. The operating condition and atmosphere give the standard conditions (temperature $T_{\text{std}}$ and pressure $p_{\text{std}}$) for a specified pressure altitude; the sea-level standard conditions (temperature $T_{0}$ and pressure $p_{0}$); and the operating temperature $T$ and pressure $p$. Here the temperatures are °R or °K. The performance characteristics depend on the temperature ratio $\theta = T/T_{0}$ and pressure ratio $\delta = p/p_{0}$. The flight Mach number $M = V/c_{s} = V/c_{s0}\sqrt{\theta}$ is obtained from the aircraft speed $V$. 
The referred or corrected performance parameters are:

\[
\begin{align*}
\text{power} & \quad \frac{P}{\delta \sqrt{\theta}} \\
\text{mass flow} & \quad \frac{\dot{m}}{\delta / \sqrt{\theta}} \\
\text{specific power} & \quad \frac{P/\dot{m}}{\theta}
\end{align*}
\]

\[
\begin{align*}
\text{fuel flow} & \quad \frac{\dot{\psi}}{\delta \sqrt{\theta}} \\
\text{force} & \quad \frac{F}{\delta} \\
\text{rotational speed} & \quad \frac{N}{\sqrt{\theta}}
\end{align*}
\]

where \( P \) is power, \( \dot{m} \) is mass flow, \( \dot{\psi} \) is fuel flow, \( F \) is a force, and \( N \) is a rotational speed. The performance at sea-level-standard static conditions is indicated by subscript 0.
Engine performance depends on the engine rating. Each engine rating has specific operating limitations, most importantly an operating time limit intended to avoid damage to the engine.

The power available from a turboshaft engine depends on the engine rating. Typical engine ratings are given in table 15-1. Engine power is generally specified in terms of sea level standard (SLS) static MCP. Takeoff typically uses MRP. CRP or ERP is restricted to use in one-engine inoperative (OEI) emergencies.

<table>
<thead>
<tr>
<th>rating</th>
<th>description</th>
<th>time limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>MCP</td>
<td>maximum continuous power</td>
<td>$\infty$</td>
</tr>
<tr>
<td>IRP</td>
<td>intermediate rated power</td>
<td>30 min</td>
</tr>
<tr>
<td>MRP</td>
<td>maximum rated power</td>
<td>10 min</td>
</tr>
<tr>
<td>CRP</td>
<td>contingency rated power</td>
<td>2.5 min</td>
</tr>
<tr>
<td>ERP</td>
<td>emergency rated power</td>
<td>1.0 min</td>
</tr>
</tbody>
</table>

The thrust available from a turbojet or turbofan engine depends on the engine rating. Typical engine ratings are given in table 15-2.

<table>
<thead>
<tr>
<th>rating</th>
<th>description</th>
<th>time limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>MCT</td>
<td>maximum continuous thrust</td>
<td>$\infty$</td>
</tr>
<tr>
<td>MTO</td>
<td>maximum takeoff thrust</td>
<td>5 min</td>
</tr>
</tbody>
</table>

15–3 Efficiency from Equivalent Circuit

The efficiency of an electrical device can be expressed in terms of its power by considering an equivalent circuit, defined by internal resistance $R$ and current $I_0$. The total voltage is $V_x = V + IR$ and the total current is $I_x = I + I_0$. Then the useful power is

$$P = VI = (V_x - IR)I = V_x I_x - I^2 R - V_x I_0 = P_{\text{total}} - P_{\text{loss}}$$

and the efficiency is

$$\eta = \frac{P}{P_{\text{total}}} = \frac{P}{P + P_{\text{loss}}} = \frac{P}{P + P^2 R/V^2 + P_0}$$

In terms of a reference power, let

$$R/V^2 = \frac{1}{P_{\text{ref}}} \left( \frac{1}{\eta_{\text{ref}}} - 1 \right)$$

$$P_0 = c P_{\text{ref}}$$

Then

$$\frac{1}{\eta} = 1 + P(R/V^2) + P_0/P = 1 + \frac{P}{P_{\text{ref}}} \left( \frac{1}{\eta_{\text{ref}}} - 1 \right) + c \frac{P_{\text{ref}}}{P}$$

So $\eta = (1/\eta_{\text{ref}} + c)^{-1}$ at $P = P_{\text{ref}}$. The efficiency decreases with $P$ because of the internal resistance, but is zero at $P = 0$ because of the internal current term.
15–4 Control and Loads

Geometry and control are defined for engine groups, jet groups, and charge groups. The group amplitude $A$ and mode $B$ are control variables:

$$A = A_0 + T_A c_{AC}$$
$$B = B_0 + T_B c_{AC}$$

with $A_0$ and $B_0$ zero, constant, or a function of flight speed (piecewise linear input). The amplitude can be power (engine group), thrust (jet group), or power (charge group). The mode can be mass flow (for convertible engines), or power flow (for generator-motor).

The group orientation is specified by selecting a nominal direction $c_{f_0}$ in body axes (positive or negative $x$-, $y$-, or $z$-axis; usually thrust forward, hence positive $x$-axis); then applying a yaw angle $\psi$; and then an incidence or tilt angle $i$ (table 15-3). The yaw and incidence angles can be connected to the aircraft controls $c_{AC}$:

$$\psi = \psi_0 + T_\psi c_{AC}$$
$$i = i_0 + T_i c_{AC}$$

with $\psi_0$ and $i_0$ zero, constant, or a function of flight speed (piecewise linear input). Hence the incidence and yaw angles can be fixed orientation or can be control variables. Optionally the lateral position of the group can be set equal to that of a designated rotor (useful for tiltrotors when the rotor hub lateral position is calculated from the clearance or wing geometry).

The group produces a force $T$, acting in the direction of the group; and a drag $D$, acting in the wind direction. The group is at location $z^F$. The force and moment acting on the aircraft in body axes are thus:

$$F^F = e_f T + e_d D$$
$$M^F = \Delta z^F F^F$$

where $\Delta z^F = z^F - z^F_{cg}$, $e_f$ is the force direction, and $e_d$ is the drag direction. The velocity relative to the air gives $e_d = -v^F / |v^F|$ (no interference). The group axes are $C_{BF} = U_i V_\psi$, where $U$ and $V$ depend on the nominal direction, as described in table 15-3. The force direction is $e_f = C_{FB} e_{f_0}$.

For a tiltrotor aircraft, one of the aircraft controls is the nacelle angle, with the convention $\alpha_{\text{tilt}} = 0$ for cruise and $\alpha_{\text{tilt}} = 90$ deg for helicopter mode. The incidence angle is then connected to $\alpha_{\text{tilt}}$ by defining the matrix $T_i$ appropriately. If the nominal direction is defined for airplane mode ($+x$), then $i = \alpha_{\text{tilt}}$ should be used; if the nominal direction is defined for helicopter mode ($-z$), then $i = \alpha_{\text{tilt}} - 90$ should be used.

Table 15-3. Group orientation.

<table>
<thead>
<tr>
<th>nominal (F axes)</th>
<th>$e_{f_0}$</th>
<th>incidence, + for force</th>
<th>yaw, + for force</th>
<th>$C_{BF} = U_i V_\psi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>forward</td>
<td>$i$</td>
<td>up</td>
<td>$Y_i Z_\psi$</td>
</tr>
<tr>
<td>$-x$</td>
<td>aft</td>
<td>$-i$</td>
<td>up</td>
<td>$Y_{-i} Z_{-\psi}$</td>
</tr>
<tr>
<td>$y$</td>
<td>right</td>
<td>$j$</td>
<td>aft</td>
<td>$Z_j X_{-\psi}$</td>
</tr>
<tr>
<td>$-y$</td>
<td>left</td>
<td>$-j$</td>
<td>up</td>
<td>$Z_{-j} X_\psi$</td>
</tr>
<tr>
<td>$z$</td>
<td>down</td>
<td>$k$</td>
<td>aft</td>
<td>$Y_{-j} X_{-\psi}$</td>
</tr>
<tr>
<td>$-z$</td>
<td>up</td>
<td>$-k$</td>
<td>right</td>
<td>$Y_j X_\psi$</td>
</tr>
</tbody>
</table>
An aerodynamic model is defined for engine groups, jet groups, and charge groups. The group includes a nacelle, which contributes to the aircraft drag. The component drag contributions must be consistent. The pylon is the rotor support and the nacelle is the engine support. The drag model for a tiltrotor aircraft with tilting engines would use the pylon drag (and no nacelle drag), since the pylon is connected to the rotor shaft axes; with non-tilting engines it would use the nacelle drag as well.

The nacelle drag acts at the group location $z^F$. The nacelle axes are the group axes, hence $C_{BF}$ is calculated as described previously (see table 15-1). For the nominal direction forward (+x-axis), the nacelle z-axis is downward and the x-axis is forward; zero incidence angle corresponds to zero angle of attack; and 90 deg incidence angle corresponds to 90 deg angle of attack (vertical drag). The velocity, angle of attack, and dynamic pressure are calculated at the nacelle (without interference). The reference area for the nacelle drag coefficient is the nacelle wetted area. The wetted area is input, or calculated from the weight $w$:

$$S_{\text{wet}} = k\left(\frac{w}{N}\right)^{2/3}$$

where $N$ is the number of engines, jets, or chargers; and the units of $k$ are $\text{ft}^2/\text{lb}^{2/3}$ or $\text{m}^2/\text{kg}^{2/3}$. The reference area is then $S_{\text{nac}} = NS_{\text{wet}}$. The nacelle area is included in the aircraft wetted area if the drag coefficient is nonzero. The drag area or drag coefficient is defined for forward flight and for vertical flight. The drag coefficient at angle of attack $\alpha$ is

$$C_D = C_{D0} + (C_{DV} - C_{D0})|\sin \alpha|^{X_d}$$

typically using $X_d = 2$. In sideward flight, $C_D = C_{D0}$ is used. The nacelle drag is $D_{\text{nac}} = qS_{\text{nac}}C_D$. 
Chapter 16
Fuel Tank

The fuel quantity stored and burned can be measured in weight or energy. Each component (engine group, jet group, charge group, or equipment) that uses or generates fuel is associated with a fuel tank system of the appropriate type. The unit of fuel energy is Mega-Joules (MJ). For reference, 1 British Thermal Unit (BTU) = 1055.056 Joule and 1 kW-hr = 3.6 MJ.

16–1 Fuel Tank System Store and Burn Weight

For fuel use measured by weight, the fuel properties are density $\rho_{\text{fuel}}$ (weight per volume, lb/gal or kg/liter) and specific energy $e_{\text{fuel}}$ (MJ/kg). Table 16-1 gives the properties of a number of aviation fuels, based on military and industry specifications (refs. 1–3). Fuels considered include jet fuel, gasoline, diesel, and hydrogen. From the fuel weight $W_{\text{fuel}}$, the energy is $E_{\text{fuel}} = e_{\text{fuel}}W_{\text{fuel}}$ (MJ) and the volume is $V_{\text{fuel}} = W_{\text{fuel}}/\rho_{\text{fuel}}$ (gallons or liters). A motive device has a fuel flow $\dot{w}$ (lb/hour or kg/hour), and its specific fuel consumption is $\text{sfc} = \dot{w}/P$ or $\text{sfc} = \dot{w}/T$.

16-1.1 Fuel Capacity

The fuel tank capacity $W_{\text{fuel-cap}}$ (maximum usable fuel weight) is determined from designated sizing missions. The maximum mission fuel required, $W_{\text{fuel-miss}}$ (excluding reserves and any fuel in auxiliary tanks), gives

$$W_{\text{fuel-cap}} = \max(f_{\text{fuel-cap}}W_{\text{fuel-miss}}, W_{\text{fuel-miss}} + W_{\text{reserve}})$$

or

$$W_{\text{fuel-cap}} = d_{\text{fuel-cap}} + f_{\text{fuel-cap}}(W_{\text{fuel-miss}} + W_{\text{reserve}})$$

where $f_{\text{fuel-cap}} \geq 1$ is an input factor. Alternatively, the fuel tank capacity $W_{\text{fuel-cap}}$ can be input. The corresponding volumetric fuel tank capacity is $V_{\text{fuel-cap}} = W_{\text{fuel-cap}}/\rho_{\text{fuel}}$.

For missions that are not used to size the fuel tank, the fuel weight may be fallout, or the fuel weight may be specified (with or without auxiliary tanks). The fuel weight for a flight condition or the start of a mission can be specified as an increment $d$, plus a fraction $f$ of the fuel tank capacity, plus auxiliary tanks:

$$W_{\text{fuel}} = \min(d_{\text{fuel}} + f_{\text{fuel}}W_{\text{fuel-cap}}, W_{\text{fuel-cap}}) + \sum N_{\text{auxtank}}W_{\text{aux-cap}}$$

where $W_{\text{aux-cap}}$ is the capacity of each auxiliary fuel tank. The fuel capacity of the wing can be estimated from

$$W_{\text{fuel-wing}} = \rho_{\text{fuel}} \sum f_{c_{tb}}t_{w}b_{w}$$

where $c_{tb}$ is the torque box chord, $t_{w}$ the wing thickness, and $b_{w}$ the wing span; and $f$ is the input fraction of the wing torque box that is filled by primary fuel tanks, for each wing. This calculation is performed in order to judge whether fuel tanks outside the wing are needed.
if $W_{\text{fuel}} > W_{\text{fuel-max}}$

for designated auxiliary tank

$N_{\text{auxtank}} = N_{\text{auxtank}} + n$

if fixed total weight: $\Delta W_{\text{fuel}} = -nf_{\text{auxtank}}W_{\text{aux-cap}}$

$\Delta W_{\text{fuel-max}} = nW_{\text{aux-cap}}$

repeat if $W_{\text{fuel}} > W_{\text{fuel-max}}$

if fixed total weight and $W_{\text{fuel}} \leq W_{\text{fuel-max}} - nW_{\text{aux-cap}}$

$N_{\text{auxtank}} = N_{\text{auxtank}} - n$

$\Delta W_{\text{fuel-max}} = -nW_{\text{aux-cap}}$

$W_{\text{fuel}} = W_{\text{fuel-max}}$ (capped)

else if $W_{\text{fuel}} < W_{\text{fuel-max}}$

for designated auxiliary tank (then for last nonzero $N_{\text{auxtank}}$)

$N_{\text{auxtank}} = N_{\text{auxtank}} - n$

if fixed total weight: $\Delta W_{\text{fuel}} = nf_{\text{auxtank}}W_{\text{aux-cap}}$

$\Delta W_{\text{fuel-max}} = -nW_{\text{aux-cap}}$

repeat if $W_{\text{fuel}} < W_{\text{fuel-max}}$

undo last increment

$N_{\text{auxtank}} = N_{\text{auxtank}} + n$

if fixed total weight: $\Delta W_{\text{fuel}} = nf_{\text{auxtank}}W_{\text{aux-cap}}$

$\Delta W_{\text{fuel-max}} = nW_{\text{aux-cap}}$

Figure 16-1. Outline of $N_{\text{auxtank}}$ calculation.

### 16-1.2 Fuel Reserves

Mission fuel reserves can be specified in several ways for each mission. Fuel reserves can be defined in terms of specific mission segments, for example 200 miles plus 20 minutes at $V_{be}$. Fuel reserves can be an input fraction of the fuel burned by all (except reserve) mission segments, so $W_{\text{fuel}} = (1 + f_{\text{res}})W_{\text{fuel-miss}}$. Fuel reserves can be an input fraction of the fuel capacity, so $W_{\text{fuel}} = W_{\text{miss-seg}} + f_{\text{res}}W_{\text{fuel-cap}}$. If more than one criterion for reserve fuel is specified, the maximum reserve is used.

### 16-1.3 Auxiliary Fuel Tank

Auxiliary fuel tanks are defined in one or more sizes. The capacity of each auxiliary fuel tank, $W_{\text{aux-cap}}$, is an input parameter. The number of auxiliary fuel tanks on the aircraft, $N_{\text{auxtank}}$ for each size, can be specified for the flight condition or mission segment. Alternatively (if the mission is not used to size the fuel tank), the number of auxiliary fuel tanks at the start of the mission can be determined from the mission fuel.

Figure 16-1 describes the process for determining $N_{\text{auxtank}}$ from the required fuel weight $W_{\text{fuel}}$ and the aircraft maximum fuel capacity $W_{\text{fuel-max}} = W_{\text{fuel-cap}} + \sum N_{\text{auxtank}}W_{\text{aux-cap}}$. The fuel weight adjustment $\Delta W_{\text{fuel}}$ is made if fuel weight is fallout from fixed gross weight and payload, accounting for the operating weight update when $N_{\text{auxtank}}$ changes. If the auxiliary tank weight is greater than the increment in fuel weight needed, then the fallout fuel weight $W_{\text{fuel}} = W_{G} - W_{O} - W_{\text{pay}}$ can not be achieved; in such a case, the fuel weight is capped at the maximum fuel capacity and the payload weight adjusted instead. The tanks changed can be the first size, the first size already used, or a designated size. The tanks can be added or dropped in groups of $n$ ($n = 2$ for pairs).
The weight and drag of \( N_{\text{auxtank}} \) tanks are included in the performance calculation. Optionally the number of auxiliary tanks required can be calculated at the beginning of designated mission segments (based on the aircraft fuel weight at that point), and tanks dropped if no longer needed. The weight of the auxiliary fuel tanks is an input fraction of the tank capacity:

\[
W_{\text{auxtank}} = \sum f_{\text{auxtank}} N_{\text{auxtank}} W_{\text{aux-cap}}.
\]

### 16-1.4 Auxiliary Fuel Tank Drag

The auxiliary fuel tanks are located at position \( z^F \). The drag area for one auxiliary tank is specified, \( (D/q)_{\text{auxtank}} \). The velocity relative to the air gives the drag direction \( e_d = -v^F/|v^F| \) and dynamic pressure \( q = 1/2 \rho |v^F|^2 \) (no interference). Then

\[
F^F = e_d q N_{\text{auxtank}} (D/q)_{\text{auxtank}}
\]

is the total drag force, calculated for each auxiliary tank size.

### 16-1.5 Weights

The fuel system consists of the tanks (including support) and the plumbing. For the fractional model, the fuel tank weight is

\[
W_{\text{tank}} = \chi_{\text{tank}} f_{\text{tank}} W_{\text{fuel-cap}}.
\]

The weight of the auxiliary fuel tanks is part of the fixed useful load; it is an input fraction of the tank capacity:

\[
W_{\text{auxtank}} = \sum f_{\text{auxtank}} N_{\text{auxtank}} W_{\text{aux-cap}}.
\]

The AFDD weight model for the plumbing requires the fuel flow rate (for all engines), calculated for the takeoff rating and conditions.

### 16-2 Fuel Tank System Store and Burn Energy

For fuel use and storage measured by energy, there is no weight change as energy is used. The energy storage (tank) is characterized by specific energy \( e_{\text{tank}} \) (MJ/kg) and energy density \( \rho_{\text{tank}} \) (MJ/liter). Table 16-2 gives the properties of a number of systems. The tank weight and volume are obtained from the fuel energy \( E_{\text{fuel-cap}} \) (MJ). The fuel weight \( W_{\text{fuel}} \) is zero. A motive device has an energy flow \( \dot{E} \) (MJ/hr), and its specific fuel consumption is \( \text{sfc} = \dot{E}/P \) (inverse of efficiency). An equivalent fuel flow is \( \dot{w}_{eq} = \dot{E}/e_{\text{ref}} \), based on the specific energy \( e_{\text{ref}} \) (MJ/kg) of the first fuel tank that burns weight (or GP-4, \( e_{\text{ref}} = 42.8 \) MJ/kg). The corresponding equivalent specific fuel consumption is \( \text{sfc}_{eq} = \dot{w}_{eq}/P \). The specific power is \( \pi_{\text{tank}} \) (kW/kg).

Storage systems considered include batteries, capacitors, and flywheels. A battery (or capacitor) stores charge (A-hr), so the capacity is expressed as energy for a nominal voltage. Variation of the voltage with operation affects the efficiency of the relation between useful power and the rate of change of the energy stored. Each fuel tank system that stores and burns energy has a battery model for computation of the charge/discharge efficiency. The components associated with a fuel tank system define the total energy flow \( \dot{E}_{\text{comp}} \) (charge or discharge). Accounting for battery capacity, efficiency, and losses gives the effective energy flow \( \dot{E}_{\text{eff}} \).

#### 16-2.1 Fuel Capacity

The fuel tank capacity \( E_{\text{fuel-cap}} \) (maximum usable fuel energy) is determined from designated sizing missions. The maximum mission fuel required, \( E_{\text{fuel-miss}} \) (excluding reserves and any fuel in auxiliary tanks), gives

\[
E_{\text{fuel-cap}} = \max(f_{\text{fuel-cap}} E_{\text{fuel-miss}}, E_{\text{fuel-miss}} + E_{\text{reserve}})
\]
if $E_{\text{fuel}} > E_{\text{fuel-max}}$

for designated auxiliary tank

$N_{\text{auxtank}} = N_{\text{auxtank}} + n$

$\Delta E_{\text{fuel-max}} = nE_{\text{aux-cap}}$

repeat if $E_{\text{fuel}} > E_{\text{fuel-max}}$

else if $E_{\text{fuel}} < E_{\text{fuel-max}}$

for designated auxiliary tank

(then for last nonzero $N_{\text{auxtank}}$)

$N_{\text{auxtank}} = N_{\text{auxtank}} - n$

$\Delta E_{\text{fuel-max}} = -nE_{\text{aux-cap}}$

repeat if $E_{\text{fuel}} < E_{\text{fuel-max}}$

undo last increment

$N_{\text{auxtank}} = N_{\text{auxtank}} + n$

$\Delta E_{\text{fuel-max}} = nE_{\text{aux-cap}}$

Figure 16-2. Outline of $N_{\text{auxtank}}$ calculation.

or

$$E_{\text{fuel-cap}} = d_{\text{fuel-cap}} + f_{\text{fuel-cap}}(E_{\text{fuel-miss}} + E_{\text{reserve}})$$

where $f_{\text{fuel-cap}} \geq 1$ is an input factor. Alternatively, the fuel tank capacity $E_{\text{fuel-cap}}$ can be input. The corresponding fuel tank weight is $W_{\text{tank}} = \chi_{\text{tank}}E_{\text{fuel-cap}}/\epsilon_{\text{tank}}$ (lb or kg) and the fuel tank volume is $V_{\text{tank}} = E_{\text{fuel-cap}}/\rho_{\text{tank}}$ (gallons or liters). The corresponding power capacity is $P_{\text{cap}} = (\pi_{\text{tank}}/\epsilon_{\text{tank}})E_{\text{fuel-cap}}$ (kW and MJ). Optionally the maximum mission battery discharge power gives $P_{\text{cap}}$, from which $E_{\text{fuel-cap}} = \max(E_{\text{fuel-cap}}, (\epsilon_{\text{tank}}/\pi_{\text{tank}})P_{\text{cap}})$ (MJ from kW).

For missions that are not used to size the fuel tank, the fuel energy may be fallout, or the fuel energy may be specified (with or without auxiliary tanks). The fuel energy for a flight condition or the start of a mission can be specified as an increment $d$, plus a fraction $f$ of the fuel tank capacity, plus auxiliary tanks:

$$E_{\text{fuel}} = \min(d_{\text{fuel}} + f_{\text{fuel}}E_{\text{fuel-cap}}, E_{\text{fuel-cap}}) + \sum N_{\text{auxtank}}E_{\text{aux-cap}}$$

where $E_{\text{aux-cap}}$ is the capacity of each auxiliary fuel tank.

16-2.2 Fuel Reserves

Mission fuel reserves can be specified in several ways for each mission. Fuel reserves can be defined in terms of specific mission segments, for example 200 miles plus 20 minutes at $V_{\text{be}}$. Fuel reserves can be an input fraction of the fuel burned by all (except reserve) mission segments, so $E_{\text{fuel}} = (1 + f_{\text{res}})E_{\text{fuel-miss}}$. Fuel reserves can be an input fraction of the fuel capacity, so $E_{\text{fuel}} = E_{\text{miss-seg}} + f_{\text{res}}E_{\text{fuel-cap}}$. If more than one criterion for reserve fuel is specified, the maximum reserve is used.

16-2.3 Auxiliary Fuel Tank

Auxiliary fuel tanks are defined in one or more sizes. The capacity of each auxiliary fuel tank, $E_{\text{aux-cap}}$, is an input parameter. The number of auxiliary fuel tanks on the aircraft, $N_{\text{auxtank}}$ for each size, can be specified for the flight condition or mission segment. Alternatively (if the mission is not used to size the fuel tank), the number of auxiliary fuel tanks at the start of the mission can be determined from the mission fuel.
Figure 16-2 describes the process for determining $N_{auxtank}$ from the required fuel energy $E_{fuel}$ and the aircraft maximum fuel capacity $E_{fuel\text{-}max} = E_{fuel\text{-}cap} + \sum N_{auxtank}E_{aux\text{-}cap}$. The tanks changed can be the first size, the first size already used, or a designated size. The tanks can be added or dropped in groups of $n$ ($n = 2$ for pairs).

The drag of $N_{auxtank}$ tanks is included in the performance calculation. Optionally the number of auxiliary tanks required can be calculated at the beginning of designated mission segments (based on the aircraft fuel energy at that point), and tanks dropped if no longer needed. The weight of the auxiliary fuel tanks is obtained from $e_{auxtank}$ (MJ/kg) and the tank capacity: $W_{auxtank} = \sum N_{auxtank}E_{aux\text{-}cap}/e_{auxtank}$.

### 16-2.4 Weights

The fuel system consists of the tanks (including support) and the plumbing. The plumbing weight here means the power distribution (wiring). The fuel tank weight is $E_{fuel\text{-}cap}/e_{tank} + W_{BMS}$ (lb or kg) and the fuel tank volume is $V_{fuel\text{-}cap} = E_{fuel\text{-}cap}/\rho_{tank}$ (gallons or liters). The battery management system (BMS) weight is a fraction of the basic tank weight: $W_{BMS} = f_{BMS}(E_{fuel\text{-}cap}/e_{tank})$. The wiring weight is a fraction of the basic tank weight: $f_{wiring}(E_{fuel\text{-}cap}/e_{tank})$. Alternatively, the wiring weight can be input as part of the electrical group weight.

The weight of the auxiliary fuel tanks is part of the fixed useful load; it is obtained from $e_{auxtank}$ (MJ/kg) and the tank capacity: $W_{auxtank} = \sum N_{auxtank}E_{aux\text{-}cap}/e_{auxtank}$.

### 16-3 Equipment Power

Systems that supply equipment power can directly use fuel. Equipment power (similar to drive train accessory losses) is calculated as the sum of an input constant; terms that scale with air density and temperature; deice power loss (if deice system is on); and an increment specified for each flight state:

$$P_{eq} = P_{eq0} + P_{eqd}\sigma + P_{eqt}\theta + P_{eqi} + dP_{eq}$$

where $\sigma = \rho/\rho_0$ is the density ratio and $\theta = T/T_0$ is the temperature ratio. The fuel flow or energy flow is then obtained from an input specific fuel consumption: $\bar{w}_{eq} = (sfc)P_{eq}$ or $\bar{E}_{eq} = (sfc)P_{eq}$. The energy $sfc$ is the inverse of an efficiency: $sfc=1/(3.67\eta)$ for energy flow in MJ/hr and power in kW.

### 16-4 References


Table 16-1. Fuel properties.

<table>
<thead>
<tr>
<th>fuel</th>
<th>specification</th>
<th>density</th>
<th>specific energy</th>
<th>energy dens.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>lb/gal</td>
<td>MJ/kg</td>
<td>BTU/lb</td>
</tr>
<tr>
<td>gasoline</td>
<td>MIL-STD-3013A</td>
<td>6.0*</td>
<td>0.719</td>
<td>43.50</td>
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<tr>
<td></td>
<td></td>
<td>7.0</td>
<td>0.839</td>
<td>43.03</td>
</tr>
<tr>
<td>diesel</td>
<td></td>
<td>6.84–</td>
<td>0.820–</td>
<td>43.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>7.05</td>
<td>0.845</td>
<td></td>
</tr>
<tr>
<td>Jet A/A-1</td>
<td>MIL-STD-3013A</td>
<td>6.7*</td>
<td>0.803</td>
<td>42.80</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6.84/</td>
<td>0.820/</td>
<td>42.8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6.71</td>
<td>0.804</td>
<td></td>
</tr>
<tr>
<td>JP-4</td>
<td>MIL-DTL-5624U</td>
<td>6.5</td>
<td>0.779</td>
<td>42.80</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6.23–</td>
<td>0.751–</td>
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<tr>
<td></td>
<td></td>
<td>6.69</td>
<td>0.802*</td>
<td></td>
</tr>
<tr>
<td>JP-5</td>
<td>MIL-STD-3013A</td>
<td>6.6*</td>
<td>0.791</td>
<td>42.57</td>
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<td>alternate design</td>
<td>MIL-DTL-5624U</td>
<td>6.8*</td>
<td>0.815</td>
<td>42.91</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6.58–</td>
<td>0.788*–</td>
<td>42.6*</td>
</tr>
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<td></td>
<td></td>
<td>7.05</td>
<td>0.845*</td>
<td></td>
</tr>
<tr>
<td>JP-8</td>
<td>MIL-STD-3013A</td>
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<td>0.779</td>
<td>42.80</td>
</tr>
<tr>
<td>alternate design</td>
<td>MIL-DTL-83133H</td>
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<td>0.775*–</td>
<td>42.8*</td>
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<td></td>
<td></td>
<td>7.01</td>
<td>0.840*</td>
<td></td>
</tr>
<tr>
<td>hydrogen (700 bar)</td>
<td></td>
<td>0.328</td>
<td>0.03930</td>
<td>120.</td>
</tr>
<tr>
<td>hydrogen (liquid)</td>
<td></td>
<td>0.592</td>
<td>0.07099</td>
<td>120.</td>
</tr>
</tbody>
</table>

*specification value

Table 16-2. Energy storage properties.

<table>
<thead>
<tr>
<th>energy storage type</th>
<th>tank specific energy</th>
<th>tank energy density</th>
<th>efficiency</th>
<th>power</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MJ/kg</td>
<td>kW-hr/kg</td>
<td>MJ/L</td>
<td>kW-hr/m³</td>
</tr>
<tr>
<td>lead-acid battery</td>
<td>0.11–0.14</td>
<td>0.03–0.04</td>
<td>0.22–0.27</td>
<td>60–75</td>
</tr>
<tr>
<td>nickel-cadmium battery</td>
<td>0.14–0.20</td>
<td>0.04–0.06</td>
<td>0.18–0.54</td>
<td>50–150</td>
</tr>
<tr>
<td>lithium-ion state-of-art +5 years</td>
<td>0.54–0.90</td>
<td>0.15–0.25</td>
<td>0.90–1.30</td>
<td>250–360</td>
</tr>
<tr>
<td></td>
<td>1.26</td>
<td>0.35</td>
<td>1.80</td>
<td>500</td>
</tr>
<tr>
<td></td>
<td>2.34</td>
<td>0.65</td>
<td>2.25</td>
<td>625</td>
</tr>
<tr>
<td>ultracapacitor</td>
<td>0.01–0.11</td>
<td>0.004–0.03</td>
<td>0.02–0.16</td>
<td>6–45</td>
</tr>
<tr>
<td>flywheel steel</td>
<td>0.11</td>
<td>0.03</td>
<td></td>
<td>~90%</td>
</tr>
<tr>
<td>graphite</td>
<td>0.90</td>
<td>0.25</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The propulsion group is a set of components and engine groups, connected by a drive system. The engine model describes a particular engine, used in one or more engine group. The components (rotors) define the power required. The engine groups define the power available. Figure 17-1 illustrates the power flow.

17–1 Drive System

The drive system defines gear ratios for all the components it connects. The gear ratio is the ratio of the component rotational speed to that of the primary rotor. There is one primary rotor per propulsion group (for which the reference tip speed is specified); other components are dependent (for which a gear ratio is specified). There can be more than one drive system state, in order to model a multiple-speed or variable-speed transmission. Each drive system state corresponds to a set of gear ratios.

For the primary rotor, a reference tip speed \( V_{\text{tip-ref}} \) is defined for each drive system state. By convention, the “hover tip speed” refers to the reference tip speed for drive state #1. If the sizing task changes the hover tip speed, then the ratios of the reference tip speeds at different engine states are kept constant. By convention, the gear ratio of the primary rotor is \( r = 1 \). For dependent rotors, either the gear ratio is specified (for each drive system state) or a tip speed is specified and the gear ratio is calculated \( r = \Omega_{\text{dep}} / \Omega_{\text{prim}}, \Omega = V_{\text{tip-ref}} / R \). For the engine group, either the gear ratio is specified (for each drive system state) or the gear ratio is calculated from the specification engine turbine speed \( \Omega_{\text{spec}} = (2\pi/60)N_{\text{spec}} \) and the reference tip speed of the primary rotor \( r = \Omega_{\text{spec}} / \Omega_{\text{prim}}, \Omega_{\text{prim}} = V_{\text{tip-ref}} / R \). The latter option means the specification engine turbine speed \( N_{\text{spec}} \) corresponds to \( V_{\text{tip-ref}} \) for all drive system states. To determine the gear ratios, the reference tip speed and radius are used, corresponding to hover.

The flight state specifies the tip speed of the primary rotor and the drive system state, for each propulsion group. The drive system state defines the gear ratio for dependent rotors and the engine groups. From the rotor radius, the rotational speed of the primary rotor is obtained \( \Omega_{\text{prim}} = V_{\text{tip}} / R \); from the gear ratios, the rotational speed of dependent rotors \( \Omega_{\text{dep}} = r \Omega_{\text{prim}} \) and the engine groups \( N = (60/2\pi)r_{\text{eng}} \Omega_{\text{prim}} \) are obtained; and from the rotor radius, the tip speed of the dependent rotor \( V_{\text{tip}} = \Omega_{\text{dep}} R \) is obtained. The flight state specification of the tip speed can be an input value, the reference tip speed, a function of flight speed or a conversion schedule, or one of several default values. These relationships between tip speed and rotational speed use the actual radius of the rotors in the flight state, which for a variable-diameter rotor may not be the same as the reference, hover radius.

A designated drive system state can have a variable speed (variable gear ratio) transmission, by introducing a factor \( f_{\text{gear}} \) on the gear ratio when the speeds of the dependent rotors and engines are
Figure 17-1. Power flow.
evaluated. The factor $f_{\text{gear}}$ is a component control, which can be connected to an aircraft control and thus set for each flight state.

An optional conversion schedule is defined in terms of two speeds: hover and helicopter mode for speeds below $V_{\text{Chover}}$, cruise mode for speeds above $V_{\text{Ccruise}}$, and conversion mode for speeds between $V_{\text{Chover}}$ and $V_{\text{Ccruise}}$. The tip speed is $V_{\text{tip-hover}}$ in helicopter and conversion mode, and $V_{\text{tip-cruise}}$ in airplane mode. Drive system states are defined for helicopter, cruise, and conversion mode flight. The flight state specifies the nacelle tilt angle, tip speeds, control state, and drive system state, including the option to obtain any or all of these quantities from the conversion schedule.

Several default values of the tip speed are defined for use by the flight state, including cruise, maneuver, one-engine inoperative, drive system limit conditions, and a function of flight speed (piecewise linear input). Optionally these default values can be input as a fraction of the hover tip speed. Optionally the tip speed can be calculated from $\mu = V/V_{\text{tip}}$, so $V_{\text{tip}} = V/\mu$; or from $M_{\text{at}} = M_{\text{tip}} \sqrt{(1 + \mu)^2 + \mu^2}$, so $V_{\text{tip}} = \sqrt{(c_s M_{\text{at}})^2 - V_{\text{z}}^2} - V$. Optionally the tip speed can be the minimum of the input value or that for $M_{\text{at}}$.

The sizing task might change the hover tip speed (reference tip speed for drive system state #1), the reference tip speed of a dependent rotor, a rotor radius, or the specification engine turbine speed $N_{\text{spec}}$. In such cases the gear ratios and other parameters are recalculated. Note that it is not consistent to change the reference tip speed of a dependent rotor if the gear ratio is a fixed input.

An increment on the primary rotor rotational speed (or primary engine group, if there are no rotors) is a control variable of the propulsion group.

