Summary Findings from the AVT-191 Project to Assess Sensitivity Analysis and Uncertainty Quantification Methods for Military Vehicle Design

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A NATO symposium held in Greece in 2008 identified many promising sensitivity analysis and uncertainty quantification technologies, but the maturity and suitability of these methods for realistic applications was not clear. The NATO Science and Technology Organization, Task Group AVT-191 was established to evaluate the maturity and suitability of various sensitivity analysis and uncertainty quantification methods for application to realistic vehicle development problems. The program ran from 2011 to 2015, and the work was organized into four discipline-centric teams: external aerodynamics, internal aerodynamics, aeroelasticity, and hydrodynamics. This paper summarizes findings and lessons learned from the task group.

Nomenclature

<table>
<thead>
<tr>
<th>Symbols</th>
<th>Description</th>
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<tr>
<td>C_A</td>
<td>axial force coefficient</td>
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<tr>
<td>C_l</td>
<td>rolling moment coefficient</td>
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<tr>
<td>C_m</td>
<td>pitching moment coefficient</td>
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<tr>
<td>C_mz</td>
<td>yawing moment coefficient</td>
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<tr>
<td>C_N</td>
<td>normal force coefficient</td>
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<tr>
<td>C_S</td>
<td>side force coefficient</td>
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<tr>
<td>C_y</td>
<td>yawing moment coefficient</td>
</tr>
<tr>
<td>L</td>
<td>reference hull length</td>
</tr>
<tr>
<td>M</td>
<td>Mach number</td>
</tr>
<tr>
<td>X</td>
<td>axial force</td>
</tr>
<tr>
<td>x,y,z</td>
<td>body axis Cartesian coordinate system</td>
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<tr>
<td>z/L</td>
<td>nondimensional heave</td>
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<tr>
<td>α</td>
<td>angle of attack, deg.</td>
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<tr>
<td>θ</td>
<td>pitch angle, rad.</td>
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</table>

Abbreviations

- AVT: Applied Vehicle Technology
- CFD: Computational Fluid Dynamics
- DLR: German Aerospace Center, Germany
- FG5: Generic Missile Configuration
- GRC: Glenn Research Center, USA
- ISNEAN: National Research Council-Marine Technology Research Institute, Italy
- LaRC: Langley Research Center, USA
- NATO: North Atlantic Treaty Organization
- NIPColM: Non-Intrusive, Probabilistic Collocation Method
- OAI: Ohio Aerospace Institute
- ONERA: Office National d’Études et de Recherches Aérospatiales, France
- POD: Proper Orthogonal Decomposition
- RTO: Research and Technology Organization

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I. Introduction

An assessment has been made of the maturity and suitability of a number of Sensitivity Analysis and Uncertainty Quantification (SA/UQ) methods on realistic problems of interest to NATO vehicle design. In particular, the NATO STO AVT-191 task group focused on variational (aleatory) uncertainties (i.e., uncertainties whose distribution functions are known). Selected methods were applied to four problems that are representative of fluid dynamic design issues associated with air and sea vehicles and their propulsion systems. Uncertainty distribution of key simulation input parameters were used with simulation methods of varying fidelity coupled with uncertainty methods to compute the distribution of selected output parameters of interest in design and analysis. The AVT-191 work had its origins in an RTO symposium held in Greece in 2008.

Fluid dynamic design issues were addressed in four problem areas that were chosen to assess the uncertainty methods. The problem areas are: external aerodynamics associated with a missile configuration, internal aerodynamics associated with a turbojet rotor configuration, aeroelasticity effects associated with a supersonic transport configuration, and hydrodynamics associated with a catamaran configuration. These problem areas cover a broad range of flow physics: incompressible and compressible flows, as well as multidisciplinary flows. They include a number of boundary conditions such as unconstrained flows with free boundaries, coupled solid-fluid and fluid-gas boundaries, and constrained flows with finite impermeable boundaries. Separate teams were formed to address each problem area.

The AVT-191 work was completed in 2015, and a final technical report is in the process of being published through NATO. An overview of the AVT-191 program, as well as technical results from the work for the four selected problem areas have been highlighted in two special sessions at this conference. Summary results from each problem area are presented in this paper and are drawn from the AVT-191 technical report.

II. Summary and Discussion of Team Results

This section highlights the methods used in each problem area with a brief discussion of lessons learned and, in some cases, recommendations. Results associated with each paper presented in the special sessions (APA-28 and APA-40) are presented in the tables.

