Application of Compressive Sensing to Gravitational Microlensing Data
- and -
Implications for Miniaturized Space Observatories

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Outline

- Gravitational Microlensing
- Compressive Sensing (CS) Motivation
- Compressive Sensing (CS) Theory
- Single Lens Microlensing Events
- Simulation Results
- Conclusion and Future work
Gravitational Microlensing

- Technique to detect exoplanets and other astrophysical entities

Credit: Space Telescope Science Institute
Current Techniques Limitations

- High rate sampling required to acquire the desired resolution
  - Miniaturized space observatories: Data bandwidth limitation
- Need high cadence for acquiring each image
  - If high cadence is not achieved, an exoplanet transition with a short period can be missed
- Miniaturized space observatories have power and on-board memory limitation
- **How do we achieve high resolution images at a high cadence by acquiring only a few samples?**
Compressive Sensing (CS) Motivation

- Acquiring each image pixel individually (sampling at the Nyquist rate) is wasteful when the information can be encoded in only a select few samples due to its sparse nature.
- Exploit sparsity in images.
- Microlensing Events are sparse in spatial domain when differenced.
  - That is, at any given time only the stars exhibiting a microlensing event vary in flux.
  - Only those stars are evident when differenced with a reference image.
CS Theory

Each sub measurement matrix gets transformed into a 1D signal representing a row in the measurement matrix.

- M sub measurement matrices
- Reconstruct original image, given y vector and the associated (sub) measurement matrix for each element in y
  - \( Y_{mx1} = \Phi_{mxn}x_{nx1} \)
  - Optimization (L1 minimization) and greedy algorithms
- A unique solution is obtained only if the original image is sparse in some domain

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Single Lens Microlensing Events

- Source star magnification only due to lensing star
- Magnification at each time is dependent on:
  \( u_0 \): lens-source separation in terms of Einstein’s ring radius
  \( t_0 \): peak magnification time
  \( t_e \): Einstein’s ring radius crossing time

**Top:** Original spatial domain image at time, \( t = 0 \)

**Bottom:** Original time domain image with magnification at center pixel plus a 3 pixel radius
Simulation Setup

All Simulations are performed in **Python**

**Gravitational Microlensing Parameters**

- Single lens event
- \( u_0 = 0.1 \)
- Total 30 time samples
  - Peak magnification at time value = 14
  - Einstein’s ring crossing time at time value = 29

**CS Parameters**

- Image size = 25x25
  - \( N = 25 \times 25 = 625 \) pixels
- Measurements, \( M \), is varied from 2\% of \( N \) to 6\% of \( N \)
  - \( \% \) Measurements = \( \frac{M}{N} \) x 100
- Sparsity: number of non-zero (or significant value) pixels = 1
- Measurement matrix, \( \theta \): Bernoulli Random with 0’s and 1’s
  - 100 Monte Carlo simulations to vary measurement matrix each time
CS Reconstruction

Green: Original signal
Blue: Reconstructed signal
Red: Error bars

Top: 2% measurements
Middle: 3% measurements
Bottom: 4% measurements

% Error at t0 over center pixel with 3 pixel radius

% Error at peak time, t0

% Measurements =

\[
\frac{\text{# of measurements}}{\text{# of total pixels}} \times 100
\]

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Resolution Accuracy

<table>
<thead>
<tr>
<th>% Measurements $\frac{M}{N} \times 100$</th>
<th>Error Difference in Reconstruction at $t_0$</th>
<th>Average Standard deviation over all $t$</th>
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</thead>
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<tr>
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<td>4.19</td>
<td>1.6</td>
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</tbody>
</table>

- Change in magnification at peak time, $t_0$, is 0.5 units of flux
  - Resolution error $<< 0.5$ to capture changes in microlensing curve

- 4% of N measurements gives optimal error, along with a low standard deviation, providing lower uncertainty
Conclusions and Future Work

• For a clean image, with very low sparsity, only 4% of Nyquist rate samples are required to reconstruct the image
  • **Significant reduction in data volume and power**
  • **Greatly benefit space flight observatories**

• Future work will include studying
  • Point spread functions and its implications for CS
  • Dense, crowded field images
  • Difference imaging for CS applications
  • Binary lens systems
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References