### 17–2 Power Required

The component power required $P_{\text{comp}}$ is evaluated for a specified flight condition, as the sum of the power required by all the components of the propulsion group. The total power required for the propulsion group is obtained by adding the transmission losses and accessory power:

$$P_{\text{reqPG}} = P_{\text{comp}} + P_{\text{xmsn}} + P_{\text{acc}}$$

The transmission losses are calculated as an input fraction $\ell_{\text{xmsn}}$ of the component power, plus windage loss:

$$P_{\text{xmsn}} = \ell_{\text{xmsn}} f_{\text{xmsn}} |P_{\text{comp}}| + P_{\text{windage}} (\Omega_{\text{prim}}/\Omega_{\text{ref}})$$

The factor $f_{\text{xmsn}}$ can equal 1, or can include a function of the drive shaft limit (increasing the losses at low power):

$$f_{\text{xmsn}} |P_{\text{comp}}| = \begin{cases} \frac{1}{2} P_{X_{\text{limit}}} & Q < \frac{1}{4} \\ \left(\frac{7}{4} \frac{Q}{2} Q \right) |P_{\text{comp}}| & \frac{1}{4} < Q < 1 \\ |P_{\text{comp}}| & 1 < Q \end{cases}$$

where $Q = |P_{\text{comp}}|/P_{X_{\text{limit}}}$, $P_{X_{\text{limit}}} = r P_{D_{\text{Slimit}}}$, and $r = N/N_{\text{spec}} = \Omega_{\text{prim}}/\Omega_{\text{ref}}$. Accessory losses are calculated as the sum of an input constant; terms that scale with air density and rotor speed; a fraction of power required (such as environmental control unit (ECU) losses); infrared suppressor fan loss (if IRS system is on); deice power loss (if deice system is on); and an increment specified for each flight state:

$$P_{\text{acc}} = P_{\text{acc0}} + P_{\text{accd}} \sigma + P_{\text{accn}} (\Omega_{\text{prim}}/\Omega_{\text{ref}}) + \ell_{\text{acc}} (|P_{\text{comp}}| + P_{\text{xmsn}}) + \ell_{\text{IRfan}} \sigma N_{\text{eng}} \rho_{\text{eng}} + P_{\text{acci}} + dP_{\text{acc}}$$

where $\sigma = \rho/\rho_0$ is the density ratio.
The power required for the propulsion group must be distributed to the engine groups. With only one engine group, \( P_{reqEG} = P_{reqPG} \). An engine group power can be fixed at \( P_{reqEG} = (N_{eng} - N_{inop})A \), where \( A \) is the input power amplitude; or fraction \( A \) of engine power available, \( P_{reqEG} = (N_{eng} - N_{inop})AP_{av} \); or fraction \( A \) of engine rated power \( P_{reqEG} = (N_{eng} - N_{inop})AP_{eng} \). The power required for the remaining (perhaps all) engine groups is distributed proportional to the engine rated power:

\[
P_{reqEG} = \left( P_{reqPG} - \sum_{\text{fixed}} P_{reqEG} \right) \frac{(N_{eng} - N_{inop})P_{eng}}{\sum_{\text{not fixed}} (N_{eng} - N_{inop})P_{eng}}
\]

omitting engine groups that do not supply shaft power. If the sum of the fixed \( P_{reqEG} \) exceeds the propulsion group power required, or if the power is fixed for all engine groups, then each is scaled by the ratio \( P_{reqPG} / \sum_{\text{fixed}} P_{reqEG} \). The fuel flow of the propulsion group is obtained from the sum over the engine groups:

\[
\dot{w}_{reqPG} = \sum \dot{w}_{reqEG}.
\]

17–3 Geometry

The length of the drive system \( \ell_{DS} \) can be input or calculated. The calculated length is the sum of the longitudinal, lateral, and vertical distances from the primary rotor hub to the other hub locations, for all rotors in the propulsion group:

\[
\ell_{DS} = f \sum (|\Delta x| + |\Delta y| + |\Delta z|),
\]

where \( f \) is an input factor.

17–4 Drive System limit

The drive system limit is defined as a power limit, \( P_{DS\text{limit}} \). The limit is properly a torque limit, \( Q_{DS\text{limit}} = P_{DS\text{limit}}/\Omega_{\text{ref}} \), but is expressed as a power limit for clarity. The drive system limit can be specified as follows (with \( f_{\text{limit}} \) an input factor):

a) Input \( P_{DS\text{limit}} \).

b) From the engine takeoff power limit, \( P_{DS\text{limit}} = f_{\text{limit}} \sum N_{eng}P_{eng} \) (summed over all engine groups).

c) From the power available at the transmission sizing conditions and missions, \( P_{DS\text{limit}} = f_{\text{limit}} (\Omega_{\text{ref}}/\Omega_{prim}) \sum N_{eng}P_{av} \) (largest of all conditions and segments).

d) From the power required at the transmission sizing conditions and missions, \( P_{DS\text{limit}} = f_{\text{limit}} (\Omega_{\text{ref}}/\Omega_{prim}) \sum N_{eng}P_{req} \) (largest of all conditions and segments).

The drive system limit is a limit on the entire propulsion system. To account for differences in the distribution of power through the drive system, limits are also used for the torque of each rotor shaft (\( P_{RS\text{limit}} \)) and of each engine group (\( P_{ES\text{limit}} \)). The engine shaft limit is calculated as for the drive system limit, without the sum over engine groups. The rotor shaft limit is either input or calculated from the rotor power required at the transmission sizing flight conditions. The power limit is associated with a reference rotational speed, and when applied the limit is scaled with the rotational speed of the flight state. The rotation speed for the drive system limit \( P_{DS\text{limit}} \) is the hover speed of the primary rotor of the propulsion group (for the first drive state). The rotation speed for the engine shaft limit \( P_{ES\text{limit}} \) is the corresponding engine turbine speed. The rotation speed for the rotor shaft limit \( P_{RS\text{limit}} \) is the corresponding speed of that rotor.

The drive system limits can be specified for several levels, analogous to engine ratings. The limit \( P_{DS\text{limit}} \) is associated with the maximum continuous rating (MCQ or MCP). An alternate rating changes the torque limit by the factor \( x \). Typically \( x > 1 \) for ratings associated with short duration operation.
The torque limit is calculated from $Q_{\text{limit}} = Q/x$ for the flight condition or mission segment. The torque limit is applied as $Q = xQ_{\text{limit}}$.

17–5 Weights

The drive system consists of gear boxes and rotor shafts, drive shafts, rotor brakes, clutches, and gas drive. The drive system weight depends on the rotor and engine rotational speeds, evaluated for the propulsion group primary rotor and a specified engine group, at a specified drive system state (gear ratio).

The AFDD drive system weight model depends on $f_Q$, the second (main or tail) rotor rated torque as a fraction of the total drive system rated torque; and on $f_P$, the second (main or tail) rotor rated power as a fraction of the total drive system rated power. These parameters are related by the rotational speeds of the two rotors: $f_P = f_Q \Omega_{\text{other}}/\Omega_{\text{main}}$. Typically $f_P = f_Q = 0.6$ for twin rotors (tandem, coaxial, and tiltrotor configurations). For the single-main-rotor and tail-rotor configuration, typically $f_Q = 0.03$ and $f_P = 0.15$ (0.18 with a two-bladed teetering main-rotor).
Chapter 18

Engine Group

The engine group consists of one or more engines of a specific type. An engine group transfers power by shaft torque, so it is associated with a propulsion group. For each engine type an engine model is defined. The engine model describes a particular engine, used in one or more engine groups.

The models include turboshaft engines (perhaps convertible, for turbojet operation or reaction drive), reciprocating engines, compressors, electric motors (perhaps with fuel cells), electric generators, and generator-motors.

18–1 Engine Group Performance

The engine size is described by the power $P_{\text{eng}}$, which is the sea-level static power available per engine at a specified takeoff rating. The number of engines $N_{\text{eng}}$ is specified for each engine group.

If the sizing task determines the engine power for a propulsion group, the power $P_{\text{eng}}$ of at least one engine group is found (including the first engine group). The total power required is $P_{\text{PG}} = r \sum N_{\text{eng}} P_{\text{eng}}$, where $r = \max(P_{\text{reqPG}}/P_{\text{avPG}})$. The sized power is $P_{\text{sized}} = P_{\text{PG}} - \sum_{\text{fixed}} N_{\text{eng}} P_{\text{eng}}$. Then the sized engine power is $P_{\text{eng}} = f_n P_{\text{sized}}/N_{\text{eng}}$ for the $n$-th engine group (with $f_1 = \sum_{n \neq 1} f_n$ for the first group). If an engine group does not consume power (compressor or generator) or does not contribute to shaft power (converted), the size is scaled with $r = \max(P_{\text{reqEG}}/P_{\text{avEG}})$.

The propulsion group power available is obtained from the sum over the engine groups: $P_{\text{avPG}} = \sum P_{\text{avEG}}$.

The propulsion group component power $P_{\text{comp}}$ includes compressor power, generator power required, and generator-motor power when it is producing energy.

The flight condition information includes the altitude, temperature, flight speed, and primary rotor speed; a power fraction $f_P$; and the states of the engine, drive system, and infrared suppressor (IRS). The engine turbine speed is $N = (60/2\pi)r_{\text{eng}}\Omega_{\text{prim}}$, where $\Omega_{\text{prim}}$ is the current rotor speed and $r_{\text{eng}}$ is the gear ratio (depending on the drive system state, including a factor $f_{\text{gear}}$ for a variable speed transmission). If the reference primary rotor speed $\Omega_{\text{prim}}$ corresponds to the specification turbine speed $N_{\text{spec}}$, then $r_{\text{eng}} = \Omega_{\text{spec}}/\Omega_{\text{prim}}$; alternatively, the engine gear ratio can be a fixed input.

The drive system limit at the flight condition is $r x P_{D\text{Slimit}}$, where $r = \Omega_{\text{prim}}/\Omega_{\text{ref}}$ and $x$ is the rating factor. Optionally this limit is applied to the propulsion group power: $P_{\text{avPG}} = \min(P_{\text{avPG}}, r x P_{D\text{Slimit}})$. Similarly the engine shaft limit at the flight condition is optionally applied to the engine group power: $P_{\text{avEG}} = \min(P_{\text{avEG}}, r x P_{E\text{Slimit}})$. 
18–2 Turboshaft Engine

Turboshaft engine performance is obtained from the Referred Parameter Turboshaft Engine Model (RPTEM).

18-2.1 Power Available

Given the flight condition and engine rating, the power available \( P_a \) is calculated from the specific power \( SP_a = P_a/\dot{m}_a \) and mass flow \( \dot{m}_a \):

\[
\frac{SP_a}{\theta} = SP_0g_\theta(\theta, M, n) \\
\frac{\dot{m}_a}{\delta/\sqrt{\theta}} = \dot{m}_0g_m(\theta, M, n) \\
\frac{P_a}{\delta/\sqrt{\theta}} = P_0g_p(\theta, M, n)
\]

as functions of temperature ratio \( \theta = T/T_0 \), Mach number \( M \), and referred engine turbine speed \( n = N/\sqrt{\theta} \).

In the engine model, installation losses \( P_{loss} \) are subtracted from \( P_a (P_{av} = P_a - P_{loss}) \), and then the mechanical limit is applied: \( P_{av} = \min(P_{av}, rP_{mech}, r = N/N_{spec}) \). The mechanical limit is properly a torque limit, \( Q_{mech} = P_{mech}/N_{spec} \), but is expressed as a power limit for clarity.

The engine model gives the performance of a single engine. The power available of the engine group is obtained by multiplying the single engine power by the number of engines operational (total number of engines less inoperable engines):

\[
P_{avEG} = f_P(N_{eng} - N_{inop})P_{av}
\]

including a specified power fraction \( f_P \).

18-2.2 Performance at Power Required

The engine performance (mass flow, fuel flow, and gross jet thrust) is calculated for a specified power required \( P_q \) and flight condition:

\[
\frac{\dot{m}_{req}}{\delta/\sqrt{\theta}} = \dot{m}_{0C}g_m(q, \theta, M, n) \\
\frac{\dot{w}_{req}}{\delta/\sqrt{\theta}} = \dot{w}_{0C}g_w(q, \theta, M, n) \\
F_g/\delta = F_{g0C}g_f(q, \theta, M, n)
\]

as functions of \( q = P_q/(P_{0C}\delta/\sqrt{\theta}) \), temperature ratio \( \theta = T/T_0 \), Mach number \( M \), and referred engine turbine speed \( n = N/(N_{spec}\sqrt{\theta}) \).

The engine model deals with a single engine. The power required of a single engine is obtained by dividing the engine group power by the number of engines operational (total number of engines less inoperable engines):

\[
P_{req} = P_{reqEG}/(N_{eng} - N_{inop})
\]

In the engine model, installation losses \( P_{loss} \) are added to \( P_{req} \): \( P_q = P_{req} + P_{loss} \).
The engine model gives the performance of a single engine. The performance of the engine group is obtained by multiplying the single engine characteristics by the number of engines operational (total number of engines less inoperable engines):

\[
\dot{m}_{reqEG} = (N_{eng} - N_{inop})\dot{m}_{req}
\]
\[
\dot{w}_{reqEG} = (N_{eng} - N_{inop})\dot{w}_{req}K_{ffd}
\]
\[
F_{NEG} = (N_{eng} - N_{inop})F_N
\]
\[
D_{auxEG} = (N_{eng} - N_{inop})D_{aux}
\]

The fuel flow has also been multiplied by a factor $K_{ffd}$ accounting for deterioration of the engine efficiency.

**18-2.3 Installation**

The difference between installed and uninstalled power is the inlet and exhaust losses $P_{loss}$: $P_{av} = P_a - P_{loss}$ and $P_{req} = P_q - P_{loss}$. The inlet ram recovery efficiency $\eta_d$ is included in the engine model calculations. The inlet and exhaust losses are modeled as fractions of power available or power required: $P_{loss} = (\ell_{in} + \ell_{ex})P_a$ or $P_{loss} = (\ell_{in} + \ell_{ex})P_q$. The installed gross jet thrust is $F_G = K_{fgr}F_g$, where $K_{fgr}$ accounts for exhaust effects. The net jet thrust is $F_N = F_G - \dot{m}_{req}V$. The momentum drag of the auxiliary air flow is a function of the mass flow $\dot{m}_{aux} = f_{aux}\dot{m}_{req}$:

\[
D_{aux} = (1 - \eta_{aux})\dot{m}_{aux}V = (1 - \eta_{aux})f_{aux}\dot{m}_{req}V
\]

where $\eta_{aux}$ is the ram recovery efficiency. Exhaust losses ($\ell_{ex}$) and auxiliary air flow parameters ($\eta_{aux}$, $f_{aux}$) are defined for IR suppressor on and off. Inlet particle separator loss is added to the inlet losses ($\ell_{in}$).

**18-2.4 Convertible Engine: Turbojet/Turbofan**

The engine mode $B$ is the mass flow fraction diverted for a convertible engine: $B = 0$ for all mass flow to the power turbine (turboshaft operation), and $B = 1$ for all mass flow to the jet exhaust or a fan (turbojet/turbofan operation).

A separate engine model defines the performance for turbojet/turbofan operation ($B = 1$). The engine group power $P_{reqEG}$ is prescribed, as a measure of the jet thrust, and this engine does not contribute to the propulsion group shaft power available. The turbojet/turbofan thrust is the engine group net jet thrust, $F_N = F_G - \dot{m}_{req}V$.

**18-2.5 Convertible Engine: Reaction Drive**

The engine mode $B$ is the mass flow fraction diverted for a convertible engine: $B = 0$ for all mass flow to the power turbine (turboshaft operation), and $B = 1$ for all mass flow to the rotor (reaction jet operation).

A separate engine model defines the engine performance for reaction jet operation ($B = 1$). The engine group power is fixed by amplitude input, or obtained from the rotor power ($P_{reqEG} = P_{react}$), and this engine does not contribute to the propulsion group shaft power available. The gross jet thrust is zero, so the net thrust is the momentum drag, $F_N = -\dot{m}_{req}V$. 
A simple tabular model of the engine performance is implemented, suitable for use with data from a limited exercise of an engine deck. The tables are for power available $P_a$, fuel flow $\dot{w}$, and net jet thrust $F_N$ as a function of altitude $h$, flight speed $V$, and rating $R$. The altitude influence is for a selected atmosphere, hence for a temperature variation with altitude. The ratings reflect engine throttle setting, including partial power. Power turbine speed variation is not considered. The tables are used with linear interpolation.

Given the altitude, speed, and rating, the power table $T_p(h, V, R)$ is interpolated. Then the power available is

$$P_a = K_p T_p$$

as a function of density ratio $\sigma = \rho / \rho_0$, temperature ratio $\theta = T / T_0$, Mach number $M$, and engine speed ratio $r = N / N_{spec}$. Installation losses $P_{loss}$ are subtracted from $P_a$ ($P_{av} = P_a - P_{loss}$), and then the mechanical limit is applied: $P_{av} = \min(P_{av}, r P_{mech} R)$. The mechanical limit is properly a torque limit, $Q_{mech} = P_{mech} / N_{spec}$, but is expressed as a power limit for clarity. The power available of the engine group is obtained by multiplying the single engine power by the number of engines operational (total number of engines less inoperable engines): $P_{avEG} = f_P(N_{eng} - N_{inop}) P_{av}$, including a specified power fraction $f_P$.

The engines are for an installed engine, including losses. Hence $P_{loss} = 0$ ($\epsilon_{in} = \epsilon_{ex} = 0$) with this model, and $P_{av} = P_a$, $P_{req} = P_q$. Mechanical limits are included in the power available data. Engine mass flow is not considered, so $D_{aux} = 0$, and the table is for net jet thrust. The fuel flow is multiplied by the factor $K_{ffd}$, accounting for deterioration of the engine efficiency. The engine is not scaled. The engine weight $W_{one eng}$ is fixed.

Reciprocating engine performance is obtained from the Reciprocating Engine Model.

### 18-4.1 Power Available

Given the flight condition and engine rating, the power available $P_a$ is calculated from:

$$P_a = P_{0gp}(\sigma, \theta, M, r)$$

as a function of density ratio $\sigma = \rho / \rho_0$, temperature ratio $\theta = T / T_0$, Mach number $M$, and engine speed ratio $r = N / N_{spec}$.

Installation losses $P_{loss}$ are subtracted from $P_a$ ($P_{av} = P_a - P_{loss}$), and then the mechanical limit is applied: $P_{av} = \min(P_{av}, r P_{mech} R)$. The mechanical limit is properly a torque limit, $Q_{mech} = P_{mech} / N_{spec}$, but is expressed as a power limit for clarity. The power available of the engine group is obtained by multiplying the single engine power by the number of engines operational (total number of engines less inoperable engines): $P_{avEG} = f_P(N_{eng} - N_{inop}) P_{av}$, including a specified power fraction $f_P$.

### 18-4.2 Performance at Power Required

The engine performance for a specified power required $P_q$ is:

$$\dot{w}_{req} = \dot{w}_{0gw}(q, r)$$
$$\dot{m}_{req} = \dot{m}_{0gm}(q, r)$$
$$F_g = F_{0gf}(q, r)$$

where $q = P_q / P_0$ and $r = N / N_{spec}$. Installation losses $P_{loss}$ are added to $P_{req}$: $P_q = P_{req} + P_{loss}$, $P_{req} = P_{reqEG} / (N_{eng} - N_{inop})$. 
The performance of the engine group is obtained by multiplying the single engine characteristics by the number of engines operational (total number of engines less inoperable engines):

\[
\dot{m}_{reqEG} = (N_{\text{eng}} - N_{\text{inop}})\dot{m}_{req}
\]

\[
w_{reqEG} = (N_{\text{eng}} - N_{\text{inop}})w_{req}K_{ffd}
\]

\[
F_{NEG} = (N_{\text{eng}} - N_{\text{inop}})F_N
\]

\[
D_{auxEG} = (N_{\text{eng}} - N_{\text{inop}})D_{aux}
\]

The fuel flow has also been multiplied by a factor \(K_{ffd}\) accounting for deterioration of the engine efficiency.

---

18-4.3 Installation

The difference between installed and uninstalled power is the installation loss: \(P_{av} = P_a - P_{loss}\) and \(P_{req} = P_q - P_{loss}\). This loss is modeled by an efficiency factor: \(P_{loss} = (1 - \eta_{loss})P_a\) or \(P_{loss} = (1 - \eta_{loss})P_q\), where \(\eta_{loss} = \ell_{in} + \ell_{ex}\). The installed gross jet thrust is \(F_G = K_{fgr}F_g\), where \(K_{fgr}\) accounts for exhaust effects. The net jet thrust is \(F_N = F_G - \dot{m}_{req}V\). The momentum drag of the auxiliary air flow is a function of the mass flow \(\dot{m}_{aux} = f_{aux}\dot{m}_{req}\):

\[
D_{aux} = (1 - \eta_{aux})\dot{m}_{aux}V = (1 - \eta_{aux})f_{aux}\dot{m}_{req}V
\]

where \(\eta_{aux}\) is the ram recovery efficiency.

---

18-5 Compressor

A compressor converts input shaft power to a jet velocity and thrust. The shaft power contributes to the propulsion group power required. The compressor does not use fuel.

18-5.1 Power Available

Given the flight condition and engine rating, the power available \(P_a\) is calculated from the specific power \(SP_a = P_a/\dot{m}_a\) and mass flow \(\dot{m}_a\):

\[
\frac{SP_a}{\theta} = SP_0g_{sp}(\theta, M, n)
\]

\[
\frac{\dot{m}_a}{\delta/\sqrt{\theta}} = \dot{m}_0g_m(\theta, M, n)
\]

\[
\frac{P_a}{\delta/\sqrt{\theta}} = P_0g_\theta(\theta, M, n)
\]

as functions of temperature ratio \(\theta = T/T_0\), Mach number \(M\), and referred compressor speed \(n = N/\sqrt{\theta}\).

Installation losses \(P_{loss}\) are subtracted from \(P_a\) \((P_{av} = P_a - P_{loss})\), and then the mechanical limit is applied: \(P_{av} = \min(P_{av}, rP_{mechR})\), \(r = N/N_{\text{spec}}\). The mechanical limit is properly a torque limit, \(Q_{mech} = P_{mech}/N_{\text{spec}}\), but is expressed as a power limit for clarity.

The compressor model gives the performance of a single compressor. The power available of the engine group is obtained by multiplying the single compressor power by the number of engines operational (total number of engines less inoperable engines):

\[
P_{avEG} = f_P(N_{\text{eng}} - N_{\text{inop}})P_{av}
\]
including a specified power fraction $f_P$.

18-5.2 Performance at Power Required

The compressor performance (mass flow and gross jet thrust) is calculated for a specified power required $P_q$ and flight condition:

$$\frac{\dot{m}_{req}}{\delta/\sqrt{\theta}} = \dot{m}_{0C} g_m(q, \theta, M, n)$$

$$\frac{ST_{req}}{\sqrt{\theta}} = ST_{0C} g_{st}(q, \theta, M, n)$$

$$\frac{F_g}{\delta} = F_{g0C} g_f(q, \theta, M, n)$$

as functions of $q = P_q/(P_{0C} \delta \sqrt{\theta})$, temperature ratio $\theta = T/T_0$, Mach number $M$, and referred compressor speed $n = N/\sqrt{\theta})$. The specific thrust gives the gross thrust, $F_g = (ST)\dot{m}$.

The power required of a single compressor is obtained by dividing the engine group power by the number of engines operational (total number of engines less inoperable engines):

$$P_{req} = P_{reqEG}/(N_{eng} - N_{inop})$$

Accounting for installation losses gives the uninstalled power required $P_q = P_{req} + P_{loss}$.

The compressor model gives the performance of a single compressor. The performance of the engine group is obtained by multiplying the single compressor characteristics by the number of engines operational:

$$\dot{m}_{reqEG} = (N_{eng} - N_{inop})\dot{m}_{req}$$

$$F_{NEG} = (N_{eng} - N_{inop})F_N$$

$$D_{auxEG} = (N_{eng} - N_{inop})D_{aux}$$

$$P_{comp} = (N_{eng} - N_{inop})P_q K_{ffd}$$

The component power is the product of the uninstalled power required and the number of operational engines, and a factor $K_{ffd}$ accounting for deterioration of the engine efficiency.

18-5.3 Installation

The difference between installed and uninstalled power is the inlet and exhaust losses $P_{loss}$: $P_{av} = P_a - P_{loss}$ and $P_{req} = P_q - P_{loss}$. The inlet and exhaust losses are modeled as fractions of power available or power required: $P_{loss} = (\ell_{in} + \ell_{ex})P_a$ or $P_{loss} = (\ell_{in} + \ell_{ex})P_q$. The installed gross jet thrust is $F_G = K_{fg} F_g$, where $K_{fg}$ accounts for exhaust effects. The net jet thrust is $F_N = F_G - \dot{m}_{req} V$. The momentum drag of the auxiliary air flow is a function of the mass flow $\dot{m}_{aux} = f_{aux} \dot{m}_{req}$:

$$D_{aux} = (1 - \eta_{aux})\dot{m}_{aux} V = (1 - \eta_{aux}) f_{aux} \dot{m}_{req} V$$

where $\eta_{aux}$ is the ram recovery efficiency.

18-5.4 Compressor for Reaction Drive

If the compressor supplies the jet force for rotor reaction drive, then the engine group power required is fixed by amplitude input, or obtained from the rotor power ($P_{reqEG} = P_{react}$). The gross jet thrust is zero, so the net thrust is the momentum drag, $F_N = -\dot{m}_{req} V$. 


18-6 Electric Motor or Generator

A motor converts electrical energy (fuel) to shaft power. A generator converts input shaft power to electrical energy, and the shaft power contributes to the propulsion group power required.

The power available is related to the size $P_{\text{eng}}$. Given the flight condition and motor rating, the uninstalled power available $P_{\text{av}}$ is calculated, including a torque limit. The motor model gives the performance of a single engine. The power available of the engine group is obtained by multiplying the single engine power by the number of engines operational:

$$P_{\text{avEG}} = f_P(N_{\text{eng}} - N_{\text{inop}})P_{\text{av}}$$

including a specified power fraction $f_P$.

From the engine group power required $P_{\text{reqEG}}$, the power required of a single engine is

$$P_{\text{req}} = P_{\text{reqEG}}/(N_{\text{eng}} - N_{\text{inop}})$$

18-6.1 Motor

The motor power required determines the energy flow from the fuel tank. The energy flow is calculated for $P_{\text{req}}$ and a specified flight condition:

$$\dot{E}_{\text{req}} = \dot{E}_{0C}g_e(q,n)$$

as a function of $q = P_{\text{req}}/P_{\text{eng}}$ and engine speed $n = N/N_{\text{spec}}$. The motor model gives the performance of a single engine. The performance of the engine group is obtained by multiplying the single engine characteristics by the number of engines operational:

$$\dot{E}_{\text{reqEG}} = (N_{\text{eng}} - N_{\text{inop}})\dot{E}_{\text{req}}K_{f\text{fd}}$$

The energy flow has also been multiplied by a factor $K_{f\text{fd}}$ accounting for deterioration of the engine efficiency.

18-6.2 Generator

The generator energy flow to the fuel tank defines the power required. The energy flow is calculated for $P_{\text{req}}$ and a specified flight condition:

$$\dot{E}_{\text{req}} = \dot{E}_{0C}g_e(q,n)$$

as a function of $q = P_{\text{req}}/P_{\text{eng}}$ and engine speed $n = N/N_{\text{spec}}$. The motor model gives the performance of a single engine. The performance of the engine group is obtained by multiplying the single engine characteristics by the number of engines operational:

$$\dot{E}_{\text{reqEG}} = (N_{\text{eng}} - N_{\text{inop}})\dot{E}_{\text{req}}K_{f\text{fd}}$$

The component power is the product of the power required and the number of operational engines, and a factor $K_{f\text{fd}}$ accounting for deterioration of the engine efficiency.
18-6.3 Generator-Motor

The engine mode \( B \) is the direction of power flow for a generator-motor: \( B \) positive for motor operation, and \( B \) negative for generator operation. Separate motor models are used for the two modes.

18-6.4 Motor and Fuel Cell

A motor with a fuel cell burns a fuel (typically hydrogen) to produce electrical energy, which is converted to shaft power. The device can also be modelled as separate motor and fuel cell components, with a battery (fuel tank) to transfer the electrical energy.

The engine performance (mass flow and fuel flow) is calculated for \( P_{req} \) and a specified flight condition:

\[
\dot{m}_{req} = \dot{m}_0 C_g m(q, n) = K_{mf} \dot{w}_{req} \\
\dot{w}_{req} = \dot{w}_0 C_g w(q, n) = \text{sfc} \ P_{req}
\]

as a function of \( q = P_{req}/P_{eng} \) and engine speed \( n = N/N_{spec} \). Inlet and exhaust losses are included in the specific fuel consumption. The net thrust is the inlet momentum drag, \( F_N = -\dot{m}_{req} V \). The motor model gives the performance of a single engine. The performance of the engine group is obtained by multiplying the single engine characteristics by the number of engines operational:

\[
\dot{m}_{reqEG} = (N_{eng} - N_{inop})\dot{m}_{req} \\
\dot{w}_{reqEG} = (N_{eng} - N_{inop})\dot{w}_{req} K_{ffd} \\
F_{NEG} = (N_{eng} - N_{inop})F_N \\
D_{auxEG} = (N_{eng} - N_{inop})D_{aux}
\]

The fuel flow has also been multiplied by a factor \( K_{ffd} \) accounting for deterioration of the engine efficiency. The momentum drag of the auxiliary air flow is a function of the mass flow \( \dot{m}_{aux} = f_{aux}\dot{m}_{req} \):

\[
D_{aux} = (1 - \eta_{aux})\dot{m}_{aux} V = (1 - \eta_{aux})f_{aux}\dot{m}_{req} V
\]

where \( \eta_{aux} \) is the ram recovery efficiency.

18-7 Control and Loads

The engine power amplitude \( A \) and mode \( B \) are control variables. The mode can be mass flow (for convertible engines), or power flow (for generator-motor). In distributing the propulsion group power required to the engine groups, an engine group power can be fixed at \( P_{reqEG} = (N_{eng} - N_{inop})A \); or fraction \( A \) of engine power available, \( P_{reqEG} = (N_{eng} - N_{inop})A P_{av} \); or fraction \( A \) of engine rated power \( P_{reqEG} = (N_{eng} - N_{inop})A P_{eng} \).

The engine orientation is specified by selecting a nominal direction \( e_f_0 \) in body axes (positive or negative \( x \)-, \( y \)-, or \( z \)-axis; usually thrust forward, hence positive \( x \)-axis); then applying a yaw angle \( \psi \); and then an incidence or tilt angle \( i \). The yaw and incidence angles can be control variables.

The engine group produces a jet thrust \( F_N \), acting in the direction of the engine (actually, \( F_N = F_G - \dot{m}V \), with the gross thrust in the direction of the engine and the momentum thrust in the wind direction); a momentum drag \( D_{aux} \) (if the component has mass flow), acting in the wind direction; and a nacelle drag \( D_{nac} \), acting in the wind direction. The engine group is at location \( z_F \). The force and moment acting on the aircraft in body axes are thus:

\[
F^F = (e_f F_G + e_d \dot{m} V) + e_d D_{aux} + e_d D_{nac} \\
M^F = \Delta z^F F^F
\]
where $\Delta z^F = z^F - z_{cg}^F$, $e_f$ is the engine thrust direction and $e_d$ is the drag direction.

### 18–8 Nacelle Drag

The engine group includes a nacelle, which contributes to the aircraft drag. The component drag contributions must be consistent. The pylon is the rotor support and the nacelle is the engine support. The drag model for a tiltrotor aircraft with tilting engines would use the pylon drag (and no nacelle drag), since the pylon is connected to the rotor shaft axes; with non-tilting engines it would use the nacelle drag as well.

The reference area for the nacelle drag coefficient is the nacelle wetted area. The wetted area per engine is input, or calculated either from the engine system (engine, exhaust, and accessories) weight or from the engine system plus drive system weight:

$$S_{nac} = N_{eng} S_{wet}$$

where $w = W_{ES}$ or $w = W_{ES} + W_{gbrs}/N_{EG}$, and the units of $k$ are $\text{ft}^2/\text{lb}^{2/3}$ or $\text{m}^2/\text{kg}^{2/3}$. The reference area is then $S_{nac} = N_{eng} S_{wet}$.

### 18–9 Weights

The component weight consists of engine system, engine section or nacelle group, and air induction group. The engine system consists of engine, exhaust system, and accessories. The engine section or nacelle group consists of engine support, engine cowling, and pylon support. These weights are for the engine group, consisting of $N_{eng}$ engines. The engine system weight $W_{ES} = W_{eng} + W_{exh} + W_{acc}$ is used for the engine nacelle wetted area, the rotor pylon wetted area, and the rotor moving weight. The rotor group includes the rotor support structural weight; this must be consistent with the use of the engine support weight and pylon support weight.
Chapter 19

Jet Group

A jet group produces a force on the aircraft, possibly used for lift, propulsion, or control. A jet model describes a particular jet, used in one or more jet groups. The models include turbojet and turbofan engines (perhaps convertible, for reaction drive), reaction drive, and a simple force. A reaction drive supplies a blade force that provides the rotor power required.

19–1 Jet Group Performance

The jet size is described by the thrust $T_{\text{jet}}$, which is the sea-level static thrust available per jet at a specified takeoff rating. The number of jets $N_{\text{jet}}$ is specified for each jet group.

If the sizing task determines the jet thrust, the total thrust required is $T_{\text{JG}} = r N_{\text{jet}} T_{\text{jet}}$, where $r = \max(T_{\text{reqJG}}/T_{\text{avJG}})$.

The flight condition information includes the altitude, temperature, and flight speed; and a thrust fraction $f_T$.

19–2 Turbojet or Turbofan

The thrust of a turbojet is

$$ T = \dot{m} ((1 + f)V_e - V) + (p_e - p_{\text{atm}}) A_e $$

where $\dot{m}$ is the mass flow; $f = \dot{w}/\dot{m}$ is the fuel-air ratio; and $V_e$, $p_e$, and $A_e$ are the velocity, pressure, and area at the exit. The pressure term is zero or small, and the fuel-air ratio is small, so the net thrust is approximately

$$ T = \dot{m} (V_e - V) $$

from the gross thrust $T_G = \dot{m} V_e$ and the inlet-momentum or ram drag $\dot{m} V$.

The thrust of a turbofan is

$$ T = \dot{m} ((1 + f)V_e - V) + \dot{m}_{\text{fan}} (V_{\text{efan}} - V) = \dot{m} ((1 + f)V_e + \beta V_{\text{efan}}) - \dot{m}(1 + \beta)V $$

with bypass ratio $\beta = \dot{m}_{\text{fan}}/\dot{m}$.

Turbojet or turbofan performance is obtained from the Referred Parameter Jet Engine Model (RP-JEM), based on references 1 and 2. The referred thrust, specific thrust $ST = T/\dot{m}$, and specific fuel con-
The specific fuel consumption $sfc = \dot{w}/T$ are functions of the flight Mach number $M$ and compressor speed $n = N/(N_0 \sqrt{\theta})$:

\[
\frac{T}{\delta} = G_t(M, n) \\
\frac{T/\dot{m}}{\sqrt{\theta}} = G_{st}(M, n) \\
\frac{\dot{w}/T}{\sqrt{\theta}} = \frac{1}{\eta_b} G_{sfc}(M, n)
\]

The independent variable can be the compressor speed, or the turbine inlet temperature, or the fuel flow. The combustion efficiency $\eta_b$ depends on the atmosphere (altitude and temperature), hence the specific fuel consumption is not a function of just $M$ and $n$.

19-2.1 Thrust Available

Given the flight condition and engine rating, the thrust available $T_a$ is calculated from the specific thrust $ST_a = T_a/\dot{m}_a$ and mass flow $\dot{m}_a$:

\[
\frac{ST_a}{\sqrt{\theta}} = ST_{0gst}(\theta, M) \\
\frac{\dot{m}_a}{\delta/\sqrt{\theta}} = \dot{m}_{0gm}(\theta, M) \\
\frac{T_a}{\delta} = T_{0gt}(\theta, M)
\]

as functions of temperature ratio $\theta = T/T_0$ and Mach number $M$.

In the jet model, installation losses $T_{loss}$ are subtracted from $T_a$ ($T_{av} = T_a - T_{loss}$), and then the mechanical limit is applied: $T_{av} = \min(T_{av}, T_{mechR})$.

The jet model gives the performance of a single jet. The thrust available of the jet group is obtained by multiplying the single jet thrust by the number of jets operational (total number of jets less inoperable jets):

$T_{avJG} = f_T(N_{jet} - N_{inop})T_{av}$

including a specified thrust fraction $f_T$.

19-2.2 Performance at Thrust Required

The thrust required of a single jet is obtained by dividing the jet group thrust by the number of jets operational (total number of jets less inoperable jets):

$T_{req} = T_{reqJG}/(N_{jet} - N_{inop})$

In the jet model, installation losses $T_{loss}$ are added to $T_{req}$ ($T_q = T_{req} + T_{loss}$).

The jet performance (mass flow and fuel flow) is calculated for a specified thrust required $T_q$ and flight condition:

\[
\frac{\dot{m}_{req}}{\delta/\sqrt{\theta}} = \dot{m}_{0c gm}(t, \theta, M) \\
\frac{\dot{w}_{req}}{\delta/\sqrt{\theta}} = \dot{w}_{0c gm}(t, \theta, M)
\]
as functions of \( t = T_q/(T_0 \delta) \) (or referred gross thrust), temperature ratio \( \theta = T/T_0 \), and Mach number \( M \).

The jet model gives the performance of a single jet. The performance of the jet group is obtained by multiplying the single jet characteristics by the number of jets operational:

\[
\dot{m}_{reqJG} = (N_{\text{jet}} - N_{\text{inop}})\dot{m}_{req}
\]

\[
\dot{w}_{reqJG} = (N_{\text{jet}} - N_{\text{inop}})\dot{w}_{req}K_{ffd}
\]

\[
D_{auxJG} = (N_{\text{jet}} - N_{\text{inop}})D_{aux}
\]

The fuel flow has also been multiplied by a factor \( K_{ffd} \) accounting for deterioration of the jet efficiency.

### 19-2.3 Installation

The difference between installed and uninstalled thrust is the inlet and exhaust losses \( T_{\text{loss}} \): \( T_{av} = T_a - T_{\text{loss}} \) and \( T_{req} = T_q - T_{\text{loss}} \). The inlet and exhaust losses are modeled as fractions of thrust available or thrust required: \( T_{\text{loss}} = (\ell_{in} + \ell_{ex})T_a \) or \( T_{\text{loss}} = (\ell_{in} + \ell_{ex})T_q \). The momentum drag of the auxiliary air flow is a function of the mass flow \( \dot{m}_{aux} = f_{aux}\dot{m}_{req} \):

\[
D_{aux} = (1 - \eta_{aux})\dot{m}_{aux}V = (1 - \eta_{aux})f_{aux}\dot{m}_{req}V
\]

where \( \eta_{aux} \) is the ram recovery efficiency. Exhaust losses (\( \ell_{ex} \)) and auxiliary air flow parameters (\( \eta_{aux}, f_{aux} \)) are defined for IR suppressor on and off.

### 19-2.4 Convertible Engine: Reaction Drive

The jet mode \( B \) is the mass flow fraction diverted for a convertible engine: \( B = 0 \) for all mass flow to the exhaust (turbojet operation), and \( B = 1 \) for all mass flow to the rotor (reaction jet operation).