A. External Aerodynamics

The external aerodynamics team’s experiences with the generic missile configuration (See, Fig. 1) were reported in these AIAA special sessions by Peter et al., Doty, and Graves, and their methods and findings are summarized in Table 1. All team members used or developed low order surrogate models on which to apply the uncertainty methods. The consensus is that such models can accurately represent the important features of the problem under investigation and reduce the computational resources required to implement the uncertainty methodology. A number of expansion and sampling methods were investigated and compared. All the uncertainty methods were employed in an uncoupled or non-intrusive manner.

An example outcome from the external aerodynamics nondeterministic analysis is shown in Fig. 2. The figure presents the output probably distribution functions (PDF) for the yawing moment coefficient analysis at $M = 0.8$, $Re_D = 600,000$, and $\alpha = 12$ deg. The figure indicates that for the range investigated, the number of samples had a small influence on the output PDF for the yawing moment. Two second order discretization schemes were studied for the analysis: an upwind and a central difference scheme. The computations indicate (See, right-hand plot of Fig. 2) that discretization method had a larger influence on the mean value of the yawing moment PDF than the sampling
size, but had little effect on the shape of the PDF. A similar analysis was performed for the side force and rolling moment coefficients with the same results. For the yawing moment coefficient, the output PDF is a slightly skewed beta distribution. This indicates that the linear effects dominate the variation caused by the fin angle and its azimuthal position over the angle-of-attack range studied.

![Figure 2](image.png)

Figure 2. Influence of number of DoE samples used to construct a surrogate model (left) and of the discretization scheme (right) on the output PDF of yawing moment, $C_{\text{mz}}$.

$M = 0.8$, $Re_D = 600,000$, and $\alpha = 12$ deg.

Figure 3 illustrates the results of a global sensitivity analysis method which gives not only the effects of the variations of the input variables on the output variables, but also, the contributions of the interactions among the input variables to the variation of the output variables. For example, consider the results of the analysis for rolling moment coefficient presented in Fig. 3. The contribution of the interaction terms to the variance budget of this aerodynamic coefficient is large and cannot be ignored. These results indicate that the usual assumption that the input variables are independent should be examined closely as the design progresses.

![Figure 3](image.png)

Figure 3. Global sensitivity analysis of FG5 missile.
There were a number lessons learned from each analysis method. A number were common to most of the team members and they will be noted here. The quality of an analysis using these methods can be greatly influenced by numerical issues such as selection of quadrature points and use of consistent (high order) numerical representation of analysis parameter values. When use is made of high-fidelity models such as Computational Fluid Dynamics (CFD) to generate databases for the development of surrogate models, the discretization scheme used by the CFD simulation and its sub-models can greatly affect computational productivity and the distribution function computed for the output parameters.

### Table 1. Summary of External Aerodynamics Team Methods and Findings

<table>
<thead>
<tr>
<th>Session</th>
<th>Paper</th>
<th>Methods</th>
<th>Findings</th>
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</table>
| 1       | Peter et al.⁵ | Expansion methods  
1. Non-intrusive polynomial chaos  
2. Non-intrusive probabilistic collocation  
Surrogate models  
1. Based on experimental measurements angle-of-attack variations (Provided in Chapter 2)  
2. CFD simulations for generation of second model for other input parameter variations | Lessons learned:  
1. Probability distribution of the three input parameter uncertainties using third order polynomials did not provide sufficient accuracy for construction of a joint probability distribution. A Kriging model fitted by a three input parameter pdfs from the product of the Kriging models produced accurate estimates of the mean and variance of the output parameter pdfs.  
2. Methods employing polynomial methods require selection of collocation quadrature points to be closely coupled to the degree of the polynomial and require more computational time than that used by a nested set of points |
| 1       | Doty⁶ | Expansion method:  
1. Polynomial chaos method on polynomial surrogate model of Chapter 4 | Lessons learned  
1. Use of standard metrics based on normal distributions leads to potentially incorrect results  
2. Confidence interval for mean and variance must use consistent critical values obtained for the input parameter pdfs  
3. Statistical comparison must use consistent and sufficient digits of precision |
| 1       | Graves⁷ | Surrogate model  
1. Developed by Design of Experiments sampling methods to generate response surface from CFD simulations  
Sampling method  
1. Non-intrusive Monte Carlo (pseudo-random) of surrogate model | Lessons learned  
1. Response surface generation using Design of Experiment (DoE) methods provides convenient and efficient means to develop surrogate models  
2. Amount of automation in grid generation for CFD simulations to produce database is a key factor in producing creditable results |

An interesting feature of the External Aerodynamics Team investigation can be found in Chapter 4 of Ref. 1. This chapter presents a very good tutorial on the development of surrogate models and the use of efficient sampling methods.
B. Internal Aerodynamics