A separate jet model defines the engine performance for \( B = 1 \). The jet thrust required is fixed by amplitude input, or obtained from the rotor power \( (T_{reqJG} = F_{\text{react}} = P_{\text{req}}/\Omega r_{\text{react}}) \). The mass flow and fuel flow follow. The jet group net thrust is the inlet momentum drag, \( F_N = -\dot{m}_{req}V \).

Thrust, specific thrust, and mass flow are defined by the jet model for turbojet operation. The specific fuel consumption of the jet model for reaction jet operation is scaled proportional to the \( sfc \) scaling for turbojet operation.

### 19–3 Reaction Drive

Rotor power can be supplied by a reaction drive, using cold or hot air ejected out of the blade tips or trailing edges. Helicopters have also been designed with ram jets on the blade tips, or with jet flaps on the blade trailing edges that use compressed air generated in the fuselage. The jet group performance includes the blade duct and nozzle, perhaps even with tip burning.

The net jet thrust required is fixed by amplitude input, or obtained from the rotor power. The rotor power required \( P_{\text{req}} \) gives the required force on the rotor blade

\[
F_{\text{react}} = P_{\text{req}}/\Omega r_{\text{react}} = \dot{m}_{\text{react}} (V_{\text{react}} - \Omega r_{\text{react}})
\]

at effective radial station \( r_{\text{react}} \). The net jet group thrust required is

\[
T_{reqJG} = F_{\text{react}} = T_G - \dot{m}_{reqJG}\Omega r_{\text{react}}
\]
The gross thrust. From $T_{reqJG}$, the jet performance (mass flow and fuel flow) is calculated. The jet group net thrust is the inlet momentum drag, $F_N = -\dot{m}_{req}V$.

19–4 Simple Force

For the simple force model, the design maximum thrust is $T_{max}$ (per jet). The thrust available is thus $T_a = T_{max}$. The force generation can use fuel as weight or as energy.

If the component burns fuel weight, the fuel flow is calculated from an input thrust-specific fuel consumption: $\dot{w}_{req} = T_q(sfc) = \dot{w}_0C_q$, where $q = T_q/T_{max}$. Units of sfc are pound/hour/pound or kilogram/hour/Newton. Then

$$\dot{w}_{reqJG} = (N_{jet} - N_{inop})\dot{w}_{req}K_{ffd}$$

The fuel flow has also been multiplied by a factor $K_{ffd}$ accounting for deterioration of the jet efficiency.

If the component uses fuel energy, the energy flow is calculated from an input thrust-specific fuel consumption: $\dot{E}_{req} = T_q(sfc) = \dot{E}_0C_q$, where $q = T_q/T_{max}$. Units of sfc are MJ/hour/pound or MJ/hour/Newton. Then

$$\dot{E}_{reqJG} = (N_{jet} - N_{inop})\dot{E}_{req}K_{ffd}$$

The energy flow has also been multiplied by a factor $K_{ffd}$ accounting for deterioration of the jet efficiency.

The simple force weight is calculated from specific weight $S$ plus a fixed increment: $W = ST_{max} + \Delta W$. This weight is identified as either engine system or propeller/fan installation weight, both of the propulsion group; or tail-rotor, of empennage group.

19–5 Control and Loads

The jet thrust amplitude $A$ and mode $B$ are control variables. The mode can be mass flow (for convertible engines). The jet group thrust can be fixed at $T_{reqJG} = (N_{jet} - N_{inop})A$; or fraction $A$ of jet thrust available, $T_{reqJG} = (N_{jet} - N_{inop})AT_{av}$; or fraction $A$ of jet rated thrust $T_{reqJG} = (N_{jet} - N_{inop})AT_{jet}$.

The jet orientation is specified by selecting a nominal direction $e_{f0}$ in body axes (positive or negative $x$-, $y$-, or $z$-axis; usually thrust forward, hence positive $x$-axis); then applying a yaw angle $\psi$; and then an incidence or tilt angle $i$. The yaw and incidence angles can be control variables.

The jet group produces a jet thrust $T$, acting in the direction of the engine (actually, the gross thrust in the direction of the engine and the momentum thrust in the wind direction); a momentum drag $D_{aux}$, acting in the wind direction; and a nacelle drag $D_{nac}$, acting in the wind direction. The jet group is at location $z^F$. The force and moment acting on the aircraft in body axes are thus:

$$F^F = (e_fT_G + e_dT_{mom}) + e_dD_{aux} + e_dD_{nac}$$

$$M^F = \Delta z^F D^F$$

where $\Delta z^F = z^F - z_{cg}^F$, $e_f$ is the jet thrust direction and $e_d$ is the drag direction.

19–6 Nacelle Drag

The jet group includes a nacelle, which contributes to the aircraft drag. The reference area for the nacelle drag coefficient is the nacelle wetted area. The wetted area per jet is input, or calculated from
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the engine system (engine, exhaust, and accessories) weight:

\[ S_{\text{wet}} = k \left( \frac{w}{N_{\text{jet}}} \right)^{2/3} \]

where \( w = W_{ES} \), and the units of \( k \) are \( \text{ft}^2/\text{lb}^{2/3} \) or \( \text{m}^2/\text{kg}^{2/3} \). The reference area is then \( S_{\text{nac}} = N_{\text{jet}} S_{\text{wet}} \).

19–7 Weights

The component weight consists of engine system, engine section or nacelle group, and air induction group. The engine system consists of engine, exhaust system, and accessories. The engine section or nacelle group consists of engine support, engine cowling, and pylon support. These weights are for the jet group, consisting of \( N_{\text{jet}} \) engines. The engine system weight \( W_{ES} = W_{\text{eng}} + W_{\text{exh}} + W_{\text{acc}} \) is used for the engine nacelle wetted area.

19–8 References

A charge group generates energy for the aircraft. A charger model describes a particular charger, used in one or more charge groups. The models include fuel cells and solar cells.

### 20–1 Charge Group Performance

The charger size is described by the power \( P_{\text{chrg}} \), which is the sea-level static power available per charger at a specified takeoff rating. The number of chargers \( N_{\text{chrg}} \) is specified for each charge group.

If the sizing task determines the charger power, the charger power required is \( rP_{\text{chrg}} \), where \( r = \max(P_{\text{reqtotal}}/P_{\text{avtotal}}) \).

The flight condition information includes the altitude, temperature, and flight speed; and a power fraction \( f_p \).

Given the flight condition and the charger rating, the cell power available is \( \dot{E}_{\text{acell}} = P_0 \). The total cell power available is obtained by multiplying the single charger power by the number of chargers operational (total number of chargers less inoperable chargers):

\[
P_{\text{avtotal}} = f_p(N_{\text{chrg}} - N_{\text{inop}}) \dot{E}_{\text{acell}}
\]

including a specified power fraction \( f_p \).

The energy flow to the fuel tank gives the charge group power required: \( P_{\text{reqCG}} = \dot{E}_{\text{reqCG}} \). The energy flow required of a single charger is

\[
\dot{E}_{\text{req}} = \dot{E}_{\text{reqCG}} K_{\text{ffd}} / (N_{\text{chrg}} - N_{\text{inop}})
\]

The energy flow has also been multiplied by a factor \( K_{\text{ffd}} \) accounting for deterioration of the charger efficiency. The cell energy flow required \( \dot{E}_{\text{qcell}} \) is obtained from the energy flow. Then

\[
P_{\text{reqtotal}} = (N_{\text{chrg}} - N_{\text{inop}}) \dot{E}_{\text{qcell}}
\]

is the total cell power required.

### 20–2 Fuel Cell

A fuel cell burns a fuel (typically hydrogen) and generates electrical energy. The cell energy flow is calculated for \( \dot{E}_{\text{req}} \) and a specified flight condition:

\[
\dot{E}_{\text{qcell}} = E_0 g_e(q)
\]
where \( q = \dot{E}_{req}/P_0 \). The fuel cell performance (mass flow and fuel flow) is obtained from \( \dot{E}_{qcell} \), or

\[
\begin{align*}
\dot{m}_{req} &= \dot{m}_{0CG} g_m(q) \\
\dot{w}_{req} &= \dot{w}_{0CG} g_w(q)
\end{align*}
\]

Installation losses are included in the specific fuel consumption. The net thrust is the inlet momentum drag, \( F_N = -\dot{m}_{req} V \). The charger model gives the performance of a single charger. The performance of the charge group is obtained by multiplying the single charger characteristic by the number of chargers operational:

\[
\begin{align*}
\dot{m}_{reqCG} &= (N_{chrg} - N_{inop}) \dot{m}_{req} \\
\dot{w}_{reqCG} &= (N_{chrg} - N_{inop}) \dot{w}_{req} \\
F_{NCG} &= (N_{chrg} - N_{inop}) F_N \\
D_{auxCG} &= (N_{chrg} - N_{inop}) D_{aux}
\end{align*}
\]

The momentum drag of the auxiliary air flow is a function of the mass flow \( \dot{m}_{aux} = f_{aux} \dot{m}_{req} \):

\[
D_{aux} = (1 - \eta_{aux}) \dot{m}_{aux} V = (1 - \eta_{aux}) f_{aux} \dot{m}_{req} V
\]

where \( \eta_{aux} \) is the ram recovery efficiency.

### 20–3 Solar Cell

The power available from solar radiation is approximately 1.36 kW/m\(^2\), reduced by atmospheric effects (absorption, reflection, and scattering) to about 1.00 kW/m\(^2\). The average solar radiation in the continental United States is 3.5–7.0 (kW/m\(^2\))(hour/day), hence approximately 15 to 25% of the available power. Typical efficiencies of solar cells are 10–35%, with some sensitivity to temperature. The solar cell is characterized by power density (W/m\(^2\)) and weight density (kg/m\(^2\)).

The cell energy flow is calculated for \( \dot{E}_{req} \) and a specified flight condition:

\[
\dot{E}_{qcell} = \dot{E}_{0CG} g_e(q)
\]

where \( q = \dot{E}_{req}/P_0 \).

### 20–4 Control and Loads

The charger power amplitude \( A \) and mode \( B \) are control variables. The charge group power can be fixed at \( \dot{E} = P_{reqCG} = (N_{chrg} - N_{inop}) A \); or fraction \( A \) of charger rated power \( \dot{E} = P_{reqCG} = (N_{chrg} - N_{inop}) A P_{chrg} \).

The charger orientation is specified by selecting a nominal direction \( e_f0 \) in body axes (positive or negative \( x-, y-, \) or \( z- \)axis); then applying a yaw angle \( \psi \); and then an incidence or tilt angle \( i \). The yaw and incidence angles can be control variables.

The charge group produces a jet thrust \( F_N \) (actually, \( F_N = F_G - \dot{m} V \), with the gross thrust in the direction of the engine and the momentum thrust in the wind direction), acting in the direction of the charger; a momentum drag \( D_{aux} \) (if the component has mass flow), acting in the wind direction; and a nacelle drag \( D_{nac} \), acting in the wind direction. The charge group is at location \( z^F \). The force and moment acting on the aircraft in body axes are thus:

\[
\begin{align*}
F^F &= (e_f F_G + e_d \dot{m} V) + e_d D_{aux} + e_d D_{nac} \\
M^F &= \Delta z^F F^F
\end{align*}
\]
where $\Delta z^F = z^F - z^F_{cg}$, $e_f$ is the charger thrust direction and $e_d$ is the drag direction.

### 20–5 Nacelle Drag

The charge group includes a nacelle, which contributes to the aircraft drag. The reference area for the nacelle drag coefficient is the nacelle wetted area. The wetted area per charger is input, or calculated from the engine system (engine, exhaust, and accessories) weight:

$$S_{\text{wet}} = k\left(\frac{w}{N_{\text{chrg}}}\right)^{2/3}$$

where $w = W_{ES}$, and the units of $k$ are $\text{ft}^2/\text{lb}^{2/3}$ or $\text{m}^2/\text{kg}^{2/3}$. The reference area is then $S_{\text{nac}} = N_{\text{chrg}}S_{\text{wet}}$.

### 20–6 Weights

The component weight consists of the engine system. The engine system consists of the charger weight. These weights are for the charge group, consisting of $N_{\text{chrg}}$ chargers. The engine system weight $W_{ES} = N_{\text{chrg}}W_{\text{one chrg}}$ is used for the charger nacelle wetted area.
Chapter 21

Referred Parameter Turboshaft Engine Model

Aircraft gas turbine engine performance capabilities are formally specified by computer programs known as engine decks, which are created by engine manufacturers in an industry-standard format. Engine decks are typically based on thermodynamic cycle analysis using real engine component performance maps. The most important performance maps for turboshaft engines are compressor, gas generator turbine, and power turbine. These component performance maps are critical to obtaining realistic off-design engine performance. Design and analysis codes calculate aircraft performance for a very wide range of operating conditions. Thus engine performance must be realistic even far from the engine design point. A simple thermodynamic cycle analysis that assumes design point component efficiencies everywhere is not realistic for such an application. Rather than developing models for component performance, the approach taken is to use a model for the total engine performance. The engine is not being designed.

The Referred Parameter Turboshaft Engine Model (RPTEM) is based on curve-fits of performance data for existing or projected engines over a range of operating conditions. The curve-fits are typically obtained by exercising an engine deck. The use of referred parameters tends to collapse the data, and provides a basis for scaling the engine. The operating condition is described by pressure altitude, ambient air temperature, flight Mach number, power turbine speed, exhaust nozzle area, and either engine rating or engine power required. These curve-fits, typically based on real engines, are scaled to the required size and adjusted to the appropriate technology level to represent a notional engine. Engine size is represented by mass flow. Engine technology is represented by specific power available and specific fuel consumption at maximum continuous power (MCP), sea level/standard day (SLS), static (zero airspeed) conditions. Engine installation effects (inlet and exhaust losses) are also modeled.

The use of referred parameters to curve-fit engine performance data was suggested by David Woodley from Boeing during the JVX program (1983). The RPTEM was developed and documented by Michael P. Scully and Henry Lee of ASRAO, U.S. Army Aeroflightdynamics Directorate (AFDD), with a subsequent implementation written by Sam Ferguson (1995).

21–1 Operating Environment

The operating condition and atmosphere give the standard conditions (temperature \( T_{\text{std}} \) and pressure \( p_{\text{std}} \)) for a specified pressure altitude; the sea-level standard conditions (temperature \( T_0 \) and pressure \( p_0 \)); and the operating temperature \( T \) and pressure \( p \). Here the temperatures are °R or °K. The engine characteristics depend on the temperature ratio \( \theta = T/T_0 \) and pressure ratio \( \delta = p/p_0 \).

The flight Mach number \( M = V/c_a = V/c_{a0}\sqrt{\theta} \) is obtained from the aircraft speed \( V \).
The inlet ram air temperature ratio and pressure ratio are obtained then from $M$ and the inlet ram recovery efficiency $\eta_d$:

$$\theta_M = \left(1 + \frac{\gamma - 1}{2} M^2\right) = (1 + 0.2M^2)$$

$$\delta_M = \left(1 + \frac{\gamma - 1}{2} \eta_d M^2\right)^{\frac{\gamma}{\gamma - 1}} = (1 + 0.2\eta_d M^2)^{3.5}$$

where the ratio of specific heats $\gamma = 1.4$.

### 21–2 Performance Characteristics

The engine performance is described by: the power available $P_a$, at each engine rating and the specification engine turbine speed $N_{\text{spec}}$; the mass flow $\dot{m}$ and fuel flow $\dot{w}$ required to produce power required $P_q$ at engine turbine speed $N$; and the gross jet thrust $F$ at a given power required $P_q$. Then, the specific power is $SP = P/\dot{m}$, and the specific fuel consumption is $sfc = \dot{w}/P$.

The reference performance is at sea-level-standard static conditions (subscript 0), and MCP (subscript $C$). For each rating R, the performance is characterized by the following quantities for sea-level-standard static conditions: power $P_{0R}$, specific power $SP_{0R}$, and mechanical power limit $P_{\text{mech}R}$. The mass flow is then $\dot{m}_{0R} = P_{0R}/SP_{0R}$. The gross jet thrust $F_{\text{g}0C}$ is given at MCP. These characteristics are at the specification turbine speed $N_{\text{spec}}$.

The installed power required $P_{\text{req}}$ and power available $P_{\text{av}} > P_{\text{req}}$ are measured at the engine output shaft. In addition to shaft power, the engine exhaust produces a net jet thrust $F_N$, from mass flow that goes through the engine core. The fuel flow and mass flow are the total required to produce the shaft power and jet thrust. The forces produced by mass flow that does not go through the engine core (such as infrared suppressor or cooling air) are treated as momentum drag $D_{\text{aux}}$.

The difference between net and gross jet thrust is the momentum drag: $F_n = F_g - \dot{m}_{\text{req}} V = \dot{m}_{\text{req}} (V_j - V)$, where $V_j$ is the engine jet exhaust velocity. Note that traditional units for mass flow are pound/sec (pps), while this equation requires slug/sec ($\dot{m}_{\text{req}}/g$ replaces $\dot{m}_{\text{req}}$).

The uninstalled power required is $P_q$, the power available $P_a$, the gross jet thrust $F_g$, and net jet thrust $F_n$. The engine model calculates $P_a$ as a function of flight condition and engine rating; or calculates engine mass flow, fuel flow, and jet thrust at $P_q$.

### 21–3 Installation

The difference between installed and uninstalled power is the inlet and exhaust losses $P_{\text{loss}}$:

$$P_{\text{av}} = P_a - P_{\text{loss}}$$

$$P_{\text{req}} = P_q - P_{\text{loss}}$$

The inlet ram recovery efficiency $\eta_d$ through $\delta_M$ is included in the engine model calculations. The inlet and exhaust losses are modeled as fractions of power available or power required: $P_{\text{loss}} = (\ell_{in} + \ell_{ex})P_a$ or $P_{\text{loss}} = (\ell_{in} + \ell_{ex})P_q$. So

$$P_{\text{av}} = P_a(1 - \ell_{in} - \ell_{ex})$$

$$P_q = P_{\text{req}}/(1 - \ell_{in} - \ell_{ex})$$
The engine model gives uninstalled power and the gross thrust $F_g$ for a nominal exhaust nozzle area. The gross jet thrust $F_G$ and exhaust power loss ($\ell_{ex}$) are both functions of the exhaust nozzle area. Smaller exhaust nozzle areas increase exhaust losses and increase gross thrust. Thus the ratio of installed to uninstalled thrust is approximated by a function of the exhaust power loss:

$$\frac{F_G}{F_g} = K_{fgr} = K_{fgr0} + K_{fgr1}\ell_{ex} + K_{fgr2}\ell_{ex}^2 + K_{fgr3}\ell_{ex}^3$$

so the net installed jet thrust is

$$F_N = K_{fgr} F_g - \dot{m}_{req} V$$

The momentum drag of the auxiliary air flow is a function of the mass flow $\dot{m}_{aux} = f_{aux}\dot{m}_{req}$:

$$D_{aux} = (1 - \eta_{aux})\dot{m}_{aux} V = (1 - \eta_{aux}) f_{aux} \dot{m}_{req} V$$

where $\eta_{aux}$ is the ram recovery efficiency. Exhaust losses ($\ell_{ex}$) and auxiliary air flow parameters ($\eta_{aux}$, $f_{aux}$) are defined for IR suppressor on and off. Inlet particle separator loss is added to the inlet losses ($\ell_{in}$).

## 21–4 Power Turbine Speed

The shaft power available is a function of the gas power available $P_G$ and the power turbine efficiency $\eta_t$: $P_a = \eta_t P_G$. Generally the power turbine speed $N$ has a significant effect on $\eta_t$, but almost no effect on $P_G$. The model used for the efficiency variation is $\eta_t \approx 1 - |(N/N_{opt}) - 1|^{X_N}$, where $N_{opt}$ is the speed for peak efficiency, hence

$$\frac{P(N)}{P(N_{spec})} = \frac{\eta_t(N)}{\eta_t(N_{spec})} = \frac{1 - |(N/N_{opt}) - 1|^{X_N}}{1 - |(N_{spec}/N_{opt}) - 1|^{X_N}}$$

Two approximations for the optimum turbine speed are used. The first is a linear expression in $p = P/(P_0 R_0 \delta \sqrt{\theta})$:

$$N_{opt} = N_{spec} \sqrt{\theta} \left( K_{N_{opt0}} \sqrt{\theta_M} + \frac{K_{N_{opt1}}}{\delta_M} p \right)$$

and the second is a cubic function of $p = P/(P_0 C \delta \sqrt{\theta})$:

$$N_{opt} = N_{opt0} \sqrt{\theta} \left( K_{N_{opt0}} + K_{N_{opt1}} p + K_{N_{opt2}} p^2 + K_{N_{opt3}} p^3 \right) \theta_M^{X_{N_{opt}}}$$

The second expression is based on a larger data sample. For power available calculations, $P = P_a(N_{spec})$; for power required calculations, $P = P_q(N)$.

## 21–5 Power Available

Given the flight condition and engine rating, the power available $P_a$ is calculated as follows. The specific power and referred mass flow (at $N_{spec}$, relative to $SP_a$ and $\dot{m}_a$ for this rating) are approximated by functions of the ambient temperature ratio $\theta$ and inlet ram air ratios:

$$SP_a(N_{spec}) = SP_0 \theta K_{spa} \left[ \delta_M \sqrt{\theta_M} \right]^{X_{spa}}$$

$$\dot{m}_a(N_{spec}) = \dot{m}_0 \left( \delta / \sqrt{\theta} \right) e^{K_{mfa}} \left[ \delta_M \sqrt{\theta_M} \right]^{X_{mfa}}$$
where the static lapse rate \((K_{spa}, K_{mfa})\) and ram air exponents \((X_{spa}, X_{mfa})\) are piecewise linear functions of \(\theta\). The power available is then

\[
P_a(N_{spec}) = P_0 \frac{SP_a(N_{spec})}{SP_0} \frac{\dot{m}_a(N_{spec})}{\dot{m}_0} = P_0 \left( \delta \sqrt{\theta} \right) K_{spa} e^{K_{mfa} \left[ \delta_M \sqrt{\theta_M} \right]}^{X_{spa} + X_{mfa}}
\]

This expression for \(\dot{m}_a\) is used only to calculate \(P_a\); elsewhere the \(\dot{m}_q\) expression below (for performance at power required) is used to obtain the mass flow at a power \(P_q\). Finally

\[
P_a(N) = P_a(N_{spec}) \frac{1 - |(N/N_{opt}) - 1|^X_{Nq}}{1 - |(N_{spec}/N_{opt}) - 1|^X_{Nq}}
\]

is the power available at turbine speed \(N\). Installation losses \(P_{loss}\) are subtracted from \(P_a\) \((P_{av} = P_a - P_{loss})\), and then the mechanical limit is applied: \(P_{av} = \min(P_{av}, rP_{mech})\), \(r = N/N_{spec}\). The mechanical limit is properly a torque limit, \(Q_{mech} = P_{mech}/N_{spec}\), but is expressed as a power limit for clarity.

### 21.5.1 Piecewise Linear Parameters

Several parameters \(K\) are input as piecewise linear functions of the temperature ratio \(\theta\). One format of the input is a set of \(I\) regions, with the function in the \(i\)-th region given by \(K = K_{0i} + K_{1i} \theta\). The break point between the \(i\) and \(i - 1\) regions is

\[
\theta_{bi} = \frac{K_{0i} - K_{0(i-1)}}{K_{1i} - K_{1(i-1)}}
K_{bi} = K_{0i} + K_{1i} \theta_{bi}
\]

for \(i = 2\) to \(I\). Another format is a set of \(I + 1\) break points, with the values at the \(i\)-th point \((\theta_{bi}, K_{bi})\). Then the coefficients between \(i\) and \(i + 1\) are

\[
K_{0i} = \frac{K_{bi} \theta_{b(i+1)} - K_{b(i+1)} \theta_{bi}}{\theta_{b(i+1)} - \theta_{bi}}
K_{1i} = \frac{-K_{bi} + K_{b(i+1)}}{\theta_{b(i+1)} - \theta_{bi}}
\]

for \(i = 1\) to \(I\). The interpolation is performed using \(K = K_{0i} + K_{1i} \theta\) for \(\theta\) in the range \(\theta_{bi}\) to \(\theta_{b(i+1)}\). Outside the defined regions, the linear expression is continued; hence \(\theta_{bi}\) is used only for \(i = 2\) to \(I\).

### 21-6 Performance at Power Required

The engine performance (mass flow, fuel flow, and gross jet thrust) is calculated for a specified power required \(P_q\) (which might equal the power available), flight condition, and engine rating. Installation losses \(P_{loss}\) are added to \(P_{req}\) \((P_q = P_{req} + P_{loss})\). The referred quantities (relative to SLS static MCP quantities) are approximated by cubic functions of \(q = P_q(N_{spec})/(P_0C\sqrt{\theta})\):

\[
w_{req} = w_{0C} \left( \delta \sqrt{\theta} \right) (K_{ffg0} + K_{ffg1}q + K_{ffg2}q^2 + K_{ffg3}q^3) [\theta_M]^{-X_{ffg}}
\]

\[
\dot{m}_{req} = \dot{m}_{0C} \left( \delta \sqrt{\theta} \right) (K_{mfg0} + K_{mfg1}q + K_{mfg2}q^2 + K_{mfg3}q^3) [\theta_M]^{X_{mfg}}
\]

\[
F_g = F_{g0C} (\delta) (K_{fgg0} + K_{fgg1}q + K_{fgg2}q^2 + K_{fgg3}q^3) [\theta_M]^{X_{fgg}}
\]
The parameters of the engine model can be defined for a specific engine, but it is also necessary to scale the parameters as part of the aircraft sizing task, in order to define an engine for a specified power. In addition, advanced technology must be represented in the model. Scaling and advanced technology are handled in terms of specific power and specific fuel consumption (at SLS static conditions, MCP, and \( N_{\text{spec}} \)). Figures 21-1 through 21-3 present historical data for engine specific fuel consumption, weight, and specific power; the variation at a given power reflects technology insertion with time.

Then the installed net jet thrust \( F_N \) and momentum drag \( D_{\text{aux}} \) are calculated.

### 21–7 Scaling

The engine technology parameters \( SP_{\text{0C}} \) and \( \text{sf}c_{\text{0C}} \) are assumed to vary linearly with mass flow \( \dot{m}_{\text{0C}} \), up to a limit \( \dot{m}_{\text{lim}} \), and constant thereafter at \( SP_{\text{lim}} \) and \( \text{sf}c_{\text{lim}} \). The mass flow at the technology condition is \( \dot{m}_{\text{tech}} = P_{\text{ref}}/SP_{\text{tech}} \), with the technology values \( SP_{\text{tech}} \) and \( \text{sf}c_{\text{tech}} \). The intercept values are projected from the technology values: \( K_{sp0} = SP_{\text{tech}} - K_{sp1}\dot{m}_{\text{tech}}, K_{sp1} = (SP_{\text{lim}} - SP_{\text{tech}})/\dot{m}_{\text{lim}} - \dot{m}_{\text{tech}}); \) and similarly for \( \text{sf}c \). Then for \( \dot{m}_{\text{0C}} < \dot{m}_{\text{lim}} \)

\[
SP_{\text{0C}} = K_{sp0} + K_{sp1}\dot{m}_{\text{0C}} \\
\text{sf}c_{\text{0C}} = K_{\text{sf}c0} + K_{\text{sf}c1}\dot{m}_{\text{0C}}
\]

and for \( \dot{m}_{\text{0C}} \geq \dot{m}_{\text{lim}} \)

\[
SP_{\text{0C}} = SP_{\text{lim}} \\
\text{sf}c_{\text{0C}} = \text{sf}c_{\text{lim}}
\]

These equations are used if \( \dot{m}_{\text{lim}} > \dot{m}_{\text{tech}} \). Otherwise the model sets \( K_{sp1} = K_{\text{sf}c1} = 0 \), so there is no variation with scale: \( SP_{\text{0C}} = SP_{\text{tech}} \) and \( \text{sf}c_{\text{0C}} = \text{sf}c_{\text{tech}} \). Usually the effect of size gives \( K_{sp2} \geq 0 \) and \( K_{\text{sf}c2} \leq 0 \). The power at the limit is \( P_{\text{lim}} = SP_{\text{lim}}\dot{m}_{\text{lim}} \). Figure 21-4 illustrates the scaling of \( SP \), \( \text{sf}c \), and \( SW \).
Using $\dot{m}_{0C} = P_{0C}/SP_{0C}$, the specific power equation can be solved for the mass flow given the power:

$$\dot{m}_{0C} = \begin{cases} \frac{P_{0C}}{K_{sp1}} & K_{sp1} = 0 \\ \sqrt{\left(\frac{P_{0C}}{K_{sp1}}\right)^2 + \frac{P_{0C}}{K_{sp1}}} & P_{0C} \geq P_{lim} \\ \frac{\sqrt{\left(\frac{P_{0C}}{K_{sp1}}\right)^2 + \frac{P_{0C}}{K_{sp1}}} - \frac{P_{0C}}{K_{sp1}}}{K_{sp1}} & \text{otherwise} \end{cases}$$

From this mass flow, $SP_{0C}$ and $sfc_{0C}$ are calculated, hence the fuel flow $\dot{w}_{0C} = sfc_{0C}P_{0C}$. The specific thrust available at MCP is assumed to be constant, and the specific power turbine speed decreases with the mass flow:

$$F_{g0C} = SF_{0C}\dot{m}_{0C}$$

$$N_{spec} = \left(N_{spec} - K_{N_{s2}}/\sqrt{\dot{m}_{0C}}\right)_{tech} + K_{N_{s2}}/\sqrt{\dot{m}_{0C}} = K_{N_{s1}} + K_{N_{s2}}/\sqrt{\dot{m}_{0C}}$$

$$N_{opt0C} = \frac{(N_{opt0C})_{ref}}{N_{tech}} N_{spec} = K_{N_{opt}}N_{spec}$$

Then the power and specific power at all ratings $R$ are obtained from the ratios:

$$P_{0R} = r_{p0R}P_{0C}, \quad SP_{0R} = r_{p0R}SP_{0C}, \quad \text{and} \quad P_{mechR} = r_{m0R}P_{0C}.$$  

The actual (perhaps scaled) values of the performance parameters are available for the engine group: $P_{0R}$, $SP_{0R}$, $P_{mechR}$, $sfc_{0C}$, $F_{g0C}$, $N_{spec}$, and $N_{opt0C}$.

### 21–8 Engine Speed

The model as described in the previous sections may not adequately account for variation of engine performance with engine speed, so it is also possible to define the parameters corresponding to a set of engine speed ratios $r = N/N_{spec}$. Then the engine performance and power available quantities are linearly interpolated to obtain the values at the required engine speed $N$. If this option is used, then the correction based on $P(N)/P(N_{spec}) = \eta(N)/\eta(N_{spec})$ is not applied.

### 21–9 Weight

The engine weight can be a fixed input value, calculated as a function of power, or scaled with engine mass flow. As a function of power, the weight of one engine is:

$$W_{one\ eng} = K_{0eng} + K_{1eng}P + K_{2eng}P^{N_{eng}}$$

where $P$ is the installed takeoff power (SLS static, specified rating) per engine. A constant weight per power $W/P$ is given by using only $K_{1eng}$. Alternatively, the specific weight $SW = P/W$ can be scaled with the mass flow $\dot{m}_{0C}$. The scaling is determined from the specific weight $SW_{ref}$ at the mass flow $\dot{m}_{tech}$, and the limit $SW_{lim}$ at $\dot{m}_{lim}$. Then

$$SW = \begin{cases} K_{sw0} + K_{sw1}\dot{m}_{0C} & \dot{m}_{0C} < \dot{m}_{lim} \\ SW_{lim} & \dot{m}_{0C} \geq \dot{m}_{lim} \end{cases}$$

(if $\dot{m}_{lim} < \dot{m}_{tech}$, then $K_{sw1} = 0$, so $SW = SW_{ref}$) and $W_{one\ eng} = P/SW$.

### 21–10 Units

In this engine model, only the reference values and scaling constants are dimensional. Conventional English units and SI units are shown in table 21-1. Units of specific power and specific fuel consumption...
21–11 Typical Parameters

Typical values of the principal parameters describing the engine and its performance are given in table 21-2 for several generic engine sizes. These values represent good current technology. Advanced technology can be introduced by reducing the specific fuel consumption and weight, and increasing the specific power. Typical ratios of the power, specific power, and mass flow to the values at MCP are given in table 21-3 for several ratings. Figure 21-5 shows typical performance characteristics: fuel flow $\dot{m}$, mass flow $\dot{m}$, and net jet thrust $F_g$ variation with power $P$ and speed (referred quantities, normalized, at $N_{\text{spec}}$). Figure 21-6 shows typical variation of the power available with engine turbine speed. Figures 21-7 to 21-12 show typical power available characteristics: specific power $SP$, mass flow $\dot{m}$, and power $P$ variation with temperature ratio $\theta$, for static and 200 knots conditions and several engine ratings (referred quantities, normalized, at $N_{\text{spec}}$).

Table 21-2. Typical engine performance parameters.

<table>
<thead>
<tr>
<th>power (MCP) $P_{0C}$</th>
<th>500</th>
<th>1000</th>
<th>2000</th>
<th>4000</th>
<th>8000</th>
<th>16000</th>
<th>hp</th>
</tr>
</thead>
<tbody>
<tr>
<td>specific power $SP_{0C}$</td>
<td>116</td>
<td>125</td>
<td>134</td>
<td>143</td>
<td>153</td>
<td>164</td>
<td>hp/lb/sec</td>
</tr>
<tr>
<td>mechanical limit $P_{\text{mech}}$</td>
<td>750</td>
<td>1500</td>
<td>3000</td>
<td>6000</td>
<td>12000</td>
<td>24000</td>
<td>hp</td>
</tr>
<tr>
<td>specific fuel cons. $SFC_{0C}$</td>
<td>0.54</td>
<td>0.48</td>
<td>0.44</td>
<td>0.40</td>
<td>0.38</td>
<td>0.35</td>
<td>lb/hp-hr</td>
</tr>
<tr>
<td>specific jet thrust $F_{\text{jet}}$</td>
<td>5.9</td>
<td>7.3</td>
<td>8.9</td>
<td>11.0</td>
<td>13.6</td>
<td>16.7</td>
<td>lb/lb/sec</td>
</tr>
<tr>
<td>gross jet thrust $F_{\text{g}}$</td>
<td>25</td>
<td>58</td>
<td>134</td>
<td>308</td>
<td>707</td>
<td>1625</td>
<td>lb</td>
</tr>
<tr>
<td>mass flow $\dot{m}_{0C}$</td>
<td>4.3</td>
<td>8.0</td>
<td>15.0</td>
<td>27.9</td>
<td>52.1</td>
<td>97.3</td>
<td>lb/sec</td>
</tr>
<tr>
<td>turbine speed $N_{\text{spec}}$</td>
<td>35600</td>
<td>26100</td>
<td>19100</td>
<td>14000</td>
<td>10200</td>
<td>7500</td>
<td>rpm</td>
</tr>
<tr>
<td>weight $W/\rho_0$</td>
<td>0.34</td>
<td>0.23</td>
<td>0.18</td>
<td>0.16</td>
<td>0.16</td>
<td>0.15</td>
<td>lb/hp</td>
</tr>
</tbody>
</table>

Table 21-3. Typical parameter ratios for various ratings (percent).

<table>
<thead>
<tr>
<th></th>
<th>IRP</th>
<th>MRP</th>
<th>CRP</th>
<th>$P_{\text{mech}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>power $P_0/P_{0C}$</td>
<td>120</td>
<td>127</td>
<td>133</td>
<td>150</td>
</tr>
<tr>
<td>specific power $SP_0/SP_{0C}$</td>
<td>117</td>
<td>123</td>
<td>128</td>
<td></td>
</tr>
<tr>
<td>mass flow $\dot{m}<em>o/\dot{m}</em>{0C}$</td>
<td>102.6</td>
<td>103.2</td>
<td>103.9</td>
<td></td>
</tr>
</tbody>
</table>
Figure 21-1. Historical data for turboshift engine specific fuel consumption.

Figure 21-2. Historical data for turboshift engine weight.
Figure 21-3. Historical data for turboshaft engine specific power.
Figure 21-4. Turboshaft engine scaling: $SP = P/\dot{m}$, $sfc = \dot{w}/P$, and $SW = P/W$. 
Figure 21-5. Fuel flow, mass flow, and net jet thrust variation with power.

Figure 21-6. Power variation with turbine speed.
Figure 21-7. Specific power variation with temperature ratio, static.

Figure 21-8. Specific power variation with temperature ratio, 200 knots.
Figure 21-9. Mass flow variation with temperature ratio, static.

Figure 21-10. Mass flow variation with temperature ratio, 200 knots.
Figure 21-11. Power variation with temperature ratio, static.

Figure 21-12. Power variation with temperature ratio, 200 knots.
A reciprocating engine generates power by means of reciprocating pistons that convert pressure into a rotating motion. Each piston is inside a cylinder, into which a gas is introduced, either already under pressure (external combustion, as in a steam engine) or heated inside the cylinder by ignition of a fuel-air mixture (internal combustion). The hot gases expand, pushing the piston to the bottom of the cylinder. Here only internal combustion engines are considered, perhaps more appropriately identified as intermittent combustion engines, to distinguish them from the continuous combustion of gas turbines and jet engines. An internal combustion engine is either a spark-ignition engine, where a spark plug initiates the combustion; or a compression-ignition engine, where the air within the cylinder is compressed, heating it and thereby igniting fuel. Internal combustion engine performance is described in reference 1.

Design and analysis codes calculate aircraft performance for a very wide range of operating conditions. Thus engine performance must be realistic even far from the engine design point. A simple thermodynamic cycle analysis is not realistic for such an application. Rather than developing high fidelity analyses, the approach taken is to use a model for the total engine performance. The engine is not being designed.

The Reciprocating Engine Model is based on curve-fits of performance data for existing or projected engines over a range of operating conditions. The operating condition is described by pressure altitude, ambient air temperature, flight Mach number, engine speed, and either engine rating or engine power required. These curve-fits, typically based on real engines, are scaled to the required size and adjusted to the appropriate technology level to represent a notional engine. Engine size is represented by power. Engine technology is represented by specific fuel consumption and power-to-weight ratio. Engine installation losses are also modeled.

### 22-1 Operating Environment

The operating condition and atmosphere give the standard conditions (temperature $T_{\text{std}}$ and pressure $p_{\text{std}}$) for a specified pressure altitude; the sea-level standard conditions (temperature $T_0$ and pressure $p_0$); and the operating temperature $T$ and pressure $p$. Here the temperatures are °R or °K. The engine characteristics depend on the density ratio $\sigma = \rho / \rho_0$ and the temperature ratio $\theta = T / T_0$.

The flight Mach number $M = V / c_s = V / c_{s0} \sqrt{\theta}$ is obtained from the aircraft speed $V$.

The inlet ram air temperature ratio and pressure ratio are obtained then from $M$ and the inlet ram
recovery efficiency $\eta_d$:

$$\theta_M = \left(1 + \frac{\gamma - 1}{2} M^2\right) = (1 + 0.2M^2)$$

$$\delta_M = \left(1 + \frac{\gamma - 1}{2} \eta_d M^2\right) = (1 + 0.2\eta_d M^2)^{3.5}$$

where the ratio of specific heats $\gamma = 1.4$. The ram density ratio is $\sigma_M = \delta_M / \theta_M$. The engine mass flow and power available depend on $(\sigma + \Delta \sigma_M)$, where $\Delta \sigma_M = \sigma_M - 1$

### 22–2 Reciprocating Engine Performance

The engine work per cycle for a reciprocating engine is $W = Pn_c/R$; where $P$ is the power, $R = N/2\pi$ the rotational speed (rev/sec, $N$ in rad/sec), $n_c$ the revolutions per cycle ($n_c = 2$ for a 4-stroke engine, $n_c = 1$ for a 2-stroke engine), and $R/n_c$ is the cycles per second. The piston area is $A_p$, bore $b$, stroke $k$; the engine displacement $V_d = A_pk$ (swept volume). The mean piston speed is defined as $s = 2Rk$ (1/2R is the time for one stroke). Thus $A_ps = 2RV_d$. The mean effective pressure (mep) is defined as

$$\text{mep} = \frac{W}{V_d} = \frac{Pn_c}{V_d R} = \frac{P}{A_ps/2n_c} = \frac{Qn_c 2\pi}{V_d}$$

from the power $P = NQ/2\pi RQ$; so mep is the specific torque. The power and torque can be written in terms of mep:

$$P = (\text{mep})V_d R/n_c = (\text{mep})A_ps/2n_c$$

$$Q = (\text{mep})V_d/2\pi n_c = (\text{mep})A_pk/2\pi n_c$$

The specific output is $P/A_p = (\text{mep})s/2n_c$.

The engine output is the brake horsepower (BHP), which equals indicated power less friction power:

$$\text{BHP} = \text{IHP} - \text{FHP} = \text{IHP} - (\text{MHP} + \text{PHP} + \text{CHP} + \text{AHP} - \text{THP})$$

The friction power is composed of losses due to mechanical friction (MHP), pumping (PHP, the work of the piston during inlet and exhaust strokes), compressor or supercharger (CHP), auxiliary or accessories (AHP, such as oil pump, water pump, cooling fan, generator), and exhaust turbine (THP, treated as negative friction). The mechanical efficiency is $\eta = \text{BHP}/\text{IHP}$.

The sum of airflow ($\dot{m} = \dot{w}_c$) and fuel flow ($\dot{w} = \dot{w}_f$) is the charge flow: $\dot{w}_c = \dot{w}_f + \dot{w}_a$. The fuel-air ratio is $F = \dot{w}_f/\dot{w}_a$. The stoichiometric (chemically correct) fuel-air ratio is about $F_{\text{stoich}} = 0.068$ for both gasoline and diesel. Rich mixture (fuel-air ratio) means extra fuel and lean mixture means extra air. The fuel-air equivalence ratio is $\phi = F/F_{\text{stoich}}$. The mass flow is

$$\dot{m} = e_v \dot{m}_{\text{ideal}} = e_v \rho_i V_d (R/n_c) = e_v \rho_i A_ps/2n_c = \sigma_i N (e_v \rho_0 V_d/(2\pi n_c))$$

where $\rho_i$ is the density at the intake and $e_v = \dot{m}/\dot{m}_{\text{ideal}}$ is the volumetric efficiency. Including the influence of forward speed, $\rho_i = \rho_0 (\sigma + \Delta \sigma_M)$. Typically $e_v = 0.8$ to 0.9. The fuel flow is then $\dot{w} = F \dot{m}$. The indicated power $P_i$ is the product of the fuel flow, fuel specific energy ($e_{\text{fuel}} = JQ_e$, from the heat of combustion $Q_c$ and Joule’s constant $J$ relating work and heat), and thermal efficiency:

$$P_i = e_{\text{fuel}} \eta_h \dot{w} = (e_{\text{fuel}} \eta_h F) \dot{m} = \sigma_i N (e_{\text{fuel}} \eta_h F)(e_v \rho_0 V_d/(2\pi n_c))$$
Reciprocating Engine Model

The specific fuel consumption is

\[ \text{sfc} = \dot{w}/P = 1/(e_{\text{fuel}}\eta_{\text{th}}) \]

and the power to mass flow ratio is

\[ P/\dot{m} = e_{\text{fuel}}\eta_{\text{th}}F = F/\text{sfc} = (\text{mep})/(e_{\alpha}\rho_i) \]

These equations all have constant factors when conventional units are used.

From \( \dot{m} \sim e_{\alpha}\rho_i \) and \( P \sim e_{\alpha}\rho_i\eta_{\text{th}}F \), it follows that mass flow and power are proportional to the density ratio \( \sigma = \delta/\theta \). For unsupercharged engines at constant fuel-air fraction and constant throttle, typically \( e_{\alpha}\rho_i \sim \rho/\sqrt{T} \) (ref. 1). Similarly, operating curves give \( \pm 1\% \) power change for \( \pm 6^\circ \text{C} \) at constant pressure altitude and manifold pressure, implying again \( P \sim 1/\sqrt{T} \). The implication is that mass flow, indicated power, and imep are proportional to \( \delta/\sqrt{\theta} = \sigma\sqrt{\theta} \).

The engine has a limit on bmep (hence torque) corresponding to detonation limits (spark-ignition) or thermal and loads limits (compression-ignition) at maximum power. This limit is defined in terms of power at a specified engine speed. Load limits associated with mean piston speed give an engine speed limit. Maximum throttle, encountered at a critical altitude or critical density, can be expressed as a power limit that varies with engine speed. These limits depend on the rating, fuel-air ratio (lean or rich), and supercharger speed.

### 22–3 Performance Characteristics

The engine performance is described by the uninstalled power available \( P_a \), as a function of flight condition and engine rating; the mass flow \( \dot{m} \) and fuel flow \( \dot{w} \) required to produce uninstalled power required \( P_q \), and the gross jet thrust. The installed power required \( P_{\text{req}} \) and power available \( P_{\text{av}} > P_{\text{req}} \) are measured at the engine output shaft. In addition to shaft power, the engine exhaust can produce a net jet thrust \( F_N \), from mass flow that goes through the engine. The difference between net and gross jet thrust is the momentum drag: \( F_n = F_g - \dot{m}_{\text{req}}V = \dot{m}_{\text{req}}(V_j - V) \), where \( V_j \) is the engine jet exhaust velocity. The forces produced by mass flow that does not go through the engine (such as cooling air) are treated as momentum drag \( D_{\text{aux}} \).

The engine performance depends on the rating. Typical ratings include maximum continuous power (MCP) and takeoff power. The rating structure of the model includes dependence on fuel-air ratio (lean or rich) and supercharger speed.

The reference performance is at sea-level-standard static conditions (subscript \( 0 \)). For each rating \( R \), the performance is characterized by the following quantities: power \( P_{\text{0R}} \) and specific fuel consumption \( \text{sfc}_{\text{0R}} \); fuel-air ratio \( F_{\text{0R}} \); specific thrust \( SF_{\text{0R}} \); mean effective pressure (mechanical) power limit \( P_{\text{mep,R}} \) and critical power \( P_{\text{crit,R}} \); and a reference engine speed \( N_{\text{0R}} \). Then the mass flow is \( \dot{m}_{\text{0R}} = \dot{w}_{\text{0R}}/F_{\text{0R}} \), and the fuel flow \( \dot{w}_{\text{0R}} = \text{sfc}_{\text{0R}}P_{\text{0R}} \). The gross jet thrust is \( F_{\text{g0R}} = SF_{\text{0R}}\dot{m}_{\text{0R}}(SF_{\text{0R}} = V_{j0R}) \). The specification engine speed is \( N_{\text{spec}} \).

### 22–4 Installation

The difference between installed and uninstalled power is the installation loss: \( P_{\text{av}} = P_a - P_{\text{loss}} \) and \( P_{\text{req}} = P_q - P_{\text{loss}} \). This loss is modeled by an efficiency factor: \( P_{\text{loss}} = (1-\eta_{\text{loss}})P_a \) or \( P_{\text{loss}} = (1-\eta_{\text{loss}})P_q \), where \( \eta_{\text{loss}} = \ell_{\text{in}} + \ell_{\text{ex}} \). The engine model gives the gross thrust \( F_g \). The installed gross jet thrust is
\[ F_G = K_{fg} F_g, \] where \( K_{fg} \) accounts for exhaust effects. The net jet thrust is \( F_N = F_G - \dot{m}_{req} V \). The momentum drag of the auxiliary air flow is a function of the mass flow \( \dot{m}_{aux} = f_{aux} \dot{m}_{req} \):

\[ D_{aux} = (1 - \eta_{aux}) \dot{m}_{aux} V = (1 - \eta_{aux}) f_{aux} \dot{m}_{req} V \]

where \( \eta_{aux} \) is the ram recovery efficiency.

### 22–5 Power Available

Given the flight condition and engine rating, the power available \( P_a \) is calculated from:

\[ P_a = P_0 K_p (\sigma + K_{ram} \Delta \theta_M) r^{X_{pN}} g^{X_{p\theta}} \]

where the engine speed ratio is \( r = N/N_0 \). The factor \( K_p \) is included so \( P_0 \) is a relevant reference power, instead of the power extrapolated to sea-level conditions. \( K_{ram} \) permits adjustment (or suppression) of the effect of flight speed. The exponents \( X_{pN} \) and \( X_{p\theta} \) are obtained by fitting the engine operating curves. The reference engine speed \( N_0 \) corresponds to \( P_0 \).

Maximum throttle, encountered at a critical altitude or critical density, is expressed as a power limit that varies with engine speed: 

\[ P_a = \min(P_{av}, P_{crit} r^{X_{crit}}), \quad r = N/N_0 \]

The exponent \( X_{crit} \) is obtained by fitting the engine operating curves.

Installation losses \( P_{loss} \) are subtracted from \( P_a \) (\( P_{av} = P_a - P_{loss} \)), and then the mechanical limit (mean effective pressure) is applied: 

\[ P_{av} = \min(P_{av}, r P_{mechR}), \quad r = N/N_{spec} \]

The mechanical limit is properly a torque limit, \( Q_{mech} = P_{mech}/N_{spec} \), but is expressed as a power limit for clarity. The \( \text{mepR} \) limit is specified at the reference engine speed \( N_{0R} \), hence \( P_{mechR} = P_{\text{mepR}}(N_{\text{spec}}/N_{0R}) \).

### 22–6 Performance at Power Required

The engine performance (fuel flow, mass flow, and gross jet thrust) is calculated for a specified power required \( P_q \) (which might equal the available), flight condition, and engine rating. Installation losses \( P_{loss} \) are added to \( P_{req} \) (\( P_q = P_{req} + P_{loss} \)). The fuel flow and fuel-air ratio are approximated by cubic functions of \( q = P_q/P_0 \):

\[
\dot{w}_{req} = \dot{w}_0 \left( K_{ffq0} + K_{fjq1} + K_{fjq2}q^2 + K_{fjq3}q^3 \right) r^{X_{ffq}} = \dot{w}_0 K_{fjq}(q) r^{X_{ffq}}
\]

\[
F_{req} = F_0 \left( K_{Fq0} + K_{Fq1} + K_{Fq2}q^2 + K_{Fq3}q^3 \right) r^{X_{Fq}} = F_0 K_{Fq}(q) r^{X_{Fq}}
\]

where the engine speed ratio is \( r = N/N_0 \). The reference engine speed \( N_0 \) corresponds to \( P_0 \). Alternatively, \( K_{fjq}(q) \) and \( K_{Fq}(q) \) can be specified as piecewise-linear functions of \( q \). The constants \( K_{ffq0}, K_{Fq0} \) and exponents \( X_{ffq}, X_{Fq} \) are obtained by fitting the engine operating curves for sfc and \( F \).

For constant specific fuel consumption, \( K_{fjq}(q) = q \). For constant fuel-air ratio, \( K_{Fq}(q) = 1 \); for fuel-air ratio proportional to power, \( K_{Fq}(q) = q \). If \( F_{stoich} \) is used for \( F_0 \), the fuel-air equivalence ratio is \( \phi = F/F_{stoich} = K_{Fq}(q) r^{X_{Fq}} \).

The mass flow is \( \dot{m} = \dot{w}/F \). The gross jet thrust is \( F_g = SF_0 \dot{m} \). The installed net jet thrust \( F_N \) and momentum drag \( D_{aux} \) are calculated from the mass flow.

### 22–7 Scaling

The parameters of the engine model can be defined for a specific engine, but it is also necessary to scale the parameters as part of the aircraft sizing task, in order to define an engine for a specified power.
The engine model includes reference values of the engine performance parameters: $P_{0R}$, $sfc_{0R}$, $F_{0R}$, $SF_{0R}$, $P_{mepR}$, $P_{critR}$, $N_{0R}$, $N_{spec}$. The actual (perhaps scaled) values are available to the engine group.

The engine size is specified as takeoff power $P_{to} = P_{eng}$, which is the power $P_{0X}$ at rating $R = X$, for SLS static conditions. If $P_{0X}$ is not equal to the reference value of the engine model, then the engine is scaled. The reference power at each engine rating $R$ defines a ratio to MCP: $r_{p0R} = P_{0R}/P_{0C}$. The reference engine speed gives the ratio $r_{N0R} = N_{0R}/N_{spec}$. These ratios are kept fixed when the engine is scaled. Hence the MCP is $P_{0C} = P_{to}/r_{p0R}$, and the power at all other ratings follows.

The size of geometrically scaled engines can be represented by the cylinder bore $b$. The specific output, mean piston speed, and specific fuel consumption are relatively weak functions of size (ref. 1). It is assumed that $P/A_p \sim b^{-X_o}$, $s \sim b^{-X_s}$, $sfc \sim b^{-X_f}$. Hence $P \sim b^{2-X_o}$, $N_{spec} \sim s/k \sim b^{-1-X_s}$, and

$$sfc_{0R} = sfc_{0Rref} p^{-X_f/(2-X_o)}$$

$$N_{spec} = N_{spec-ref} p^{-1+X_s)/(2-X_o)}$$

where $p = P_{eng}/P_{0Xref}$. For example, $X_o \approx 0.2$, $X_s \approx 0.3$, $X_f \approx 0.1$ for diesel engines (ref. 1).

$P_{mep}$ and $P_{crit}$ scale with power:

$$P_{0R} = P_{0CRp0R}$$

$$P_{critR} = P_{0Rr_{critR}}$$

$$P_{mechR} = P_{mepR}(N_{spec}/N_{0R}) = P_{0Rr_{mepR}}$$

where $r_{critR} = P_{crit-refR}/P_{0Rref}$ and $r_{mepR} = (P_{mep-ref}/P_{0Rref})(N_{spec-ref}/N_{0Rref})$. The engine speed scales with $N_{spec}$: $N_{0R} = N_{spec}r_{N0R}$.

### 22–8 Weight

The engine weight can be a fixed input value, or calculated as a function of power. As a function of power, the weight of one engine is:

$$W_{one\,eng} = K_{0eng} + K_{1eng}P + K_{2eng}P^{X_{eng}}$$

where $P$ is the installed takeoff power (SLS static, specified rating) per engine. A constant weight per power $W/P$ is given by using only $K_{1eng}$.

### 22–9 Units

In this engine model, only the reference values and scaling constants are dimensional. Conventional English units and SI units are shown in table 22-1. Units of specific power and specific fuel consumption follow from these conventions.

<table>
<thead>
<tr>
<th></th>
<th>power $P$</th>
<th>mass flow $\dot{m}$</th>
<th>fuel flow $\dot{w}$</th>
<th>force $F$</th>
<th>engine speed $N$</th>
<th>mep</th>
</tr>
</thead>
<tbody>
<tr>
<td>English:</td>
<td>horsepower</td>
<td>pound/sec</td>
<td>pound/hour</td>
<td>point</td>
<td>rpm</td>
<td>pound/in²</td>
</tr>
<tr>
<td>SI:</td>
<td>kiloWatt</td>
<td>kilogram/sec</td>
<td>kilogram/hour</td>
<td>Newton</td>
<td>rpm</td>
<td>kN/m²</td>
</tr>
</tbody>
</table>

Table 22-1. Conventional units.
Reciprocating Engine Model

22–10 Typical Parameters

The maximum brake mean effective pressure is 125–300 lb/in² (850–2050 kN/m²). Reciprocating aircraft engine mean piston speed at maximum power is typically 10-16 m/sec for geared engines. Direct drive aircraft engine speeds are typically limited by propeller tip speed. Stoichiometric (exact chemical) fuel-air ratios are given in table 22-2 for a number of fuels. The fuel-air equivalence ratio is \( \phi = \frac{F}{F_{stoich}} \).

Spark-ignition engines can run at stoichiometric fuel-air ratio (\( \phi = 1 \)). Typical operating fuel-air ratios are \( \phi = 0.8 \) to 1.5, with rich values up to \( \phi = 1.8 \) for takeoff power. Modern “lean-burn” spark-ignition engines use Gasoline Direct Injection (GDI) to get \( \phi \) below 0.3 at low power. Highly supercharged engines at maximum power run rich. From reference 1, the highest power for each throttle setting is at \( \phi \approx 1.1 \), with best break economy at \( \phi = 0.9 \) (full load) to \( \phi = 1.1 \) (zero load).

Compression-ignition engines run lean, limited by incomplete combustion. Typical operating fuel-air ratios are \( \phi = 0.2 \) to 0.8, with bmep increasing with \( \phi \). Best break economy is near \( \phi = 0.5 \) (ref. 1).

<table>
<thead>
<tr>
<th>fuel</th>
<th>( F_{stoich} )</th>
<th>fuel</th>
<th>( F_{stoich} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>gasoline</td>
<td>0.0678–0.685</td>
<td>propane</td>
<td>0.0638</td>
</tr>
<tr>
<td>diesel</td>
<td>0.0690–0.697</td>
<td>ethanol</td>
<td>0.111</td>
</tr>
<tr>
<td>hydrogen</td>
<td>0.0292</td>
<td>methanol</td>
<td>0.155</td>
</tr>
<tr>
<td>natural gas</td>
<td>0.058</td>
<td>methane</td>
<td>0.0581</td>
</tr>
</tbody>
</table>

22–11 References

A compressor converts input shaft power to a jet velocity and thrust. The shaft power contributes to the propulsion group power required. The compressor does not use fuel.

The operating condition is described by pressure altitude, ambient air temperature, flight Mach number, and either compressor rating or power required. The parametric model is scaled to the required size and adjusted to the appropriate technology level to represent a notional compressor. Compressor size is represented by mass flow. Technology is represented by specific power available at maximum continuous power (MCP), sea level/standard day (SLS), static (zero airspeed) conditions. Installation effects are also modeled.

### 23–1 Operating Environment

The operating condition and atmosphere give the standard conditions (temperature $T_{\text{std}}$ and pressure $p_{\text{std}}$) for a specified pressure altitude; the sea-level standard conditions (temperature $T_0$ and pressure $p_0$); and the operating temperature $T$ and pressure $p$. Here the temperatures are °R or °K. The characteristics depend on the temperature ratio $\theta = T/T_0$ and pressure ratio $\delta = p/p_0$.

The flight Mach number $M = V/c_s = V/c_{s0}\sqrt{\theta}$ is obtained from the aircraft speed $V$.

The inlet ram air temperature ratio and pressure ratio are obtained then from $M$ and the inlet ram recovery efficiency $\eta_d$:

$$\theta_M = \left(1 + \frac{\gamma - 1}{2} M^2\right) = (1 + 0.2M^2)$$
$$\delta_M = \left(1 + \frac{\gamma - 1}{2} \eta_d M^2\right)^{\gamma/\gamma - 1} = (1 + 0.2\eta_d M^2)^{3.5}$$

where the ratio of specific heats $\gamma = 1.4$.

### 23–2 Performance Characteristics

The uninstalled power required is $P_q$, the power available $P_a$, the gross jet thrust $F_G$, and net jet thrust $F_N$. The compressor model calculates $P_a$ as a function of flight condition and rating; or calculates jet thrust and mass flow at $P_q$. The specific power is $SP = P/\dot{m}$, the specific thrust is $ST = F_G/\dot{m}$. The forces produced by mass flow that does not go through the core are treated as momentum drag $D_{\text{aux}}$.

The reference performance is at sea-level-standard static conditions (subscript 0), and MCP (subscript C). For each rating R, the performance is characterized by the following quantities for sea-level-standard static conditions: Power $P_{0R}$, specific power $SP_{0R}$, and mechanical power limit $P_{\text{mech}R}$. The mass flow is then $\dot{m}_{0R} = P_{0R}/SP_{0R}$. 
The difference between net and gross jet thrust is the momentum drag: \( F_N = F_G - \dot{m}_{req} V \). Note that traditional units for mass flow are pound/sec (pps), while this equation requires slug/sec (\( \dot{m}_{req}/g \) replaces \( \dot{m}_{req} \)).

### 23–3 Installation

The difference between installed and uninstalled power is the inlet and exhaust losses \( P_{loss} \):

\[
P_{av} = P_a - P_{loss} \\
P_{req} = P_q - P_{loss}
\]

The inlet and exhaust losses are modeled as fractions of power available or power required: \( P_{loss} = (\ell_{in} + \ell_{ex}) P_a \) or \( P_{loss} = (\ell_{in} + \ell_{ex}) P_q \). So

\[
P_{av} = P_a (1 - \ell_{in} - \ell_{ex}) \\
P_q = P_{req} (1 - \ell_{in} - \ell_{ex})
\]

The momentum drag of the auxiliary air flow is a function of the mass flow \( \dot{m}_{aux} = f_{aux} \dot{m}_{req} \):

\[
D_{aux} = (1 - \eta_{aux}) \dot{m}_{aux} V = (1 - \eta_{aux}) f_{aux} \dot{m}_{req} V
\]

where \( \eta_{aux} \) is the ram recovery efficiency.

### 23–4 Power Available

Given the flight condition and engine rating, the power available \( P_a \) is calculated as follows. The specific power and referred mass flow (relative to \( SP_0 \) and \( \dot{m}_0 \) for this rating) are approximated by functions of the ambient temperature ratio \( \theta \), here just:

\[
SP_a = SP_0 \theta \left[ \delta M \sqrt{\theta_M} \right] X_{spa} \\
\dot{m}_a = \dot{m}_0 \left[ \delta/\sqrt{\theta} \right] \left[ \delta M \sqrt{\theta_M} \right] X_{mfa}
\]

The power available is then

\[
P_a = SP_a \dot{m}_a = P_0 \left[ \delta \sqrt{\theta} \right] \left[ \delta M \sqrt{\theta_M} \right] X_{spa} + X_{mfa}
\]

This expression for \( \dot{m}_a \) is used only to calculate \( P_a \); elsewhere the \( \dot{m}_q \) expression below (for performance at power required) is used to obtain the mass flow at a power \( P_q \). The influence of compressor rotational speed is not considered.

### 23–5 Performance at Power Required

The compressor performance (mass flow and gross jet thrust) is calculated for a specified power required \( P_q \), flight condition, and rating. Installation losses \( P_{loss} \) are added to \( P_{req} \) (\( P_q = P_{req} + P_{loss} \)). The referred quantities (relative to SLS static MCP quantities) are approximated by functions of \( q = P_q/(P_0 C \delta \sqrt{\theta}) \):

\[
\dot{m}_{req} = \dot{m}_{0C} \left( \delta/\sqrt{\theta} \right) (K_{mfq0} + K_{mfq1} q + K_{mfq2} q^2 + K_{mfq3} q^3) \left[ \theta_M \right] X_{mjq} \\
ST_{req} = ST_{0C} \sqrt{\theta} \left[ \theta_M \right] X_{stq}
\]
Compressor Model

The gross jet thrust is then

\[
F_G = ST_{req} \dot{m}_{req} = F_{g0C} \delta \left( K_{mfq0} + K_{mfq1}q + K_{mfq2}q^2 + K_{mfq3}q^3 \right) \left[ \theta_M \right]^{X_{stq} + X_{mfq}}
\]

Then the installed net jet thrust \( F_N \) and momentum drag \( D_{aux} \) are calculated. The influence of compressor rotational speed is not considered.

## 23–6 Scaling

The parameters of the compressor model can be defined for a specific compressor, but it is also necessary to scale the parameters as part of the aircraft sizing task, in order to define a compressor for a specified power. In addition, advanced technology must be represented in the model. Scaling and advanced technology are handled in terms of specific power and specific thrust (at SLS static conditions and MCP).

The compressor model includes reference values of the performance parameters: \( P_{0R}, SP_{0R}, \) and \( P_{mechR} \). Mass flow is obtained from \( \dot{m}_{0R} = \frac{P_{0R}}{SP_{0R}} \). The reference power at each rating \( R \) defines a ratio to MCP: \( r_{p0R} = \frac{P_{0R}}{P_{0C}} \). Similarly for specific power and mechanical limits: \( r_{s0R} = \frac{SP_{0R}}{SP_{0C}} \) and \( r_{m0R} = \frac{P_{mechR}}{P_{0C}} \). These ratios are kept fixed when the compressor is scaled.

The compressor size is specified as power \( P_{eng} \), which is the power at takeoff rating \( R \), for SLS static conditions. Hence the MCP power is \( P_{0C} = P_{to}/r_{p0R} \), and the power at all other ratings follows. If \( P_{0C} \) is not equal to the reference value of the compressor model, then the compressor is scaled.

## 23–7 Weight

The compressor weight can be a fixed input value, or calculated as a function of power. As a function of power, the weight of one compressor is:

\[
W_{one\ eng} = K_{0comp} + K_{1comp}P + K_{2comp}P^{X_{comp}}
\]

where \( P \) is the installed power (SLS static, specified rating) per compressor. A constant weight per power \( W/P \) is given by using only \( K_{1comp} \).
A motor converts electrical energy (fuel) to shaft power. A generator converts input shaft power to electrical energy, and the shaft power contributes to the propulsion group power required. The model follows references 1 to 4.

24–1 Motor Characteristics

The motor or generator size is defined by the maximum power available, \( P_{\text{max}} = P_{\text{eng}} \), and a peak torque \( Q_{\text{peak}} \). For clarity, the torque limit is expressed as a power limit \( Q_{\text{peak}} = P_{\text{peak}}/N_{\text{spec}} \). The ratio of maximum power and peak torque is the base rotational speed: \( P_{\text{max}}/Q_{\text{peak}} = N_{\text{base}} \), at \( N_{\text{base}} \), the power and torque limits coincide. Figure 24-1 shows the maximum power and peak torque for a number of motors. The base rotational speed is typically \( N_{\text{base}} = 700–7000 \) rpm, lower for high-torque motors. The ratio of maximum power to continuous power is typically \( \text{MRP}/\text{MCP} = 1.25–3 \) (fig. 24-2).

Motor weight depends primarily on the peak torque. Figure 24-3 shows the weight for a number of motors. For a given size, there is a wide range of \( Q/W \) values. Here high torque-to-weight is defined as \( Q/W > 3 \text{ ft-lb/lb} \). Motor electrical controller weight is typically 10–30% of the basic motor weight, up to 70% for small motors. The density is typically 100–250 lb/ft\(^3\) (fig. 24-4).

The engine performance is described by the power available \( P_{\text{av}} \) at each engine rating, and the energy flow \( \dot{E} \) required to produce the power required \( P_{\text{req}} \). The specific fuel consumption is \( \text{sfc} = \dot{E}/P \) (inverse of efficiency). For each rating \( R \), the maximum power is \( P_{0R} \), and the torque limit is \( P_{\text{peak}R} \).

24–2 Power Available and Performance at Power Required

Given the flight condition and engine rating \( R \), the power available is

\[
P_{\text{av}} = \min(P_{0R}, rP_{\text{peak}R})
\]

where \( r = N/N_{\text{spec}} \), and \( N \) is the motor rotational speed.

The performance is calculated for a specified power required (which might equal the power available), flight condition, and engine rating. The motor power required determines the energy flow:

\[
\dot{E}_{\text{req}} = \dot{E}_{0C}g_e(q, n) = P_{\text{req}}/\eta_{\text{motor}}
\]

where \( q = P_{\text{req}}/P_{\text{eng}} \), \( n = N/N_{\text{spec}} \), and \( \eta_{\text{motor}} \) is the motor efficiency. The generator energy flow to the fuel tank is related to the power required:

\[
\dot{E}_{\text{req}} = \dot{E}_{0C}g_e(q, n) = P_{\text{req}}\eta_{\text{motor}}
\]

where \( \eta_{\text{motor}} \) is the generator efficiency.
For a motor and fuel cell, the performance (mass flow and fuel flow) is calculated for a specified power required:

\[ \dot{w}_{\text{req}} = \dot{w}_{0C}(q/\eta) = sfc_{0C}(P_{\text{req}}/\eta) \]

\[ \dot{m}_{\text{req}} = \dot{m}_{0C}(q/\eta) = K_{mf}\dot{w}_{\text{req}} \]

Inlet and exhaust installation losses are included in the specific fuel consumption. The efficiency \( \eta = \eta_{\text{cell}}\eta_{\text{motor}} \) includes both motor and fuel cell losses. The fuel cell efficiency as a function of power is

\[ \frac{1}{\eta_{\text{cell}}} = 1 + \frac{P}{P_{\text{eng}}} \left( \frac{1}{\eta_{\text{ref}}} - 1 \right) + c \frac{P_{\text{eng}}}{P} \]

from \( \eta_{\text{ref}} \) and \( c \) for the fuel cell. The ratio of mass flow and fuel flow \( (K_{mf}) \) follows from the chemistry of the reaction.

### 24–3 Efficiency

Motor loss sources include copper (internal resistance, proportional to current-squared hence torque-squared), iron core (eddy current and hysteresis, proportional to rotational speed), and mechanical (friction, proportional to speed, and windage, proportional to speed-cubed). Equivalent circuit and efficiency map models are implemented. The efficiency can also be a fixed value.

#### 24-3.1 Equivalent Circuit

The motor or generator efficiency as a function of power is estimated considering an equivalent circuit, defined by internal resistance \( R \) and current \( I_0 \). The voltage is \( V = V_o - IR \). The efficiency is \( \eta_{\text{motor}} = P/(P + P_{\text{loss}}) \), from the power loss \( P_{\text{loss}} = I^2R + P_0 \). For small loss, \( I \approx P/V_o \). In terms of \( P_{\text{eng}} \), let

\[ \frac{R}{V_o^2} = \frac{1}{P_{\text{eng}}} \left( \frac{1}{\eta_{\text{ref}}} - 1 \right) \]

\[ P_0 = cP_{\text{eng}} \]

Then

\[ \frac{1}{\eta_{\text{motor}}} = 1 + P \frac{R}{V_o^2} + P_0/P = 1 + \frac{P}{P_{\text{eng}}} \left( \frac{1}{\eta_{\text{ref}}} - 1 \right) + c \frac{P_{\text{eng}}}{P} \]

So \( \eta_{\text{motor}} = (1/\eta_{\text{ref}}+c)^{-1} \) at \( P = P_{\text{eng}} \). The efficiency decreases with \( P \) because of the internal resistance, but is zero at \( P = 0 \) because of the internal current term.

#### 24-3.2 Efficiency Map

The motor or generator power loss is described as a polynomial in the motor torque and rotational speed:

\[ P_{\text{loss}} = P_{\text{eng}}f_{\text{loss}} \sum_{i=0}^{3} \sum_{j=0}^{3} C_{ij}t^i n^j \]

where \( q = P_{\text{req}}/P_{\text{eng}}, n = N/N_{\text{spec}} \), and \( t = q/n \). The factor \( f_{\text{loss}} \) allows adjustment of the peak efficiency. Controller losses, including power conversion and conditioning, are represented by an efficiency \( \eta_{\text{cont}} \). Then

\[ \eta_{\text{motor}} = \eta_{\text{cont}} \frac{P_{\text{req}}}{P_{\text{req}} + P_{\text{loss}}} = \eta_{\text{cont}} \frac{q}{q + P_{\text{loss}}/P_{\text{eng}}} \]

is the motor or generator efficiency. Constant efficiency implies just \( C_{11} = 1/\eta - 1 \). The copper, iron, and windage losses imply \( P_{\text{loss}} = K_cQ^2 + K_iN + K_wN^3 + K_0 = P_{\text{eng}}(C_{20}t^2 + C_{01}n + C_{03}n^3 + C_{00}) \).
Consider an efficiency map with a peak \( \eta_0 \) at \( t_0 = Q_0/Q_{\text{eng}} \) and \( n_0 = N_0/N_{\text{spec}} \). Taking the derivatives of \( (C_{20}t^2 + C_{01}n + C_{03}n^3 + C_{00}) = nt(1 - \eta)/\eta \) with respect to \( t \) and \( n \), and evaluating at \( t_0 \) and \( n_0 \) so \( \partial \eta/\partial t = \partial \eta/\partial n = 0 \), gives

\[
C_{01} = \frac{t_0}{4} \frac{1 - \eta_0}{\eta_0} - \frac{3C_{00}}{2n_0}, \\
C_{03} = \frac{t_0}{4n_0^2} \frac{1 - \eta_0}{\eta_0} + \frac{C_{00}}{2n_0^3}, \\
C_{20} = \frac{n_0}{2t_0} \frac{1 - \eta_0}{\eta_0}
\]

(ref. 2). Note that \( C_{01} > 0 \) requires \( C_{00} < (n_0t_0/6)(1 - \eta_0)/\eta_0 \).

In general, expanding the efficiency in \( t \) and \( n \) about the peak gives

\[
\frac{1}{\eta} = \frac{P_{\text{loss}}}{QN} + 1 = \frac{P_{\text{loss}}}{P_{\text{eng}} tn} + 1 \approx \frac{1}{\eta_0} \left( 1 + a(\bar{t} - 1)^2 + b(\bar{t} - 1)(\bar{n} - 1) + c(\bar{n} - 1)^2 \right)
\]

where \( \bar{t} = t/t_0 \) and \( \bar{n} = n/n_0 \). Hence

\[
\frac{P_{\text{loss}}}{P_{\text{eng}}} = tn \left( \frac{1}{\eta} - 1 \right) = tn \left[ \frac{1}{\eta_0} \left( 1 + a(\bar{t} - 1)^2 + b(\bar{t} - 1)(\bar{n} - 1) + c(\bar{n} - 1)^2 \right) - 1 \right] + d(\bar{t} - 1)^2(\bar{n} - 1)^2
\]

The last term is introduced so the efficiency is zero at zero torque or zero speed.

Alternatively, consider an equivalent circuit defined by internal resistance \( R \), no load current \( I_0 \), motor constant \( K_v \), and friction \( K_f \). So \( V = V_m + IR \) (back emf \( V_m = N/K_v \)), \( I = I_m + I_0 \) (\( I_m = Q_m/N/V_m = K_vQ_m \)), \( Q = Q_m - Q_f \) (output torque, \( Q_f = K_f N \)). The input power is \( VI = (N/K_v+IR)I \), with \( I = K_vQ_m + I_0 = K_v(Q + K_f N) + I_0 \). The shaft power is \( QN \). Then the power loss is

\[
P_{\text{loss}} = VI - QN = \left( N + (I/K_v)K_v^2R \right)(I/K_v) - QN
\]

\[
= N \left( Q + K_f N + I_0/K_v \right) + \left( Q + K_f N + I_0/K_v \right)^2 K_v^2R - QN
\]

\[
= N \left( K_f N + I_0/K_v \right) + \left( Q + K_f N + I_0/K_v \right)^2 K_v^2R
\]

This expression does not show a peak efficiency as a function of torque and speed.

### 24–4 Weight

The motor or generator weight can be a fixed input value, or calculated as a function of power or torque. As a function of power, the weight of one engine is:

\[
W_{\text{eng}} = K_{0\text{motor}} + K_{1\text{motor}}P + K_{2\text{motor}}P^{X_{\text{motor}}}
\]

where \( P = P_{\text{eng}} \).

Motor weight depends primarily on the peak torque. For the NASA15 model, the weight is:

\[
W_{\text{eng}} = 0.5382f_{\text{design}}Q^{0.8129}
\]

where \( Q = P_{\text{peak}}/N_{\text{spec}} \) (ft-lb), for takeoff rating \( R \). The structural design factor \( f_{\text{design}} = 1.0 \) for high torque-to-weight motors \( (Q/W > 3.5 \text{ ft-lb/lb}) \), and \( f_{\text{design}} = 2.5606 \) for others. Based on 64 motors, the
average error is 27.5% (fig. 24-5); 25% for high torque-to-weight and 29% for others. Including either
maximum rotational speed or maximum power does not reduce the weight estimation error significantly.
Considering only the high torque-to-weight motors, the weight is

\[ W_{\text{one eng}} = 0.3928Q^{0.8587} \]

Based on 25 motors, the average error is 21.8% (fig. 24-5). This sensitivity of motor weight to output
torque capability is somewhat greater than the drive system trend.