The internal aerodynamics team’s experience with the NASA Rotor 37 configuration (See, Figure 4) were reported in these AIAA special sessions by Nigro et al., and their methods and findings are summarized in Table 2. The team used high-fidelity, physics-based flow models (i.e., CFD) on which to apply the uncertainty methods. These methods were of the expansion type and consisted of the non-intrusive, probabilistic collocation method (NIPColM) and the proper orthogonal decomposition (POD) method. These methods were coupled with efficiency approaches such as sensitivity analysis to reduce the number of uncertain parameters used in the analysis and sparse grid sampling techniques to minimize the number of CFD simulations required. They found that the use of efficiency approaches kept the amount of computing resources to a tractable level. They also determined that the NIPColM worked best for discrete uncertainty variables and the POD method worked best for estimates involving a discrete random field with a number of random variables, e.g., surface geometry variations caused by the manufacturing process.

An example outcome from the internal aerodynamics nondeterministic analysis is shown in Figure 5. The figure compares non-deterministic and deterministic mean values of the total pressure ratio as a function of mass flow; it shows that the results are not the same. Both the deterministic and the nondeterministic simulations are close to the experimental results for the NASA Rotor 37 configuration. However, the nondeterministic simulations include quantified uncertainty bands in both mass flow and total pressure ratio, and these bands are larger than the differences among the two simulations and the experiment. Nondeterministic simulations, such as these, can alter the focus of a numerical simulation process from matching experimental trends to understanding the sources of the uncertainty in the numerical outcomes.

Figure 4. NASA Rotor 37 configuration.

Figure 5. Comparison of deterministic and non-deterministic performance curves of the pressure ratio as a function of the mass flow.
### Table 2. Summary of Internal Aerodynamics Team Methods and Findings

<table>
<thead>
<tr>
<th>Session</th>
<th>Paper</th>
<th>Methods</th>
<th>Findings</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Nigro, et al.⁹</td>
<td>Models</td>
<td>Lessons learned</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1. High-fidelity, physics-based (CFD)</td>
<td>1. Number of computations depends on required statistical accuracy of analysis</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Expansion Methods</td>
<td>2. Manufacturing process uncertainties cannot be represented as independent random variables. They require use of random fields. An example of such variables are surface geometric variations.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1. Non-intrusive Probabilistic Collocation method</td>
<td>Recommendations</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2. Proper Orthogonal Decomposition</td>
<td>1. Sensitivity analysis is recommended to be used early in the analysis to reduce the number of uncertainty parameters to be included in analysis</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Sampling methods</td>
<td>2. Sparse grid techniques should be used to minimize the number of high fidelity simulations required for an analysis</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1. Space grid technique</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>2. Sensitivity Analysis</td>
<td></td>
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</tbody>
</table>
C. Aeroelasticity

The Aeroelasticity Team’s experience on the supersonic transport model\(^\text{10}\) (See, Figure 6.) were reported in these AIAA special sessions by Cunningham and Holman\(^\text{11}\), Nikbay and Heeg\(^\text{12}\), and Tartaruga et al.\(^\text{13}\), and their methods and findings are summarized in Table 3. All the team members used or developed surrogate models to which they applied the uncertainty methods because they found that such models represented the features of interest in the analysis with sufficient accuracy and they significantly reduced the computational resources required for the analysis. They found that the type of aeroelastic model used has a significant effect on the output parameter distribution function. In particular, linear models with a Gaussian input parameter distribution generated Gaussian output parameter distributions. Whereas, if a non-linear model was used, Gaussian input distributions generated Beta output distributions.

An example outcome from the aeroelasticities nondeterministic analysis is shown in Figure 7. In this figure, probability density functions for the flutter dynamic pressure were determined at Mach numbers spanning subsonic to supersonic values. Minimum flutter dynamic pressure occurs at transonic conditions, as is often the case, but the nondeterministic simulations demonstrated that the uncertainty in flutter dynamic pressure was smallest at transonic speeds. The results also show that uncertainty in flutter dynamic pressure increased significantly at supersonic speeds and became biased toward the lower bound.