### 24–5 Scaling

The parameters of the motor model can be defined for a specific motor, but it is also necessary to
to scale the parameters as part of the aircraft sizing task, in order to define a motor for a specified power.
In addition, advanced technology must be represented in the model.

The motor model includes reference values of the performance parameters: \( P_{0R}, P_{\text{peak}R} \). The reference power at each rating \( R \) defines a ratio to MCP: \( r_{p0R} = P_{0R}/P_{0C} \). Similarly for torque limits:

\[ r_{m0R} = P_{\text{peak}R}/P_{0C} \]

These ratios are kept fixed when the motor is scaled. The specification motor speed is scaled with the power:

\[ N_{\text{spec}} = N_{\text{spec-ref}} \left( \frac{P_{\text{ref}}}{P_{\text{eng}}} \right)^{K_{N_s}} \]

The speed ratio \( N/N_{\text{spec}} \) influences the efficiency map and the torque limit. If the base rotational speed
\( N_{\text{base}} = P_{\text{max}}/Q_{\text{peak}} \) scales with power as \( K_{N_b} \) instead of \( K_{N_s} \), then the scaled values of \( P_{\text{peak}R} \) require
a factor of \( (P_{\text{eng}}/P_{\text{ref}})^{K_{N_b}} \) as well as \( r_{m0R} \). Depending on the design approach, \( K_{N_b} \) ranges from
about 0.8 to zero (constant \( N_{\text{spec}} \)) to about −1.2 (\( N_{\text{spec}} \) increase with power).

The motor size is specified as power \( P_{\text{eng}} \), which is the power at takeoff rating \( R \), for sea level
standard (SLS) static conditions. Hence the MCP power is \( P_{0C} = P_{00}/r_{p0R} \), and the power at all other
ratings follows. If \( P_{0C} \) is not equal to the reference value of the motor model, then the motor is scaled.

### 24–6 References


3) Sinsay, J.D.; Alonso, J.J.; Kontinos, D.A.; Melton, J.E.; and Grabbe, S. “Air Vehicle Design
   and Technology Considerations for an Electric VTOL Metro-Regional Public Transportation System.”

4) Datta, A., and Johnson, W. “Requirements for a Hydrogen Powered All-Electric Manned Helicopter.”
Figure 24-1. Motor maximum power and peak torque.

Figure 24-2. Motor maximum power and continuous power (MRP/MCP).
Figure 24-3. Motor weight.

Figure 24-4. Motor density.
Figure 24-5. Motor weight (NASA15).
Design and analysis codes calculate aircraft performance for a very wide range of operating conditions. Thus the jet performance model must be realistic even far from the design point. A simple thermodynamic cycle analysis that assumes design point component efficiencies everywhere is not realistic for such an application. Rather than developing models for component performance, the approach taken is to use a model for the total turbojet or turbofan performance. The jet is not being designed.

The Referred Parameter Jet Engine Model (RPJEM) is based on curve-fits of performance data for existing or projected jets over a range of operating conditions. The use of referred parameters tends to collapse the data and provides a basis for scaling the jet. The operating condition is described by pressure altitude, ambient air temperature, flight Mach number, and either jet rating or jet thrust required. The parametric model is scaled to the required size and adjusted to the appropriate technology level to represent a notional jet. Jet size is represented by mass flow. Jet technology is represented by specific thrust available and specific fuel consumption at maximum continuous thrust (MCT), sea level/standard day (SLS), static (zero airspeed) conditions. Jet installation effects are also modeled.

25–1 Operating Environment

The operating condition and atmosphere give the standard conditions (temperature $T_{\text{std}}$ and pressure $p_{\text{std}}$) for a specified pressure altitude; the sea-level standard conditions (temperature $T_0$ and pressure $p_0$); and the operating temperature $T$ and pressure $p$. Here the temperatures are °R or °K. The characteristics depend on the temperature ratio $\theta = T/T_0$ and pressure ratio $\delta = p/p_0$.

The flight Mach number $M = V/c_s = V/c_{s0}\sqrt{\theta}$ is obtained from the aircraft speed $V$.

The inlet ram air temperature ratio and pressure ratio are obtained then from $M$ and the inlet ram recovery efficiency $\eta_d$:

$$\theta_M = \left(1 + \frac{\gamma - 1}{2} M^2\right) = (1 + 0.2M^2)$$

$$\delta_M = \left(1 + \frac{\gamma - 1}{2} \eta_d M^2\right)^{\frac{\gamma}{\gamma - 1}} = (1 + 0.2\eta_d M^2)^{3.5}$$

where the ratio of specific heats $\gamma = 1.4$.

25–2 Performance Characteristics

The uninstalled thrust required is $T_q$, the thrust available $T_a$. The jet model calculates $T_a$ as a function of flight condition and engine rating; or calculates mass flow and fuel flow at $T_q$. The specific thrust is $ST = T/\dot{m}$, and the specific fuel consumption is $sfc = \dot{w}/T$. A turbofan engine has a bypass
ratio $\beta = \dot{m}\text{fan}/\dot{m}$. The forces produced by mass flow that does not go through the core or fan are treated as momentum drag $D_{\text{aux}}$.

The reference performance is at sea-level-standard static conditions (subscript 0), and MCT (subscript $C$). For each rating $R$, the performance is characterized by the following quantities for sea-level-standard static conditions: thrust $T_{0R}$, specific thrust $ST_{0R}$, and mechanical thrust limit $T_{\text{mech}R}$. The mass flow is then $\dot{m}_{0R} = T_{0R}/ST_{0R}$.

The difference between net and gross thrust is the momentum drag: $T_N = T_G - \dot{m}(ST)_{\text{mom}}$, where $(ST)_{\text{mom}} = (1 + \beta)V$ for turbojet or turbofan, or $(ST)_{\text{mom}} = \Omega\text{r}\text{react}$ for reaction drive. Note that traditional units for mass flow are pound/sec (pps), while this equation requires slug/sec ($\dot{m}_{\text{req}}/g$ replaces $\dot{m}_{\text{req}}$).

### 25–3 Installation

The difference between installed and uninstalled thrust is the inlet and exhaust losses $T_{\text{loss}}$: $T_{\text{av}} = T_{a} - T_{\text{loss}}$ and $T_{\text{req}} = T_{q} - T_{\text{loss}}$. The inlet and exhaust losses are modeled as fractions of thrust available or thrust required: $T_{\text{loss}} = (\ell_{\text{in}} + \ell_{\text{ex}})T_{a}$ or $T_{\text{loss}} = (\ell_{\text{in}} + \ell_{\text{ex}})T_{q}$. The momentum drag of the auxiliary air flow is a function of the mass flow $\dot{m}_{\text{aux}} = f_{\text{aux}}\dot{m}_{\text{req}}$:

$$D_{\text{aux}} = (1 - \eta_{\text{aux}})\dot{m}_{\text{aux}}V = (1 - \eta_{\text{aux}})f_{\text{aux}}\dot{m}_{\text{req}}V$$

where $\eta_{\text{aux}}$ is the ram recovery efficiency. Exhaust losses ($\ell_{\text{ex}}$) and auxiliary air flow parameters ($\eta_{\text{aux}}$, $f_{\text{aux}}$) are defined for infrared suppressor on and off.

### 25–4 Thrust Available

Given the flight condition and jet rating, the thrust available $T_{a}$ is calculated as follows. The gross specific thrust and referred mass flow (relative to $ST_{0}$ and $\dot{m}_{0}$ for this rating) are approximated by functions of the ambient temperature ratio $\theta$, here just:

$$ST_{a} = ST_{0} \sqrt{\theta} \left[ \delta M \sqrt{\theta M} \right]^{X_{\text{sta}}}$$

$$\dot{m}_{a} = \dot{m}_{0} \left( \delta/\sqrt{\theta} \right) \left[ \delta M \sqrt{\theta M} \right]^{X_{\text{mf}}}$$

The thrust available is then

$$T_{a} = ST_{a}\dot{m}_{a} - \dot{m}_{a}(ST)_{\text{mom}} = T_{0}\delta \left[ \delta M \sqrt{\theta M} \right]^{X_{\text{sta}}+X_{\text{mf}}} - \dot{m}_{a}(ST)_{\text{mom}}$$

This expression for $\dot{m}_{a}$ is used only to calculate $T_{a}$; elsewhere the $\dot{m}_{q}$ expression below (for performance at thrust required) is used to obtain the mass flow at a thrust $T_{q}$.

### 25–5 Performance at Thrust Required

The jet performance (mass flow and fuel flow) is calculated for a specified thrust required $T_{q}$, flight condition, and jet rating. The referred quantities (relative to SLS static MCT quantities) are approximated by functions of referred gross thrust $t = (T_{q} + \dot{m}(ST)_{\text{mom}})/(T_{0C}\delta)$:

$$\dot{w}_{\text{req}} = \dot{w}_{0C} \left( \delta/\sqrt{\theta} \right) (K_{ffq0} + K_{ffq1}t + K_{ffq2}t^{2}) [\theta M]^{-X_{ffq}}$$

$$\dot{m}_{\text{req}} = \dot{m}_{0C} \left( \delta/\sqrt{\theta} \right) t^{K_{mfq}} [\theta M]^{X_{mfq}}$$
The mass flow solution is

\[
\frac{\dot{m}_{\text{req}}}{\dot{m}_{\text{OC}}(\delta/\sqrt{\theta})[\theta_M]^{X_{mfq}}} = \begin{cases} 
1 & K_{mfq} = 0 \\
\frac{b}{(1 - a)} & K_{mfq} = 1 \\
-(a/2) + \sqrt{(a/2)^2 + b} & K_{mfq} = 1/2
\end{cases}
\]

where \( a = (ST)_{\text{mom}}\dot{m}_{\text{OC}}[\theta_M]^{X_{mfq}}/T_{\text{OC}}\delta \) and \( b = T_q/T_{\text{OC}}\delta \).

### 25–6 Scaling

The parameters of the jet model can be defined for a specific turbojet or turbofan, but it is also necessary to scale the parameters as part of the aircraft sizing task, in order to define a jet for a specified thrust. In addition, advanced technology must be represented in the model. Scaling and advanced technology are handled in terms of specific thrust and specific fuel consumption (at SLS static conditions and MCT). Figures 25-1 through 25-3 present historical data for jet specific fuel consumption, weight, and specific thrust.

The jet model includes reference values of the performance parameters: \( T_{0R}, ST_{0R}, T_{\text{mech}R}, \) and \( sfc_{\text{OC}} \). Mass flow and fuel flow are obtained from \( \dot{m}_{0R} = T_{0R}/ST_{0R} \) and \( \dot{w}_{\text{OC}} = sfc_{\text{OC}}T_{\text{OC}} \). The reference thrust at each rating \( R \) defines a ratio to MCT: \( r_{0R} = T_{0R}/T_{\text{OC}} \). The reference thrust at each rating \( R \) defines a ratio to MCT: \( r_{0R} = T_{0R}/T_{\text{OC}} \). Similarly for specific thrust and mechanical limits: \( r_{s0R} = ST_{0R}/ST_{\text{OC}} \) and \( r_{m0R} = T_{\text{mech}R}/T_{\text{OC}} \). These ratios are kept fixed when the jet is scaled.

The jet size is specified as takeoff thrust \( T_{\text{to}} = T_{\text{eng}}, \) which is the thrust at rating \( R, \) for SLS static conditions. Hence the MCT is \( T_{\text{OC}} = T_{\text{to}}/r_{0R}, \) and the thrust at all other ratings follows. If \( T_{\text{OC}} \) is not equal to the reference value of the jet model, then the jet is scaled. To reflect advanced technology, the specific thrust and specific fuel consumption can be specified: \( ST_{\text{OC}} = ST_{\text{tech}} \) and \( sfc_{\text{OC}} = sfc_{\text{tech}} \) (replacing the jet model reference values). The default values are the reference values of the jet model. In the following paragraph, the subscript “tech” refers to these quantities; the subscript “ref” means the jet model reference values.

The jet technology parameters \( ST_{\text{OC}} \) and \( sfc_{\text{OC}} \) are assumed to vary linearly with mass flow \( \dot{m}_{\text{OC}} \) up to a limit \( \dot{m}_{\text{lim}} \), and constant thereafter at \( ST_{\text{lim}} \) and \( sfc_{\text{lim}} \). The mass flow at the technology condition is \( \dot{m}_{\text{tech}} = T_{\text{ref}}/ST_{\text{tech}}, \) with the technology values \( ST_{\text{tech}} \) and \( sfc_{\text{tech}} \). The intercept values are projected from the technology values: \( K_{st0} = ST_{\text{tech}} - K_{st1}\dot{m}_{\text{tech}}, K_{st1} = (ST_{\text{lim}} - ST_{\text{tech}})/(\dot{m}_{\text{lim}} - \dot{m}_{\text{tech}}) \); and similarly for \( sfc \). Then for \( \dot{m}_{\text{OC}} < \dot{m}_{\text{lim}} \)

\[
ST_{\text{OC}} = K_{st0} + K_{st1}\dot{m}_{\text{OC}}
\]

\[
sfc_{\text{OC}} = K_{sf0} + K_{sf1}\dot{m}_{\text{OC}}
\]

and for \( \dot{m}_{\text{OC}} \geq \dot{m}_{\text{lim}} \)

\[
ST_{\text{OC}} = ST_{\text{lim}}
\]

\[
sfc_{\text{OC}} = sfc_{\text{lim}}
\]

These equations are used if \( \dot{m}_{\text{lim}} > \dot{m}_{\text{tech}} \). Otherwise the model sets \( K_{st1} = K_{sf1} = 0 \), so there is no variation with scale: \( ST_{\text{OC}} = ST_{\text{tech}} \) and \( sfc_{\text{OC}} = sfc_{\text{tech}} \). Usually the effect of size gives \( K_{sv2} \geq 0 \) and \( K_{sfc2} \leq 0 \). The thrust at the limit is \( T_{\text{lim}} = ST_{\text{lim}}\dot{m}_{\text{lim}} \). Figure 25-4 illustrates the scaling of \( SP, Sfc, \) and \( SW \).
Figure 25-1. Historical data for turbojet and turbofan specific fuel consumption.

Figure 25-2. Historical data for turbojet and turbofan weight.
Using \( \dot{m}_{0C} = T_{0C}/ST_{0C} \), the specific thrust equation can be solved for the mass flow given the thrust:

\[
\dot{m}_{0C} = \begin{cases}
T_{0C}/K_{st0} & K_{st1} = 0 \\
T_{0C}/ST_{lim} & T_{0C} \geq T_{lim} \\
\sqrt{\left(\frac{K_{0}}{2K_{1}}\right)^2 + \frac{T_{0C}}{K_{1}}} - \frac{K_{0}}{2K_{1}} & \text{otherwise}
\end{cases}
\]

From this mass flow, \( ST_{0C} \) and \( sfc_{0C} \) are calculated, hence the fuel flow \( \dot{w}_{0C} = sfc_{0C}T_{0C} \). Then the thrust and specific thrust at all ratings \( R \) are obtained from the ratios: \( T_{0R} = r_{t0R}T_{0C} \), \( ST_{0R} = r_{s0R}ST_{0C} \), and \( T_{mechR} = r_{m0R}T_{0C} \).

The actual (perhaps scaled) values of the performance parameters are available for the jet group: \( T_{0R}, ST_{0R}, T_{mechR}, \) and \( sfc_{0C} \).

### 25–7 Weight

The jet weight can be a fixed input value, or calculated as a function of thrust. As a function of thrust, the weight of one jet is:

\[
W_{\text{one jet}} = K_{0\text{jet}} + K_{1\text{jet}}T + K_{2\text{jet}}T^{X_{\text{jet}}}
\]

where \( T \) is the installed takeoff thrust (SLS static, specified rating) per jet. A constant weight per thrust \( W/T \) is given by using only \( K_{1\text{jet}} \).
In this jet model, only the reference values and scaling constants are dimensional. Conventional English units and SI units are shown in Table 25-1. Units of specific thrust and specific fuel consumption follow from these conventions.

<table>
<thead>
<tr>
<th>Table 25-1. Conventional units.</th>
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<tr>
<td>thrust $T$</td>
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<td>English:</td>
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<td>SI:</td>
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</table>
A fuel cell burns a fuel (typically hydrogen) and generates electrical energy, which is stored in a fuel tank system or used directly by a motor. The energy flow defines the power required. The power available is related to the size $P_{\text{chrg}}$. The model follows reference 1.

Given the flight condition and the charger rating, the cell power available is $\dot{E}_{\text{cell}} = P_0$. The power required $\dot{E}_{\text{req}}$ is defined by the charge group energy flow. The cell power required is

$$\dot{E}_{\text{cell}} = \dot{E}_{\text{req}} / \eta_{\text{chrg}}$$

where $q = \dot{E}_{\text{req}} / P_0$, and $\eta_{\text{chrg}}$ is the fuel cell efficiency. The fuel cell performance (mass flow and fuel flow) is calculated from the cell power required:

$$\dot{w}_{\text{req}} = sfc_0 \dot{E}_{\text{cell}} = \dot{w}_{\text{req}} / \eta_{\text{chrg}}$$

$$\dot{m}_{\text{req}} = K_{\text{mf}} \dot{w}_{\text{req}} = \dot{m}_{\text{req}} / \eta_{\text{chrg}}$$

where $q = \dot{E}_{\text{req}} / P_0$, $sfc_0 = sfc_0 \dot{E}_{\text{cell}}$, and $K_{\text{mf}} = K_{\text{mf}} \dot{w}_{\text{req}}$. Inlet and exhaust installation losses are included in the specific fuel consumption. The specific fuel consumption is given by the fuel specific energy and the fuel cell thermal efficiency: $sfc = e_{\text{fuel}} / \eta_{\theta}$. The ratio of mass flow and fuel flow follows from the chemistry of the reaction. For hydrogen and air

$$K_{\text{mf}} = \frac{\lambda_A}{\lambda_H} \frac{m_A}{x_O m_H} = \frac{\lambda_A}{\lambda_H} 68.59$$

The molar masses of hydrogen and air are $m_H = 2.016$ and $m_A = 28.97$ g/mole; $x_O = 0.2095$ is the molar fraction of oxygen in air. The supply ratio $\lambda_A / \lambda_H = 1/2$ from stoichiometry, and typically $\lambda_A / \lambda_H \approx 1.25$ in practice.

The fuel cell efficiency as a function of power is estimated considering an equivalent circuit, defined by internal resistance $R$ and current $I_0$. The voltage is $V = V_o - IR$. The efficiency is $\eta_{\text{chrg}} = P / (P + P_{\text{loss}})$, from the power loss $P_{\text{loss}} = I^2 R + P_0$. For small loss, $I \approx P / V_o$. In terms of $P_{\text{chrg}}$, let

$$\frac{R}{V_o^2} = \frac{1}{P_{\text{chrg}}} \left( \frac{1}{\eta_{\text{ref}}} - 1 \right)$$

$$P_0 = c P_{\text{chrg}}$$

Then

$$\frac{1}{\eta_{\text{chrg}}} = 1 + P(R/V_o^2) + P_0 / P = 1 + \frac{P}{P_{\text{chrg}}} \left( \frac{1}{\eta_{\text{ref}}} - 1 \right) + c \frac{P_{\text{chrg}}}{P}$$

So $\eta_{\text{chrg}} = (1/\eta_{\text{ref}} + c)^{-1}$ at $P = P_{\text{chrg}}$. The efficiency decreases with $P$ because of the internal resistance, but is zero at $P = 0$ because of the internal current term. Alternatively, the efficiency can be a fixed value.
The fuel cell weight can be a fixed input value or calculated as a function of power. The weight of one charger is:

$$W_{\text{one chrg}} = K_{0\text{cell}} + K_{1\text{cell}} P + K_{2\text{cell}} P^{X_{\text{cell}}}$$

where $P = P_{\text{chrg}}$.

## 26–1 References

A solar cell generates electrical energy, which is stored in a fuel tank system. The energy flow defines the power required. The power available is related to the size $P_{\text{chrg}}$. The solar cell is characterized by power density $\varepsilon_{\text{solar}}$ (W/m$^2$) and weight density $\sigma_{\text{solar}}$ (kg/m$^2$). From the size $P_{\text{chrg}}$, the area is $A_{\text{solar}} = P_{\text{chrg}}/\varepsilon_{\text{solar}}$; and then the weight is $W_{\text{one chrg}} = W_{\text{solar}} = A_{\text{solar}}\sigma_{\text{solar}}$.

Given the flight condition and the charger rating, the cell power available is $\dot{E}_{\text{cell}} = P_0$. The power required $\dot{E}_{\text{req}}$ is defined by the charge group energy flow. The cell power required is

$$\dot{E}_{\text{qcell}} = \dot{E}_{0C} g_c(q) = \dot{E}_{\text{req}}/\eta_{\text{chrg}}$$

where $q = \dot{E}_{\text{req}}/P_0$, and $\eta_{\text{chrg}}$ is the solar cell efficiency.

The solar cell efficiency as a function of power is estimated considering an equivalent circuit, defined by internal resistance $R$ and current $I_0$. The voltage is $V = V_o - IR$. The efficiency is $\eta_{\text{chrg}} = P/(P + P_{\text{loss}})$, from the power loss $P_{\text{loss}} = I^2R + P_0$. For small loss, $I \approx P/V_o$. In terms of $P_{\text{chrg}}$, let

$$\frac{R}{V_o^2} = \frac{1}{P_{\text{chrg}}} \left( \frac{1}{\eta_{\text{ref}}} - 1 \right)$$

$$P_0 = c P_{\text{chrg}}$$

Then

$$\frac{1}{\eta_{\text{chrg}}} = 1 + P(R/V_o^2) + P_0/P = 1 + \frac{P}{P_{\text{chrg}}} \left( \frac{1}{\eta_{\text{ref}}} - 1 \right) + c \frac{P_{\text{chrg}}}{P}$$

So $\eta_{\text{chrg}} = (1/\eta_{\text{ref}} + c)^{-1}$ at $P = P_{\text{chrg}}$. The efficiency decreases with $P$ because of the internal resistance, but is zero at $P = 0$ because of the internal current term. Alternatively, the efficiency can be a fixed value.
A battery is a fuel tank system for which the fuel quantity stored and burned is measured in energy. The unit of fuel energy is Mega-Joules (MJ). For reference, 1 kW-hr = 3.6 MJ. The operating state affects the efficiency of the relation between useful power and the rate of change of the energy stored. The battery model produces the charge/discharge efficiency. The battery model can be used for capacitors and flywheels as well. References 1–6 provide background for the model.

The components associated with a fuel tank system (motor, generator, fuel cell, and solar cell) define the total energy flow $\dot{E}_{\text{comp}}$ (charge or discharge). Accounting for efficiency and losses gives the battery energy flow $\dot{E}_{\text{batt}}$. Accounting for battery capacity then gives the effective energy flow $\dot{E}_{\text{eff}}$. The change in stored energy is calculated from $\dot{E}_{\text{eff}}$ and time. The convention is that energy flow $\dot{E} > 0$ for discharge, and $\dot{E} < 0$ for charge. Power and current are positive for both discharge and charge.

The battery capacity is $E_{\text{fuel-cap}}$ (maximum usable fuel energy). The battery is characterized by specific energy $e_{\text{tank}}$ (MJ/kg) and energy density $\rho_{\text{tank}}$ (MJ/liter), so the tank weight and volume are obtained from the capacity. The current amount of energy stored is $E_{\text{fuel}}$. The state-of-charge is $s = E_{\text{fuel}} / E_{\text{fuel-cap}}$; the depth-of-discharge is $d = 1 - s$.

The charge capacity is $C$ (A-hr). $C$ is the usable capacity at low current and a reference temperature. Typically 20–30% of the stored charge is not usable at nominal conditions, which is accounted for in the capacity through the specific energy and energy density values. The corresponding energy capacity is obtained for a reference voltage, $E = CV_{\text{ref}}$ (W-hr, expressed in MJ). Then the specific energy (conventional units W-hr/kg) gives the weight, and the energy density (conventional units W-hr/liter) gives the volume. The current is measured in terms of the charge capacity: $I = xC$ (A), corresponding to a discharge time of $1/x$ hours. The units of $x$ are 1/hr. The maximum burst discharge current ($mbd$) is $x_{mbd}$.

The usable capacity decreases with time and duty cycles, depending on the temperature and current. This factor is assumed to be constant during a mission or flight condition. Capacity fade is accounted for using a factor $f_{\text{fade}} < 1$ on the available capacity or on the energy flow. Thus the effective energy flow is obtained by dividing the actual energy flow by $f_{\text{fade}}$.

The capacity depends on the discharge current. The Peukert model assumes $I^k T = \text{constant}$, where $T$ is the discharge time for current $I = xC_{\text{ref}}$. The Peukert coefficient $k = 1.2$ to 1.3 for lead-acid batteries, and $k = 1.01$ to 1.05 for lithium-ion batteries (weak dependence). Thus the capacity $C = IT \sim 1/x^{k-1}$. An increase in current by a factor of 10 means a 2–11% reduction of capacity for lithium-ion batteries. For a larger current, the battery reaches a specified discharge voltage sooner, hence effectively has a reduced capacity.
The specific power is $\pi_{\text{batt}}$ (kW/kg). The power capacity is obtained at the maximum burst discharge current:

$$P_{\text{cap}} = I_{mbd}V_{\text{ref}} = x_{mbd}CV_{\text{ref}} = x_{mbd}E_{\text{fuel-cap}}$$

(for $P_{\text{cap}}$ in W and $E_{\text{fuel-cap}}$ in W-hr). Hence given $x_{mbd}$, $\pi_{\text{batt}} = x_{mbd}e_{\text{tank}}/3.6$ (kW/kg from MJ/kg). Typically $x_{mbd}$ is smaller for large capacity, implying a trade between high specific power and high specific energy.

### 28–1 Equivalent Circuit Model

The discharge or charge efficiency as a function of power is estimated considering an equivalent circuit, defined by internal resistance $R$ and current $I_0$. The voltage is $V = V_o - IR$. The efficiency is

$$\eta_{\text{batt}} = P/(P + P_{\text{loss}}),$$

from the power loss $P_{\text{loss}} = I^2R + P_0$. For small loss, $I \cong P/V_o$. In terms of a reference power $P_{\text{ref}} = P_{\text{cap}}$, and battery power $P = |\dot{E}_{\text{comp}}|$, let

$$R/V_o^2 = \frac{1}{P_{\text{ref}}} \left( \frac{1}{\eta_{\text{ref}}} - 1 \right),$$

$$P_0 = cP_{\text{ref}}$$

Then

$$\frac{1}{\eta_{\text{batt}}} = 1 + P(R/V_o^2) + P_0/P = 1 + \frac{P}{P_{\text{ref}}} \left( \frac{1}{\eta_{\text{ref}}} - 1 \right) + c \frac{P_{\text{ref}}}{P}$$

So $\eta_{\text{batt}} = (1/\eta_{\text{ref}} + c)^{-1}$ at $P = P_{\text{ref}}$. The efficiency decreases with $P$ because of the internal resistance, but is zero at $P = 0$ because of the internal current term. Alternatively, the efficiency can be a fixed value.

The current is $x = I/C \cong P/V_oC \cong x_{mbd}P/P_{\text{ref}} = x_{mbd}|\dot{E}_{\text{comp}}|/P_{\text{cap}}$. Given the maximum current $x_{\text{max}}$, the maximum battery power is $P_{\text{max}} = (x_{\text{max}}/x_{mbd})P_{\text{cap}}$.

For discharge, the battery energy flow is $\dot{E}_{\text{batt}} = \dot{E}_{\text{comp}}/\eta_{\text{batt}} = \dot{E}_{\text{comp}} + P_{\text{loss}}$. Then the effective energy flow is $\dot{E}_{\text{eff}} = \dot{E}_{\text{batt}}/f_{\text{fade}}$. For charge (negative energy flow), the battery energy flow is $\dot{E}_{\text{batt}} = \dot{E}_{\text{comp}}\eta_{\text{batt}} = \dot{E}_{\text{comp}} + P_{\text{loss}}$. Then the effective energy flow is $\dot{E}_{\text{eff}} = \dot{E}_{\text{batt}}/f_{\text{fade}}$. The battery power margin is $P_{\text{max}} - |\dot{E}_{\text{batt}}|$.

### 28–2 Lithium-Ion Battery Model

The capacity and power of currently available lithium-ion batteries are shown in figure 28-1. The power shown corresponds to the maximum burst discharge current, $P_{\text{cap}} = x_{mbd}E_{\text{fuel-cap}}$. The cell reference voltage is typically $V_{\text{ref}} = 4.2$ V. The maximum continuous discharge current is in the range $x_{\text{med}} = 1–30$ (fig. 28-2). The maximum burst discharge current (which equals the power divided by capacity) is in the range $x_{mbd} = 10–50$ (fig. 28-3), with $x_{mbd} \approx 10$ for large capacity. Values of $x_{mbd}$ above 100 are achievable, but such high-power/short-duration capability will not have much application to aircraft. The maximum charge current is typically $x_{\text{Ccapmax}} = 1–5$ (fig. 28-4). Figure 28-5 shows the corresponding specific energy $e_{\text{tank}}$ and specific power $\pi_{\text{batt}} = x_{mbd}e_{\text{tank}}$. The specific energy is typically $e_{\text{tank}} = 50–200$ W-hr/kg = 0.2–0.7 MJ/kg. Figure 28-6 shows the specific energy $e_{\text{tank}}$ (W-hr/kg) and the energy density $\rho_{\text{tank}}$ (W-hr/liter). The battery density is 2–3 kg/liter for all configurations (cylindrical, pouch, prismatic).

The battery temperature is likely controlled, for safety and efficiency. The operating temperature depends on the environment ambient temperature, internal heat production, and active heating or cooling. Internal heat production includes that due to internal resistance ($I^2R$), and radiation and convection.
that this loss is a fraction $f_{TC}$ of the total power.

28-2.1 Discharge Characteristics

The lithium-ion battery voltage during discharge depends on the discharge capacity (depth-of-discharge $d$), current ($I = xC$), and cell temperature ($T_c$). Figure 28-7 shows typical variation of the voltage $V$ at constant current and constant temperature. The nominal energy capacity $E = CV_{\text{ref}}$ is obtained from the nominal charge capacity $C$ and a reference voltage $V_{\text{ref}}$. The open-circuit voltage $V_d$ is the voltage at zero current, a reference temperature $T_{\text{ref}}$, and 100% charge. Typically the cell voltage is $V_d = 4.2$ V, and $V_d$ is used as the reference $V_{\text{ref}}$. The actual battery capacity is determined by the voltage reaching a critical value $V_{\text{crit}}$ (typically 2–3 V). Usually $V$ decreases gradually with $d$, so the voltage provides a measure of depth-of-discharge. The decrease of $V$ with current is primarily due to internal resistance. The voltage also decreases as temperature decreases (fig. 28-7). There is a maximum discharge current (continuous or burst), which gives the rated power: $P_{\text{cap}} = I_{\text{mbd}}V_{\text{ref}} = x_{\text{mbd}}CV_{\text{ref}}$. The actual battery power supplied is $P = IV$.

The voltage variation with current is primarily due to the internal resistance: $V = V_o - IR$. The open-circuit voltage decreases with depth-of-discharge: $V_o = V_d F_V(d)$, such that $F_V = 1$ at $d = 0$ and $F_V = f_{\text{crit}} = V_{\text{crit}}/V_d$ at $d = 1$; and $V_d = f_d V_{\text{ref}}$. For large discharge ($d$ approaching 1), there is a secondary influence of current, which can be modelled by adjusting the discharge by a factor $k$. Then the capacity is given by $F_V(kd) = f_{\text{crit}}$. Variations of the voltage with current are scaled using $I = xC = (x_{\text{mbd}}C)(x/x_{\text{mbd}})$. The influence of temperature on the open circuit voltage $V_o$ is reasonably accounted for by an increment proportional to the cube of the temperature difference $\Delta T = T_c - T_{\text{ref}}$.

The model for the discharge voltage is

$$V = V_d F_V(kd) + k_{VT} \Delta T^3 - IR = V_d F_V(kd) + k_{VT} \Delta T^3 - V_{\text{ref}} \frac{x_{\text{mbd}}CR}{V_{\text{ref}}} \left(\frac{x}{x_{\text{mbd}}}\right)$$

with

$$k = 1 + k_{dl}I - k_{dT} \Delta T = 1 + k_{dl}x_{\text{mbd}}C \frac{x}{x_{\text{mbd}}} - k_{dT} \Delta T$$

The open circuit voltage function $F_V(d)$ is derived from $V(d)$ as a function of $I$ and $T_c$: $F_V = (V - k_{VT} \Delta T^3 + IR)/V_d$ at $kd$. The model parameters $R$, $k_{VT}$, $k_{dl}$, and $k_{dT}$ are adjusted for a good fit to the measured discharge characteristics. Figure 28-8 shows $F_V(d)$ obtained for a number of lithium-ion batteries. Typically $f_{\text{crit}} \cong 0.6$. That $d \neq 1$ at $F_V = f_{\text{crit}}$ reflects the definition of nominal capacity for the battery characteristics. Table 28-1 gives a typical $F_V(d)$ function. The function extends below $f_{\text{crit}}$ since high temperature can increase the voltage. Figure 28-9 shows the current parameters $x_{\text{mbd}}CR/V_{\text{ref}}$ and $k_{dT}x_{\text{mbd}}C$ for a number of batteries. Generally $x_{\text{mbd}}CR/V_{\text{ref}} = 0.05–0.20$ and $k_{dT}x_{\text{mbd}}C = 0–0.25$. Figure 28-10 shows the temperature parameters $k_{VT}$ and $k_{dT}$ for a number of batteries.

The battery power available to the components is

$$\dot{E}_{\text{comp}} = P = IV = I(V_d F_V(kd) + k_{VT} \Delta T^3) - I^2 R - P_{TC} = \dot{E}_{\text{batt}} - P_{\text{loss}}$$
including power loss for temperature control. Hence

\[ \dot{E}_{\text{batt}} = P_{\text{cap}} \xi \left( f_d F_V(kd) + (k_{VT}/V_{\text{ref}})\Delta T^3 \right) \]
\[ P_{\text{loss}} = P_{\text{cap}} \hat{R} \xi^2 + f_T C |\dot{E}_{\text{comp}}| \]
\[ k = 1 + \hat{k}_{dT} \xi - k_{dT} \Delta T \]

where \( \xi = x/x_{\text{mbd}} \), \( \hat{R} = x_{\text{mbd}} CR/V_{\text{ref}} \), and \( \hat{k}_{dT} = k_{dT} x_{\text{mbd}} C \). The battery capacity for a given current and temperature is obtained by finding \( kd_{\text{crit}} \) such that

\[ V = V_d F_V(kd_{\text{crit}}) + k_{VT} \Delta T^3 - IR = V_d f_{\text{crit}} \]

or

\[ F_V(kd_{\text{crit}}) = f_{\text{crit}} + (\hat{R} \xi - (k_{VT}/V_{\text{ref}})\Delta T^3)/f_d \]

Then \( d_{\text{crit}} = (kd_{\text{crit}})/k \), and the effective capacity is \( d_{\text{crit}} f_{\text{fade}} E_{\text{fuel-cap}} \). Rather than change the fuel tank capacity, an effective fuel flow is used: \( \dot{E}_{\text{eff}} = \dot{E}_{\text{batt}}/(d_{\text{crit}} f_{\text{fade}}) \).

For a flight condition or mission segment, the total energy flow required is \( \dot{E}_{\text{comp}} \). The temperature \( T_c \) is specified. The depth-of-discharge is \( d = 1 - s = 1 - E_{\text{fuel}}/E_{\text{fuel-cap}} \). The discharge current \( x = x_{\text{mbd}} \xi \) is calculated from the required energy flow:

\[ \xi = \frac{(\dot{E}_{\text{comp}} + P_{\text{loss}})/P_{\text{cap}}}{f_d F_V(kd) + (k_{VT}/V_{\text{ref}})\Delta T^3} \]

Successive-substitution solution for \( \xi \), with \( f_{\text{crit}} \) as the minimum for \( F_V \), converges in about 10 iterations. Since \( F_V \) is a monotonically decreasing function of \( d \), \( d_{\text{crit}} \) is obtained from \( x \) by interpolating \( d(F_V) \). Then \( \dot{E}_{\text{batt}} = \dot{E}_{\text{comp}} + P_{\text{loss}} \), and the battery efficiency is \( \eta_{\text{batt}} = \dot{E}_{\text{comp}}/\dot{E}_{\text{batt}} \). The effective energy flow is \( \dot{E}_{\text{eff}} = \dot{E}_{\text{batt}}/(d_{\text{crit}} f_{\text{fade}}) \). The maximum battery discharge power is \( P_{\text{max}} = \dot{E}_{\text{batt}} \) evaluated at \( x_{\text{mbd}} \), or at a specified discharge current. The battery power margin is \( P_{\text{max}} - |\dot{E}_{\text{batt}}| \).

### Table 28-1. Lithium-ion battery open circuit voltage.

<table>
<thead>
<tr>
<th>depth-of-charge ( d )</th>
<th>( F_V )</th>
<th>depth-of-charge ( d )</th>
<th>( F_V )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>1.000</td>
<td>0.92</td>
<td>0.842</td>
</tr>
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<td>0.10</td>
<td>0.970</td>
<td>0.93</td>
<td>0.835</td>
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<td>0.40</td>
<td>0.915</td>
<td>0.96</td>
<td>0.800</td>
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<tr>
<td>0.70</td>
<td>0.880</td>
<td>0.99</td>
<td>0.700</td>
</tr>
<tr>
<td>0.80</td>
<td>0.870</td>
<td>1.00</td>
<td>0.600</td>
</tr>
<tr>
<td>0.90</td>
<td>0.850</td>
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</tr>
<tr>
<td>0.91</td>
<td>0.847</td>
<td>1.02</td>
<td>0.000</td>
</tr>
</tbody>
</table>

#### 28-2.2 Charge Characteristics

Lithium-ion battery charging is controlled for safety, with limits on minimum and maximum voltage, maximum current, and minimum and maximum temperature. Standard charging procedure consists of
constant current (CC) followed by constant voltage (CV). Figure 28-11 shows the voltage, current, and capacity variation with charging time. The charging current is $I_{CC} = x_{CC}C$; the maximum charging current is typically $x_{CC\text{max}} = 1–5$ (fig. 28-4). The charging voltage is $V_c = f_eV_{\text{ref}}$; typically $V_c = V_{\text{ref}} = 4.2$ V. The state-of-charge is $s$.

During the CC phase, the voltage starts at a fraction $k_{CV}$ (typically 0.05–0.20) below $V_c$ and increases with time, reaching $V_c$ at time $t_c = \sigma/x_{CC}$. The charging model assumes linear variation of the voltage. During the CV phase, the voltage is fixed at $V_c$, while the current decreases rapidly. The charging model assumes the current varies inversely with the square of the time increment since $t_c$. The charge is then the integral of the current, $Q = \int_0^t I \, dt$; and the state-of-charge $s = Q/C$. The charging power is $P = IV$.

Table 28-2 gives the charging model as a function of normalized time $\tau = t/t_c - 1 = tx_{CC}/\sigma - 1$.

<table>
<thead>
<tr>
<th>Table 28-2. Lithium-ion battery charging model.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>CC phase, $t &lt; t_c$, $\tau = -1$ to $0$, $s &lt; \sigma$</strong></td>
</tr>
<tr>
<td>current</td>
</tr>
<tr>
<td>voltage</td>
</tr>
<tr>
<td>charge</td>
</tr>
<tr>
<td>state-of-charge</td>
</tr>
<tr>
<td>power</td>
</tr>
<tr>
<td>time</td>
</tr>
</tbody>
</table>

At $t = t_c (\tau = 0)$, the current and voltage have the CC/CV values ($I = x_{CC}C$, $V = V_c$), the state-of-charge $s = \sigma$, and the charging power is at the peak ($P = x_{CC}CV_c = P_{\text{cap}}(x_{CC}/x_{mbd})f_e$). For $s = 1$ at large time, $k_{cl}$ must be related to $\sigma$: $1/\sigma = 1 + 1/k_{cl}$; the measured CV capacity is matched better with smaller values for $k_{cl}$. Figure 28-12 shows the charging parameters $k_{CV}$ and $k_{cl}$ for a number of batteries.

For small charging current $x_{CC}$, the voltage $V_c$ is reached at the nominal charging time $t_c = 1/x_{CC}$. So the CC phase is the entire process, and $s = x_{CC}t$. For higher current, the voltage $V_c$ is reached sooner, at $t_c = \sigma/x_{CC}$ with $\sigma < 1$. Then the CV phase is required to complete the charging. For a given battery, $\sigma = 1$ at low charge current, and $\sigma$ decreases with increasing $x_{CC}$. Figure 28-13 shows $1/\sigma$ and $1 + 1/k_{cl}$ for a number of batteries. An approximation for the variation of $\sigma$ is

$$\frac{1}{\sigma} = 1 + \frac{1}{k_{cl}} = 1 + k_{c}(x_{CC} - 0.2)$$

above $x_{CC} = 0.2$, with $k_{c}$ depending on the battery.

For a flight condition or mission segment, the total energy flow delivered to the battery is $\dot{E}_{\text{comp}}$. The temperature $T_e$ is specified. The state-of-charge is $s = E_{\text{fuel}}/E_{\text{fuel-cap}}$. The charge current $x = x_{mbd}$.
Battery Model

is calculated from the required energy flow, $|\dot{E}_{\text{batt}}| = P = IV = \xi x_{mbd} CV = P_{\text{cap}} \xi V/V_{\text{ref}}$. In the CC phase ($\tau < 0, s < \sigma$):

$$\xi = \left(\left(|\dot{E}_{\text{comp}}| - P_{\text{loss}}\right)/P_{\text{cap}}\right)/f_c (1 + k_c V \tau)$$

and $x_{CC} = x$. In the CV phase ($\tau > 0, s > \sigma$):

$$\xi = \left(\left(|\dot{E}_{\text{comp}}| - P_{\text{loss}}\right)/P_{\text{cap}}\right)/f_c$$

and $x_{CC} = x(1 + k_c \tau)^2$. Successive-substitution solution for $\xi$ converges in about 10 iterations. First the CC solution is found, then if $\tau > 0$ the CV solution is found (with an upper limit on $x_{CC}$). Then $\dot{E}_{\text{batt}} = \dot{E}_{\text{comp}} + P_{\text{loss}}$, and the battery efficiency is $\eta_{\text{batt}} = \dot{E}_{\text{batt}}/\dot{E}_{\text{comp}}$. The effective energy flow is $\dot{E}_{\text{eff}} = \dot{E}_{\text{batt}}/f_{\text{fade}}$. The maximum battery charging power is $P_{\text{max}} = P$ evaluated at current $x_{CC_{\text{max}}}$, or at a specified charge current $x_{CC}$. The battery power margin is $P_{\text{max}} - |\dot{E}_{\text{batt}}|$.

28–3 References


Figure 28-1. Lithium-ion battery capacity (W-hr) and power (W).

Figure 28-2. Lithium-ion battery maximum continuous discharge current.
Figure 28-3. Lithium-ion battery maximum burst discharge current.

Figure 28-4. Lithium-ion battery maximum charge current.
Figure 28-5. Lithium-ion battery specific energy (W-hr/kg) and specific power (W/kg).

Figure 28-6. Lithium-ion battery specific energy (W-hr/kg) and energy density (W-hr/liter).
Figure 28-7. Lithium-ion battery discharge characteristics.
Figure 28-8. Lithium-ion battery open circuit voltage function.

Figure 28-9. Lithium-ion battery discharge model, current parameters.
Figure 28-10. Lithium-ion battery discharge model, temperature parameters.
Figure 28-11. Lithium-ion battery charge characteristics.
Figure 28-12. Lithium-ion battery charge model parameters.

Figure 28-13. Lithium-ion battery charge model, CV parameters.
Chapter 29

AFDD Weight Models

This chapter presents the rotorcraft weight models developed by the U.S. Army Aeroflightdynamics Directorate (AFDD). For some weight groups several models are available, designated AFDDnn. The weights are estimated from parametric equations, based on the weights of existing turbine-powered helicopters and tiltrotors. The figures of this chapter compare the weights calculated from these equations with the actual weights. The results of these equations are the weight in pounds, and the units of the parameters are noted in the tables. Technology factors $\chi$ are included in the weight equations. The input usually includes a weight increment $dW$ that can be added to the results of the weight model. Thus typically a component or element weight is obtained from $W = \chi w_{\text{model}} + dW$. Weight of individual elements in a group can be fixed by using $dW$ and setting the corresponding technology factor $\chi = 0$. With $\chi \neq 0$, the increment $dW$ can account for something not included in the parametric model.

The weight models are implemented as part of the aircraft components. The weights are entered into the weight statement data structure (extended RP8A format) for each component, reflected in the organization of this chapter.

29–1 Wing Group

The wing group consists of: basic structure (primary structure, consisting of torque box and spars, plus extensions); fairings (leading edge and trailing edge); fittings (non-structural); fold/tilt structure; and control surfaces (flaps, ailerons, flaperons, and spoilers). There are separate models for a tiltrotor or tiltwing configuration and for other configurations (including compound helicopter).

29-1.1 Tiltrotor or Tiltwing Wing

Wing weight equations for a tiltrotor or tiltwing aircraft are based on methodology developed by Chappell and Peyran (refs. 1 and 2). The wing is sized primarily to meet torsional stiffness requirements. The primary structure weight is calculated from torque box and spar weights:

\[
W_{\text{box}} = A_{tb}\rho_{tb}b_w/e_{tb}
\]

\[
W_{\text{spar}} = C_tA_{sp}\rho_{sp}b_w/e_{sp}
\]

\[
w_{\text{prim}} = (W_{\text{box}} + W_{\text{spar}})f_{\text{units}}
\]

\[
W_{\text{prim}} = \chi_{\text{prim}}w_{\text{prim}}
\]

A consistent mass-length-time system is used in the equations for $W_{\text{box}}$ and $W_{\text{spar}}$, which therefore have units of slug or kg. The primary structure weight $W_{\text{prim}}$ however has units of lb or kg, hence a conversion factor $f_{\text{units}} = g$ is required for English units. The wing fairing (leading edge and trailing edge), control
The weight on wing tip is \( M \), the weight on the wing tips; which is the sum of rotor group, engine section or nacelle group, air induction group, engine system, drive system (except drive shaft), rotary wing and conversion flight controls, hydraulic group, trapped fluids, and wing extensions. An adjustment for beam bending, chord bending, torsion, and spar cap vertical/horizontal bending. The ideal shape for beam bending is two caps \( \frac{t}{b} \) apart, so

\[
F_B = 0.073 \sin(2\pi(t/t_w - 0.151)/0.1365) + 0.14598t_w + 0.610 \sin(2\pi(w_{tb} + 0.080)/2.1560) - (0.4126 - 1.6309t_w)(w_{tb} - 0.131) + 0.0081
\]

\[
F_C = 0.640424w_{tb}^3 - 0.89717w_{tb} + 0.4615t_w + 0.655317
\]

\[
F_T = ((0.27 - t_w)/0.12)0.12739 \left(-0.96 + \sqrt{3.32 + 94.6788w_{tb} - (w_{tb}/0.08344)^2}\right) - 2.7545w_{tb}^2 + 5.1799w_{tb} - 0.2683
\]

\[
F_{VH} = 0.25 \sin(5.2366w_{tb}) + 0.325
\]

for beam bending, chord bending, torsion, and spar cap vertical/horizontal bending. The ideal shape for torsional stiffness is a tube of radius \( t_w \), so the torsional stiffness \( J = F_T A_{tb} t_w^2 / 4 \). The ideal shape for chord bending is two caps \( c_{tb} \) apart, so \( I_{Ctb} = F_C A_{tb} c_{tb}^2 / 4 \). The ideal shape for beam bending is two caps \( t_w \) apart, so \( I_{Bsp} = F_{VH} A_{sp} t_w^2 / 4 \) and \( I_{Rtb} = F_B A_{tb} t_w^2 / 4 \). The torque box cross-sectional area is obtained from the wing torsion frequency;

\[
GJ = (\omega_T \Omega)^2 \frac{1}{2} (b_w - w_{attach}) \frac{1}{2} M_{tip} r_{pylon}^2
\]

\[
A_{tb} = 4GJ/(G_{tb} F_T t_w^2)
\]
It is assumed that between the points of attachment to the fuselage, the torque box is very stiff in torsion, but does deflect in bending. So the effective length is $\frac{1}{2}(bw - w_{attach})$ for torsion, and $\frac{1}{2}bw$ for bending.

The spar cap cross-sectional area (in addition to torque box material) is obtained from beam and chord bending frequencies:

$$EI_C = (\omega_C \Omega)^2 \frac{1}{24} b_w^3 \frac{1}{2} M_{tip} f_{mode}$$
$$EI_B = (\omega_B \Omega)^2 \frac{1}{24} b_w^3 \frac{1}{2} M_{tip} f_{mode}$$

$$EI_{Ctb} = E_{tb} F_{C} A_{tb} c_{tb}^2 / 4$$
$$EI_{Csp} = EI_C - EI_{Ctb}$$
$$A_{Csp} = EI_{Csp} / (E_{sp} c_{tb}^2 / 4)$$
$$EI_{Btb} = E_{tb} F_{B} A_{tb}^2 / 4$$
$$EI_{VH} = E_{sp} F_{VH} A_{Csp} / 2$$
$$EI_{Bsp} = EI_{B} - EI_{Btb} - EI_{VH}$$
$$A_{Bsp} = EI_{Bsp} / (E_{sp} t^2_w / 4)$$
$$EI_s = EI_{VH} + EI_{Bsp}$$
$$A_s = A_{Csp} + A_{Bsp}$$

where $EI_{Csp}$ and $EI_{Bsp}$ are replaced by zero if negative (no additional spar material required). The factor $f_{mode} = 1 - f_{tip}$ is a mode shape correction for fuselage motion. Next the primary structure, fairing, flap, and fitting weights are calculated as before; and the sum $W_{wing} = W_{prim} + W_{fair} + W_{flap} + W_{fit}$.

Additional spar cap material for a jump takeoff condition is obtained from the ultimate applied bending moment at the wing root:

$$M_U = T_{cap} \ell_W \left( 0.75(1 - f_{tip}) - 0.375(\ell_w / bw)(W_{wing} / W_{SD}) \right)$$

where $T_{cap}$ is the maximum thrust capability of one rotor, equal to the greater of $n_{jump} W_{SD} / N_{rotor}$ or $(C_T / \sigma) \rho A_{tip} V_{tip}^2$ (from an input $C_T / \sigma$ at the jump takeoff condition, sea level standard, and hover rotor speed). The bending moment capacity of the wing is

$$M_{tb} = 2 EI_{Btb} \ell_U / t_w$$
$$M_{sp} = 2 C_m EI_{sp} \ell_U / t_w$$

Then the additional cross-section area is obtained from the moment deficit:

$$\Delta M = M_U - (M_{tb} + M_{sp})$$
$$\Delta A_s = 2 \Delta M / (\ell_U E_{sp} t_w)$$
$$\Delta W_{spar} = C_j \Delta A_s \rho_{sp} b_w / \ell_{sp}$$

where $\Delta M$ is replaced by zero if negative (no additional spar material required). If $\Delta W_{spar}$ is positive, it is added to $W_{spar}$, and the primary structure, fairing, flap, and fitting weights are recalculated. Parameters are defined in table 29-1, including units as used in these equations. Here a consistent mass-length-time system is used, producing $W_{box}$ and $W_{spar}$ in slug or kg. Typically the input uses conventional English units for density ($lb/in^3$) and modulus ($lb/in^2$).
Table 29-1. Parameters for tiltrotor wing weight.

<table>
<thead>
<tr>
<th>parameter</th>
<th>definition</th>
<th>units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_{\text{box}}$, $W_{\text{spar}}$</td>
<td>wing torque box and spar weights</td>
<td>slug or kg</td>
</tr>
<tr>
<td>$W_{SD}$</td>
<td>structural design gross weight</td>
<td>lb</td>
</tr>
<tr>
<td>$(C_T/\sigma)_{\text{jump}}$</td>
<td>rotor maximum thrust capability (jump takeoff)</td>
<td>lb</td>
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<tr>
<td>$n_{\text{jump}}$</td>
<td>load factor at $W_{SD}$ (jump takeoff)</td>
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<td>$N_{\text{rotor}}$</td>
<td>number of rotors</td>
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<tr>
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</tr>
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</tr>
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<td>rotor speed for wing weight design condition</td>
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<td>$\rho_{sp}$</td>
<td>density of spar cap material</td>
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<tr>
<td>$U_{\text{fair}}$</td>
<td>unit weight of leading and trailing-edge fairings</td>
<td>lb/ft$^2$ or kg/m$^2$</td>
</tr>
<tr>
<td>$U_{\text{flap}}$</td>
<td>unit weight of control surfaces</td>
<td>lb/ft$^2$ or kg/m$^2$</td>
</tr>
<tr>
<td>$S_{\text{fair}}$</td>
<td>area of leading and trailing-edge fairings</td>
<td>ft$^2$ or m$^2$</td>
</tr>
<tr>
<td>$S_{\text{flap}}$</td>
<td>area of control surfaces</td>
<td>ft$^2$ or m$^2$</td>
</tr>
<tr>
<td>$f_{\text{fit}}$</td>
<td>wing fittings and brackets (fraction maximum thrust of one rotor)</td>
<td></td>
</tr>
<tr>
<td>$f_{\text{fold}}$</td>
<td>wing fold/tilt (fraction total weight excluding fold, including weight on tips)</td>
<td></td>
</tr>
<tr>
<td>$w_{\text{attach}}$</td>
<td>width of wing structural attachments to body</td>
<td>ft or m</td>
</tr>
<tr>
<td>$U_{\text{ext}}$</td>
<td>unit weight of wing extension</td>
<td>lb/ft$^2$ or kg/m$^2$</td>
</tr>
<tr>
<td>$S_{\text{ext}}$</td>
<td>area of wing extensions (span $b_{\text{ext}}$ times mean chord $c_{\text{ext}}$)</td>
<td>ft$^2$ or m$^2$</td>
</tr>
<tr>
<td>$f_{\text{efold}}$</td>
<td>wing extension fold/tilt (fraction extension weight)</td>
<td></td>
</tr>
</tbody>
</table>
29-1.2 Aircraft Wing

There are two models intended for the wing of a compound helicopter: area method and parametric method. For the area method (based on weight per unit area), the total wing weight excluding folding is:

\[ w_{\text{wing}} = S_w U_w \]

and \( f_{\text{prim}} = 1 - f_{\text{fair}} - f_{\text{flap}} - f_{\text{fit}} \). Typically \( U_w = 5 \) to 9 lb/ft\(^2\). For the parametric method (AFDD93), the total wing weight including folding is:

\[
w_{\text{wing}} = 5.66411 f_{\text{LGloc}} \left( \frac{W_{SD} f_L}{1000 \cos \Lambda_w} \right)^{0.847} n_z^{0.39579} S_w^{0.21754} A_w^{0.50016} \]

\[
(1 + \lambda_w) \left( \frac{f_{\text{LGloc}}}{\tau_w} \right)^{0.09359} (1 - b_{\text{fold}})^{-0.14356}
\]

with \( f_{\text{prim}} = 1 - f_{\text{fair}} - f_{\text{flap}} - f_{\text{fit}} - f_{\text{fold}} \); here \( f_{\text{LGloc}} = 1.7247 \) if the landing gear is on the wing, and 1.0 otherwise. The lift factor \( f_L \) accounts for wing relief factor and wing/rotor lift share. Based on 25 fixed-wing aircraft, the average error of the aircraft wing equation is 3.4% (fig. 29-1). Then the primary structure, secondary structure, and control surface weights are:

\[
W_{\text{prim}} = \chi_{\text{prim}} f_{\text{prim}} w_{\text{wing}}
\]

\[
W_{\text{fair}} = \chi_{\text{fair}} f_{\text{fair}} w_{\text{wing}}
\]

\[
W_{\text{flap}} = \chi_{\text{flap}} f_{\text{flap}} w_{\text{wing}}
\]

\[
W_{\text{fit}} = \chi_{\text{fit}} f_{\text{fit}} w_{\text{wing}}
\]

\[
W_{\text{fold}} = \chi_{\text{fold}} f_{\text{fold}} w_{\text{wing}}
\]

The wing extension weight is:

\[
w_{\text{ext}} = S_{\text{ext}} U_{\text{ext}} \quad \quad W_{\text{ext}} = \chi_{\text{ext}} w_{\text{ext}}
\]

\[
w_{\text{efold}} = f_{\text{efold}} W_{\text{ext}} \quad \quad W_{\text{efold}} = \chi_{\text{efold}} w_{\text{efold}}
\]

and these terms are added to \( W_{\text{prim}} \) and \( W_{\text{fold}} \). Parameters are defined in table 29-2, including units as used in these equations.

Table 29-2. Parameters for aircraft wing weight.

<table>
<thead>
<tr>
<th>parameter</th>
<th>definition</th>
<th>units</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S_w )</td>
<td>wing planform area (theoretical)</td>
<td>ft(^2)</td>
</tr>
<tr>
<td>( U_w )</td>
<td>unit weight of wing planform</td>
<td>lb/ft(^2) or kg/m(^2)</td>
</tr>
<tr>
<td>( W_{SD} )</td>
<td>structural design gross weight</td>
<td>lb</td>
</tr>
<tr>
<td>( f_L )</td>
<td>lift factor</td>
<td></td>
</tr>
<tr>
<td>( n_z )</td>
<td>design ultimate flight load factor at ( W_{SD} )</td>
<td>g</td>
</tr>
<tr>
<td>( \Lambda_w )</td>
<td>wing sweep angle</td>
<td>deg</td>
</tr>
<tr>
<td>( A_w )</td>
<td>wing aspect ratio</td>
<td></td>
</tr>
<tr>
<td>( \lambda_w )</td>
<td>wing taper ratio (tip chord/root chord)</td>
<td></td>
</tr>
<tr>
<td>( \tau_w )</td>
<td>wing airfoil thickness-to-chord ratio</td>
<td></td>
</tr>
<tr>
<td>( b_{\text{fold}} )</td>
<td>fraction wing span that folds (0 to 1)</td>
<td></td>
</tr>
<tr>
<td>( f_{\text{fair}} )</td>
<td>fairings (fraction total wing weight)</td>
<td></td>
</tr>
<tr>
<td>( f_{\text{flap}} )</td>
<td>control surfaces (fraction total wing weight)</td>
<td></td>
</tr>
<tr>
<td>( f_{\text{fit}} )</td>
<td>fittings (fraction total wing weight)</td>
<td></td>
</tr>
<tr>
<td>( f_{\text{fold}} )</td>
<td>fold/tilt (fraction total wing weight)</td>
<td></td>
</tr>
<tr>
<td>( U_{\text{ext}} )</td>
<td>unit weight of wing extension</td>
<td>lb/ft(^2) or kg/m(^2)</td>
</tr>
<tr>
<td>( S_{\text{ext}} )</td>
<td>area of wing extensions (span ( b_{\text{ext}} ) times mean chord ( c_{\text{ext}} ))</td>
<td>ft(^2)</td>
</tr>
<tr>
<td>( f_{\text{efold}} )</td>
<td>wing extension fold/tilt (fraction extension weight)</td>
<td></td>
</tr>
</tbody>
</table>
The rotor group consists of: blades, hub and hinge, spinner, and blade fold structure. The blade and hub-hinge weights for the AFDD82 model are:

\[ w_{\text{blade}} = 0.02606N_{\text{rotor}}^{0.6592} R^{1.3371} c_{\text{tip}}^{0.6682} v_{\text{tip}}^{2.5279} \]
\[ w_{\text{hub}} = 0.003722N_{\text{rotor}}^{0.8207} R^{1.5377} c_{\text{hub}}^{0.4290} v_{\text{tip}}^{2.1414} (w_{\text{blade}}/N_{\text{rotor}})^{0.5505} \]

\[ W_{\text{blade}} = \chi_{\text{blade}} w_{\text{blade}} \]
\[ W_{\text{hub}} = \chi_{\text{hub}} w_{\text{hub}} \]

Based on 37 aircraft, the average error of the blade equation is 7.7% (fig. 29-2). Based on 35 aircraft, the average error of the hub equation is 10.2% (fig. 29-3). The blade and hub-hinge weights for the AFDD00 model are:

\[ w_{\text{blade}} = 0.0024419f_{\text{tilt}}N_{\text{rotor}}^{0.53479} R^{1.74231} c_{\text{tip}}^{0.87562} v_{\text{tip}}^{2.51048} \]
\[ w_{\text{hub}} = 0.0061182N_{\text{rotor}}^{0.20373} R^{0.60406} c_{\text{hub}}^{0.52803} (w_{\text{blade}}/N_{\text{rotor}})^{0.87127} \]

\[ W_{\text{blade}} = \chi_{\text{blade}} w_{\text{blade}} \]
\[ W_{\text{hub}} = \chi_{\text{hub}} w_{\text{hub}} \]

where \( f_{\text{tilt}} = 1.17940 \) for tilting rotors; 1.0 otherwise. Based on 51 aircraft, the average error of the blade equation is 7.9% (fig. 29-4) and the average error of the hub equation is 12.2% (fig. 29-5). This equation for hub weight was developed using the actual blade weight. Using the blade weight equation instead gives

\[ w_{\text{hub}} = 0.18370N_{\text{rotor}}^{0.16383} R^{1.19337} c_{\text{hub}}^{0.06171} v_{\text{tip}}^{0.46203} (w_{\text{blade}}/N_{\text{rotor}})^{1.02958} \]
\[ = 0.00037547f_{\text{tilt}}^{1.10258} N_{\text{rotor}}^{0.71443} R^{1.19332} c_{\text{hub}}^{0.79577} v_{\text{hub}}^{0.46203} v_{\text{tip}}^{2.58473} \]

which really just adds chord and \( f_{\text{tilt}} \) to the regression parameters. Based on 51 aircraft, the average error of this equation is 9.2% (fig. 29-5). For teetering and gimballed rotors, the flap frequency \( \nu \) should be the coning frequency. Thus using the blade weight equation results in a lower average error, and best represents legacy rotor systems. The hub weight equation using the actual blade weight is best for advanced technology rotors with blades lighter than trend.

The fairing/spinner and blade fold weights are:

\[ W_{\text{spin}} = \chi_{\text{spin}} 7.386N_{\text{rotor}} D_{\text{spin}}^2 \]
\[ W_{\text{fold}} = \chi_{\text{fold}} f_{\text{fold}} W_{\text{blade}} \]

The blade weight is for all blades of the rotors. If the weight is evaluated separately for each rotor, then \( N_{\text{rotor}} = 1 \) should be used in the equations. Typically \( f_{\text{fold}} = 0.04 \) for manual fold and \( f_{\text{fold}} = 0.28 \) for automatic fold. Parameters are defined in table 29-3, including units as used in these equations. The rotor support and duct weights are:

\[ W_{\text{supt}} = \chi_{\text{supt}} f_{\text{supt}} W_{\text{MTO}} \]
\[ W_{\text{duct}} = \chi_{\text{duct}} N_{\text{rotor}} S_{\text{duct}} U_{\text{duct}} \]
For lift-offset rotors, the blade and hub weights can be calculated based on the methodology of reference 3. The blade weight is estimated based on the beam stiffness required to maintain the clearance $s$ when the blade is loaded by the lift offset. The blade tip deflection is proportional to $\delta \propto PR^3/EI$, where $EI$ is the bending stiffness. The beam loading is $P \propto n_z W_{SD}/N_{blade}$. With $A_{xz}$, the blade cross-section area, the moment of inertia $I \propto A_{xz}t^2$. The criterion is $\delta = h - s$. Hence the blade weight is $W_{blade} \propto \rho N_{blade} R A_{xz} \propto (\rho/E)n_z W_{SD}LR^4/(t^2(h-s))$ with $E$ the elastic modulus, and $\rho$ here the material density. The hub weight is estimated based on the structure to react the hub moment caused by lift offset. The hub moment $M_{hub} \propto n_z W_{SD}R$. The centrifugal force $C_{cent} \propto (W_{blade}/N_{blade})V_{tip}^2/R$. The bending moment $M_{bend} \propto n_z W_{SD}R$. The limit tensile stress $\sigma \propto F/A_{xz}$ gives a criterion for the total hub arm area $A_{xz}$. The radius of the hub $\ell$ scales with the blade thickness $t$. Hence the hub weight is $W_{hub} \propto \rho N_{blade} \ell A_{xz} \propto (\rho/\sigma)(W_{blade} V_{tip}^2 t/R + Kn_z W_{SD} R N_{blade})$. The distribution factor $K$ is determined from the calibration weights. The inter-rotor shaft weight is estimated based on the structure to react the hub moment caused by lift offset. The hub moment $M \propto n_z W_{SD}LR$. The shaft diameter $d$ scales with the blade thickness $t$. The shaft length $\ell$ scales with the rotor separation $h$. The ultimate bending stress $\sigma = M/(I/c)$ gives a criterion for the area moment $I/c \propto d^2w$, hence for the shaft wall thickness $w$. Then $W_{shaft} \propto \rho \ell dw \propto (\rho/\sigma)n_z W_{SD} LR h/t$ is the shaft weight.

The blade, hub-hinge, and inter-rotor shaft weights are:

\[
\begin{align*}
    w_{blade} &= N_{rotor} \cdot 0.000083770 \cdot wL R^3/(2(h-s)t_{2R}^2) \\
    w_{hub} &= N_{rotor} \cdot \left(0.17153 w \cdot RN_{blade} + 0.000010534(W_{blade}/N_{rotor})V_{tip}^2 t_{.2R}/R\right) \\
    w_{shaft} &= N_{rotor} \cdot 0.081304 \cdot wL R^2 2h/t_{.2R}
\end{align*}
\]

where $w = n_z W_{SD}/1000$, and

\[
t_{.2R} = \tau_{.2R} \begin{pmatrix} 0.8 + 0.2\lambda \\ 0.5 + 0.5\lambda \end{pmatrix}
\]
is the blade thickness at $20\% R$. These equations were developed for the coaxial rotor configuration ($N_{rotor} = 2$), and calibrated to the XH-59A weights. The material factors ($\rho/E$ and $\rho/\sigma$) are included in the technology factors $\chi$. These weight terms can be used separately, with other hub and blade weight models. Parameters are defined in table 29-4, including units as used in these equations.

Table 29-4. Parameters for lift-offset rotor weight.

<table>
<thead>
<tr>
<th>parameter</th>
<th>definition</th>
<th>units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_{rotor}$</td>
<td>number of rotors</td>
<td></td>
</tr>
<tr>
<td>$N_{blade}$</td>
<td>number of blades per rotor</td>
<td></td>
</tr>
<tr>
<td>$W_{SD}$</td>
<td>structural design gross weight</td>
<td>lb</td>
</tr>
<tr>
<td>$n_z$</td>
<td>design ultimate flight load factor at $W_{SD}$</td>
<td>g</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>blade taper ratio (tip chord/root chord)</td>
<td></td>
</tr>
<tr>
<td>$\tau_{2R}$</td>
<td>blade airfoil thickness-to-chord ratio (at $20% R$)</td>
<td></td>
</tr>
<tr>
<td>$R$</td>
<td>rotor radius</td>
<td>ft</td>
</tr>
<tr>
<td>$c$</td>
<td>blade mean chord</td>
<td>ft</td>
</tr>
<tr>
<td>$h$</td>
<td>coaxial rotor separation (fraction rotor diameter)</td>
<td></td>
</tr>
<tr>
<td>$s$</td>
<td>coaxial rotor tip clearance (fraction rotor diameter)</td>
<td></td>
</tr>
<tr>
<td>$L$</td>
<td>lift offset ($M_{roll}/TR$)</td>
<td></td>
</tr>
<tr>
<td>$V_{tip}$</td>
<td>rotor hover tip velocity</td>
<td>ft/sec</td>
</tr>
</tbody>
</table>

29–3 Empennage Group

The empennage group consists of: horizontal tail, vertical tail, and tail-rotor. The tail plane weight consists of the basic structure and fold structure. There are two models for tail plane basic weight: area method and parametric method. For the area method (based on weight per unit area), the weight is $W_{tail} = S_{tail} U_{tail}$. The parametric weight model depends on the aircraft configuration. The helicopter or compound model is AFDD82. The horizontal tail weight is:

$$W_{ht} = \chi_{ht} S_{ht} (0.00395 S_{ht}^{0.2} V_{dive} - 0.4885)$$

where $f_{tr} = 1.6311$ if the tail-rotor is located on the vertical tail; 1.0 otherwise. Based on 12 aircraft, the average error of the helicopter horizontal tail equation is 22.4% (fig. 29-6). The vertical tail weight is:

$$W_{vt} = \chi_{vt} S_{vt} (0.00395 S_{vt}^{0.2} V_{dive} - 0.4885)$$

where $f_{tr} = 1.6311$ if the tail-rotor is located on the vertical tail; 1.0 otherwise. Based on 12 aircraft, the average error of the helicopter vertical tail equation is 23.3% (fig. 29-7). $V_{dive}$ is the design dive speed, calculated or input; $V_{dive} = 1.25 V_{max}$, where $V_{max}$ is the maximum speed at design gross weight and sea level standard conditions. The fold weight is a fraction of the basic weight: $W_{fold} = \chi_{fold} f_{fold} W_{basic}$, where $W_{basic} = W_{ht}$ or $W_{vt}$.

The tail-rotor weight is:

$$W_{tr} = \chi_{tr} 1.3778 P_{tr}^{0.0897} (P_{Dlimit} R/V_{tip})^{0.8951}$$
Based on 19 aircraft, the average error of the helicopter tail-rotor equation is 16.7% (fig. 29-8). The tail-rotor weight is calculated by the rotor component model, including rotor support and duct weights. Rotor weight models can be used for the tail-rotor. Parameters are defined in table 29-5, including units as used in these equations.

<table>
<thead>
<tr>
<th>parameter</th>
<th>definition</th>
<th>units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{ht}$</td>
<td>horizontal tail planform area</td>
<td>ft²</td>
</tr>
<tr>
<td>$S_{vt}$</td>
<td>vertical tail planform area</td>
<td>ft²</td>
</tr>
<tr>
<td>$A_{ht}$</td>
<td>horizontal tail aspect ratio</td>
<td></td>
</tr>
<tr>
<td>$A_{vt}$</td>
<td>vertical tail aspect ratio</td>
<td></td>
</tr>
<tr>
<td>$V_{dive}$</td>
<td>design dive speed at sea level</td>
<td>kts</td>
</tr>
<tr>
<td>$R_{tr}$</td>
<td>tail-rotor radius</td>
<td>ft</td>
</tr>
<tr>
<td>$P_{DS\text{limit}}$</td>
<td>drive system power limit (MCP)</td>
<td>hp</td>
</tr>
<tr>
<td>$R$</td>
<td>main-rotor radius in hover</td>
<td>ft</td>
</tr>
<tr>
<td>$V_{tip}$</td>
<td>main-rotor hover tip velocity</td>
<td>ft/sec</td>
</tr>
<tr>
<td>$f_{fold}$</td>
<td>fold (fraction basic weight excluding fold)</td>
<td></td>
</tr>
<tr>
<td>$S_{tail}$</td>
<td>horizontal or vertical tail planform area</td>
<td>ft² or m²</td>
</tr>
<tr>
<td>$U_{Tail}$</td>
<td>unit weight of tail planform</td>
<td>lb/ft² or kg/m²</td>
</tr>
</tbody>
</table>

### 29–4 Fuselage Group

The fuselage group consists of: basic structure; wing and rotor fold/retraction; tail fold/tilt; and marinization, pressurization, and crashworthiness structure. The AFDD84 model is a universal body weight equation, used for tiltrotor and tiltwing as well as for helicopter configurations. The AFDD82 model is a helicopter body weight equation, not used for tiltrotor or tiltwing configuration.

Typically $f_{tfold} = 0.30$ for a folding tail, and $f_{cw} = 0.06$. For wing folding the weight on the wing tip ($W_{tip}$) is required (calculated as for the wing group). Parameters are defined in table 29-6, including units as used in these equations.
For the AFDD82 (HELO) model, the basic structure weight is

\[ w_{\text{basic}} = 5.896 f_{\text{ramp}} \left( \frac{W_{\text{MTO}}}{1000} \right)^{0.4908} n_z^{0.1323} S_{\text{body}}^{0.2544} \ell^{0.6100} \]

\[ W_{\text{basic}} = \chi_{\text{basic}} w_{\text{basic}} \]

where \( f_{\text{ramp}} = 1.3939 \) if there is a cargo ramp, and 1.0 otherwise. Based on 30 aircraft, the average error of the body equation is 8.7% (fig. 29-10). The tail fold, wing and rotor fold, marinization, pressurization, and crashworthiness weights are:

\[ w_{\text{tfold}} = f_{\text{tfold}} W_{\text{basic}} \]

\[ W_{\text{tfold}} = \chi_{\text{tfold}} w_{\text{tfold}} \]

\[ w_{\text{wfold}} = f_{\text{wfold}} (W_{\text{basic}} + W_{\text{tfold}}) \]

\[ W_{\text{wfold}} = \chi_{\text{wfold}} w_{\text{wfold}} \]

\[ w_{\text{mar}} = f_{\text{mar}} W_{\text{basic}} \]

\[ W_{\text{mar}} = \chi_{\text{mar}} w_{\text{mar}} \]

\[ w_{\text{press}} = f_{\text{press}} W_{\text{basic}} \]

\[ W_{\text{press}} = \chi_{\text{press}} w_{\text{press}} \]

\[ w_{\text{cw}} = f_{\text{cw}} (W_{\text{basic}} + W_{\text{tfold}} + W_{\text{wfold}} + W_{\text{mar}} + W_{\text{press}}) \]

\[ W_{\text{cw}} = \chi_{\text{cw}} w_{\text{cw}} \]

Typically \( f_{\text{tfold}} = 0.05 \) for a folding tail, and \( f_{\text{cw}} = 0.06 \). Parameters are defined in table 29-7, including units as used in these equations.
29–5 Alighting Gear Group

The alighting gear group consists of: basic structure, retraction, and crashworthiness structure. There are two models, parametric (AFDD82) and fractional. The basic landing gear weight is:

- **parametric**
  \[ w_{LG} = 0.4013 W_{MTO}^{0.6662} N_{LG}^{0.5360} (W/S)^{0.1525} \]

- **fractional**
  \[ w_{LG} = f_{LG} W_{MTO} \]

and \( W_{LG} = \chi_{LG} w_{LG} \). Typically \( f_{LG} = 0.0325 \) (fractional method). Based on 28 aircraft, the average error of the parametric equation is 8.4% (fig. 29-11). The retraction and crashworthiness weights are:

- **retraction**
  \[ w_{LG_{ret}} = f_{LG_{ret}} W_{LG} \]
  \[ W_{LG_{ret}} = \chi_{LG_{ret}} w_{LG_{ret}} \]

- **crashworthiness**
  \[ w_{LG_{cw}} = f_{LG_{cw}} (W_{LG} + W_{LG_{ret}}) \]
  \[ W_{LG_{cw}} = \chi_{LG_{cw}} w_{LG_{cw}} \]

Typically \( f_{LG_{ret}} = 0.08 \), and \( f_{LG_{cw}} = 0.14 \). Parameters are defined in table 29-8, including units as used in these equations.