![Figure 6. S4T aeroelastic wind tunnel model.](image)

![Figure 7. Nondeterministic analysis for flutter dynamic pressure of the S4T configuration.](image)
Table 3. Summary of Aeroelastic Team Methods and Findings

<table>
<thead>
<tr>
<th>Session</th>
<th>Paper</th>
<th>Methods</th>
<th>Findings</th>
</tr>
</thead>
</table>
| 2       | Cunningham and Holman\textsuperscript{11} | Models:  
1. MSC NASTRAN  
2. Surrogate model generated from CFD  
   – Euler simulations and expansions  
   a. Polynomial Chaos  
   b. Proper Orthogonal Decomposition  
3. Surrogate model sampling  
   a. Monte Carlo  
   b. Latin Hypercube  
Sampling  
1. Polynomial Chaos  
2. Proper Orthogonal Decomposition  
3. Monte Carlo  
4. Latin Hypercube | Lessons learned:  
1. Aeroelastic model input-output relationships  
   a. Linear: Gaussian input pdf gives Gaussian output pdf  
   b. Nonlinear: Gaussian input pdf gives Beta output pdf  
2. Use of surrogate models significantly reduced computational time for associated uncertainty analyses made here |
| 2       | Nikbay and Heeg\textsuperscript{12} | Models:  
1. CFD – Euler simulations  
Sampling  
1. Latin Hypercube for CFD simulations | Lessons learned  
1. Complete geometric modeling of test configuration is very important to obtain results that match wind tunnel data  
2. Measurement and geometric modeling for CFD provide uncertainties in average aeroelastic slopes with angle of attack that are sufficiently large so that they justify using deviations from mean slopes as a basis of comparison of results |
| 2       | Tartaruga et al.\textsuperscript{13} | Models  
1. Surrogate models developed from Singular Value Decomposition method  
Sampling  
1. Monte Carlo w/ Latin Hypercube  
Expansion  
1. Polynomial Chaos  
2. Fuzzy Logic | Lessons learned  
1. Use of surrogate models significantly reduced computational time required for analysis  
2. All methods provide efficient and accurate results |
D. Hydrodynamic

The Hydrodynamics Team’s experience with the Delft Catamaran configuration\textsuperscript{14} (See, Figure 8.) were reported in these AIAA special sessions by Stern et al.\textsuperscript{15}, and Diez et al.\textsuperscript{16}, and their methods and findings are summarized in Table 4. The team members used both high-fidelity, physics models (CFD) directly with Monte Carlo sampling methods and surrogate models with Monte Carlo and expansion methods. They found that stochastic design methods had minimal impact on hydrodynamic design decisions when the stochastic and deterministic optima were in the same neighborhood of the design space. However, stochastic methods were found to have a significant impact when optimization includes environmental uncertainty parameters. The team recommended the use of polynomial based surrogate models coupled with Latin Hypercube or Markov-chain sampling methods. They found that surrogate models provide a simple means to estimate confidence intervals and uncertainty bands and they can be easily extended from uncertainty domains to design space for optimization.

An example outcome from the hydrodynamics nondeterministic analysis is shown in Figure 9. The figure presents a comparison of Empirical and Normal time history distributions. An analysis of the convergence and accuracy of the two distributions was made and revealed that accuracy converged except of heave amplitude and pitch. The results indicated that future efforts should experiment with increasing the sample size to improve convergence.

![Figure 8. Delft high speed catamaran.](image)

![Figure 9. Comparison of time history PDF for irregular wave benchmark and variable regular wave uncertainty quantification. Empirical and Normal density functions are shown.](image)
Table 4. Summary of Hydrodynamics Team Methods and Findings

<table>
<thead>
<tr>
<th>Session</th>
<th>Papers</th>
<th>Methods</th>
<th>Findings</th>
</tr>
</thead>
</table>
| 2       | Stern et al. 15, Diez et al. 16 | Models  
1. Surrogate  
2. High-fidelity, physics-based (CFD)  
Sampling Methods  
1. Latin Hypercube  
2. Markov Chain  
Expansion Methods  
1. Polynomial chaos | Lessons learned  
1. UQ methods impact on design is negligible when deterministic and stochastic optima are in same region of design space  
2. UQ methods impact on design is significant when optimization involves environmental factors  
3. Uncertainty in standard deviation in CFD results is larger than in expected value  
4. Least-square support vector machine is most efficient for ship-hydrodynamics problems  
5. Dynamic surrogate models are most effective overall  
Recommendations  
1. Recommend Monte Carlo methods using Latin Hypercube and Markov chain sampling coupled with polynomial based surrogate models |

III. Summary

All four problem areas found that the use of surrogate (i.e., reduced order) models coupled with either sampling or expansion methods were the most efficient approach to implement stochastic design procedures. Several methods were used to develop surrogate models from either high-fidelity simulations such as CFD or from measurements. There is a consensus that polynomial based surrogate models are the most efficient.

Finally, all of the assessments made in this study agree that sensitivity analysis and uncertainty quantification methods, especially, those based on surrogate (reduced order) models are sufficiently mature to use in the design of vehicles.

IV. References

4 FG5 generic missile configuration data supplied to AVT 191 by Applied Aerodynamics Department of ONERA.