Table 29-8. Parameters for landing gear weight.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>( W_{MTO} )</td>
<td>maximum takeoff weight</td>
<td>lb</td>
</tr>
<tr>
<td>( f_{LG} )</td>
<td>landing gear weight (fraction maximum takeoff weight)</td>
<td></td>
</tr>
<tr>
<td>( W/S )</td>
<td>wing loading (1.0 for helicopter)</td>
<td>lb/ft²</td>
</tr>
<tr>
<td>( N_{LG} )</td>
<td>number of landing gear assemblies</td>
<td></td>
</tr>
<tr>
<td>( f_{LG_{ret}} )</td>
<td>retraction weight (fraction basic weight)</td>
<td></td>
</tr>
<tr>
<td>( f_{LG_{cw}} )</td>
<td>crashworthiness weight (fraction basic and retraction weight)</td>
<td></td>
</tr>
</tbody>
</table>

29–6 Engine Section or Nacelle Group and Air Induction Group

The engine section or nacelle group consists of: engine support structure, engine cowling, and pylon support structure. The weights (AFDD82 model) are:

- **engine group**
  \[ W_{supt} = \chi_{supt} 0.0412 (1 - f_{airind})(W_{eng}/N_{eng})^{1.1433} N_{eng}^{1.3762} \]

- **cowling**
  \[ W_{cowl} = \chi_{cowl} 0.2315 S_{nac}^{1.3476} \]

- **pylon**
  \[ W_{pylon} = \chi_{pylon} f_{pylon} W_{MTO} \]

Based on 12 aircraft, the average error of the engine support equation is 11.0% (fig. 29-12). Based on 12 aircraft, the average error of the engine cowling equation is 17.9% (fig. 29-13). The air induction group weight (AFDD82 model) is:

- **air induction**
  \[ W_{airind} = \chi_{airind} 0.0412 f_{airind}(W_{eng}/N_{eng})^{1.1433} N_{eng}^{1.3762} \]

Typically \( f_{airind} = 0.3 \) (range 0.1 to 0.6). Based on 12 aircraft, the average error of the air induction equation is 11.0% (fig. 29-14).

Alternatively, the nacelle group weight (including support, cowling, and pylon) scales with power or thrust, and the air induction group weight scales with the nacelle area:

- **engine group**
  \[ W_{cowl} = \chi_{cowl} K_{nac} P X_{nac} \]

- **jet group**
  \[ W_{cowl} = \chi_{cowl} K_{nac} T X_{nac} \]
\[ W_{\text{airind}} = \chi_{\text{airind}} S_{\text{nac}} U_{\text{airind}} \]

Parameters are defined in table 29-9, including units as used in these equations.

<table>
<thead>
<tr>
<th>parameter</th>
<th>definition</th>
<th>units</th>
</tr>
</thead>
<tbody>
<tr>
<td>( W_{\text{MTO}} )</td>
<td>maximum takeoff weight</td>
<td>lb</td>
</tr>
<tr>
<td>( W_{\text{eng}} )</td>
<td>weight all main engines</td>
<td>lb</td>
</tr>
<tr>
<td>( N_{\text{eng}} )</td>
<td>number of main engines</td>
<td></td>
</tr>
<tr>
<td>( S_{\text{nac}} )</td>
<td>wetted area of nacelles and pylon (less spinner)</td>
<td>ft²</td>
</tr>
<tr>
<td>( f_{\text{airind}} )</td>
<td>air induction weight (fraction nacelle plus air induction)</td>
<td></td>
</tr>
<tr>
<td>( f_{\text{pylon}} )</td>
<td>pylon support structure weight (fraction ( W_{\text{MTO}} ))</td>
<td></td>
</tr>
<tr>
<td>( P )</td>
<td>engine group power (SLS static, specified rating)</td>
<td>hp</td>
</tr>
<tr>
<td>( T )</td>
<td>jet group thrust (SLS static, specified rating)</td>
<td>lb</td>
</tr>
<tr>
<td>( U_{\text{airind}} )</td>
<td>unit weight of air induction group</td>
<td>lb/ft² or kg/m²</td>
</tr>
</tbody>
</table>

### 29–7 Propulsion Group

The propulsion group consists of the engine system, fuel system, and drive system.

#### 29-7.1 Engine System

The engine system consists of the main engines, the engine exhaust system, and the engine accessories. The engine system weights are:

**engine group:**

\[
W_{\text{eng}} = \chi_{\text{eng}} N_{\text{eng}} W_{\text{one eng}}
\]

\[
W_{\text{exh}} = \chi_{\text{exh}} N_{\text{eng}} \left( K_{0\text{exh}} + K_{1\text{exh}} P \right)
\]

**jet group:**

\[
W_{\text{eng}} = \chi_{\text{jet}} N_{\text{jet}} W_{\text{one jet}}
\]

\[
W_{\text{exh}} = \chi_{\text{exh}} N_{\text{jet}} \left( K_{0\text{exh}} + K_{1\text{exh}} T \right)
\]

\[
W_{\text{acc}} = \chi_{\text{acc}} 2.0088 f_{\text{lub}} \left( W_{\text{eng}}/N_{\text{eng}} \right)^{0.5919} N_{\text{eng}}^{0.7858}
\]

where \( f_{\text{lub}} = 1.4799 \) if the accessory weight includes the lubrication system weight, 1.0 if the lubrication system weight is in the engine weight. The exhaust system weight is per engine, including any infrared suppressor. The accessory weight equation is the AFDD82 model. Based on 16 aircraft, the average error of the accessories equation is 11.5% (fig. 29-15). The engine system weight \( W_{\text{ES}} = W_{\text{eng}} + W_{\text{exh}} + W_{\text{acc}} \).

Parameters are defined in table 29-10, including units as used in these equations.

<table>
<thead>
<tr>
<th>parameter</th>
<th>definition</th>
<th>units</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N_{\text{eng}} )</td>
<td>number of main engines</td>
<td></td>
</tr>
<tr>
<td>( N_{\text{jet}} )</td>
<td>number of jets</td>
<td></td>
</tr>
<tr>
<td>( P )</td>
<td>installed takeoff power (SLS static, specified rating) per engine</td>
<td>hp</td>
</tr>
<tr>
<td>( T )</td>
<td>installed takeoff thrust (SLS static, specified rating) per jet</td>
<td>lb</td>
</tr>
<tr>
<td>( K_{0\text{exh}}, K_{1\text{exh}} )</td>
<td>engine exhaust weight vs. power or thrust, constants</td>
<td></td>
</tr>
</tbody>
</table>
29-7.2 Propeller/Fan Installation

The auxiliary propulsion or propeller weight is:

\[
W_{at} = \chi_{at} 0.0809484 N_{at} T_{at}^{0.04771} (T_{at}/A_{at})^{-0.07821}
\]

\[
W_{at} = \chi_{at} 9.90350 N_{at} P_{at}^{0.91996} N_{blade}^{-0.48578} \Omega_{prop}^{-0.45904} D_{prop}^{0.15690} f_m
\]

where \(T_{at}\) is at maximum speed, design gross weight, and SLS conditions, calculated or input. The material factor \(f_m = 1\) for composite construction; 1.20 for wood; 1.31 for aluminum spar; and 1.44 for aluminum construction. Based on 16 aircraft, the average error of the AFDD10 equation is 10.5%. The propeller weight is calculated by the rotor component model, including rotor support and duct weights. Rotor weight models can be used for the propeller. Parameters are defined in table 29-11, including units as used in these equations.

<table>
<thead>
<tr>
<th>parameter</th>
<th>definition</th>
<th>units</th>
</tr>
</thead>
<tbody>
<tr>
<td>(N_{at})</td>
<td>number of auxiliary thrusters</td>
<td></td>
</tr>
<tr>
<td>(T_{at})</td>
<td>thrust per propeller</td>
<td>lb</td>
</tr>
<tr>
<td>(A_{at})</td>
<td>auxiliary thruster disk area</td>
<td>ft(^2)</td>
</tr>
<tr>
<td>(P_{at})</td>
<td>power per propeller</td>
<td>hp</td>
</tr>
<tr>
<td>(N_{blade})</td>
<td>number of blades per propeller</td>
<td></td>
</tr>
<tr>
<td>(\Omega_{prop})</td>
<td>propeller rotation speed at (P_{at})</td>
<td>rpm</td>
</tr>
<tr>
<td>(D_{prop})</td>
<td>propeller diameter</td>
<td>ft</td>
</tr>
</tbody>
</table>

29-7.3 Fuel System

The fuel system consists of tanks and support structure (including fuel tanks, bladders, supporting structure, filler caps, tank covers, and filler material for void and ullage), and fuel plumbing (including fuel system weight not covered by tank weight). There are two models for tank weight, parametric (AFDD82) and fractional. The fuel system weights (AFDD82 model) are:

\[
W_{tank} = \chi_{tank} 0.4341 C_{int}^{0.7717} N_{int}^{0.5897} f_{cw} f_{bt}^{1.9491}
\]

\[
W_{plumb} = \chi_{plumb} [K_{plumb} + K_{plumb}(0.01 N_{plumb} + 0.06 N_{eng})(F/N_{eng})^{0.866}]
\]

where \(f_{cw} = 1.3131\) for ballistically survivable (UTTAS/AAH level) and 1.0 otherwise. The ballistic tolerance factor \(f_{bt} = 1.0\) to 2.5. Based on 15 aircraft, the average error of the fuel tank equation is 4.6% (fig. 29-16). The fuel flow rate \(F\) is calculated for the takeoff power rating at static SLS conditions. \(K_{plumb}\) is a crashworthiness and survivability factor; typically \(K_{plumb} = 2\). \(K_{plumb}\) is the sum of weights for auxiliary fuel (typically up to 120 lb), in-flight refueling (up to 150 lb), pressure refueling (up to 150 lb), inerting system (up to 20 lb), etc.; typically \(K_{plumb} = 50\) to 250 lb. Alternatively, for the fractional model the fuel tank weight is \(W_{tank} = \chi_{tank} f_{tank} W_{fuel\_cap}\). The fractional model for the fuel plumbing weight is \(W_{plumb} = \chi_{plumb} (W_{tank}/\chi_{tank}) f_{plumb}/(1 - f_{plumb})\). Parameters are defined in table 29-12, including units as used in these equations.
256 AFDD Weight Models

Table 29-12. Parameters for fuel system weight.

<table>
<thead>
<tr>
<th>parameter</th>
<th>definition</th>
<th>units</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N_{\text{int}} )</td>
<td>number of internal fuel tanks</td>
<td></td>
</tr>
<tr>
<td>( C_{\text{int}} )</td>
<td>internal fuel tank capacity</td>
<td>gallons</td>
</tr>
<tr>
<td>( f_{\text{bd}} )</td>
<td>ballistic tolerance factor</td>
<td></td>
</tr>
<tr>
<td>( N_{\text{plumb}} )</td>
<td>total number of fuel tanks (internal and auxiliary) for plumbing</td>
<td></td>
</tr>
<tr>
<td>( N_{\text{eng}} )</td>
<td>number of main engines</td>
<td></td>
</tr>
<tr>
<td>( K_{1\text{plumb}}, K_{1\text{plumb}} )</td>
<td>plumbing weight, constants</td>
<td>lb</td>
</tr>
<tr>
<td>( F )</td>
<td>fuel flow rate</td>
<td>lb/hr</td>
</tr>
<tr>
<td>( f_{\text{tank}} )</td>
<td>fuel tank weight (fraction fuel capacity)</td>
<td></td>
</tr>
<tr>
<td>( f_{\text{plumb}} )</td>
<td>fuel plumbing weight (fraction total fuel system weight)</td>
<td></td>
</tr>
</tbody>
</table>

29-7.4 Drive System

The drive system consists of gear boxes and rotor shafts, drive shafts, and rotor brake. This distribution of drive system weights is based on the following functional definitions. Gearboxes are parts of the drive system that transmit power by gear trains, and the structure that encloses them. Rotor shafts are the structure (typically a shaft) that transmits power to the rotor. Drive shafts are the structure (typically a shaft) that transmits power in the propulsion system, but not directly to the rotor or by a gear train. The rotor brake weight encompasses components that can prevent the rotor from freely turning.

The gear box and rotor shaft weights for the AFDD83 model are:

\[
\begin{align*}
W_{\text{gbrs}} &= 57.72 P_{\text{DSlim}}^{0.8195} Q^{0.0680} N_{\text{gb}}^{0.0663} (\Omega_{\text{eng}}/1000)^{0.0369} / \Omega_{\text{rotor}}^{0.6379} \\
W_{\text{gb}} &= \chi_{\text{gb}} (1 - f_{rs}) W_{\text{gbrs}} \\
W_{\text{rs}} &= \chi_{\text{rs}} f_{rs} W_{\text{gbrs}}
\end{align*}
\]

Based on 30 aircraft, the average error of the gear box and rotor shaft equation is 7.7% (fig. 29-17). The gear box and rotor shaft weights for the AFDD00 model are:

\[
\begin{align*}
W_{\text{gbrs}} &= 95.7634 \chi_{\text{rotor}}^{0.38553} P_{\text{DSlim}}^{0.78137} Q^{0.09899} N_{\text{ds}}^{0.03909} / \Omega_{\text{rotor}}^{0.80686} \\
W_{\text{gb}} &= \chi_{\text{gb}} (1 - f_{rs}) W_{\text{gbrs}} \\
W_{\text{rs}} &= \chi_{\text{rs}} f_{rs} W_{\text{gbrs}}
\end{align*}
\]

Based on 52 aircraft, the average error of the gear box and rotor shaft equation is 8.6% (fig. 29-18). Typically \( f_{rs} = 0.13 \) (range 0.06 to 0.20). Parameters are defined in table 29-13, including units as used in these equations.

The drive shaft (AFDD82 model) and rotor brake weights are:

\[
\begin{align*}
W_{\text{ds}} &= \chi_{\text{ds}} 1.166 Q_{\text{DSlim}}^{0.3828} N_{\text{ds}}^{1.0455} f_{P}^{0.2693} \\
W_{\text{rb}} &= \chi_{\text{rb}} 0.000871 W_{\text{blade}} (0.01 V_{\text{tip}})^2
\end{align*}
\]

where \( f_{P} = f_{Q} \Omega_{\text{other}} / \Omega_{\text{main}} \). Based on 28 aircraft, the average error of the drive shaft equation is 16.0% (fig. 29-19). Based on 23 aircraft, the average error of the rotor brake equation is 25.1% (fig. 29-20). The clutch weight in the weight statement is associated with an auxiliary power unit, and is a fixed input.
value. The conventional rotor drive system clutch and free wheeling device weights are included in the gear box and rotor shaft weight equations. Parameters are defined in table 29-14, including units as used in these equations.

Typically $f_P = f_Q = 60\%$ for twin main-rotors (tandem, coaxial, and tiltrotor); for a single main-rotor and tail-rotor, $f_Q = 3\%$ and $f_P = 15\%$ (18\% for 2-bladed rotors).

Table 29-13. Parameters for drive system weight.

<table>
<thead>
<tr>
<th>parameter</th>
<th>definition</th>
<th>units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{DS\text{Limit}}$</td>
<td>drive system power limit (MCP)</td>
<td>hp</td>
</tr>
<tr>
<td>$N_{\text{rotor}}$</td>
<td>number of main-rotors</td>
<td></td>
</tr>
<tr>
<td>$N_{gb}$</td>
<td>number of gear boxes</td>
<td></td>
</tr>
<tr>
<td>$\Omega_{\text{rotor}}$</td>
<td>main-rotor rotation speed</td>
<td>rpm</td>
</tr>
<tr>
<td>$\Omega_{\text{eng}}$</td>
<td>engine output speed</td>
<td>rpm</td>
</tr>
<tr>
<td>$f_Q$</td>
<td>second (main or tail) rotor torque limit*</td>
<td>%</td>
</tr>
<tr>
<td>$f_{rs}$</td>
<td>rotor shaft weight (fraction gear box and rotor shaft)</td>
<td></td>
</tr>
</tbody>
</table>

*input as fraction of total drive system torque limit

Table 29-14. Parameters for drive shaft and rotor brake weight.

<table>
<thead>
<tr>
<th>parameter</th>
<th>definition</th>
<th>units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_{DS\text{Limit}}$</td>
<td>$P_{DS\text{Limit}}/\Omega_{\text{rotor}}$ (MCP)</td>
<td>hp/rpm</td>
</tr>
<tr>
<td>$N_{\text{ds}}$</td>
<td>number of intermediate drive shafts</td>
<td></td>
</tr>
<tr>
<td>$x_{\text{hub}}$</td>
<td>length of drive shaft between rotors</td>
<td>ft</td>
</tr>
<tr>
<td>$f_P$</td>
<td>second (main or tail) rotor power limit*</td>
<td>%</td>
</tr>
<tr>
<td>$V_{\text{tip}}$</td>
<td>main-rotor tip speed</td>
<td>ft/sec</td>
</tr>
</tbody>
</table>

*input as fraction of total drive system power limit

29–8 Flight Controls Group

The flight controls group consists of cockpit controls, automatic flight control system, and system controls. System controls consist of fixed wing flight controls, rotary wing flight controls, and conversion (rotor tilt) flight controls. The weight equations model separately non-boosted controls (which do not see aerodynamic surface or rotor loads), boost mechanisms (actuators), and boosted controls (which are affected by aerodynamic surface or rotor loads). The load path goes from pilot, to cockpit controls, to non-boosted controls, to boost mechanisms, to boosted controls, and finally to the component.

The cockpit controls weight $W_{cc}$ is fixed (input), or scaled with design gross weight: $W_{cc} = K_{cc}W_D^{N_{cc}}$. The automatic flight control system weight $W_{afcs}$ is fixed (input).

Fixed wing flight controls consist of non-boosted flight controls and flight control boost mechanisms. The weights are:

- full controls $w = 0.91000W_{MTO}^{0.6}$
- only stabilizer controls $w = 0.01735W_{MTO}^{0.64345}C_{ht}^{0.40952}$

and then

$$W_{FW\text{nb}} = \chi_{FW\text{nb}} f_{FW\text{nb}} w$$
$$W_{FW\text{mb}} = \chi_{FW\text{mb}} (1 - f_{FW\text{nb}}) w$$
For a helicopter, the stabilizer control equation is used. Parameters are defined in table 29-15, including units as used in these equations.

Table 29-15. Parameters for fixed wing flight control weight.

<table>
<thead>
<tr>
<th>parameter</th>
<th>definition</th>
<th>units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{ht}$</td>
<td>horizontal tail planform area</td>
<td>ft$^2$</td>
</tr>
<tr>
<td>$W_{MTO}$</td>
<td>maximum takeoff weight</td>
<td>lb</td>
</tr>
<tr>
<td>$f_{FW_{nb}}$</td>
<td>fixed wing non-boosted weight</td>
<td>(fraction total fixed wing flight control weight)</td>
</tr>
</tbody>
</table>

Rotary wing flight controls consist of non-boosted flight controls, flight control boost mechanisms, and boosted flight controls. The non-boosted flight control weight (AFDD82 model) is:

\[
W_{RW_{nb}} = \chi_{RW_{nb}} f_{RW_{nb}} (1 - f_{RW_{hyd}}) w_{fc}
\]

fraction method

\[
W_{RW_{nb}} = \chi_{RW_{nb}} \frac{0.3999}{N_{rotor}^{1.3855}}
\]

parametric method

where $f_{nbsv} = 1.8084$ for ballistically survivable (UTTAS/AAH level), 1.0 otherwise. The parametric method assumes the rotor flight controls are boosted and computes the weight of the non-boosted portion up to the control actuators. Based on 20 aircraft, the average error of the non-boosted flight controls equation is 10.4% (fig. 29-21). The flight control boost mechanism weight and boosted flight control weight (AFDD82 model) are:

\[
w_{fc} = 0.2873 f_{mbsv} (N_{rotor} N_{blade})^{0.6257} c^{1.3286} (0.01 V_{tip})^{2.1129} f_{RW_{red}}
\]

\[
W_{RW_{nb}} = \chi_{RW_{nb}} (1 - f_{RW_{hyd}}) w_{fc}
\]

\[
W_{RW_b} = \chi_{RW_b} 0.02324 f_{bsv} (N_{rotor} N_{blade})^{1.0042} c^{2.2296} (0.01 V_{tip})^{3.1877}
\]

where $f_{mbsv} = 1.3029$ and $f_{bsv} = 1.1171$ for ballistically survivable (UTTAS/AAH level); 1.0 otherwise; and $f_{RW_{red}} = 1.0$ to 3.0. Typically $f_{RW_{nb}} = 0.6$ (range 0.3 to 1.8); $f_{RW_{hyd}} = 0.4$. Based on 21 aircraft, the average error of the boost mechanisms equation is 6.5% (fig. 29-22). Based on 20 aircraft, the average error of the boosted flight controls equation is 9.7% (fig. 29-23). Parameters are defined in table 29-16, including units as used in these equations.

Table 29-16. Parameters for rotary wing flight control weight.

<table>
<thead>
<tr>
<th>parameter</th>
<th>definition</th>
<th>units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_{MTO}$</td>
<td>maximum takeoff weight</td>
<td>lb</td>
</tr>
<tr>
<td>$N_{rotor}$</td>
<td>number of main-rotors</td>
<td></td>
</tr>
<tr>
<td>$N_{blade}$</td>
<td>number of blades per rotor</td>
<td></td>
</tr>
<tr>
<td>$c$</td>
<td>rotor mean blade chord</td>
<td>ft</td>
</tr>
<tr>
<td>$V_{tip}$</td>
<td>rotor hover tip velocity</td>
<td>ft/sec</td>
</tr>
<tr>
<td>$f_{RW_{nb}}$</td>
<td>rotary wing non-boosted weight (fraction boost mechanisms weight)</td>
<td></td>
</tr>
<tr>
<td>$f_{RW_{hyd}}$</td>
<td>rotary wing hydraulics weight</td>
<td></td>
</tr>
<tr>
<td>$f_{RW_{red}}$</td>
<td>flight control hydraulic system redundancy factor</td>
<td></td>
</tr>
</tbody>
</table>
The conversion controls consist of non-boosted tilt controls, and tilt control boost mechanisms; they are used only for tilting rotor configurations. The weights are:

\[
\begin{align*}
    w_{CV \text{mb}} &= f_{CV \text{mb}} W_{MTO} \\
    w_{CV \text{nb}} &= f_{CV \text{nb}} W_{CV \text{mb}} \\
    W_{CV \text{mb}} &= \chi_{CV \text{mb}} w_{CV \text{mb}} \\
    W_{CV \text{nb}} &= \chi_{CV \text{nb}} w_{CV \text{nb}}
\end{align*}
\]

Parameters are defined in table 29-17, including units as used in these equations.

### Table 29-17. Parameters for conversion control weight.

<table>
<thead>
<tr>
<th>parameter</th>
<th>definition</th>
<th>units</th>
</tr>
</thead>
<tbody>
<tr>
<td>(W_{MTO})</td>
<td>maximum takeoff weight</td>
<td>lb</td>
</tr>
<tr>
<td>(f_{CV \text{nb}})</td>
<td>conversion non-boosted weight (fraction boost mechanisms weight)</td>
<td></td>
</tr>
<tr>
<td>(f_{CV \text{mb}})</td>
<td>conversion boost mechanisms weight (fraction maximum takeoff weight)</td>
<td></td>
</tr>
</tbody>
</table>

#### 29–9 Hydraulic Group

The hydraulic group consists of hydraulics for fixed wing flight controls, rotary wing flight controls, conversion (rotor tilt) flight controls, and equipment. The hydraulic weight for equipment, \(W_{EQ\text{hyd}}\), is fixed (input). The weights (AFDD82 model) are

\[
\begin{align*}
    W_{FW\text{hyd}} &= \chi_{FW\text{hyd}} f_{FW\text{hyd}} W_{FW\text{mb}} \\
    W_{RW\text{hyd}} &= \chi_{RW\text{hyd}} f_{RW\text{hyd}} w_{fc} \\
    W_{CV\text{hyd}} &= \chi_{CV\text{hyd}} f_{CV\text{hyd}} W_{CV\text{mb}}
\end{align*}
\]

Typically \(f_{RW\text{hyd}} = 0.4\). Parameters are defined in table 29-18, including units as used in these equations.

### Table 29-18. Parameters for hydraulic group weight.

<table>
<thead>
<tr>
<th>parameter</th>
<th>definition</th>
<th>units</th>
</tr>
</thead>
<tbody>
<tr>
<td>(f_{FW\text{hyd}})</td>
<td>fixed wing hydraulics weight (fraction boost mechanisms weight)</td>
<td></td>
</tr>
<tr>
<td>(f_{RW\text{hyd}})</td>
<td>rotary wing hydraulics weight (fraction hydraulics plus boost mechanisms weight)</td>
<td></td>
</tr>
<tr>
<td>(f_{CV\text{hyd}})</td>
<td>conversion hydraulics weight (fraction boost mechanisms weight)</td>
<td></td>
</tr>
</tbody>
</table>

#### 29–10 Anti-Icing Group

The anti-icing group consists of the anti-ice system. The electrical system for anti-icing is part of the electrical group. The weights are obtained from the sum over all rotors, all wings, and all engines:

\[
\begin{align*}
    W_{DI\text{elect}} &= \chi_{DI\text{elect}} \sum k_{\text{elec}} A_{\text{blade}} \\
    W_{DI\text{sys}} &= \chi_{DI\text{sys}} \left( \sum k_{\text{rotor}} A_{\text{blade}} + \sum k_{\text{wing}} l_{\text{wing}} + \sum k_{\text{air}} W_{\text{eng}} + \sum k_{\text{jet}} W_{\text{jet}} \right)
\end{align*}
\]

Parameters are defined in table 29-19, including units as used in these equations.
Table 29-19. Parameters for anti-icing group weight.

<table>
<thead>
<tr>
<th>parameter</th>
<th>definition</th>
<th>units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{\text{blade}}$</td>
<td>total blade area of rotor, from geometric solidity</td>
<td>ft$^2$ or m$^2$</td>
</tr>
<tr>
<td>$\ell_{\text{wing}}$</td>
<td>wing length (wing span less fuselage width)</td>
<td>ft or m</td>
</tr>
<tr>
<td>$k_{\text{elect}}$</td>
<td>electrical system weight factor</td>
<td></td>
</tr>
<tr>
<td>$k_{\text{rotor}}$</td>
<td>rotor deice system weight factor</td>
<td></td>
</tr>
<tr>
<td>$k_{\text{wing}}$</td>
<td>wing deice system weight factor</td>
<td></td>
</tr>
<tr>
<td>$k_{\text{air}}$</td>
<td>engine air intake deice system weight factor</td>
<td></td>
</tr>
<tr>
<td>$k_{\text{jet}}$</td>
<td>jet air intake deice system weight factor</td>
<td></td>
</tr>
</tbody>
</table>

29–11 Other Systems and Equipment

The following weights are fixed (input) in this model: auxiliary power group; instruments group; pneumatic group; electrical group (aircraft); avionics group (mission equipment); armament group (armament provisions and armor); furnishing and equipment group; environmental control group; and load and handling group. Typical fixed weights are given in table 29-20, based on medium to heavy helicopters and tiltrotors.

Table 29-20. Other systems and equipment weight.

<table>
<thead>
<tr>
<th>SYSTEMS AND EQUIPMENT</th>
<th>typical weight (lb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>flight controls group</td>
<td></td>
</tr>
<tr>
<td>cockpit controls</td>
<td>100–125</td>
</tr>
<tr>
<td>automatic flight control system</td>
<td>35–200</td>
</tr>
<tr>
<td>flight control electronics, mechanical</td>
<td>35–100</td>
</tr>
<tr>
<td>flight control electronics, fly-by-wire</td>
<td>250</td>
</tr>
<tr>
<td>auxiliary power group</td>
<td>130–300</td>
</tr>
<tr>
<td>instruments group</td>
<td>150–250</td>
</tr>
<tr>
<td>hydraulic group equipment</td>
<td>50–300</td>
</tr>
<tr>
<td>equipment</td>
<td></td>
</tr>
<tr>
<td>electrical group</td>
<td>400–1000</td>
</tr>
<tr>
<td>avionics group (mission equipment)</td>
<td>400–1500</td>
</tr>
<tr>
<td>furnishings &amp; equipment group</td>
<td>600–1000</td>
</tr>
<tr>
<td>crew only</td>
<td>100–200</td>
</tr>
<tr>
<td>environmental control group</td>
<td>50–250</td>
</tr>
<tr>
<td>anti-icing group</td>
<td>50–300</td>
</tr>
<tr>
<td>load &amp; handling group</td>
<td></td>
</tr>
<tr>
<td>internal</td>
<td>200–400</td>
</tr>
<tr>
<td>external</td>
<td>150–300</td>
</tr>
<tr>
<td>FIXED USEFUL LOAD</td>
<td></td>
</tr>
<tr>
<td>crew</td>
<td>500–800</td>
</tr>
<tr>
<td>fluids (oil, unusable fuel)</td>
<td>50–150</td>
</tr>
</tbody>
</table>
29–12 Folding Weight

Folding weights are calculated in a number of groups: wing $W_{\text{fold}}$ (including extensions), rotor $W_{\text{fold}}$, tail $W_{\text{fold}}$, fuselage $W_{\text{fold}}$ and $W_{\text{wfold}}$. These are the total weights for folding and the impact of folding on the group. A fraction $f_{\text{foldkit}}$ of these weights can be in a kit, hence optionally removable. Thus of the total folding weight, the fraction $f_{\text{foldkit}}$ is a kit weight in the fixed useful load of the weight statement, while the remainder is kept in the wing, rotor, or fuselage group weight.

29–13 Parametric Weight Correlation

Table 29-21 summarizes the statistics of the parametric weight estimation equations. Figure 29-24 shows the error of the calculated weight for the sum of all parametric weight, accounting on average for 42\% of the empty weight. This sum is composed of the structural group (based on the AFDD00 equation for rotor blade and hub weights, and the AFDD84 equation for body weight), the propulsion group (based on the AFDD00 equation for drive system weight), and the flight controls group. Based on 42 aircraft, the average error of the sum of all parametric weight is 5.3\%. The corresponding average error is 6.1\% for the structural group (8.6\% for the rotor group alone), 10.9\% for the propulsion group, and 8.7\% for the flight controls group.


<table>
<thead>
<tr>
<th>group</th>
<th>number of aircraft</th>
<th>average error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>wing</td>
<td>25</td>
<td>3.4</td>
</tr>
<tr>
<td>rotor blade</td>
<td>AFDD82</td>
<td>37</td>
</tr>
<tr>
<td>rotor hub</td>
<td>AFDD82</td>
<td>37</td>
</tr>
<tr>
<td>rotor blade</td>
<td>AFDD00</td>
<td>51</td>
</tr>
<tr>
<td>rotor hub</td>
<td>AFDD00</td>
<td>51</td>
</tr>
<tr>
<td>horizontal tail</td>
<td>13</td>
<td>22.4</td>
</tr>
<tr>
<td>vertical tail</td>
<td>12</td>
<td>23.3</td>
</tr>
<tr>
<td>tail-rotor</td>
<td>19</td>
<td>16.7</td>
</tr>
<tr>
<td>fuselage</td>
<td>AFDD82</td>
<td>30</td>
</tr>
<tr>
<td>fuselage</td>
<td>AFDD84</td>
<td>35</td>
</tr>
<tr>
<td>alighting gear</td>
<td>28</td>
<td>8.4</td>
</tr>
<tr>
<td>engine support</td>
<td>12</td>
<td>11.0</td>
</tr>
<tr>
<td>engine cowling</td>
<td>12</td>
<td>17.9</td>
</tr>
<tr>
<td>air induction</td>
<td>12</td>
<td>11.0</td>
</tr>
<tr>
<td>accessory</td>
<td>16</td>
<td>11.5</td>
</tr>
<tr>
<td>propeller</td>
<td>AFDD10</td>
<td>16</td>
</tr>
<tr>
<td>fuel tank</td>
<td>15</td>
<td>4.6</td>
</tr>
<tr>
<td>gear box + rotor shaft</td>
<td>AFDD83</td>
<td>30</td>
</tr>
<tr>
<td>gear box + rotor shaft</td>
<td>AFDD00</td>
<td>52</td>
</tr>
<tr>
<td>drive shaft</td>
<td>28</td>
<td>16.0</td>
</tr>
<tr>
<td>rotor brake</td>
<td>23</td>
<td>25.1</td>
</tr>
<tr>
<td>rotary wing flight controls</td>
<td>non-boosted</td>
<td>20</td>
</tr>
<tr>
<td>rotary wing flight controls</td>
<td>boost mechanisms</td>
<td>21</td>
</tr>
<tr>
<td>rotary wing flight controls</td>
<td>boosted</td>
<td>20</td>
</tr>
</tbody>
</table>
29–14 References


Figure 29-1. Wing group (AFDD93).

Figure 29-2. Rotor group, blade weight (AFDD82).
Figure 29-3. Rotor group, hub weight (AFDD82).

Figure 29-4. Rotor group, blade weight (AFDD00).
Figure 29-5. Rotor group, hub weight (AFDD00).

Figure 29-6. Empennage group, horizontal tail weight (AFDD82).
Figure 29-7. Empennage group, vertical tail weight (AFDD82).

Figure 29-8. Empennage group, tail-rotor weight (AFDD82).
Figure 29-9. Fuselage group, fuselage weight (AFDD84).

Figure 29-10. Fuselage group, fuselage weight (AFDD82).
Figure 29-11. Alighting gear group, landing gear weight (AFDD82).

Figure 29-12. Engine section or nacelle group, engine support weight (AFDD82).
Figure 29-13. Engine section or nacelle group, cowling weight (AFDD82).

Figure 29-14. Air induction group, air induction weight (AFDD82).
Figure 29-15. Propulsion group, accessories weight (AFDD82).

Figure 29-16. Propulsion group, fuel tank weight (AFDD82).
Figure 29-17. Propulsion group, gear box and rotor shaft weight (AFDD83).

Figure 29-18. Propulsion group, gear box and rotor shaft weight (AFDD00).
Figure 29-19. Propulsion group, drive shaft weight (AFDD82).

Figure 29-20. Propulsion group, rotor brake weight.
Figure 29-21. Flight controls group, rotor non-boosted control weight (AFDD82).

Figure 29-22. Flight controls group, rotor boost mechanisms weight (AFDD82).
Figure 29-23. Flight controls group, rotor boosted control weight (AFDD82).

Figure 29-24. Sum of all parametric weight.
This chapter presents aircraft weight models developed by a number of organizations: Boeing (refs. 1 to 4), GARTEUR (Aerospatiale and MBB, ref. 5), Tishchenko (refs. 6 and 3), Torenbeek (ref. 7), and Raymer (ref. 8). The weights are estimated from parametric equations, based on the weights of existing aircraft.

The results of these equations are the weight in pounds, and the units of the parameters are noted in the tables. Technology factors $\chi$ are included in the weight equations. The input usually includes a weight increment $dW$ that can be added to the results of the weight model. Thus typically a component or element weight is obtained from $W = \chi w_{\text{model}} + dW$. Weight of individual elements in a group can be fixed by using $dW$ and setting the corresponding technology factor $\chi = 0$. With $\chi \neq 0$, the increment $dW$ can account for something not included in the parametric model.

### 30–1 Wing Group

The wing group consists of: basic structure (primary structure, consisting of torque box and spars, plus extensions); fairings (leading edge and trailing edge); fittings (non-structural); fold/tilt structure; and control surfaces (flaps, ailerons, flaperons, and spoilers).

The total wing weight is:

**Boeing**

$$w_{\text{wing}} = 0.0680 \left[ \sqrt{n_z W_D f_L S_w \log(b_w/w_f)} \sqrt{\frac{1 + \lambda_w}{2 \tau_w}} \log V_{\text{dive}} \log A_w \right]^{0.585}$$

**GARTEUR**

$$w_{\text{wing}} = 0.00715 f_{TR} (1 + 0.12 f_{\text{flap}})^{0.4} \left(n_z W_D f_L \right)^{0.52} S_w^{0.7} A_w^{0.47} \left(\frac{2}{\tau_w}\right)^{0.4} \left(1.1 + \lambda_w/2\right)^{0.695}$$

The factor $f_{TR} = 1.116$ for tiltrotors, 1.0 otherwise.

**Torenbeek (light aircraft, up to 12500 lb)**

$$w_{\text{wing}} = 0.00125 \left(n_z^{0.55} W_{MTO f_L} \right) \left(\frac{b_w}{\cos \Lambda_w}\right)^{0.75} \left(1 + \sqrt{6.25 \cos \Lambda_w/b_w}\right) \left(\frac{b_w/\cos \Lambda_w t_w}{W_{MTO f_L}/S_w}\right)^{0.3}$$

**Torenbeek (transport category aircraft)**

$$w_{\text{wing}} = 0.0017 \left(n_z^{0.55} W_{MZF f_L} \right) \left(\frac{b_w}{\cos \Lambda_w}\right)^{0.75} \left(1 + \sqrt{6.25 \cos \Lambda_w/b_w}\right) \left(\frac{b_w/\cos \Lambda_w t_w}{W_{MZF f_L}/S_w}\right)^{0.3}$$
Raymer (cargo/transport)
\[ w_{\text{wing}} = 0.0051(n_z W_D f_L)^{0.557} S_w^{0.649} A_w^{0.5} \left(1 + \lambda_w\right)^{0.1} S_e^{0.1} \frac{n_z}{t_w^{0.3}} \cos \Lambda_w \]

Raymer (general aviation)
\[ w_{\text{wing}} = 0.0087(n_z W_D f_L)^{0.49} S_w^{0.758} A_w^{0.6} \lambda_w^{0.041} V_{\text{dive}}^{0.012} \]

The lift factor \( f_L \) accounts for wing relief factor and wing/rotor lift share.

Then the primary structure, secondary structure, and control surface weights are calculated as for the AFDD aircraft wing model. The wing extension weights are calculated as for the AFDD model. Parameters are defined in table 30-1, including units as used in these equations.

<table>
<thead>
<tr>
<th>parameter</th>
<th>definition</th>
<th>units</th>
</tr>
</thead>
<tbody>
<tr>
<td>( W_D )</td>
<td>design gross weight</td>
<td>lb</td>
</tr>
<tr>
<td>( W_{MTO} )</td>
<td>maximum takeoff weight</td>
<td>lb</td>
</tr>
<tr>
<td>( W_{MZ\n} )</td>
<td>maximum zero fuel weight</td>
<td>lb</td>
</tr>
<tr>
<td>( f_L )</td>
<td>lift factor</td>
<td></td>
</tr>
<tr>
<td>( S_w )</td>
<td>wing planform area (theoretical)</td>
<td>ft(^2)</td>
</tr>
<tr>
<td>( b_w )</td>
<td>wing span</td>
<td>ft</td>
</tr>
<tr>
<td>( A_w )</td>
<td>wing aspect ratio</td>
<td></td>
</tr>
<tr>
<td>( w_f )</td>
<td>maximum fuselage width</td>
<td>ft</td>
</tr>
<tr>
<td>( \lambda_w )</td>
<td>wing taper ratio (tip chord/root chord)</td>
<td></td>
</tr>
<tr>
<td>( \Lambda_w )</td>
<td>wing sweep angle</td>
<td>deg</td>
</tr>
<tr>
<td>( \tau_w )</td>
<td>wing root airfoil thickness-to-chord ratio</td>
<td></td>
</tr>
<tr>
<td>( t_w )</td>
<td>wing root airfoil thickness</td>
<td>ft</td>
</tr>
<tr>
<td>( n_z )</td>
<td>design ultimate flight load factor</td>
<td>g</td>
</tr>
<tr>
<td>( V_{\text{dive}} )</td>
<td>dive speed</td>
<td>kts</td>
</tr>
<tr>
<td>( S_c )</td>
<td>control surface area</td>
<td>ft(^2)</td>
</tr>
<tr>
<td>( f_{\text{flap}} )</td>
<td>ratio maximum lift coefficient with and without flaps</td>
<td></td>
</tr>
</tbody>
</table>

### 30–2 Rotor Group

The rotor group consists of: blades, hub and hinge, spinner, and blade fold structure. The blade and hub-hinge weights are:

Boeing
\[
\begin{align*}
\text{w}_{\text{blade}} &= 0.0378 N_{\text{rotor}} \left(n_z W_D f_{tb} / N_{\text{rotor}}\right) R^2 (R - r) N_{\text{blade}} c f_b f_{dr} \right]^{0.438} \\
\text{w}_{\text{hub}} &= 0.00704 N_{\text{rotor}} \left(W_{\text{blade}} / N_{\text{rotor}}\right) R t_l^2 (P_{to} f_{th} / N_{\text{rotor}}) r^{1.82} N_{\text{blade}}^{1.5} f_{amd} \right]^{0.358}
\end{align*}
\]

with \( f_b = 1.0 \) for articulated rotor, 2.2 hingeless or teetering rotor; \( f_{tb} = 0.247 \) and \( f_{th} = 0.577 \) for tandem, 1.0 single main rotor; \( f_{amd} = f_s f_m f_d \), design concept factor \( f_s = 0.53 \) for hingeless rotor and 1.0 otherwise, material factor \( f_m = 1.0 \) for steel and 0.56 titanium and 0.354 composite, development factor \( f_d = 1.0 \) early, 0.62 developed. The droop factor is \( f_{dr} = \max(1, R^{1.6} / (1200 t_{0.25})) \). The blade attachment radius is typically \( r / R = 0.08 \) to 0.11.
GARTEUR

\[ w_{\text{blade}} = 4.199 N_{\text{rotor}} f_{\text{root}} f_{cb} (R N_{\text{blade}} c) \]
\[ w_{\text{hub}} = 0.004111 N_{\text{rotor}} \left( \left( W_{\text{blade}} / N_{\text{rotor}} \right) V_{\text{tip}}^2 \left( P_{to} / N_{\text{rotor}} \right) R^{0.82} N_{\text{blade}}^{1.5} \right)^{0.36} \]

with \( f_{\text{root}} = 1.17 \) for hingeless and teetering blades, 0.88 for composite articulated blades. The original model is extended to other configurations by introducing \( f_{cb} = 0.56 \) for tandem helicopter, 0.65 for tiltrotor.

Tishchenko

\[ w_{\text{blade}} = 1.23 N_{\text{rotor}} f_{cb} \left( \sigma R^{2.7} / \bar{\lambda}^{0.7} \right) \left[ 1 + 0.011 \bar{R} (\lambda - \lambda_0) \right] \]
\[ w_{\text{hub}} = 0.0000777 N_{\text{rotor}} f_{cb} N_{\text{blade}} (C F_{\text{blade}})^{1.35} \left[ 1 + 0.05 (N_{\text{blade}} - 4) \right] \]

where \( \bar{\lambda} = \lambda / 18 \), blade aspect ratio \( \lambda = R / c_{0.7} \), \( \bar{R} = R / 52.5 \), \( \bar{\lambda}_0 = 12.4 \), and the last factor in the blade weight equation has a minimum value of 1. For the hub weight, the centrifugal force per blade is \( C F_{\text{blade}} = \int_0^R \Omega^2 r m dr = \Omega^2 \frac{1}{2} R (M_{\text{blade}} / (N_{\text{blade}} N_{\text{rotor}})) \). The original model is extended to other configurations by introducing \( f_{cb} = 0.45 \) for tandem helicopter, 0.80 for tiltrotor; and \( f_{ch} = 1.11 \) for tandem helicopter, 0.34 for tiltrotor, 0.53 for teetering rotor, 0.67 for hingeless rotor.

The blade weight is for all blades of the rotors. If the weight is evaluated separately for each rotor, then the leading \( N_{\text{rotor}} = 1 \) in these equations (with blade weight, design gross weight, and takeoff power per rotor). The fairing/spinner, blade fold, rotor support, and duct weights are calculated as for the AFDD model. Parameters are defined in table 30-2, including units as used in these equations.

Table 30-2. Parameters for rotor weight.

<table>
<thead>
<tr>
<th>parameter</th>
<th>definition</th>
<th>units</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N_{\text{rotor}} )</td>
<td>number of rotors</td>
<td></td>
</tr>
<tr>
<td>( N_{\text{blade}} )</td>
<td>number of blades per rotor</td>
<td></td>
</tr>
<tr>
<td>( R )</td>
<td>rotor radius in hover</td>
<td>ft</td>
</tr>
<tr>
<td>( r )</td>
<td>center of rotation to blade attachment</td>
<td>ft</td>
</tr>
<tr>
<td>( c )</td>
<td>rotor mean geometric blade chord</td>
<td>ft</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>rotor solidity</td>
<td></td>
</tr>
<tr>
<td>( t_{0.25} )</td>
<td>blade thickness at 25%R</td>
<td>ft</td>
</tr>
<tr>
<td>( W_D )</td>
<td>design gross weight</td>
<td>lb</td>
</tr>
<tr>
<td>( n_z )</td>
<td>design limit flight load factor</td>
<td>g</td>
</tr>
<tr>
<td>( \Omega )</td>
<td>rotor rotation speed</td>
<td>rpm</td>
</tr>
<tr>
<td>( V_{\text{tip}} )</td>
<td>hover tip speed</td>
<td>ft/sec</td>
</tr>
<tr>
<td>( P_{to} )</td>
<td>takeoff power</td>
<td>hp</td>
</tr>
</tbody>
</table>
30–3 Empennage Group

The empennage group consists of: horizontal tail, vertical tail, and tail-rotor. The tail plane weight consists of the basic structure and fold structure. The horizontal tail weight is:

GARTEUR

\[ W_{ht} = \chi_{ht} 0.00000518 V_{\text{max}}^{2.44} W_{D}^{0.35} \]

Torenbeek (\(V_{\text{dive, up to 250 knots}}\))

\[ W_{\text{tail}} = \chi_{ht} 0.04 \left(n_z S_{\text{tail}}^2 \right)^{0.75} \]

Torenbeek (transport category aircraft)

\[ W_{ht} = \chi_{ht} f_h S_{ht} 0.00346 \left( S_{ht}^{0.2} V_{\text{dive}} / \sqrt{\cos \Lambda_{ht}} \right) \]

where \( f_h = 1.0 \) for fixed stabilizer, 1.1 for variable-incidence tail.

Raymer (cargo/transport)

\[ W_{ht} = \chi_{ht} 0.0379 f_h n_z 0.10 W_{D}^{0.639} S_{ht}^{0.75} A_{ht}^{0.166} K_y^{0.704} \frac{(1 + S_z / S_{ht})^{0.1}}{(1 + w_f / b_h)^{0.25} L_t \cos \Lambda_{ht}} \]

Raymer (general aviation)

\[ W_{ht} = \chi_{ht} f_h S_{ht} 0.00354 (n_z W_{D})^{0.414} S_{ht}^{0.896} A_{ht}^{0.043} V_{\text{dive}}^{0.336} \cos \Lambda_{ht}^{0.034} \]

\[ \frac{1}{\lambda_{ht}^{0.02} r_{ht}^{0.12}} \]

where \( f_h = 1.143 \) for all-moving horizontal tail, 1.0 otherwise.

The vertical tail weight is:

GARTEUR

\[ W_{vt} = \chi_{vt} 0.0253 S_{vt}^{0.508} W_{D}^{0.647} \]

Torenbeek (\(V_{\text{dive, up to 250 knots}}\))

\[ W_{\text{tail}} = \chi_{vt} 0.04 \left(n_z S_{\text{tail}}^2 \right)^{0.75} \]

Torenbeek (transport category aircraft)

\[ W_{vt} = \chi_{vt} f_v S_{vt} 0.00346 \left( S_{vt}^{0.2} V_{\text{dive}} / \sqrt{\cos \Lambda_{vt}} \right) \]

where \( f_v = 1.0 \) for fuselage-mounted horizontal tail, \( f_v = 1 + 0.15(S_{ht} h_t) / (S_{vt} b_{vt}) \) for fin-mounted stabilizer (T-tail).

Raymer (cargo/transport)

\[ W_{vt} = \chi_{vt} 0.0026 f_v n_z 0.536 W_{D}^{0.556} S_{vt}^{0.5} A_{vt}^{0.35} K_{z}^{0.875} \frac{1}{r_{\text{root}}^{0.5} r_{t}^{0.5} \cos \Lambda_{vt}} \]

Raymer (general aviation)

\[ W_{vt} = \chi_{vt} 0.00382 f_v n_z 0.376 W_{D}^{0.376} S_{vt}^{0.873} A_{vt}^{0.357} V_{\text{dive}}^{0.244} \]

\[ \frac{1}{\lambda_{vt}^{0.039} r_{vt}^{0.49} \cos \Lambda_{vt}^{0.224}} \]

where \( f_v = 1.0 \) for conventional tail, 1.169 for T-tail; \( f_{vt} = 1.0 \) for conventional tail, 1.2 for T-tail.

The tail-rotor weight is:

Boeing

\[ W_{tr} = \chi_{tr} 0.0297 \left[ \rho^{0.25} \rho^{0.5} R N_{\text{blade}} \right]^{0.67} \]

GARTEUR

\[ W_{tr} = \chi_{tr} 12.48 W_{D}^{0.556} \left( R \sigma \tau^{0.7} \right)^{1.372} \]
Fold weight is calculated as for the AFDD model. Parameters are defined in Table 30-3, including units as used in these equations.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{\text{max}}$</td>
<td>Maximum speed</td>
<td>kts</td>
</tr>
<tr>
<td>$V_{\text{dive}}$</td>
<td>Dive speed</td>
<td>kts</td>
</tr>
<tr>
<td>$S$</td>
<td>Tail planform area</td>
<td>ft$^2$</td>
</tr>
<tr>
<td>$b$</td>
<td>Tail span</td>
<td></td>
</tr>
<tr>
<td>$A$</td>
<td>Tail aspect ratio</td>
<td></td>
</tr>
<tr>
<td>$\Lambda$</td>
<td>Sweep angle</td>
<td>deg</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Taper ratio (tip chord/root chord)</td>
<td></td>
</tr>
<tr>
<td>$h_t/b_h$</td>
<td>Height horizontal tail above fin root (fraction span)</td>
<td></td>
</tr>
<tr>
<td>$P_{tr}$</td>
<td>Tail rotor transmission power limit</td>
<td>hp</td>
</tr>
<tr>
<td>$V_{\text{tip}}$</td>
<td>Hover tip speed</td>
<td>ft/sec</td>
</tr>
<tr>
<td>$R$</td>
<td>Radius</td>
<td>ft</td>
</tr>
<tr>
<td>$N_b$</td>
<td>Number of blades</td>
<td></td>
</tr>
<tr>
<td>$c$</td>
<td>Mean chord</td>
<td>ft</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Solidity</td>
<td></td>
</tr>
<tr>
<td>$\tau_{0.7}$</td>
<td>Airfoil thickness-to-chord ratio at 0.7R</td>
<td>lb</td>
</tr>
<tr>
<td>$W_D$</td>
<td>Design gross weight</td>
<td></td>
</tr>
<tr>
<td>$n_z$</td>
<td>Design ultimate flight load factor</td>
<td>g</td>
</tr>
<tr>
<td>$w_f/b_h$</td>
<td>Fuselage width at horizontal tail (fraction span)</td>
<td></td>
</tr>
<tr>
<td>$L_t$</td>
<td>Tail length (wing to horizontal tail)</td>
<td>ft</td>
</tr>
<tr>
<td>$S_c$</td>
<td>Control surface area</td>
<td>ft$^2$</td>
</tr>
<tr>
<td>$K_y$</td>
<td>Aircraft pitch radius of gyration</td>
<td>ft</td>
</tr>
<tr>
<td>$K_z$</td>
<td>Aircraft yaw radius of gyration</td>
<td>ft</td>
</tr>
</tbody>
</table>

### 30–4 Fuselage Group

The fuselage group consists of: basic structure; wing and rotor fold/retraction; tail fold/tilt; and marinization, pressurization, and crashworthiness structure. The basic structure weight is

**Boeing**

$$w_{\text{basic}} = 0.198 \left[ (n_z W_{SD} S_{\text{body}} (\ell_c + \ell_r + \Delta CG))^{0.5} \log V_{\text{dive}} \right]^{0.8}$$

where $(\ell_c + \ell_r + \Delta CG)/\ell_{\text{body}} \approx 1.0$ for tandem, and 0.3–0.6 for single main rotor (0.7–0.8 with ramp).

**GARTEUR (airplane)**

$$w_{\text{basic}} = 0.149 \left( n_z W_{SD} S_{\text{body}} \ell_{\text{body}} \right)^{0.44}$$

**GARTEUR (helicopter)**

$$w_{\text{basic}} = 0.0280 R W_{SD}^{0.67} n_z^{0.335}$$

**Tishchenko**

$$w_{\text{basic}} = 0.251 W_{SD}^{0.25} S_{\text{body}}^{0.88} L_{rr}^{0.16(1+\alpha)}$$
where $\alpha = 0.0$ for single main rotor, 0.2 for tandem helicopter, 0.05 for side-by-side.

**Torenbeek**

$$w_{\text{basic}} = 0.021 S_{\text{body}}^{1.2} \left( V_{\text{dive}} \frac{L_t}{w_{\text{body}} + h_{\text{body}}} \right)^{0.5}$$

**Raymer (cargo/transport)**

$$w_{\text{basic}} = 0.3280 f_{\text{door}} f_{\text{LG}} (n_z W_D)^{0.5} S_{\text{body}}^{0.302} \ell_{\text{body}}^{0.25} x_w^{0.04} (\ell_{\text{body}} / h_{\text{body}})^{0.10}$$

**Raymer (general aviation)**

$$w_{\text{basic}} = 0.0103 (n_z W_D)^{0.177} S_{\text{body}}^{0.086} \frac{V_{\text{dive}}^{0.482}}{L_t^{0.051} (\ell_{\text{body}} / h_{\text{body}})^{0.072}}$$

$$w_{\text{press}} = 11.9 (V_{\text{press}} P_{\text{delta}})^{0.271}$$

where $f_{\text{door}} = 1.0$ for no cargo door, 1.06 for one side cargo door, 1.12 for two side cargo doors or aft clamshell door, 1.25 for two side cargo doors and aft clamshell door ($f_{\text{door}} = 1 + 0.06 n_{\text{door}}$); $f_{\text{LG}} = 1.12$ for fuselage-mounted main landing gear, 1.0 otherwise; and $x_w = 1 + 0.75 [(1 + 2 \lambda_w) / (1 + \lambda_w)] h_{\text{w}} \tan \Lambda_w / \ell_{\text{body}}$.

For the weight penalty due to pressurization, $V_{\text{press}} \cong \ell_{\text{body}} w_{\text{body}} h_{\text{body}}$, and typically $P_{\text{delta}} = 8$ psi.

The tail fold, wing and rotor fold, marinization, pressurization, and crashworthiness weights are calculated as for the AFDD84 model. Parameters are defined in table 30-4, including units as used in these equations.

**Table 30-4. Parameters for fuselage weight.**

<table>
<thead>
<tr>
<th>parameter</th>
<th>definition</th>
<th>units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_{SD}$</td>
<td>structural design gross weight</td>
<td>lb</td>
</tr>
<tr>
<td>$W_D$</td>
<td>design gross weight</td>
<td>lb</td>
</tr>
<tr>
<td>$n_z$</td>
<td>design ultimate flight load factor</td>
<td>g</td>
</tr>
<tr>
<td>$S_{\text{body}}$</td>
<td>wetted area of body</td>
<td>ft$^2$</td>
</tr>
<tr>
<td>$\ell_c$</td>
<td>length of cabin (nose to end of cabin floor)</td>
<td>ft</td>
</tr>
<tr>
<td>$\ell_r$</td>
<td>length of rampwell</td>
<td>ft</td>
</tr>
<tr>
<td>$\Delta CG$</td>
<td>center of gravity range at $W_D$</td>
<td>ft</td>
</tr>
<tr>
<td>$V_{\text{dive}}$</td>
<td>dive speed</td>
<td>kts</td>
</tr>
<tr>
<td>$\ell_{\text{body}}$</td>
<td>fuselage length</td>
<td>ft</td>
</tr>
<tr>
<td>$w_{\text{body}}$</td>
<td>fuselage width</td>
<td>ft</td>
</tr>
<tr>
<td>$h_{\text{body}}$</td>
<td>fuselage height</td>
<td>ft</td>
</tr>
<tr>
<td>$L_{rr}$</td>
<td>rotor-rotor longitudinal separation</td>
<td>ft</td>
</tr>
<tr>
<td>$L_t$</td>
<td>tail length (wing to horizontal tail)</td>
<td>ft</td>
</tr>
<tr>
<td>$n_{\text{door}}$</td>
<td>number of cargo doors</td>
<td></td>
</tr>
<tr>
<td>$\Lambda_w$</td>
<td>wing sweep angle</td>
<td>deg</td>
</tr>
<tr>
<td>$\lambda_w$</td>
<td>wing taper ratio (tip chord/root chord)</td>
<td></td>
</tr>
<tr>
<td>$b_w$</td>
<td>wing span</td>
<td></td>
</tr>
<tr>
<td>$V_{\text{press}}$</td>
<td>pressurized volume</td>
<td>ft$^3$</td>
</tr>
<tr>
<td>$P_{\text{delta}}$</td>
<td>cabin pressure differential</td>
<td>psi</td>
</tr>
</tbody>
</table>
### 30–5 Engine Section or Nacelle Group and Air Induction Group

The engine section or nacelle group consists of: engine support structure, engine cowling, and pylon support structure. The weights (engine mounts and nacelle structure) are:

Boeing

\[ W_{\text{supt}} = \chi_{\text{supt}} N_{\text{eng}} \left( \frac{W_{\text{eng}}}{N_{\text{eng}}} \right)^{0.41} \]
\[ W_{\text{cowl}} = \chi_{\text{cowl}} 1.25 S_{\text{nac}} \]

The crash load factor is typically \( n_{clf} = 8g \) for civil rotorcraft, \( 20g \) for military rotorcraft.

Raymer (cargo/transport)

\[ W_{\text{cowl}} = \chi_{\text{cowl}} 0.6724 f_{n}^{0.10} S_{\text{nac}}^{0.294} \left( \frac{w_{nac}}{\ell_{nac}} \right)^{0.119} \]
where \( f_{n} = 1.017 \) for pylon-mounted nacelle, 1.0 otherwise. The Raymer model includes the air induction group weight. The nacelle length and width are calculated from \( f = \frac{w_{nac}}{\ell_{nac}} \), assuming \( S_{\text{nac}} = \pi w_{nac} \ell_{nac} \). Parameters are defined in table 30-5, including units as used in these equations.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>( W_{\text{eng}} )</td>
<td>weight all main engines</td>
<td>lb</td>
</tr>
<tr>
<td>( N_{\text{eng}} )</td>
<td>number of main engines</td>
<td></td>
</tr>
<tr>
<td>( S_{\text{nac}} )</td>
<td>wetted area of nacelles and pylon (less spinner)</td>
<td>( \text{ft}^2 )</td>
</tr>
<tr>
<td>( \ell_{nac} )</td>
<td>nacelle length</td>
<td>ft</td>
</tr>
<tr>
<td>( w_{nac} )</td>
<td>nacelle width</td>
<td>ft</td>
</tr>
<tr>
<td>( n_z )</td>
<td>design ultimate flight load factor</td>
<td>g</td>
</tr>
</tbody>
</table>

### 30–6 Propulsion Group

The propulsion group consists of the engine system, fuel system, and drive system.

#### 30-6.1 Propeller/Fan Installation

The auxiliary propulsion or propeller weight is:

Boeing

\[ W_{at} = \chi_{at} 0.0297 N_{at} \left[ r^{0.25} P_{at}^{0.5} V_{tip} R N_{\text{blade}} \right]^{0.67} \]

GARTEUR

\[ W_{at} = \chi_{at} 0.5320 N_{at}^{1.544} N_{\text{blade}}^{0.471} P_{at}^{0.376} \]

Torenbeek

\[ W_{at} = \chi_{at} 0.186 N_{at} f_{e} \left( R P_{at} N_{\text{blade}}^{0.5} \right)^{0.78174} \]

where \( f_{e} = 1.0 \) for turboprop, 1.333 for reciprocating engine. The propeller weight is calculated by the rotor component model, including rotor support and duct weights. Rotor weight models can be used for
the propeller. Parameters are defined in table 30-6, including units as used in these equations.

<table>
<thead>
<tr>
<th>parameter</th>
<th>definition</th>
<th>units</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N_{at} )</td>
<td>number of auxiliary thrusters</td>
<td></td>
</tr>
<tr>
<td>( P_{at} )</td>
<td>power per propeller</td>
<td>hp</td>
</tr>
<tr>
<td>( V_{tip} )</td>
<td>hover tip speed</td>
<td>ft/sec</td>
</tr>
<tr>
<td>( R )</td>
<td>radius</td>
<td>ft</td>
</tr>
<tr>
<td>( N_{blade} )</td>
<td>number of blades</td>
<td></td>
</tr>
<tr>
<td>( c )</td>
<td>mean chord</td>
<td>ft</td>
</tr>
<tr>
<td>( r )</td>
<td>rotor blade attachment radius</td>
<td>ft</td>
</tr>
</tbody>
</table>

### 30-6.2 Fuel System

The fuel system consists of tanks and support structure (including fuel tanks, bladders, supporting structure, filler caps, tank covers, and filler material for void and ullage), and fuel plumbing (including fuel system weight not covered by tank weight). The fuel system weights are:

**Torenbeek (integral tanks)**

\[
W_{\text{tank}} = \chi_{\text{tank}} \left[ 80(N_{\text{eng}} + N_{\text{int}} - 1) + 15C_{\text{int}}^{0.333}N_{\text{int}}^{0.5} \right]
\]

**Torenbeek (generic)**

\[
W_{\text{tank}} = \chi_{\text{tank}}K_{\text{tank}}C_{\text{int}}^{X_{\text{tank}}}
\]

with \( K_{\text{tank}} = 2 \) and \( X_{\text{tank}} = 0.667 \) for single-engine reciprocating; \( K_{\text{tank}} = 4.5 \) and \( X_{\text{tank}} = 0.60 \) for multi-engine reciprocating; \( K_{\text{tank}} = 3.2 \) and \( X_{\text{tank}} = 0.727 \) for bladder tanks, turbofan or turboprop.

**Raymer (cargo/transport)**

\[
W_{\text{tank}} = \chi_{\text{tank}}2.405C_{\text{int}}^{0.606}N_{\text{int}}^{0.5}\frac{1 + f_p}{1 + f_i}
\]

**Raymer (general aviation)**

\[
W_{\text{tank}} = \chi_{\text{tank}}2.49C_{\text{int}}^{0.720}N_{\text{int}}^{0.242}N_{\text{eng}}^{0.157}(1 + f_i)^{-0.363}
\]

Parameters are defined in table 30-7, including units as used in these equations.

<table>
<thead>
<tr>
<th>parameter</th>
<th>definition</th>
<th>units</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N_{\text{int}} )</td>
<td>number of internal fuel tanks</td>
<td></td>
</tr>
<tr>
<td>( C_{\text{int}} )</td>
<td>internal fuel tank capacity</td>
<td>gallons</td>
</tr>
<tr>
<td>( f_i )</td>
<td>integral tank capacity (fraction total)</td>
<td></td>
</tr>
<tr>
<td>( f_p )</td>
<td>protected (self-sealing) tank capacity (fraction total)</td>
<td></td>
</tr>
<tr>
<td>( N_{\text{eng}} )</td>
<td>number of main engines</td>
<td></td>
</tr>
</tbody>
</table>
### 30-6.3 Drive System

The drive system consists of gear boxes and rotor shafts, drive shafts, and rotor brake. This distribution of drive system weights is based on the following functional definitions. Gearboxes are parts of the drive system that transmit power by gear trains, and the structure that encloses them. Rotor shafts are the structure (typically a shaft) that transmits power to the rotor. Drive shafts are the structure (typically a shaft) that transmits power in the propulsion system, but not directly to the rotor or by a gear train. The rotor brake weight encompasses components that can prevent the rotor from freely turning. The gear box and rotor shaft weights are:

**Boeing**

\[ w_{gbrs} = 250 \left( \frac{P_{DS\text{\ limit}}}{\Omega_{mr}} \right)^{0.25} f_t^{0.67} \]

**Boeing (alternate)**

\[ w_{gbrs} = 300 \left[ 1.1 \frac{P}{\Omega} \right]^{0.8} \]

where \( f_t = 1.0 \) for single main rotor, \( f_t = 1.3 \times 1.2 = 1.56 \) for tandem main rotors. The number of drive stages is typically \( z_{mr} \approx W_D/5000 \). For the alternate model applied to the entire drive system, \( P/\Omega = P_{DS\text{\ limit}}/\Omega_{mr} \). The alternate model is also applied to the tail rotor only, with \( P/\Omega = P_{tr}/\Omega_{tr} \).

**GARTEUR (helicopter)**

\[ w_{gbrs} = 252.3 \left( \frac{P_{DS\text{\ limit}}}{\Omega_{mr}} \right)^{0.74} \]

**GARTEUR (tiltrotor)**

\[ w_{gbrs} = 345.6 \left( \frac{P_{DS\text{\ limit}}}{\Omega_{mr}} \right)^{0.8} \]

**Tishchenko**

\[ w_{gbrs} = 199.3 \left( \frac{P_{DS\text{\ limit}}}{\Omega_{mr}} \right)^{0.8} + 643 \left( \frac{P_{tr}}{\Omega_{tr}} \right)^{0.8} \]

with \( P_{tr} = f_{tr} P_{DS\text{\ limit}} \), typically \( f_{tr} \approx 10\% \). These models are all implemented with an optional tail rotor drive weight increment \( \Delta w_{gbrs} = K_{trgb} 270 (P_{tr}/\Omega_{tr})^{0.8} \). Then

\[ W_{gb} = \chi_{gb} (1 - f_{rs}) w_{gbrs} \]

\[ W_{rs} = \chi_{rs} f_{rs} w_{gbrs} \]

Parameters are defined in table 30-8, including units as used in these equations.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_{DS\text{\ limit}} )</td>
<td>drive system power limit</td>
<td>hp</td>
</tr>
<tr>
<td>( \Omega_{mr} )</td>
<td>main-rotor rotation speed</td>
<td>rpm</td>
</tr>
<tr>
<td>( z_{mr} )</td>
<td>number of stages in main-rotor drive</td>
<td></td>
</tr>
<tr>
<td>( P_{tr} )</td>
<td>tail rotor drive system power limit</td>
<td>hp</td>
</tr>
<tr>
<td>( \Omega_{tr} )</td>
<td>tail-rotor rotation speed</td>
<td>rpm</td>
</tr>
<tr>
<td>( f_{tr} )</td>
<td>tail-rotor power (fraction main-rotor power)</td>
<td></td>
</tr>
</tbody>
</table>
The flight controls group consists of cockpit controls, automatic flight control system, and system controls. System controls consist of fixed wing flight controls, rotary wing flight controls, and conversion (rotor tilt) flight controls. The weight equations model separately non-boosted controls (which do not see aerodynamic surface or rotor loads), boost mechanisms (actuators), and boosted controls (which are affected by aerodynamic surface or rotor loads). The load path goes from pilot, to cockpit controls, to non-boosted controls, to boost mechanisms, to boosted controls, and finally to the component. The hydraulic group consists of hydraulics for fixed wing flight controls, rotary wing flight controls, and equipment.

Fixed wing flight controls consist of non-boosted flight controls and flight control boost mechanisms. The weights are:

GARTEUR

$$w = 0.0350W_D \left( \frac{W_D}{S_w} \right)^{-0.36}$$

Raymer (cargo/transport)

$$w = 55.47 \frac{N_f^{0.554}}{1 + f_m} S_{cs}^{0.20} I_y^{0.07}$$

$$W_{FW_{hyd}} = \chi_{FW_{hyd}}^{0.2673} N_f (\ell_{body} + b_w)^{0.937}$$

Raymer (general aviation)

$$w = 0.053(\ell_{body}/10000)^{0.8} f_{body}^{1.536} f_1^{0.371}$$

with typically $N_f = 4$ to $7$, $f_m N_f = 0$ to $2$. Then

$$W_{FW_{nb}} = \chi_{FW_{nb}} f_{FW_{nb}} w$$

$$W_{FW_{mb}} = \chi_{FW_{mb}} (1 - f_{FW_{nb}}) w$$

$$W_{FW_{hyd}} = \chi_{FW_{hyd}} f_{FW_{hyd}} W_{FW_{mb}}$$

as for the AFDD model. Parameters are defined in table 30-9, including units as used in these equations.

<table>
<thead>
<tr>
<th>parameter</th>
<th>definition</th>
<th>units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_D$</td>
<td>design gross weight</td>
<td>lb</td>
</tr>
<tr>
<td>$S_w$</td>
<td>wing planform area (theoretical)</td>
<td>ft$^2$</td>
</tr>
<tr>
<td>$N_f$</td>
<td>number of functions performed by controls</td>
<td></td>
</tr>
<tr>
<td>$f_m$</td>
<td>number of mechanical functions (fraction $N_f$)</td>
<td></td>
</tr>
<tr>
<td>$S_{cs}$</td>
<td>total area of control surfaces</td>
<td>ft$^2$</td>
</tr>
<tr>
<td>$I_y$</td>
<td>yaw moment of inertia ($W_D K_y^2$)</td>
<td>lb-ft$^2$</td>
</tr>
<tr>
<td>$\ell_{body}$</td>
<td>fuselage length</td>
<td>ft</td>
</tr>
<tr>
<td>$b_w$</td>
<td>wing span</td>
<td>ft</td>
</tr>
</tbody>
</table>

Rotary wing flight controls consist of non-boosted flight controls, flight control boost mechanisms, and boosted flight controls. The Boeing weight equations model the main rotor controls (from and including the power actuators, swashplate, up through the pitch links; identified as boost mechanisms
plus boosted controls) and the rotor systems (between cockpit controls and rotor controls; identified as non-boosted controls) and hydraulics. The weights are:

Boeing

\[
W_{cc} = \chi_{cc} 1.7 \left[ W_D \right]^{0.41}
\]

\[
w_{mb+b} = 0.562 N_{\text{rotor}} \left[ c \left( RW_{\text{blade}}/N_{\text{rotor}} \right)^{0.5} \right]^{1.11}
\]

\[
w_{nb+hyd} = 0.0906 N_{\text{rotor}} \left[ W_D/N_{\text{rotor}} \right]^{0.84}
\]

\[
W_{RW_{mb}} = \chi_{RW_{mb}} (1 - f_{RW_{b}}) w_{mb+b}
\]

\[
W_{RW_{b}} = \chi_{RW_{b}} f_{RW_{b}} w_{mb+b}
\]

\[
W_{RW_{nb}} = \chi_{RW_{nb}} (1 - f_{RW_{hyd}}) w_{nb+hyd}
\]

\[
W_{RW_{hyd}} = \chi_{RW_{hyd}} f_{RW_{hyd}} w_{nb+hyd}
\]

or

GARTEUR

\[
w_{fc} = 0.0324 W_D^{1.0136}
\]

Tishchenko

\[
w_{fc} = 0.824 N_{\text{rotor}} N_{\text{blade}} c^2 R
\]

\[
W_{RW_{mb}} = \chi_{RW_{mb}} f_{RW_{mb}} w_{fc}
\]

\[
W_{RW_{b}} = \chi_{RW_{b}} f_{RW_{b}} w_{fc}
\]

\[
W_{RW_{nb}} = \chi_{RW_{nb}} (1 - f_{RW_{mb}} - f_{RW_{b}} - f_{RW_{hyd}}) w_{fc}
\]

\[
W_{RW_{hyd}} = \chi_{RW_{hyd}} f_{RW_{hyd}} w_{fc}
\]

Parameters are defined in table 30-10, including units as used in these equations.

<table>
<thead>
<tr>
<th>parameter</th>
<th>definition</th>
<th>units</th>
</tr>
</thead>
<tbody>
<tr>
<td>( W_D )</td>
<td>design gross weight</td>
<td>lb</td>
</tr>
<tr>
<td>( c )</td>
<td>rotor mean blade chord</td>
<td>ft</td>
</tr>
<tr>
<td>( R )</td>
<td>rotor radius</td>
<td>ft</td>
</tr>
<tr>
<td>( W_{\text{blade}} )</td>
<td>weight rotor blades</td>
<td>lb</td>
</tr>
<tr>
<td>( N_{\text{rotor}} )</td>
<td>number of main rotors</td>
<td></td>
</tr>
<tr>
<td>( N_{\text{blade}} )</td>
<td>number of blades</td>
<td></td>
</tr>
</tbody>
</table>

### 30–8 Parametric Weight Correlation

Tables 30-11 and 30-12 summarize the statistics of the parametric weight estimation equations, for the AFDD data base and a data base from Boeing (ref. 4), respectively. The technology factor \( \chi \) was adjusted for each equation to minimize the average error. Except for the wing group, the aircraft in the data base are all helicopters, in particular for the fuselage group and empennage group. The fuel system weight is for just the tanks; the technology factors are about 70% larger for both tanks and plumbing. The drive system weight includes the tail rotor.
<table>
<thead>
<tr>
<th>Model</th>
<th>Number of Aircraft</th>
<th>Average Error (%)</th>
<th>Technology Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wing Group</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Boeing</td>
<td>23</td>
<td>14.1</td>
<td>$\chi = 0.73$</td>
</tr>
<tr>
<td>GARTEUR</td>
<td>23</td>
<td>20.9</td>
<td>$\chi = 1.07$</td>
</tr>
<tr>
<td>Torenbeek</td>
<td>23</td>
<td>20.3</td>
<td>$\chi = 1.04$</td>
</tr>
<tr>
<td>Raymer (cargo/transport)</td>
<td>23</td>
<td>16.9</td>
<td>$\chi = 1.24$</td>
</tr>
<tr>
<td>Raymer (general aviation)</td>
<td>23</td>
<td>16.8</td>
<td>$\chi = 1.31$</td>
</tr>
<tr>
<td>Rotor Group, Blade</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Boeing</td>
<td>51</td>
<td>10.1</td>
<td>$\chi = 0.90$</td>
</tr>
<tr>
<td>GARTEUR</td>
<td>51</td>
<td>11.8</td>
<td>$\chi = 0.98$</td>
</tr>
<tr>
<td>Torenbeek</td>
<td>51</td>
<td>18.8</td>
<td>$\chi = 0.99$</td>
</tr>
<tr>
<td>Rotor Group, Hub</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Boeing</td>
<td>45</td>
<td>27.5</td>
<td>$\chi = 1.07$</td>
</tr>
<tr>
<td>GARTEUR</td>
<td>45</td>
<td>27.6</td>
<td>$\chi = 1.34$</td>
</tr>
<tr>
<td>Torenbeek</td>
<td>51</td>
<td>22.8</td>
<td>$\chi = 0.89$</td>
</tr>
<tr>
<td>Horizontal Tail</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GARTEUR</td>
<td>13</td>
<td>37.3</td>
<td>$\chi = 0.65$</td>
</tr>
<tr>
<td>Torenbeek (transport)</td>
<td>13</td>
<td>24.7</td>
<td>$\chi = 1.80$</td>
</tr>
<tr>
<td>Vertical Tail</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GARTEUR</td>
<td>12</td>
<td>26.7</td>
<td>$\chi = 0.79$</td>
</tr>
<tr>
<td>Torenbeek (transport)</td>
<td>12</td>
<td>29.2</td>
<td>$\chi = 1.92$</td>
</tr>
<tr>
<td>Tail Rotor</td>
<td></td>
<td></td>
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<td>22</td>
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### Table 30-11. Statistics of parametric weight equations, Boeing data base.

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### 30–9 References


